Two-loop resummation in (F)APT

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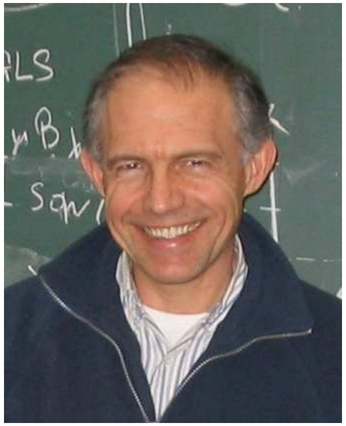
OUTLINE

- **Intro: Asymptotic Series in Perturbative QFT**
- **APT and FAPT**
- **Resummation in APT and FAPT**
- **Applications: Resummation for Adler function $D(Q^2)$**
- **Applications: Higgs decay $H^0 \to b\bar{b}$**
- **Conclusions**

Collaborators & Publications

Collaborators:



S. Mikhailov (Dubna) **D. Shirkov (Dubna)** **N. Stefanis (Bochum)**

Publications:

- **A. B.&Mikhailov — Solovtsov Memorial Seminar, Dubna, Jan. 17–18, 2008, Dubna: JINR (2008) pp. 119–133**
- **A. B. — Phys. Part. Nucl. 40 (2009) 715**
- **A. B., Mikhailov, Stefanis — JHEP 1006 (2010) 085**
- **A. B.&Shirkov — ArXiv:1102.2380[hep-ph]**

Asymptotic Series in Perturbative QFT

Strength and Weakness of Pert. QFT

A lot of successive pert. calculations in QM and QFT.
Practically, it is synonym of Quantum Theory.
Feynman diagrams became a symbol of QFT.

Nevertheless, power expansion of the quantum amplitude $C(\alpha)$ is not convergent.

Feynman Series $\sum c_k \alpha^k$ is not Convergent !

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Nevertheless, power expansion of the quantum amplitude $C(\alpha)$ is not convergent.

Feynman Series $\sum c_k \alpha^k$ is not Convergent !

Due to

- Essential singularity at $\alpha = 0$
- Factorial growth of coefficients $c_k \sim k!$

Series $\sum c_k \alpha^k$ is not Convergent!

- **Dyson argument (1952)**

  In QED, change $\alpha(=\frac{e^2}{4\pi}) \to -\alpha$ is equivalent to $e \to i\,e$.
  As $S = T(e^{i\int L_{\text{int}}(x)\,dx}) = T(e^{i\,e\int j_\mu A^\mu\,dx})$,
  this change destroys Unitarity.
  Hence, in the complex α plane, the origin $\alpha = 0$ can not be a regular point.

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  Hence, in the complex α plane, the origin $\alpha = 0$ can not be a regular point.

- **The ill-posed Problem**
  Small parameter g at highest nonlinearity — indispensable attribute of Quantum Perturbation:
  - **First, one quantizes linear system (as a set of oscillators).**
  - **Second, one takes into account non-linear term(s) $\sim g \ll 1$ as a small perturbation.**

  Non-linearity change equation seriously — new solutions appear.

Singularity at $g = 0$*, factorial growth* $c_k \sim k!$

For illustration, take the 0**-dim analog** $I(g) = \int\limits_{-\infty}^{\infty} e^{-x^2 - g x^4}\, dx$

Expanding it in power-in-g **series:**

$$I(g) \sim \sum_{k=0} (-g)^k I_k \qquad \textbf{with} \qquad I_k = \frac{\Gamma(2k + 1/2)}{\Gamma(k + 1)} \to 2^k\, k!$$

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Meanwhile, $I(g)$ can be expressed via MacDonald function

$$I(g) = \frac{1}{\sqrt{2g}}\, e^{1/8g}\, K_{1/4}\left(\frac{1}{8g}\right)$$

with known analytic properties in complex g plane.

Essential Singularity at $g = 0$

The $I(g)$ is a 4-sheeted function of the complex variable g, analytical in the whole complex plane with a cut from the origin $g = 0$.

There it has an essential singularity $e^{-1/8g}$ and can be written down in the Cauchy integral form

$$I(g) = \sqrt{\pi} - \frac{g}{\sqrt{2\pi}} \int_0^\infty \frac{d\gamma\, \mathbf{exp}(-1/4\gamma)}{\gamma(g+\gamma)}$$

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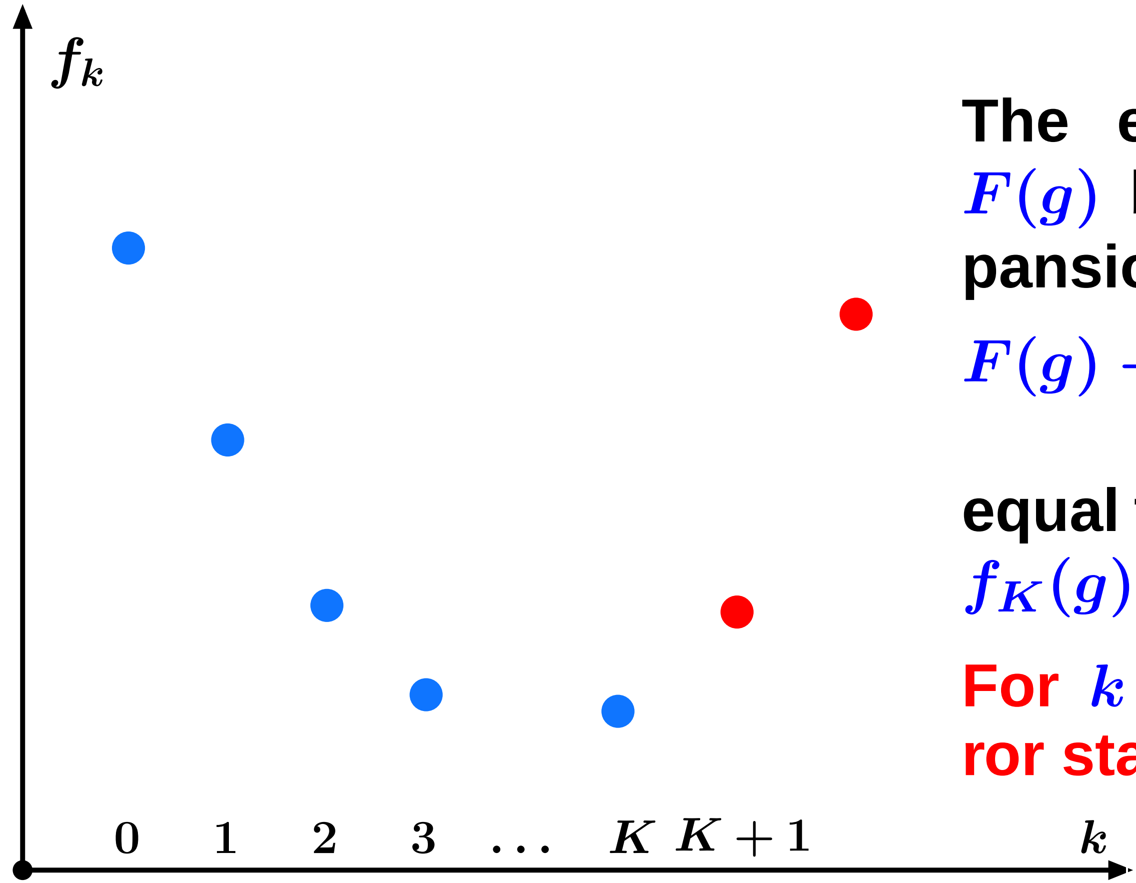
As far as the origin is not an analytical point, the power Taylor series has no convergence domain for real positive g values — in concert with factorial growth of power expansion.

Also, the power series is not valid for negative g values — in accordance with Dyson's reasoning.

Asymptotic Series and 'Practic. Convergence'

The Henry Poincaré (end of XIX) analysis of Asymptotic Series (AS) can be summed as follows:
AS can be used for obtaining quantitative information on expanded function.



The error of approximating $F(g)$ by first K terms of expansion, $F_K(g)$,

$$F(g) \to F_K(g) = \sum_{k \leq K} f_k(g)$$ is

equal to the last detained term $f_K(g)$.

For $k \geq K+1$ truncation error starts to grow!

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For the power AS, $f_k(g) = f_k\, g^k$ with factorial growth $f_k \sim k!$ absolute values of expansion terms $f_k(g)$ cease to diminish at $k \sim 1/g$.

This yields to the natural best possible accuracy of a given AS (in contrast to convergent series!)

Asymptotic Series and ‘Practic. Convergence’

$$I(g) = \int_{-\infty}^{\infty} e^{-x^2-gx^4}\,dx \quad ?=? \quad \sum_{k\geq 0} I_k\,(-g)^k$$

g	K	$(-g)^K\,I_K$	$(-g)^{K+1}\,I_{K+1}$	$\Delta_K I(g)$
0.07	7	−0.04(2%)	+0.07(4.4%)	1.4%
0.07	9	−0.17(10%)	+0.42(25%)	7%

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0.15	2	$+0.13(8\%)$	$-0.16(10\%)$	4%
0.15	4	$+0.30(18\%)$	$-0.72(44\%)$	12%

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Thus, one has $K_*(g=0.07)=7$ and $K_*(g=0.15)=2$.
It is not possible at all to get the 1% accuracy at $g=0.15$ for $I(g)$.

Analytic Perturbation Theory

in

QCD

History of APT

Euclidean	Minkowskian
$Q^2 = \vec{q}^2 - q_0^2 \geq 0$	$s = q_0^2 - \vec{q}^2 \geq 0$

RG+Analyticity
ghost-free $\overline{\alpha}_{\text{QED}}(Q^2)$
Bogoliubov et al. 1959

pQCD+RG: resum π^2-terms
Arctg(s), UV Non-Power Series
Radyush., Krasn. & Pivov. 1982

DispRel+renormalons
IR finite $\alpha_s^{\text{eff}}(Q^2)$
Dokshitzer et al. 1995

pQCD+renormalons
Arctg(s) at **LE** region
Ball, Beneke & Braun 1994-95

RG+Analyticity
ghost-free $\alpha_{\text{E}}(Q^2)$
Shirkov & Solovtsov 1996

Integral Transformation:
$\mathcal{R}\left[\overline{\alpha}_s\right] \to$ **Arctg**(s)
Jones & Solovtsov 1995

History of APT

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Jones & Solovtsov 1995

pQCD+RG+Analyticity

Transforms: $\hat{\mathcal{D}} = \hat{\mathcal{R}}^{-1}$

Couplings: $\alpha_{\mathsf{E}}(Q^2) \Leftrightarrow \alpha_{\mathsf{M}}(s)$

Milton & Solovtsov 1996–97

Analytic (global) pQCD+Analyticity

Global couplings: $\mathcal{A}_n(Q^2) \Leftrightarrow \mathfrak{A}_n(s)$

Non-Power perturbative expansions

Shirkov 1999–2001

History of F(ractional)APT

Euclidean $Q^2 = \vec{q}^2 - q_0^2 \geq 0$

Minkowskian $s = q_0^2 - \vec{q}^2 \geq 0$

Analytization of α_s^ν: $\mathcal{A}_\nu(Q^2) \Leftrightarrow \mathfrak{A}_\nu(s)$

Analytization of $\alpha_s^\nu \times$ **Log**^{m}: $\mathcal{L}_{\nu,m}(Q^2) \Leftrightarrow \mathfrak{L}_{\nu,m}(s)$

A. B. & Mikhailov & Stefanis 2005–2006

Resummation in 1-loop global FAPT

A. B. & Mikhailov 2008

Analytization of $\alpha_s^\nu(1 + c_1\alpha_s)^{\nu'}$: $\mathcal{B}_{\nu,\nu'}(Q^2) \Leftrightarrow \mathfrak{B}_{\nu,\nu'}(s)$

A. B. 2008–2009

Resummation in 2-loop global FAPT

with 2-loop evolution factors $\mathcal{B}_{\nu,\nu'}(Q^2) \Leftrightarrow \mathfrak{B}_{\nu,\nu'}(s)$

A. B. & Mikhailov & Stefanis 2010

PT in QCD

- coupling $\alpha_s(\mu^2) = (4\pi/b_0)\, a_s[L]$ with $L = \ln(\mu^2/\Lambda^2)$
- RG equation $\frac{d\, a_s[L]}{d\, L} = -a_s^2 - c_1\, a_s^3 - \ldots$
- 1-loop solution generates Landau pole singularity: $a_s[L] = 1/L$
- 2-loop solution generates square-root singularity: $a_s[L] \sim 1/\sqrt{L + c_1 \ln c_1}$
- PT series: $D[L] = 1 + d_1 a_s[L] + d_2 a_s^2[L] + \ldots$
- RG evolution: $B(Q^2) = \left[Z(Q^2)/Z(\mu^2)\right]\, B(\mu^2)$ reduces in 1-loop approximation to

$$Z \sim a^{\nu}[L]\Big|_{\nu\, =\, \nu_0\, \equiv\, \gamma_0/(2b_0)}$$

Basics of APT

- Different effective couplings in Euclidean (S&S) and Minkowskian (R&K&P) regions
- Based on RG $\oplus$ Causality

  RG $\Downarrow$ UV asymptotics

  Causality $\Downarrow$ Spectrality

- Euclidean: $-q^2 = Q^2,\ \ L = \ln Q^2/\Lambda^2,\ \ \{\mathcal{A}_n(L)\}_{n\in\mathbb{N}}$
- Minkowskian: $q^2 = s,\ \ L_s = \ln s/\Lambda^2,\ \ \{\mathfrak{A}_n(L_s)\}_{n\in\mathbb{N}}$
- PT $\quad \sum_m d_m a_s^m(Q^2) \quad \Rightarrow \quad \sum_m d_m \mathcal{A}_m(Q^2) \quad$ APT

Spectral representation

By analytization we mean "Källen–Lehmann" representation

$$\left[f(Q^2)\right]_{\text{an}} = \int_0^\infty \frac{\rho_f(\sigma)}{\sigma + Q^2 - i\epsilon}\, d\sigma$$

Then (note here pole remover):

$$\rho(\sigma) = \frac{1}{L_\sigma^2 + \pi^2}$$

$$\mathcal{A}_1[L] = \int_0^\infty \frac{\rho(\sigma)}{\sigma + Q^2}\, d\sigma = \frac{1}{L} - \frac{1}{e^L - 1}$$

$$\mathfrak{A}_1[L_s] = \int_s^\infty \frac{\rho(\sigma)}{\sigma}\, d\sigma = \frac{1}{\pi} \arccos \frac{L_s}{\sqrt{\pi^2 + L_s^2}}$$

Spectral representation

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$$\left[f(Q^2)\right]_{\text{an}} = \int_0^\infty \frac{\rho_f(\sigma)}{\sigma + Q^2 - i\epsilon}\, d\sigma$$

with spectral density $\rho_f(\sigma) = \textbf{Im}\left[f(-\sigma)\right]/\pi$**. Then:**

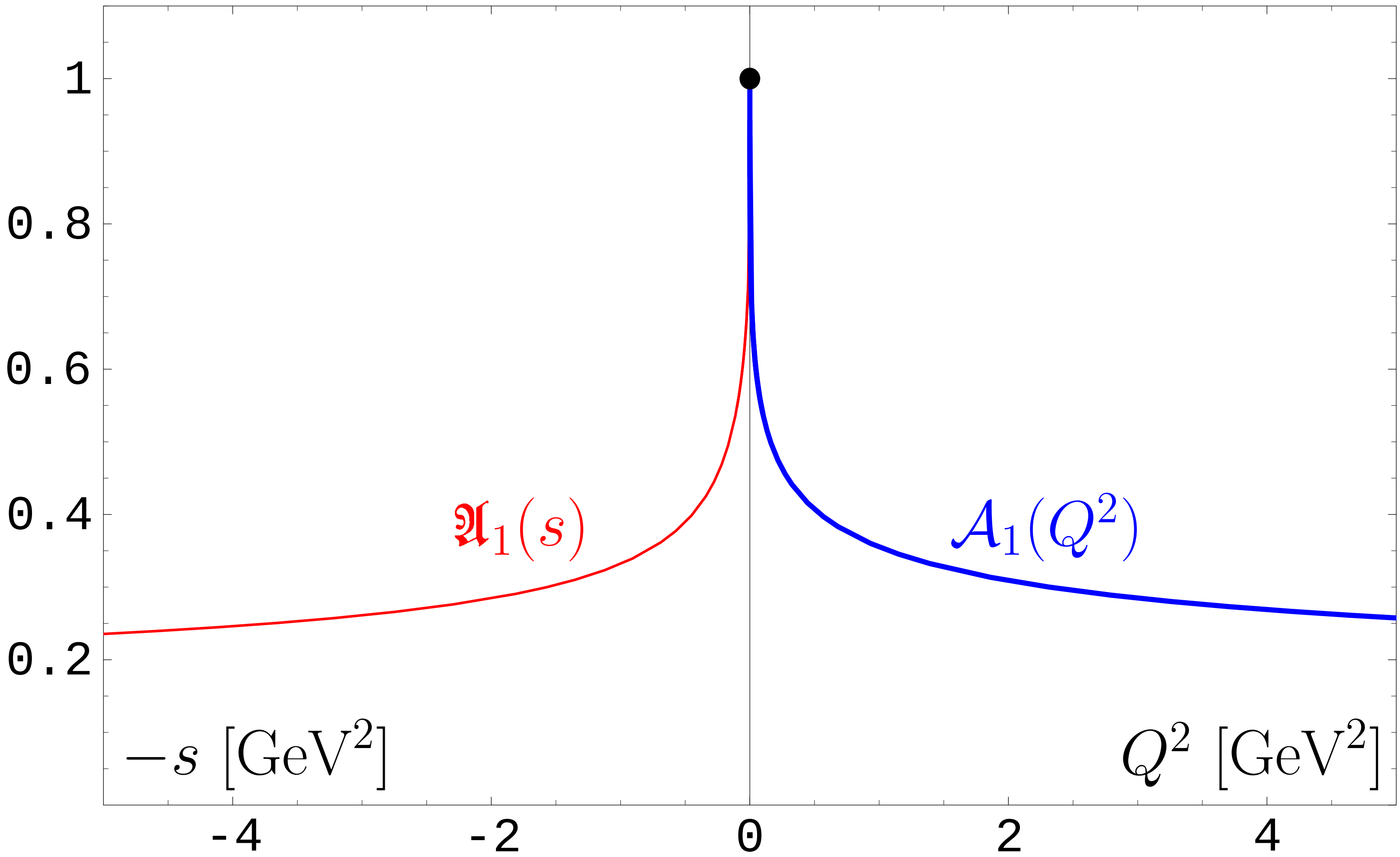
$$\mathcal{A}_n[L] = \int_0^\infty \frac{\rho_n(\sigma)}{\sigma + Q^2}\, d\sigma = \frac{1}{(n-1)!}\left(-\frac{d}{dL}\right)^{n-1} \mathcal{A}_1[L]$$

$$\mathfrak{A}_n[L_s] = \int_s^\infty \frac{\rho_n(\sigma)}{\sigma}\, d\sigma = \frac{1}{(n-1)!}\left(-\frac{d}{dL_s}\right)^{n-1} \mathfrak{A}_1[L_s]$$

$$a_s^n[L] = \frac{1}{(n-1)!}\left(-\frac{d}{dL}\right)^{n-1} a_s[L]$$

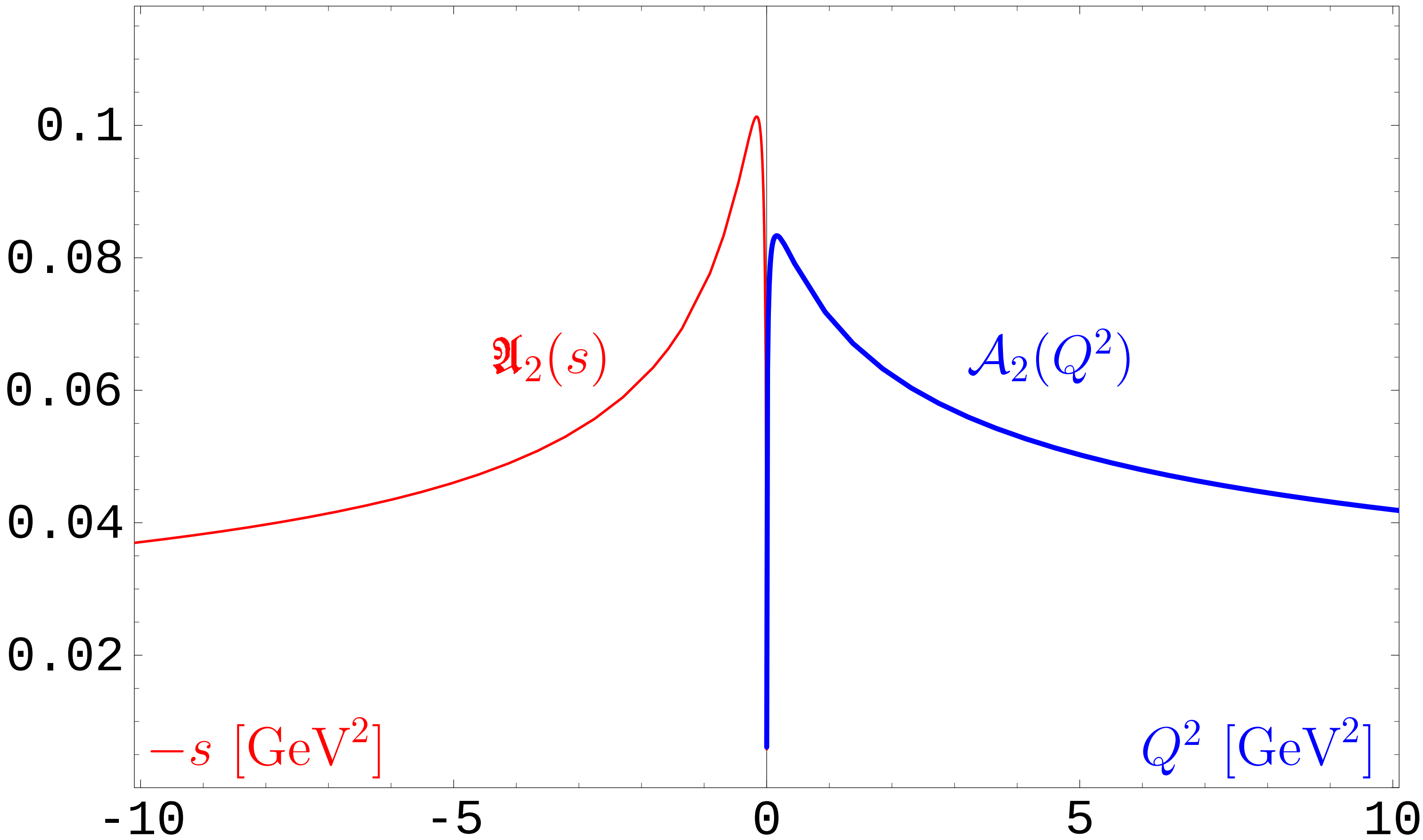
APT graphics: Distorting mirror

First, couplings: $\mathfrak{A}_1(s)$ **and** $\mathcal{A}_1(Q^2)$



APT graphics: Distorting mirror

Second, square-images: $\mathfrak{A}_2(s)$ **and** $\mathcal{A}_2(Q^2)$



Non-power APT: Loop and RS Stability

Instead of universal power-in-α_s expansion:

$$D_{\text{PT}}(Q^2) = d_0 + d_1\,\alpha_s(Q^2) + d_2\,\alpha_s^2(Q^2) + d_3\,\alpha_s^3(Q^2)$$

in APTone should use non-power functional expansions:

$$\mathcal{D}_{\text{APT}}(Q^2) = d_0 + d_1\,\mathcal{A}_1(Q^2) + d_2\,\mathcal{A}_2(Q^2) + d_3\,\mathcal{A}_3(Q^2) \quad \textbf{(*E)}$$

$$\mathcal{R}_{\text{APT}}(s) = d_0 + d_1\,\mathfrak{A}_1(s) + d_2\,\mathfrak{A}_2(s) + d_3\,\mathfrak{A}_3(s) \quad \textbf{(*M)}$$

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$$\mathcal{R}_{\mathsf{APT}}(s) = d_0 + d_1\,\mathfrak{A}_1(s) + d_2\,\mathfrak{A}_2(s) + d_3\,\mathfrak{A}_3(s) \quad \textbf{(*M)}$$

This provides

- Better loop convergence and practical RS independence of observables;
- The d_3 terms in (*E) and (*M) contribute less than 5%. Again the 2-loop (N^{2}LO) level is sufficient.

Relative size of N^kLO terms

Standard pQCD:

Observable	Scale	LO	NLO	N^{2}LO	N^{3}LO	Δ_{exp}
$R_{e^+e^-\to\text{hadrons}}$	10 GeV	92%	7.6%	1.0%	-0.6%	12–30%
R_τ in τ-decay	2 GeV	51%	27%	14%	8%	5%
Bjorken SR	2 GeV	56%	21%	12%	11%	6%

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Bjorken SR	2 GeV	75%	21%	4.1%	-0.1%	6%

Need

to use

Fractional APT

Problems of APT

In standard QCD PT we have not only power series $F[L] = \sum_m f_m\, a_s^m[L]$, but also:

- RG-improvement to account for higher-orders $\to$
$$Z[L] = \textbf{exp}\left\{\int^{a_s[L]} \frac{\gamma(a)}{\beta(a)}\, da\right\} \xrightarrow{\text{1-loop}} [a_s[L]]^{\gamma_0/(2\beta_0)}$$
- Factorization $\to [a_s[L]]^n\, L^m$
- Sudakov resummation $\to \textbf{exp}\,[-a_s[L]\cdot f(x)]$

New functions: $(a_s)^\nu\,,\ (a_s)^\nu\,\textbf{ln}(a_s),\ (a_s)^\nu\, L^m,\ e^{-a_s},\ \ldots$

Constructing one-loop FAPT

In one-loop APT we have a very nice recurrence relation

$$\mathcal{A}_n[L] = \frac{1}{(n-1)!}\left(-\frac{d}{dL}\right)^{n-1}\mathcal{A}_1[L]$$

and the same in Minkowski domain

$$\mathfrak{A}_n[L] = \frac{1}{(n-1)!}\left(-\frac{d}{dL}\right)^{n-1}\mathfrak{A}_1[L]\,.$$

We can use it to construct FAPT.

FAPT(E): Properties of $\mathcal{A}_\nu[L]$

First, Euclidean coupling ($L = L(Q^2)$):

$$\mathcal{A}_\nu[L] = \frac{1}{L^\nu} - \frac{F(e^{-L}, 1-\nu)}{\Gamma(\nu)}$$

Here $F(z, \nu)$ is reduced Lerch transcendent. function. It is analytic function in ν. Properties:

- $\mathcal{A}_0[L] = 1$;
- $\mathcal{A}_{-m}[L] = L^m$ for $m \in \mathbb{N}$;
- $\mathcal{A}_m[L] = (-1)^m \mathcal{A}_m[-L]$ for $m \geq 2\,,\ m \in \mathbb{N}$;
- $\mathcal{A}_m[\pm\infty] = 0$ for $m \geq 2\,,\ m \in \mathbb{N}$;

FAPT(M): Properties of $\mathfrak{A}_\nu[L]$

Now, Minkowskian coupling ($L = L(s)$):

$$\mathfrak{A}_\nu[L] = \frac{\sin\left[(\nu-1)\arccos\left(L/\sqrt{\pi^2+L^2}\right)\right]}{\pi(\nu-1)\left(\pi^2+L^2\right)^{(\nu-1)/2}}$$

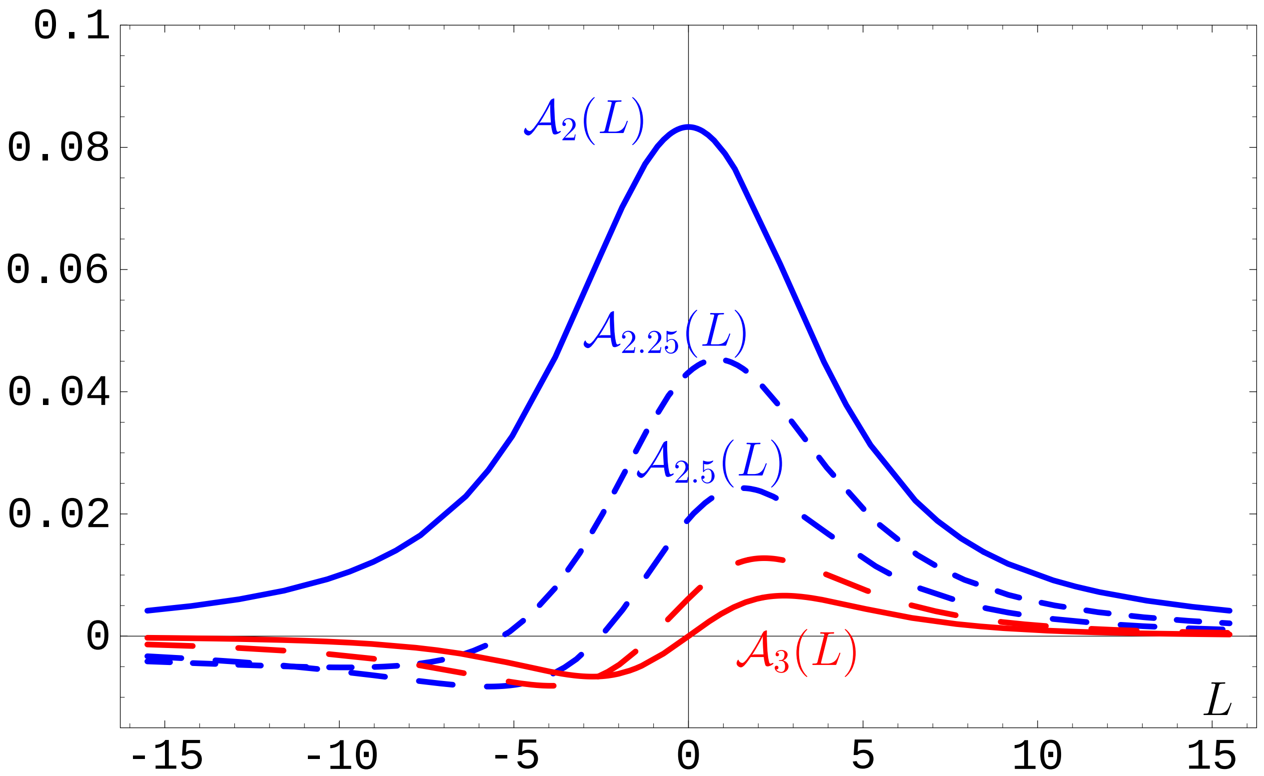
Here we need only elementary functions. Properties:

- $\mathfrak{A}_0[L] = 1$;
- $\mathfrak{A}_{-1}[L] = L$;
- $\mathfrak{A}_{-2}[L] = L^2 - \dfrac{\pi^2}{3}$, $\mathfrak{A}_{-3}[L] = L\left(L^2 - \pi^2\right)$, … ;
- $\mathfrak{A}_m[L] = (-1)^m \mathfrak{A}_m[-L]$ for $m \geq 2\,,\ m \in \mathbb{N}$;
- $\mathfrak{A}_m[\pm\infty] = 0$ for $m \geq 2\,,\ m \in \mathbb{N}$

FAPT(E): Graphics of $\mathcal{A}_\nu[L]$ vs. L

$$\mathcal{A}_\nu[L] = \frac{1}{L^\nu} - \frac{F(e^{-L}, 1-\nu)}{\Gamma(\nu)}$$

Graphics for fractional $\nu \in [2, 3]$:



FAPT(M): Graphics of $\mathfrak{A}_\nu[L]$ vs. L

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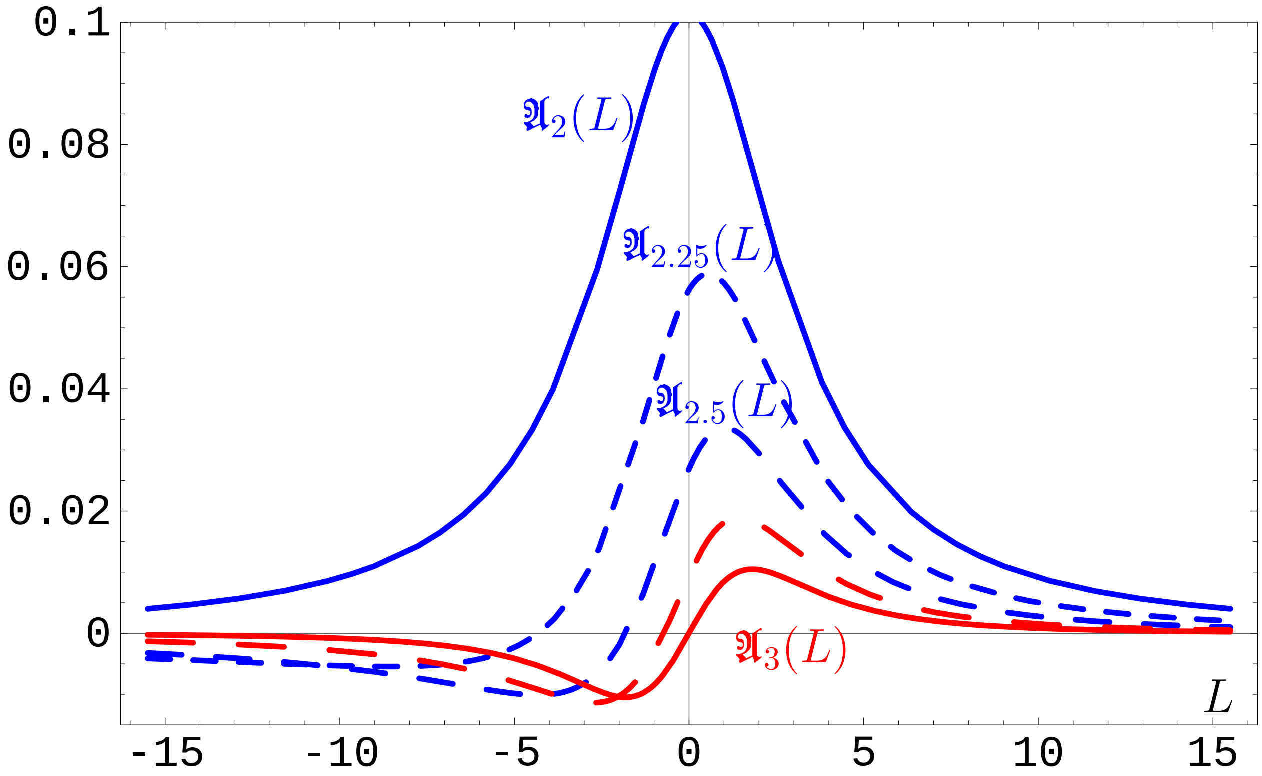
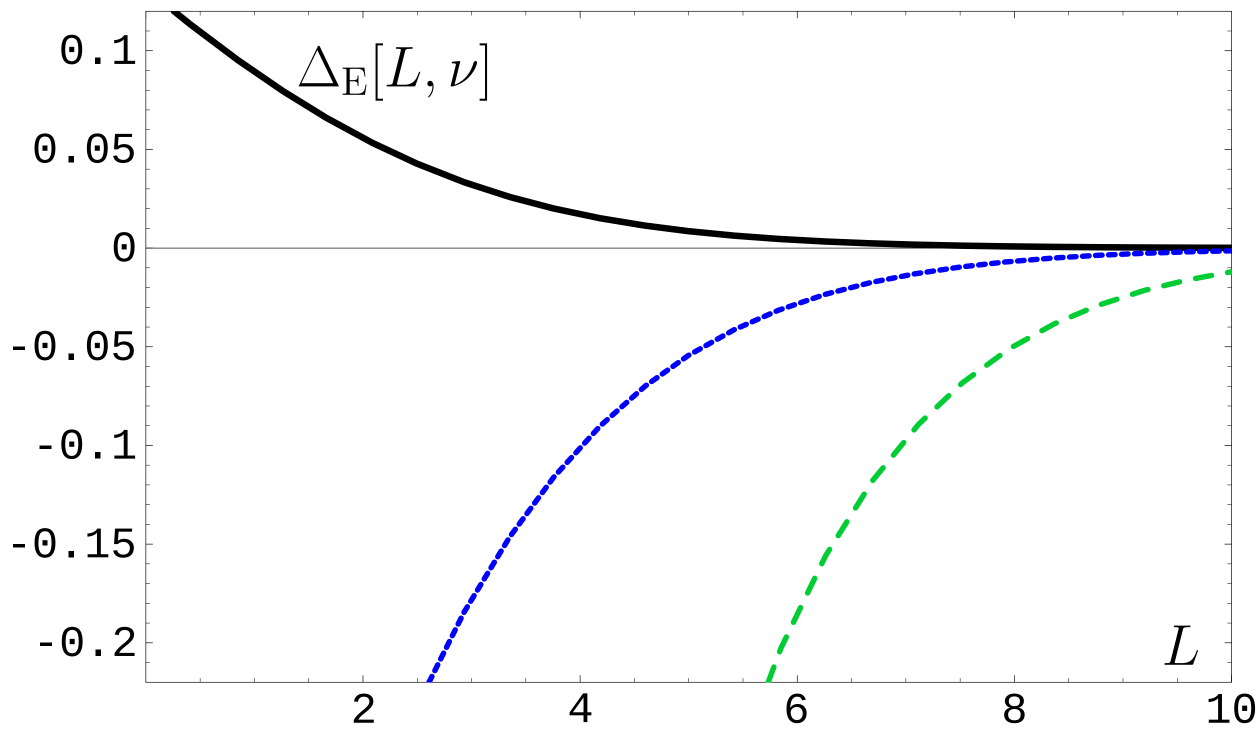
Compare with graphics in Minkowskian region :



FAPT(E): Comparing $\mathcal{A}_\nu$ with $(\mathcal{A}_1)^\nu$

$$\Delta_{\mathrm{E}}(L,\nu) = \frac{\mathcal{A}_\nu[L] - (\mathcal{A}_1[L])^\nu}{\mathcal{A}_\nu[L]}$$

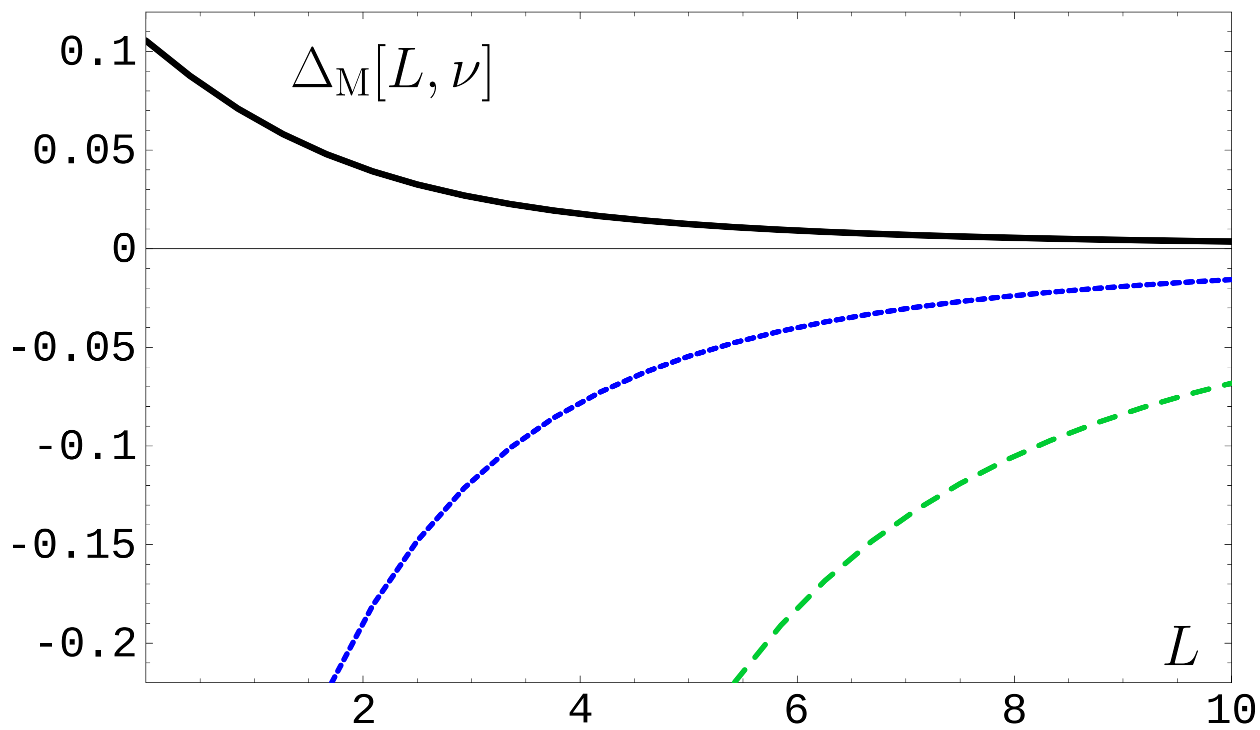
Graphics for fractional $\nu =$0.62, 1.62 and 2.62:



FAPT(M): Comparing $\mathfrak{A}_\nu$ with $\left(\mathfrak{A}_1\right)^\nu$

$$\Delta_{\mathrm{M}}(L,\nu) = \frac{\mathfrak{A}_\nu[L] - \left(\mathfrak{A}_1[L]\right)^\nu}{\mathfrak{A}_\nu[L]}$$

Minkowskian graphics for $\nu =$0.62, 1.62 and 2.62:



Resummation in one-loop APT and FAPT

Resummation in one-loop APT

Consider series $$\mathcal{D}[L] = d_0 + \sum_{n=1}^{\infty} d_n \, \mathcal{A}_n[L]$$

Resummation in one-loop APT

Consider series $\mathcal{D}[L] = d_0 + \sum_{n=1}^{\infty} d_n \, \mathcal{A}_n[L]$

Let exist the generating function $P(t)$ for coefficients:

$$d_n = d_1 \int_0^\infty P(t)\, t^{n-1} dt \quad \text{with} \quad \int_0^\infty P(t)\, dt = 1\,.$$

We define a shorthand notation

$$\langle\langle f(t) \rangle\rangle_{P(t)} \equiv \int_0^\infty f(t)\, P(t)\, dt\,.$$

Then coefficients $d_n = d_1 \,\langle\langle t^{n-1} \rangle\rangle_{P(t)}$.

Resummation in one-loop APT

Consider series $\mathcal{D}[L] = d_0 + \sum_{n=1}^{\infty} d_n \, \mathcal{A}_n[L]$

with coefficients $d_n = d_1 \, \langle\langle t^{n-1} \rangle\rangle_{P(t)}$.

We have one-loop recurrence relation:

$$\mathcal{A}_{n+1}[L] = \frac{1}{\Gamma(n+1)} \left(-\frac{d}{dL}\right)^n \mathcal{A}_1[L] \,.$$

Resummation in one-loop APT

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$$\mathcal{A}_{n+1}[L] = \frac{1}{\Gamma(n+1)} \left(-\frac{d}{dL}\right)^n \mathcal{A}_1[L]\,.$$

Result:

$$\mathcal{D}[L] = d_0 + d_1 \,\langle\langle \mathcal{A}_1[L-t] \rangle\rangle_{P(t)}$$

Resummation in one-loop APT

Consider series $\mathcal{D}[L] = d_0 + \sum_{n=1}^{\infty} d_n\, \mathcal{A}_n[L]$

with coefficients $d_n = d_1 \, \langle\langle t^{n-1} \rangle\rangle_{P(t)}$.

We have one-loop recurrence relation:

$$\mathcal{A}_{n+1}[L] = \frac{1}{\Gamma(n+1)} \left(-\frac{d}{dL}\right)^n \mathcal{A}_1[L]\,.$$

Result:

$$\mathcal{D}[L] = d_0 + d_1 \, \langle\langle \mathcal{A}_1[L-t] \rangle\rangle_{P(t)}$$

and for Minkowski region:

$$\mathcal{R}[L] = d_0 + d_1 \, \langle\langle \mathfrak{A}_1[L-t] \rangle\rangle_{P(t)}$$

Resummation in one-loop FAPT

Consider seria $$\mathcal{R}_\nu[L] = d_0\,\mathfrak{A}_\nu[L] + \sum_{n=1}^{\infty} d_n\,\mathfrak{A}_{n+\nu}[L]$$

and $$\mathcal{D}_\nu[L] = d_0\,\mathcal{A}_\nu[L] + \sum_{n=1}^{\infty} d_n\,\mathcal{A}_{n+\nu}[L]$$

with coefficients $d_n = d_1\,\langle\langle t^{n-1}\rangle\rangle_{P(t)}$.

Result:

$$\begin{aligned} \mathcal{R}_\nu[L] &= d_0\,\mathfrak{A}_\nu[L] + d_1\,\langle\langle\mathfrak{A}_{1+\nu}[L-t]\rangle\rangle_{P_\nu(t)}\,; \\ \mathcal{D}_\nu[L] &= d_0\,\mathcal{A}_\nu[L] + d_1\,\langle\langle\mathcal{A}_{1+\nu}[L-t]\rangle\rangle_{P_\nu(t)}\,. \end{aligned}$$

where $$P_\nu(t) = \int_0^1 P\left(\frac{t}{1-z}\right)\nu\,z^{\nu-1}\frac{dz}{1-z}\,.$$

Resummation in two-loop APT and FAPT

Resummation in two-loop APT

Consider series $\mathcal{S}[L] = \sum_{n=1}^{\infty} \langle\langle t^{n-1} \rangle\rangle_{P(t)} \mathcal{F}_n[L]$.

Here $\mathcal{F}_n[L] = \mathcal{A}_n^{(2)}[L]$ or $\mathfrak{A}_n^{(2)}[L]$.

Resummation in two-loop APT

Consider series $\mathcal{S}[L] = \sum_{n=1}^{\infty} \langle\langle t^{n-1} \rangle\rangle_{P(t)} \mathcal{F}_n[L]$.

Here $\mathcal{F}_n[L] = \mathcal{A}_n^{(2)}[L]$ or $\mathfrak{A}_n^{(2)}[L]$.

We have two-loop recurrence relation ($c_1 = b_1/b_0^2$):

$$-\frac{1}{n}\frac{d}{dL}\mathcal{F}_n[L] = \mathcal{F}_{n+1}[L] + c_1\,\mathcal{F}_{n+2}[L]$$

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Result ($\tau(t) = t - c_1 \ln(1 + t/c_1)$):

$$\mathcal{S}[L] = \left\langle\!\!\left\langle \frac{c_1\,\mathcal{F}_1[L] + t\,\mathcal{F}_1[L-\tau(t)]}{c_1+t} + \frac{c_1\,t}{c_1+t}\,\mathcal{F}_2[L-\tau(t)] \right\rangle\!\!\right\rangle_{P(t)}$$
$$- \left\langle\!\!\left\langle \frac{c_1\,t}{c_1+t}\int_0^t \frac{dt'}{c_1+t'}\,\frac{d\mathcal{F}_1[L+\tau(t')-\tau(t)]}{dL} \right\rangle\!\!\right\rangle_{P(t)} .$$

Resummation in two-loop (global) FAPT

Consider series $$\mathcal{S}_\nu[L] = \sum_{n=1}^{\infty} \langle\langle t^{n-1} \rangle\rangle_{P(t)} \mathcal{F}_{n+\nu}[L].$$

Here $\mathcal{F}_\nu[L] = \mathcal{A}_\nu^{(2)}[L]$ or $\mathfrak{A}_\nu^{(2)}[L]$ (or $\rho_\nu^{(2)}[L]$ — for global).

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Result ($\tau(t) = t - c_1 \ln(1 + t/c_1)$**):**

$$\mathcal{S}[L] = \left\langle\!\!\left\langle \mathcal{F}_{1+\nu}[L] - \frac{t^2}{c_1+t}\int_0^1 z^\nu dz\,\dot{\mathcal{F}}_{1+\nu}[L+\tau(t\,z)-\tau(t)] \right.\right.$$

$$\left.\left. + \frac{c_1\,t}{c_1+t}\left\{\mathcal{F}_{2+\nu}[L] - \int_0^1 dz\,\frac{t^2\,z^{\nu+1}}{c_1+t\,z}\dot{\mathcal{F}}_{2+\nu}[L+\tau(t\,z)-\tau(t)]\right\}\right\rangle\!\!\right\rangle_{P(t)}$$

Resummation in two-loop (global) FAPT

Consider series $\mathcal{S}_{\nu_0,\nu_1}[L] = \sum_{n=1}^{\infty} \langle\langle t^{n-1} \rangle\rangle_{P(t)} \mathcal{F}_{n+\nu_0,\nu_1}[L]$.

Here $\mathcal{F}_{n+\nu_0,\nu_1}[L] = \mathcal{B}^{(2)}_{n+\nu_0,\nu_1}[L]$ **or** $\mathfrak{B}^{(2)}_{n+\nu_0,\nu_1}[L]$

(or $\rho^{(2)}_{n+\nu_0,\nu_1}[L]$ **— for global),**

where

$$\mathcal{B}_{\nu;\nu_1}[L] = \mathbf{A}_{\mathrm{E,M}} \left[a^{\nu}_{(2)}[L] \left(1 + c_1\, a_{(2)}\right)^{\nu_1}[L] \right]$$

is the analytic image of the two-loop evolution factor.

We have constructed formulas of resummation for $\mathcal{S}_{\nu_0,\nu_1}[L]$

as well.

Resummation for Adler function $D(Q^2)$

Adler function $D(Q^2)$ in vector channel

Adler function $D(Q^2)$ can be expressed in QCD by means of the correlator of quark vector currents

$$\Pi_{\mathsf{V}}(Q^2) = \frac{(4\pi)^2}{3q^2}\, i \int dx\, e^{iqx} \langle 0|\; T[\, J_\mu(x) J^\mu(0)\,]\; |0\rangle$$

in terms of discontinuity of its imaginary part

$$R_{\mathsf{V}}(s) = \frac{1}{\pi}\,\mathrm{Im}\,\Pi_{\mathsf{V}}(-s - i\epsilon)\,,$$

so that

$$D(Q^2) = Q^2 \int_0^\infty \frac{R_{\mathsf{V}}(\sigma)}{(\sigma + Q^2)^2}\, d\sigma\,.$$

APT analysis of $D(Q^2)$ *and* $R_V(s)$

QCD PT gives us

$$D(Q^2) = 1 + \sum_{m>0} \frac{d_m}{\pi^m} \left(\alpha_s(Q^2)\right)^m .$$

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$$\mathcal{D}_N(Q^2) = 1 + \sum_{m>0}^{N} \frac{d_m}{\pi^m} \mathcal{A}_m^{\text{glob}}(Q^2)$$

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and in APT(M)

$$\mathcal{R}_{\text{V};N}(s) = 1 + \sum_{m>0}^{N} \frac{d_m}{\pi^m} \mathfrak{A}_m^{\text{glob}}(s)$$

Model for perturbative coefficients

Coefficients d_m of the PT series:

Model	d_1	d_2	d_3	d_4	d_5
pQCD with $N_f = 4$	1	1.52	2.59		—

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$c = 3.467,\ \beta = 1.325$	1	1.50	2.62		

We use model $\tilde{d}_n^{\text{mod}} = \dfrac{c^{n-1}(\beta^{n+1} - n)}{\beta^2 - 1}\,\Gamma(n)$

with parameters β and c estimated by known $\tilde{d}_n$

that possesses the Lipatov asymptotics $\tilde{d}_n^{\text{mod}} \sim b^n n!$ at $n \gg 1$.

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$c = 3.456,\ \beta = 1.325$	1	1.49	2.60	27.5	

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"INNA" model	1	1.44	$[3, 9]$	$[20, 48]$	$[674, 2786]$

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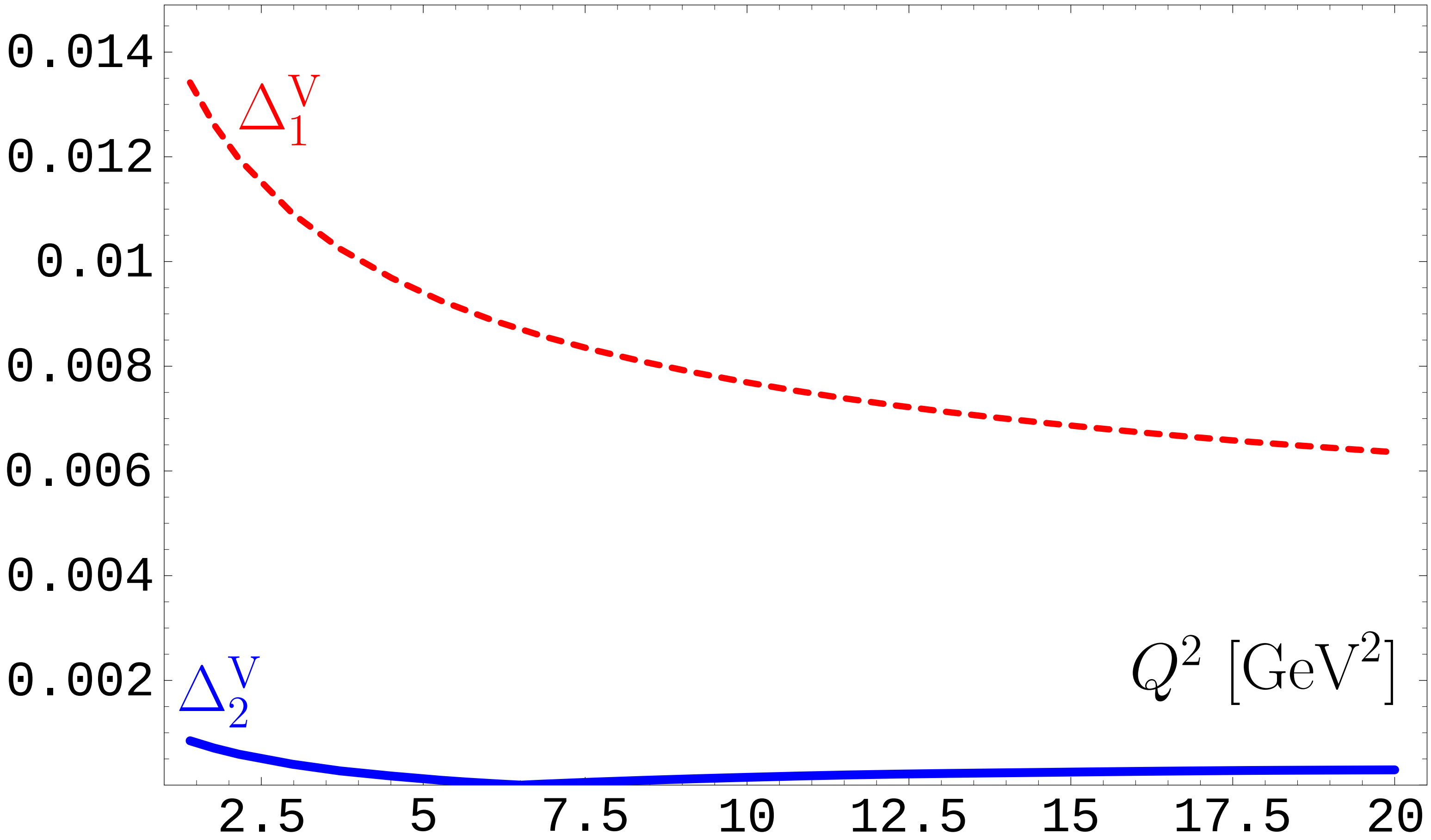
APT(E) for $\mathcal{D}(Q^2)$: Truncation errors

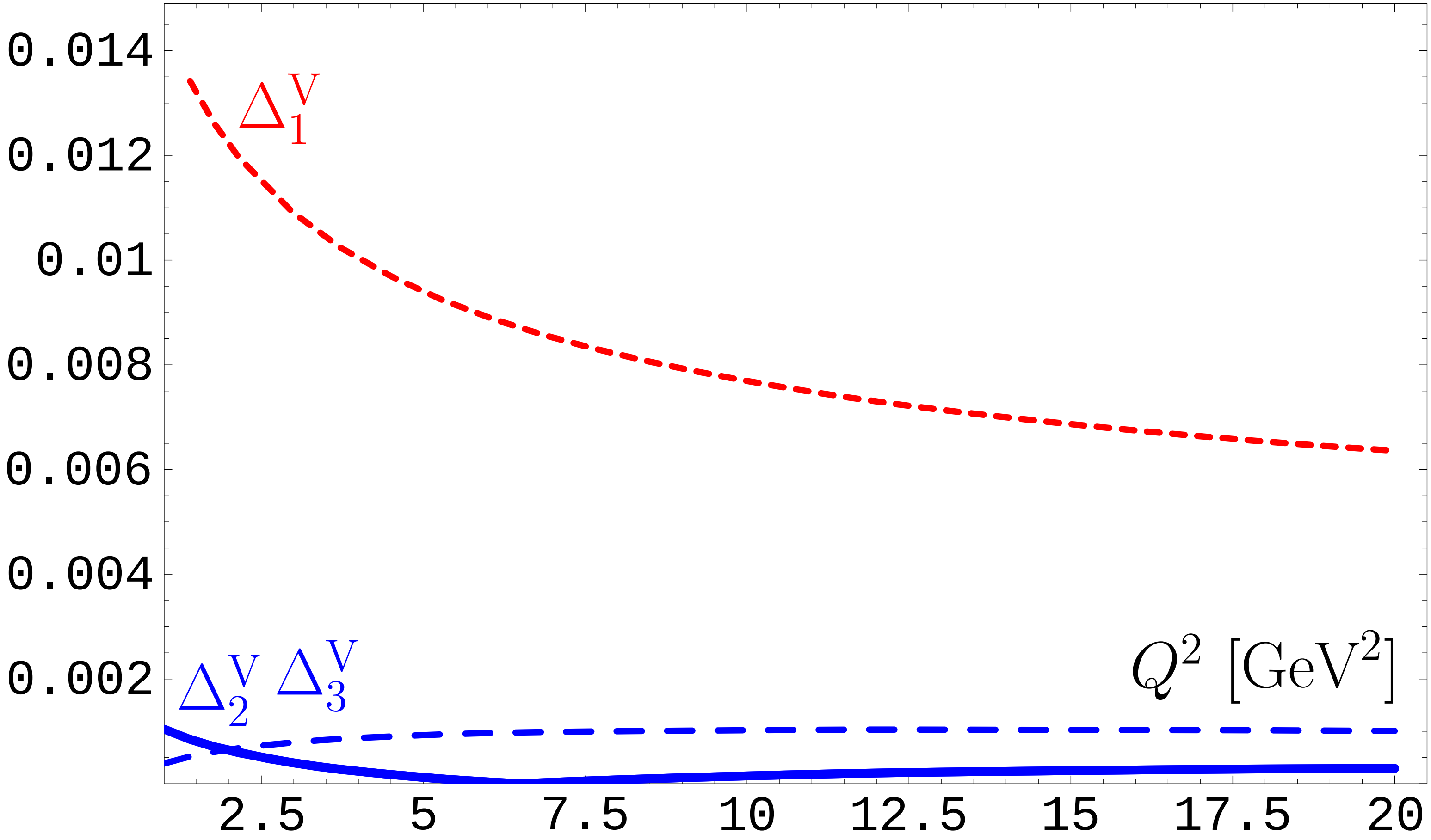
We define relative errors of series truncation at Nth term:

$$\Delta_N^{\mathrm{V}}[L] = 1 - \mathcal{D}_N[L]/\mathcal{D}_\infty[L]$$



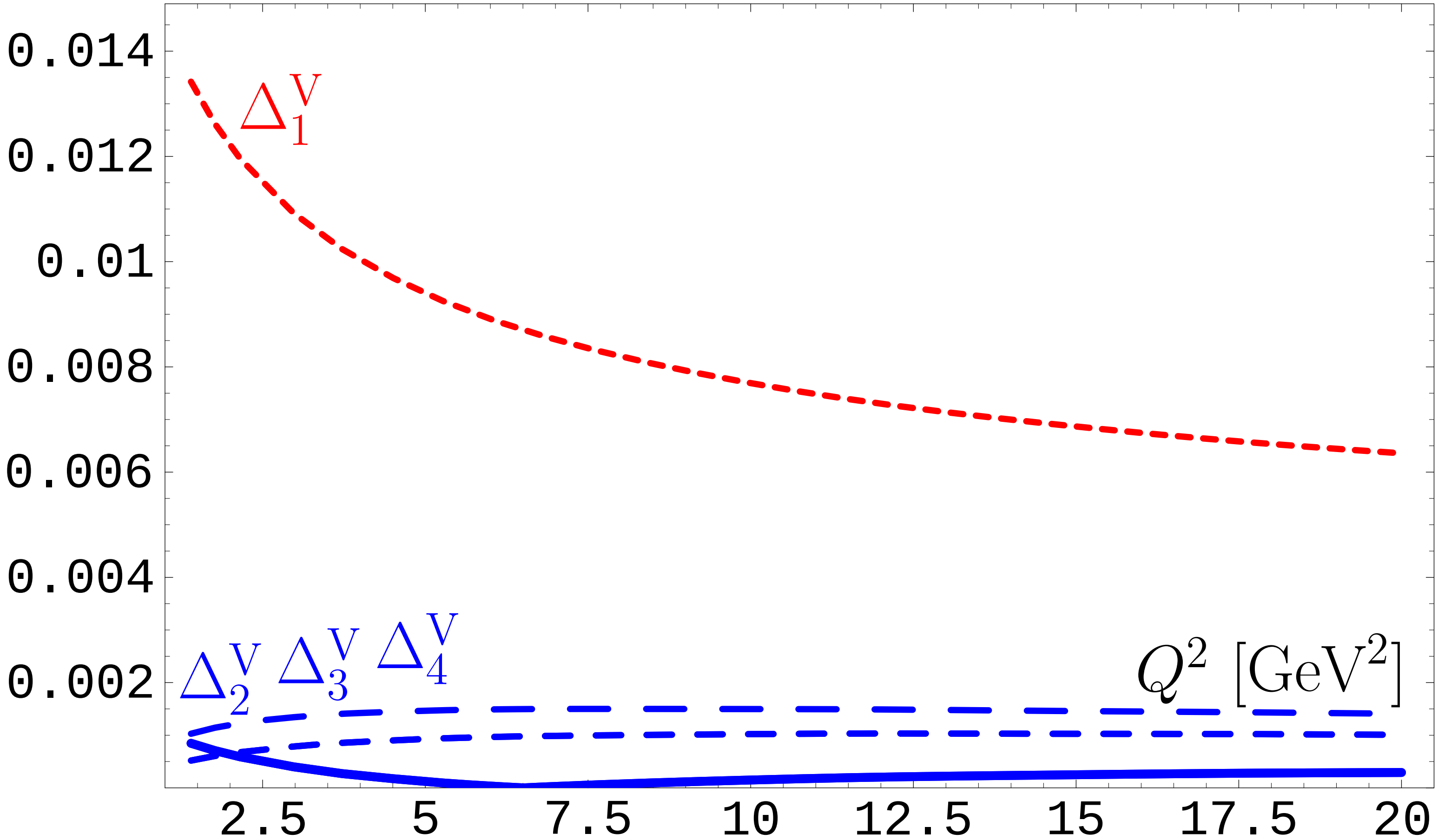
APT(E) for $\mathcal{D}(Q^2)$: Truncation errors

Conclusion: **The best accuracy (better than 0.1%) is achieved for N^{2}LO approximation.**



APT(E) for $\mathcal{D}(Q^2)$: Truncation errors

Conclusion: If we add more terms N^{3}LO — truncation error increases.



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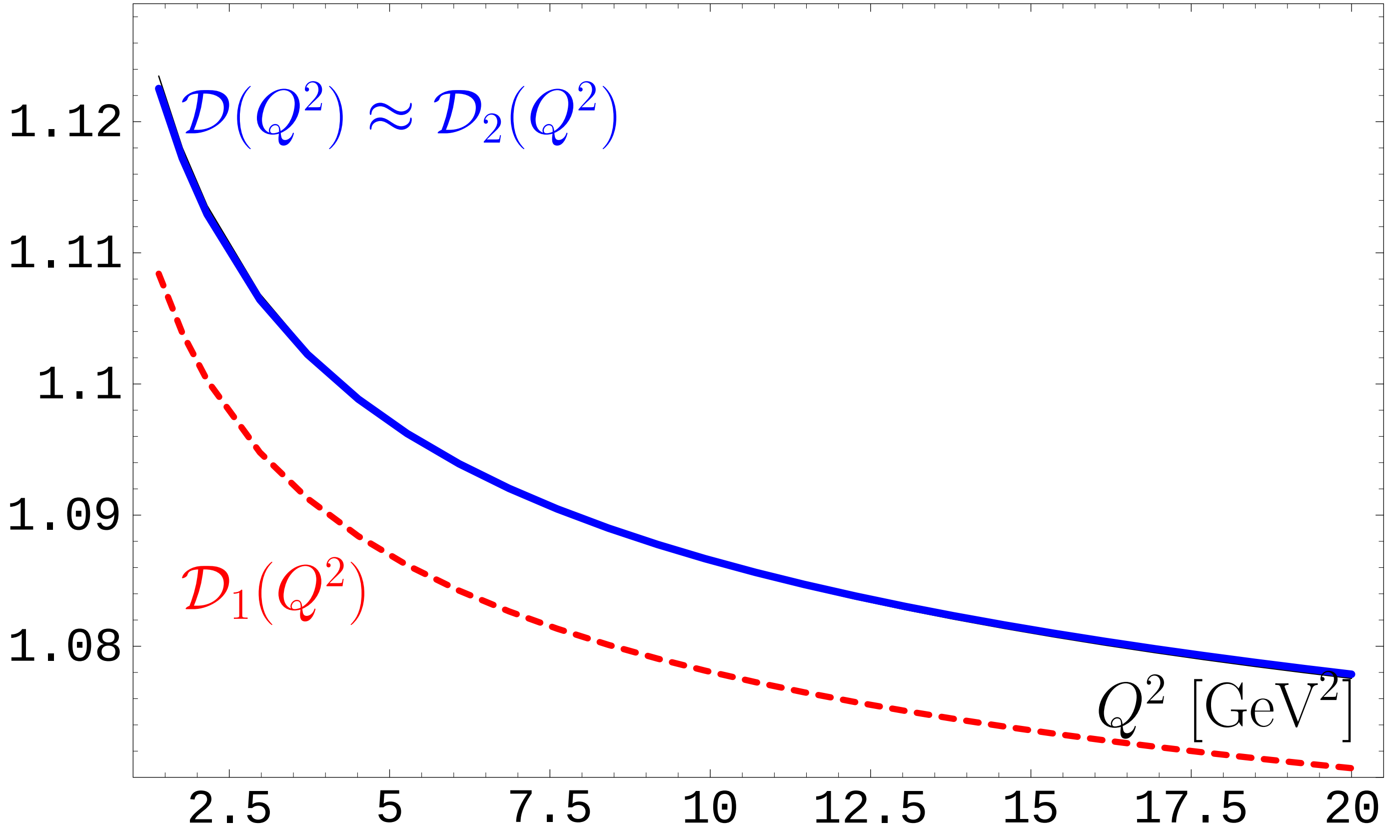

APT(E) for $\mathcal{D}(Q^2)$: Errors of modelling $P(t)$

We use model $d_n^{\text{mod}} = \dfrac{c^{n-1}(\beta^{n+1} - n)}{\beta^2 - 1}\,\Gamma(n)$

with parameters $\beta = 1.325$ and $c = 3.456$ estimated by known $\tilde{d}_n$ and with use of Lipatov asymptotics.

We apply it to resum APT series and obtain $\mathcal{D}(Q^2)$.

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We deform our model for d_n by using coefficients $\beta_{\text{NNA}} = 1.322$ and $c_{\text{NNA}} = 3.885$

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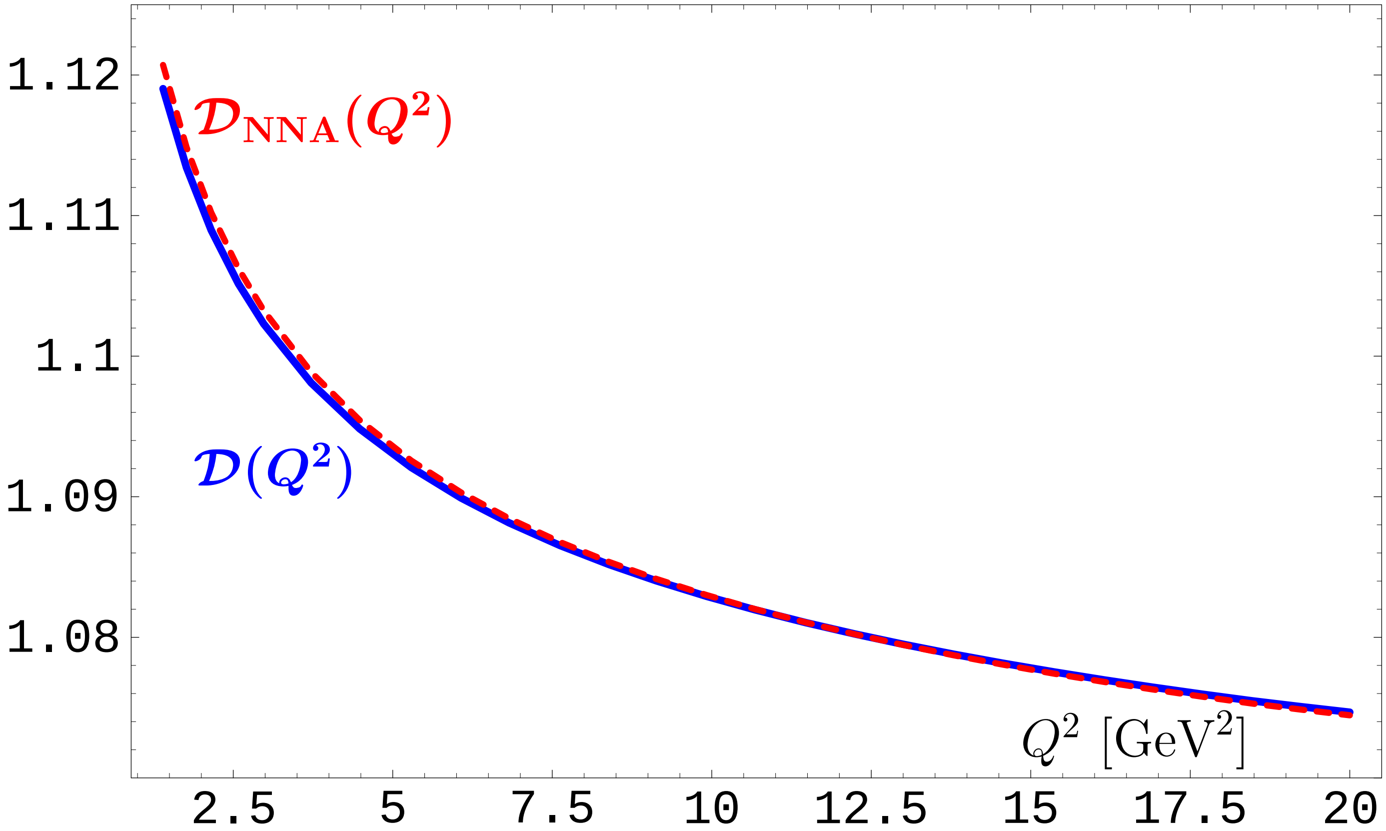
We apply it to resum APT series and obtain $\mathcal{D}_{\text{NNA}}(Q^2)$.

APT(E) for $\mathcal{D}(Q^2)$: Errors of modelling $P(t)$

Conclusion: **The result of resummation is stable to the variations of higher-order coefficients: deviation is of the order of 0.1%.**



Higgs boson decay

$$H^0 \to b\bar{b}$$

Higgs boson decay into $b\bar{b}$-pair

This decay can be expressed in QCD by means of the correlator of quark scalar currents $J_{\mathrm{S}}(x) = :\bar{b}(x)b(x):$

$$\Pi(Q^2) = (4\pi)^2 i \int dx e^{iqx} \langle 0 | \, T[\, J_{\mathrm{S}}(x) J_{\mathrm{S}}(0)\,] \, | 0 \rangle$$

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in terms of discontinuity of its imaginary part

$$R_{\mathsf{S}}(s) = \mathrm{Im}\,\Pi(-s - i\epsilon)/(2\pi\, s)\,,$$

so that

$$\Gamma_{\mathsf{H}\to b\bar{b}}(M_{\mathsf{H}}) = \frac{G_F}{4\sqrt{2}\pi} M_{\mathsf{H}}\, m_b^2(M_{\mathsf{H}})\, R_{\mathsf{S}}(s = M_{\mathsf{H}}^2)\,.$$

FAPT(M) analysis of R_S

Running mass $m(Q^2)$ is described by the RG equation

$$m^2(Q^2) = \hat{m}^2 \alpha_s^{\nu_0}(Q^2) \left[1 + \frac{c_1 b_0 \alpha_s(Q^2)}{4\pi^2}\right]^{\nu_1} .$$

with RG-invariant mass $\hat{m}^2$ (for b-quark $\hat{m}_b \approx 8.53$ GeV) and $\nu_0 = 1.04$, $\nu_1 = 1.86$.

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$$\left[3\,\hat{m}_b^2\right]^{-1}\,\widetilde{D}_\mathrm{S}(Q^2) = \alpha_s^{\nu_0}(Q^2) + \sum_{m>0} \frac{d_m}{\pi^m}\,\alpha_s^{m+\nu_0}(Q^2)\,.$$

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In 1-loop FAPT(M) we obtain

$$\widetilde{\mathcal{R}}_{\text{S}}^{(1);N}[L] = 3\hat{m}^2 \left[\mathfrak{A}_{\nu_0}^{(1);\text{glob}}[L] + \sum_{m>0}^{N} \frac{d_m}{\pi^m}\,\mathfrak{A}_{m+\nu_0}^{(1);\text{glob}}[L]\right]$$

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with RG-invariant mass $\hat{m}^2$ (for b-quark $\hat{m}_b \approx 8.53$ GeV) and $\nu_0 = 1.04$, $\nu_1 = 1.86$. This gives us

$$\left[3\,\hat{m}_b^2\right]^{-1} \widetilde{D}_S(Q^2) = \alpha_s^{\nu_0}(Q^2) + \sum_{m>0} \frac{d_m}{\pi^m}\, \alpha_s^{m+\nu_0}(Q^2)\,.$$

In 2-loop FAPT(M) we obtain

$$\widetilde{\mathcal{R}}_S^{(2);N}[L] = 3\hat{m}^2 \left[\mathfrak{B}_{\nu_0,\nu_1}^{(2);\text{glob}}[L] + \sum_{m>0}^{N} \frac{d_m}{\pi^m}\, \mathfrak{B}_{m+\nu_0,\nu_1}^{(2);\text{glob}}[L]\right]$$

Model for perturbative coefficients

Coefficients of our series, $\tilde{d}_m = d_m/d_1$, with $d_1 = 17/3$:

Model	$\tilde{d}_1$	$\tilde{d}_2$	$\tilde{d}_3$	$\tilde{d}_4$	$\tilde{d}_5$
pQCD	1	7.42	62.3	620	—
$c = 2.5,\ \beta = -0.48$	1	7.42	62.3	662	—
$c = 2.4,\ \beta = -0.52$	1	7.50	61.1	625	7826
"PMS" model	—	—	64.8	547	7782

We use model $\tilde{d}_n^{\text{mod}} = \dfrac{c^{n-1}(\beta\,\Gamma(n) + \Gamma(n+1))}{\beta + 1}$

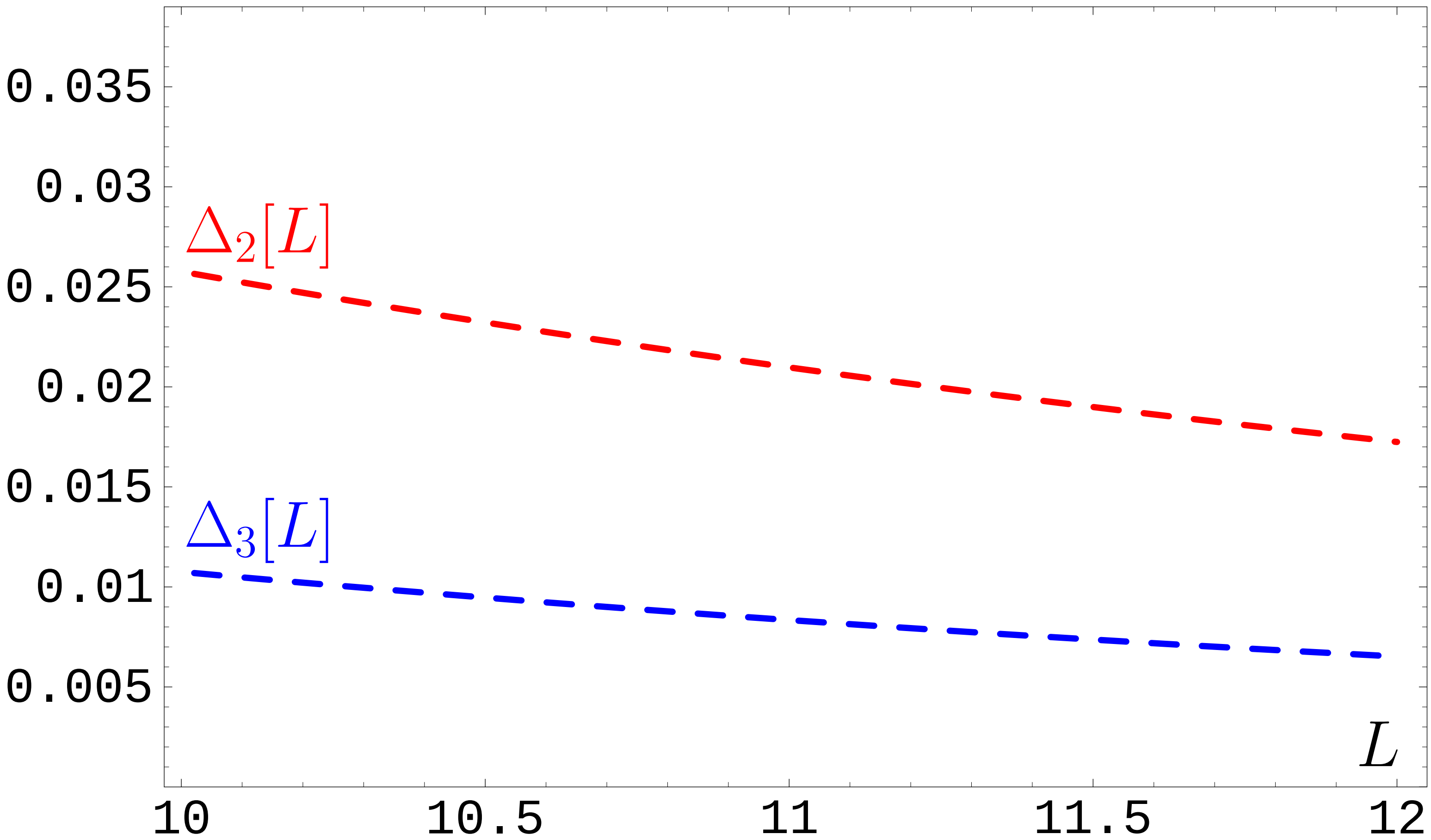
with parameters β and c estimated by known $\tilde{d}_n$

that possesses the Lipatov asymptotics $\tilde{d}_n^{\text{mod}} \sim c^n\, n!$ at $n \gg 1$.

FAPT(M) for $\Gamma_{H\to\bar{b}b}(m_H)$: Truncation errors

We define relative errors of series truncation at Nth term:

$$\Delta_N[L] = 1 - \widetilde{\mathcal{R}}_{\mathrm{S}}^{(2;N)}[L] / \widetilde{\mathcal{R}}_{\mathrm{S}}^{(2;\infty)}[L]$$



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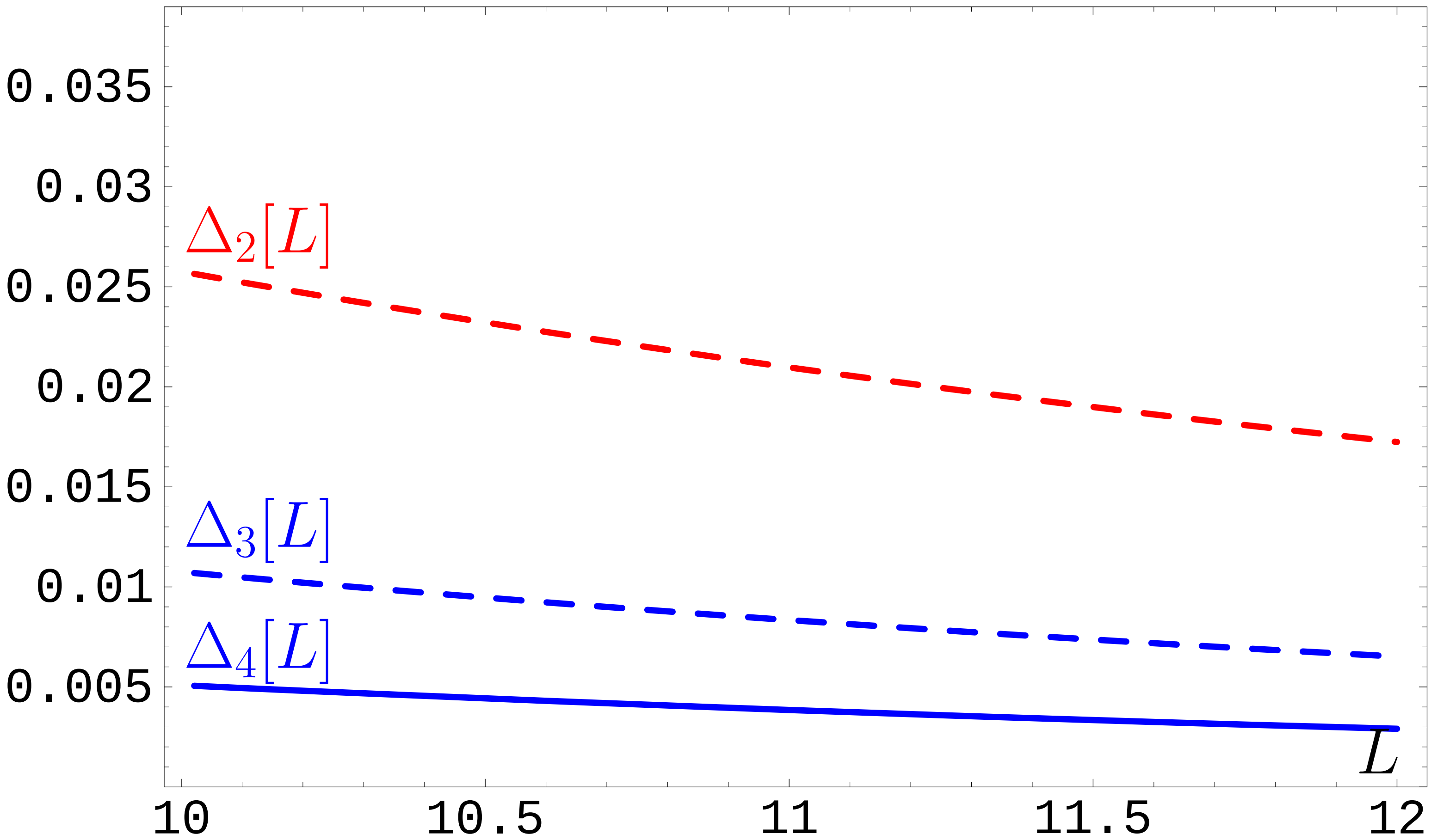

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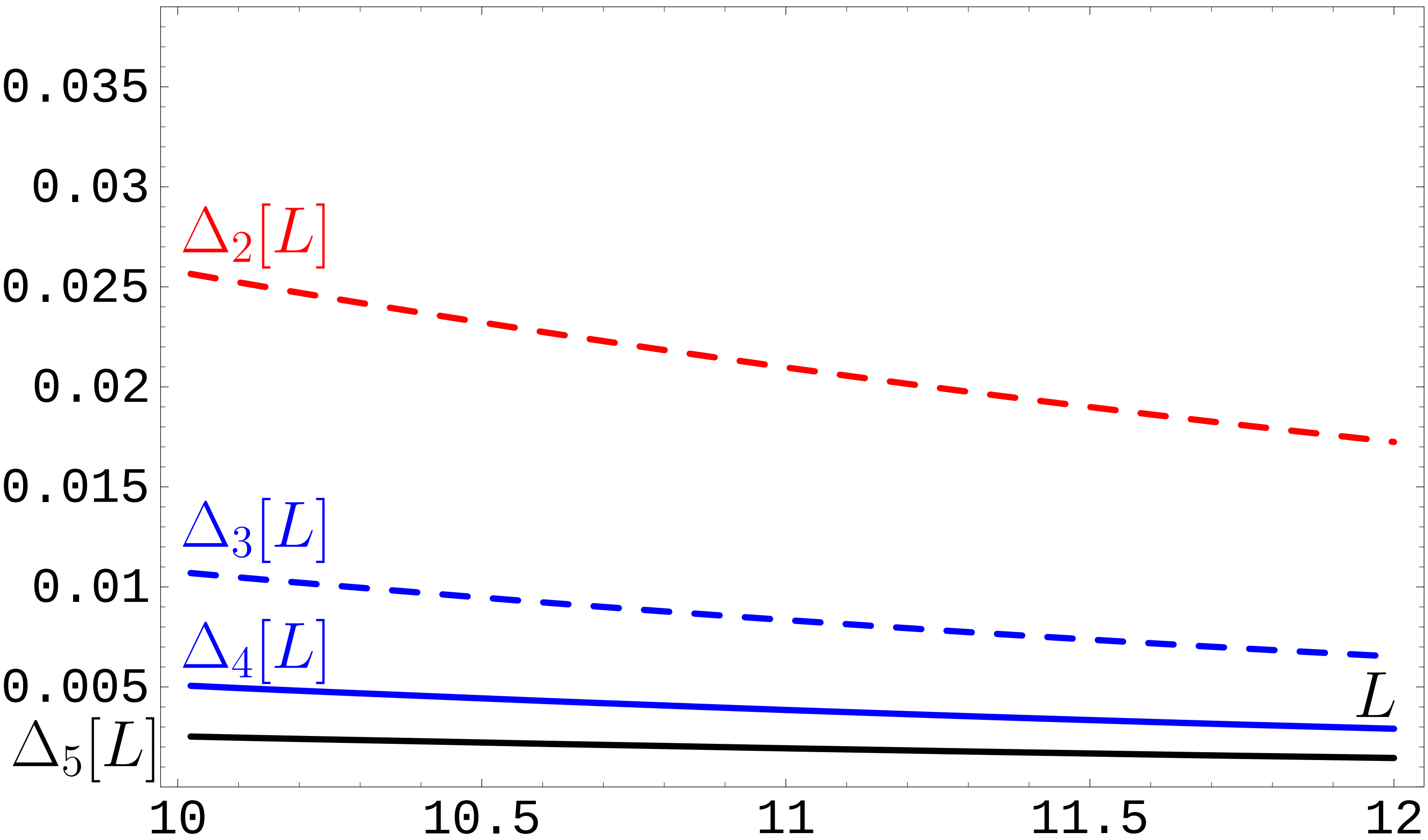

FAPT(M) for $\Gamma_{H\to\bar{b}b}(m_H)$: Truncation errors

Conclusion: **If we need accuracy better than 0.5% — only then we need to calculate the 5-th correction.**

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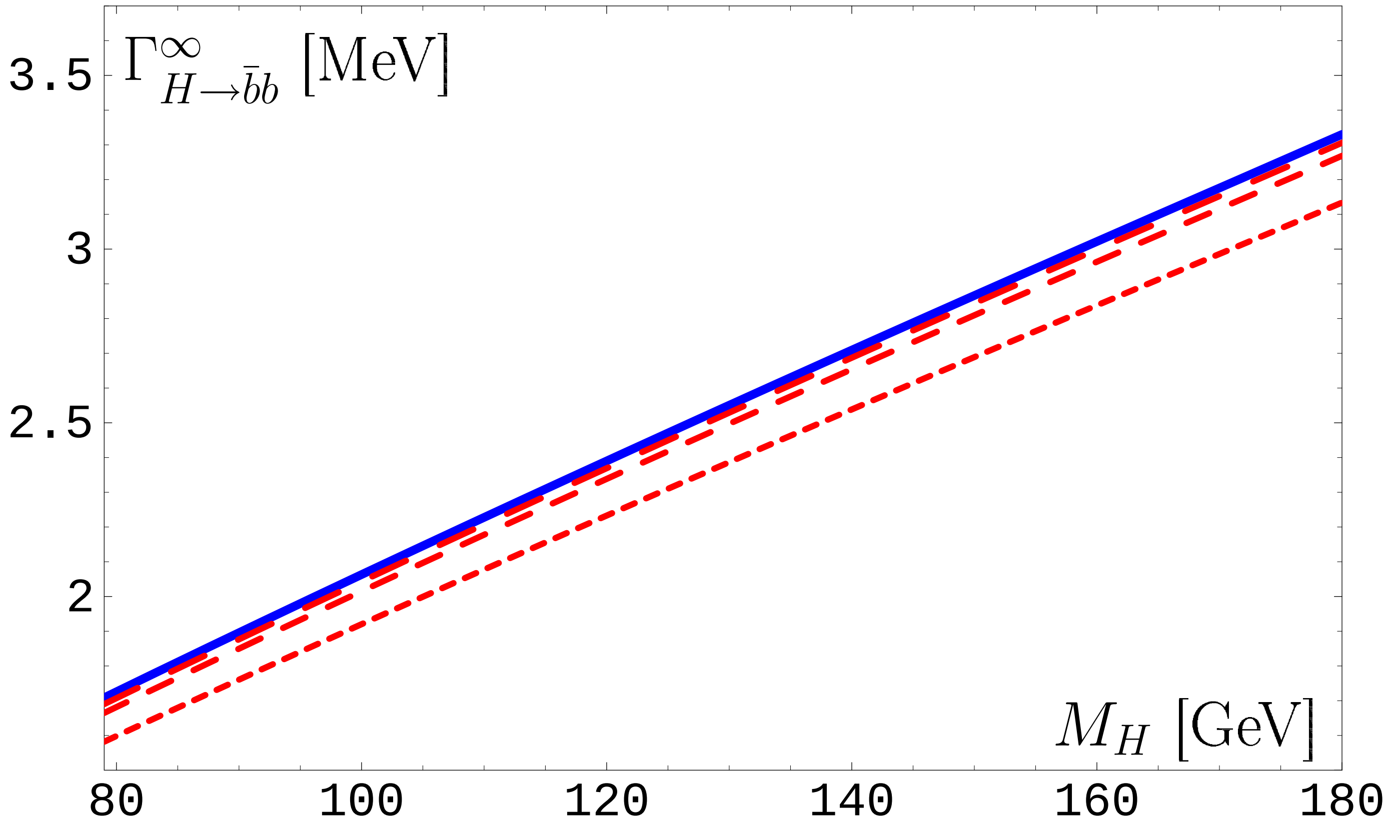
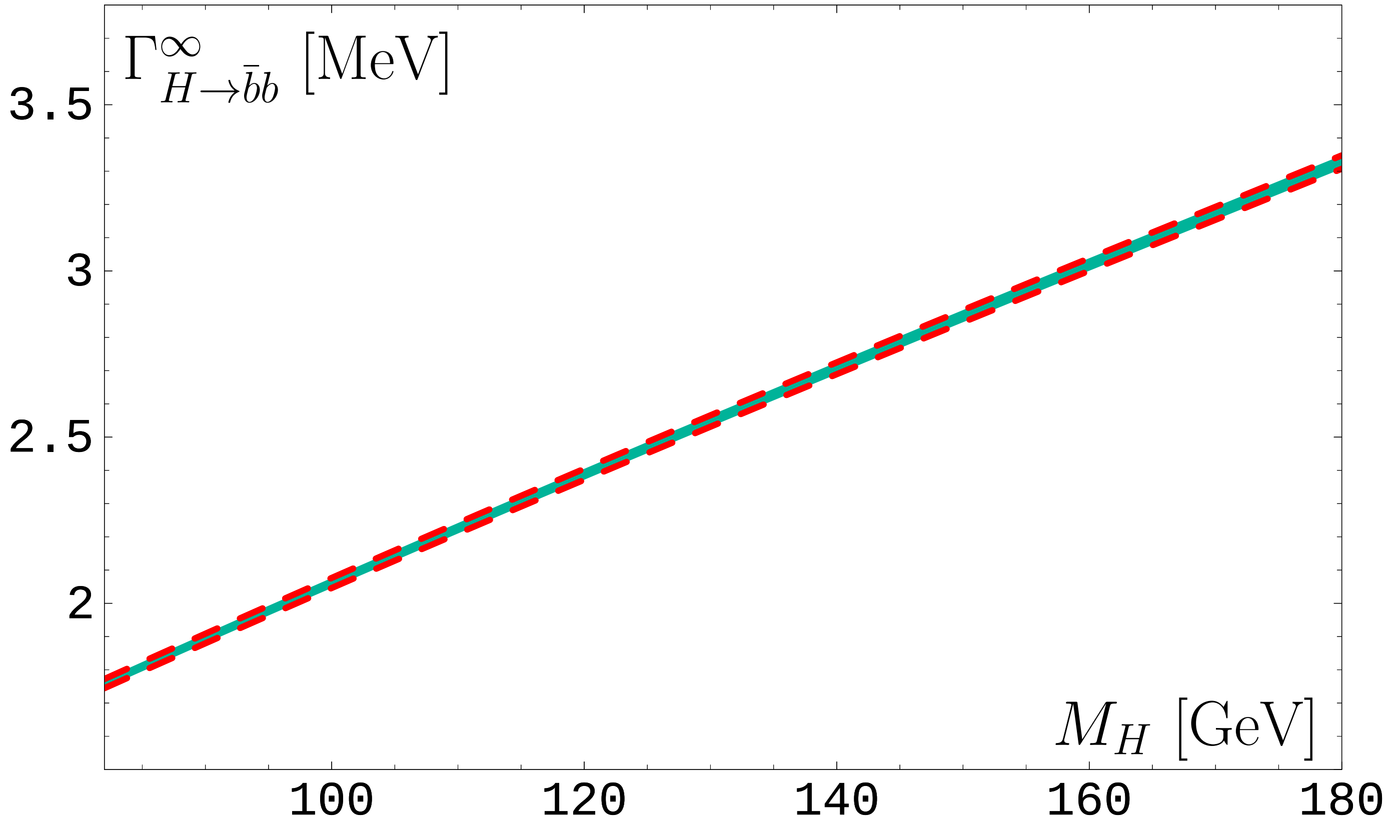
But **profit will be tiny — instead of** 0.5% **one'll obtain** 0.3%**!**



FAPT(M) for $\Gamma_{H\to\bar{b}b}(m_H)$: Truncation errors

Conclusion: If we need accuracy of the order 0.5% — then we need to take into account up to the 4-th correction.

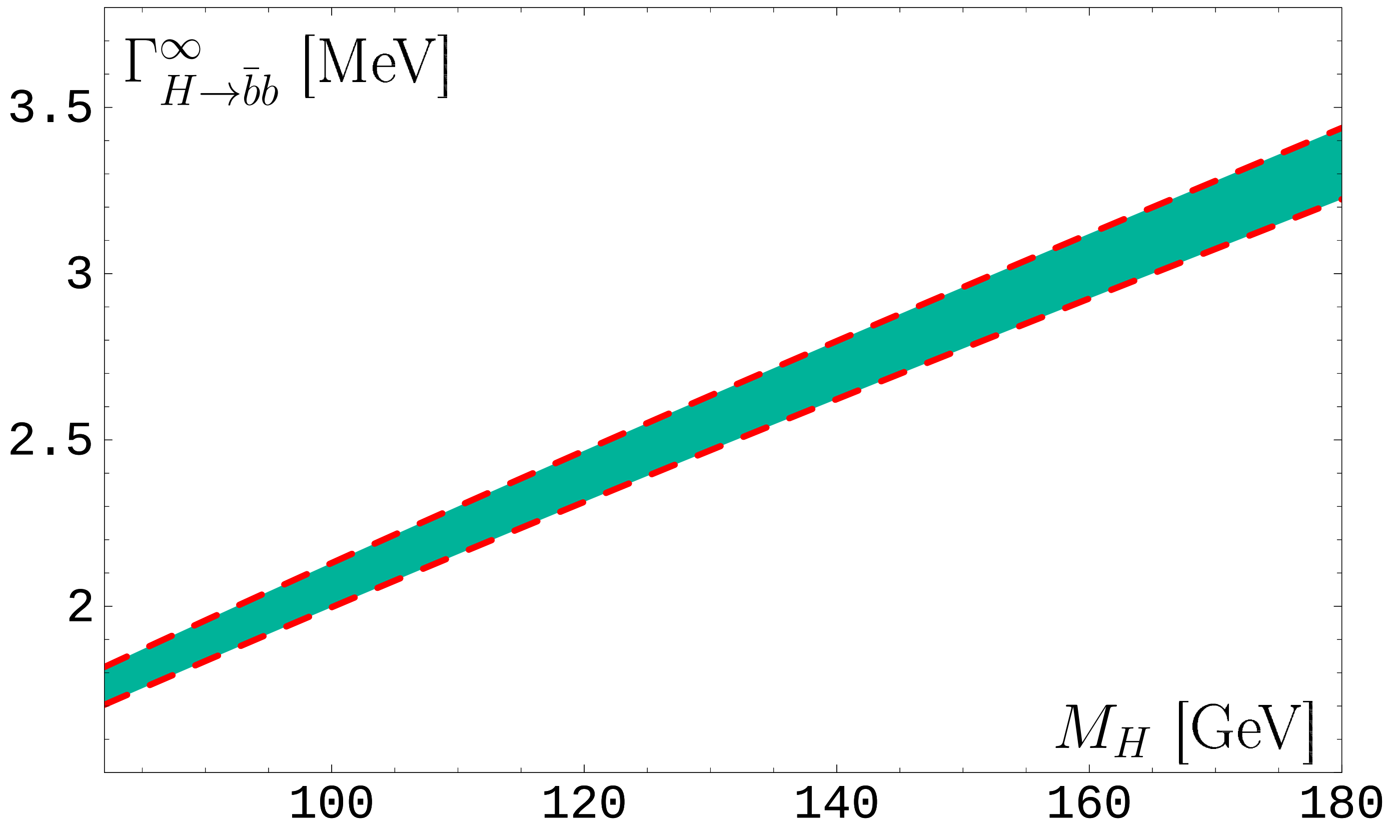
Note: uncertainty due to $P(t)$-modelling is small $\lesssim 0.6\%$.



FAPT(M) for $\Gamma_{H\to\bar{b}b}(m_H)$: Truncation errors

Conclusion: **If we need accuracy of the order 1% — then we need to take into account up to the 3-rd correction — in agreement with Kataev&Kim [0902.1442].**

Note: **RG-invariant mass uncertainty $\sim 2\%$.**



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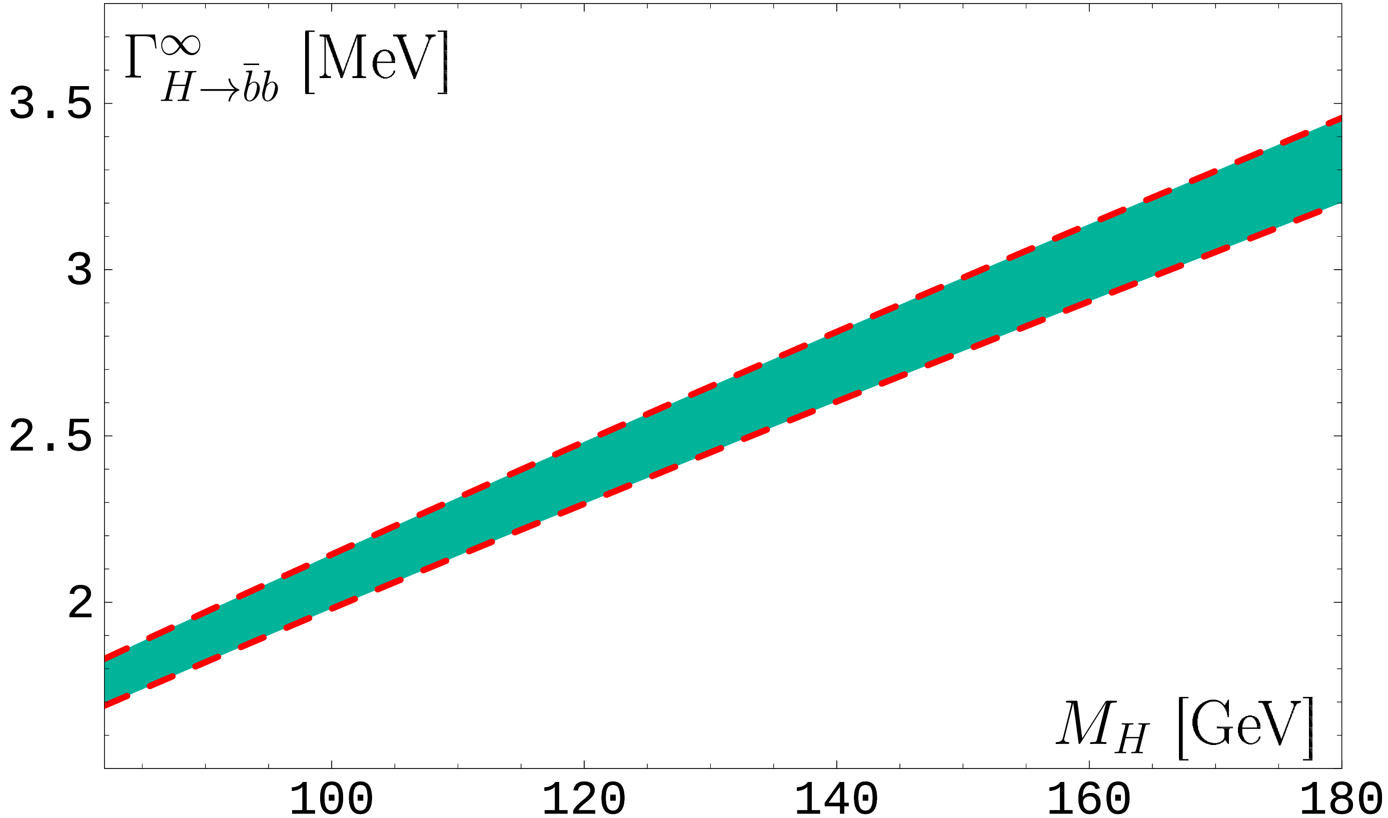
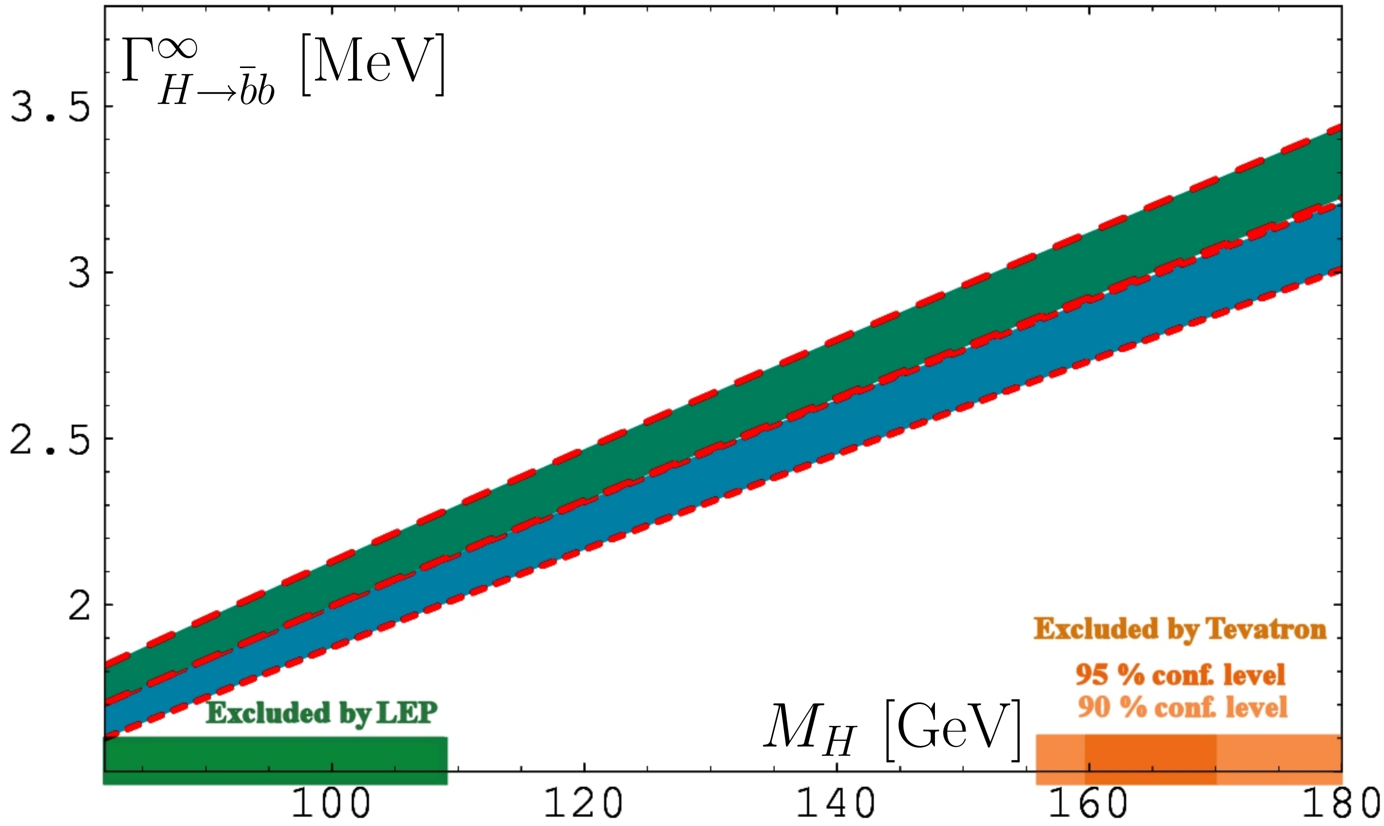
Conclusion: **If we need accuracy of the order 1% — then we need to take into account up to the 3-rd correction — in agreement with Kataev&Kim [0902.1442].**
Note: **overall uncertainty** $\sim 3\%$ **.**



Resummation for $\Gamma_{H\to\bar{b}b}(m_H)$: Loop orders

Comparison of 1- (upper strip) and 2- (lower strip) loop results. We observe a 5% reduction of the two-loop estimate.



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  Agreement with Kataev&Kim [PoS, ACAT08 (2009) 004].