QCD's Partner in needed for Hadronic Mass Spectra and Parton Structure Functions

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Question:

Can QCD alone produce mass spectra and parton structure functions?

How about QED

- In 1965, Freeman Dyson received Heinemann prize at the APS metting in Washington.

- He said about QED.

Quantum Field Theory alone does not solve all the problems in the world. It can be more effective if combined with other branches of physics.

As an example, he mentioned the *np* mass difference calculation using both QED and “Bootstrap dynamics” based on the S-matrix theory, where bound states correspond to poles on the complex plane.

It had been and still is believed that the neutron-proton mass difference comes from an electromagnetic perturbation.

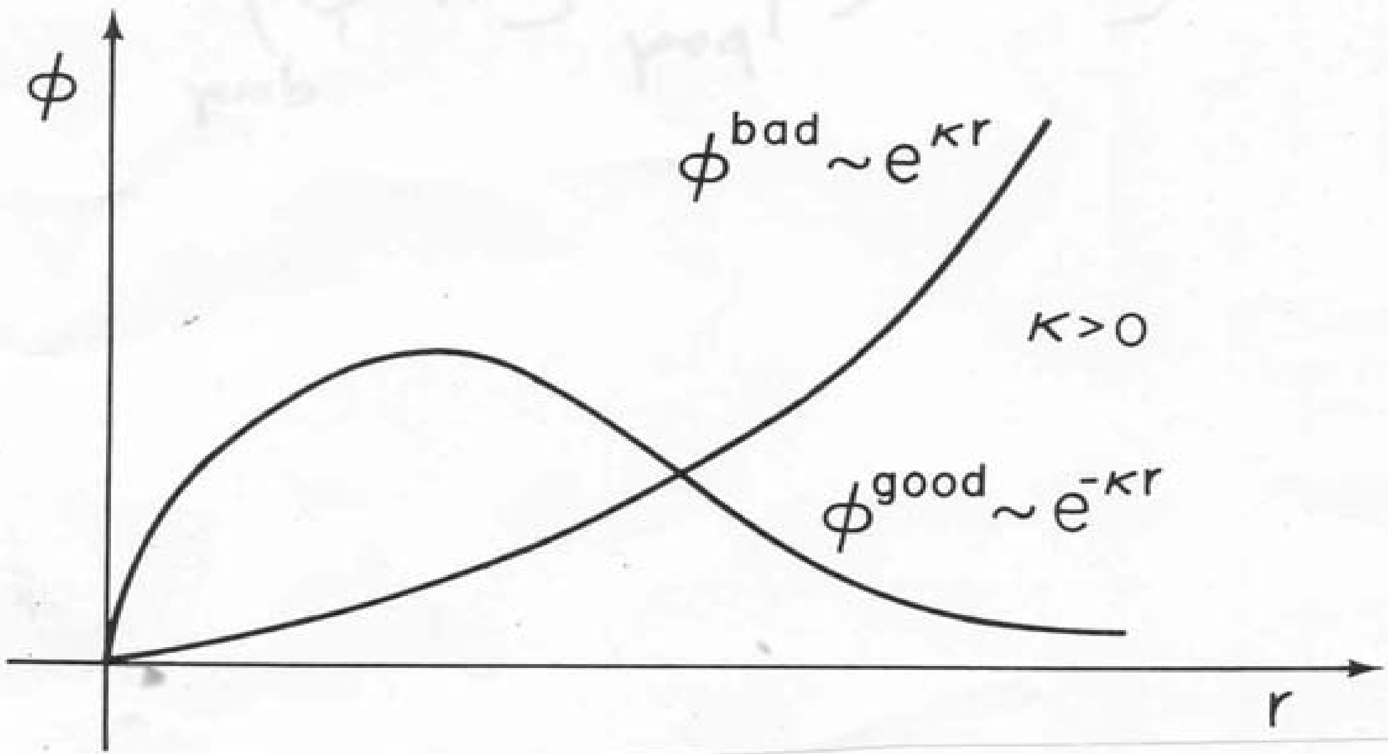
The question is how to calculate it starting from the equal mass for both neutron and proton.

Roger Dashen's Bootstrap Calculation of the n-p Mass Differernce

Dyson said it was a history-making calculation, and Dashen became the genius of the century.

Appointed as a permanent member of the Institute for Advanced Study (1965).

In the Schroedinger Picture, Dashen's Calculation is



$$\delta E^{bad} = (\phi^{bad}, \delta V \phi^{good})$$

QED’s Old Partner

Schroedinger Picture of the Hydrogen Atom

Hydrogen Wave Functions satisfying the localization boundary condition.

I learned this when I was a senior at Carnegie Tech (CMU), from

Michel Baranger (Feynman’s student at Cornell) Worked on Lamb shift.

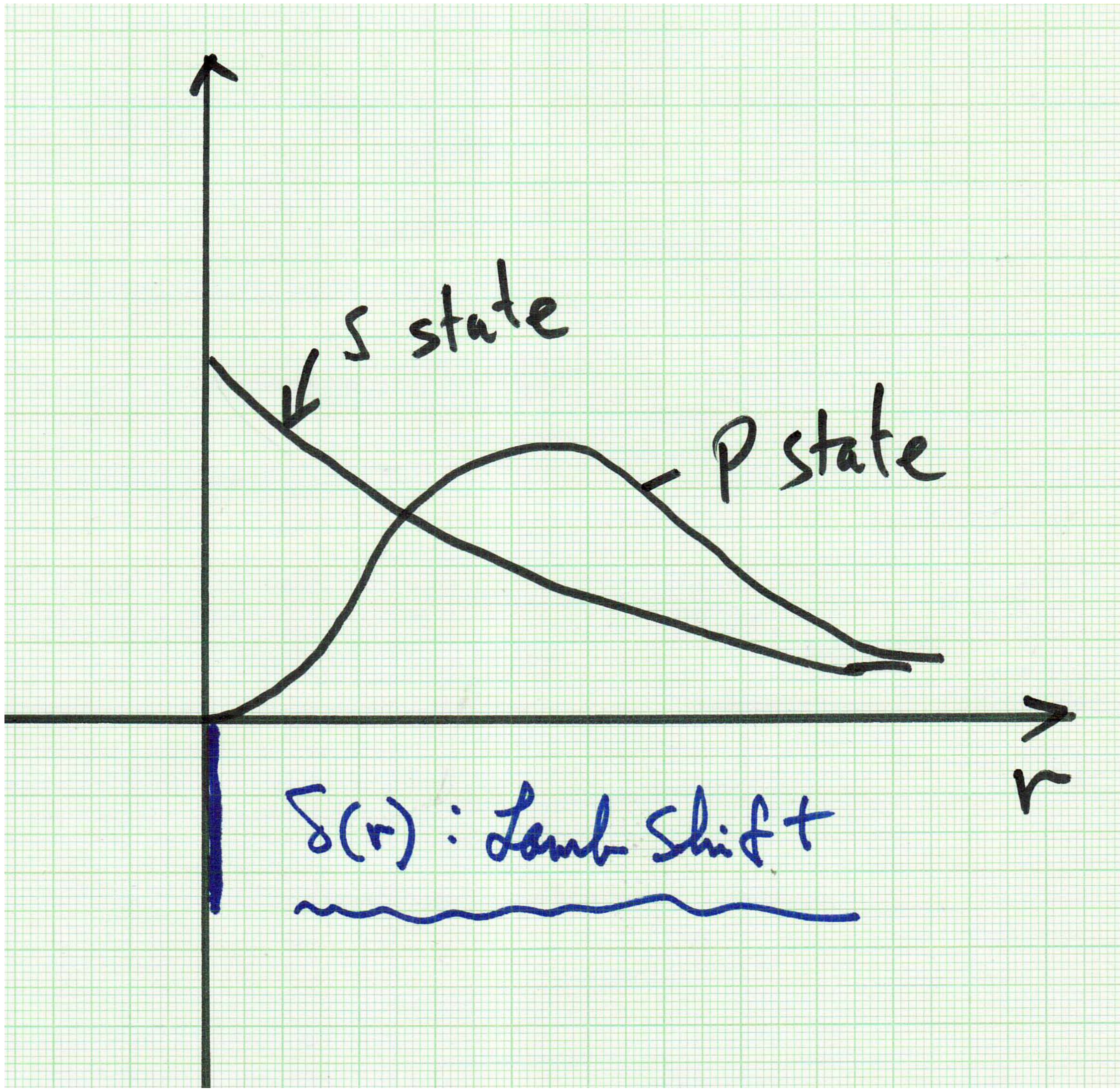
Photo with Michel Baranger

Dyson was right in saying Field Theory needs a Partner.

Who is the Partner?

Wave function! Without it, you cannot calculate the Lamb Shift.

Lamb Shift



Schroedinger Nonrelativic, thus Bethe-Salpeter Equation

G. C. Wick (1954) introduced Wick rotation. However, Wick said he does not like Bethe-Salpeter equation. The title of his was is "Problems with B-S equations."

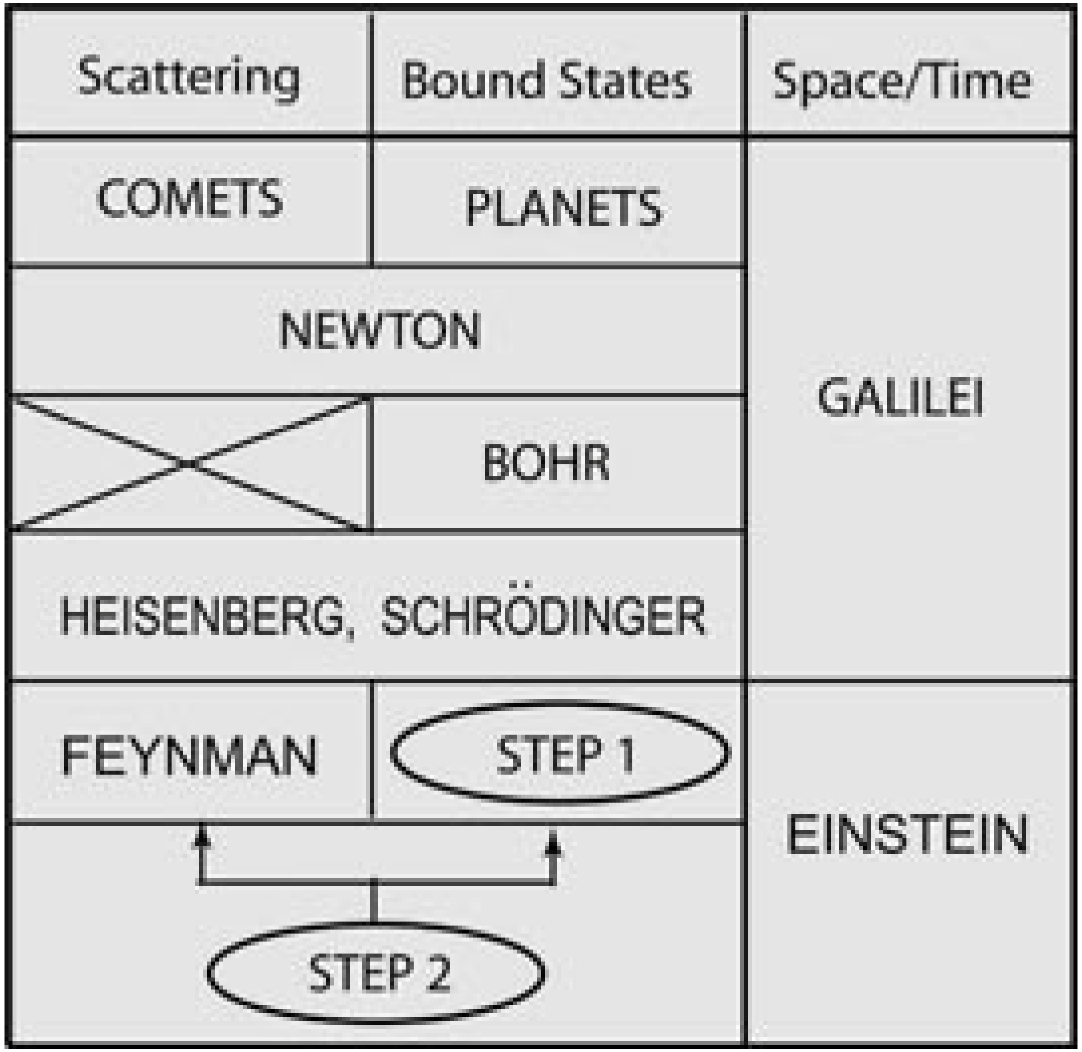
- **Boundary conditions on imaginary time variable. Wave functions can not be given probability interpretation.**
- **I published severlal papers on this subjects, especially on its Lorentz covariance, I could not continue,**
- **As far as I can see, lattice QCD has the same problem as B-S equation,**

Feynman, Washington, 1970

Scattering and Bound States are Partners in Physics

Feynman said we should not use Feynman diagrams for bound states.

Use harmonic oscillators for bound states.



Feynman published his talk.

R. P. Feynman, M. Kislinger, and F. Ravndal, Phys. Rev. D {\bf 3}, 2706 (1971).

From the mathematical point of view, this paper is a joke, but contains many original ideas from the physical point of view.

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PHYSICAL REVIEW D VOLUME 3, NUMBER 11 1 JUNE 1971

Current Matrix Elements from a Relativistic Quark Model*

R. P. Feynman, M. Kislinger, and F. Ravndal

Lauritsen Laboratory of Physics, California Institute of Technology, Pasadena, California 91109

(Received 17 December 1970)

A relativistic equation to represent the symmetric quark model of hadrons with harmonic interaction is used to define and calculate matrix elements of vector and axial-vector currents. Elements between states with large mass differences are too big compared to experiment, so a factor whose functional form involves one arbitrary constant is introduced to compensate this. The vector elements are compared with experiments on photoelectric meson production, K_{l3} decay, and $\omega \rightarrow \pi\gamma$. Pseudoscalar-meson decay widths of hadrons are calculated supposing the amplitude is proportional (with one new scale constant) to the divergence of the axial-vector current matrix elements. Starting only from these two constants, the slope of the Regge trajectories, and the masses of the particles, 75 matrix elements are calculated, of which more than $\frac{2}{3}$ agree with the experimental values within 40%. The problems of extending this calculational scheme to a viable physical theory are discussed.

INTRODUCTION

The symmetric, nonrelativistic harmonic-oscillator quark model has been shown by a number of people[1,2] to offer considerable promise of helping to organize the wealth of data in the resonance region for high-energy phenomena. We intend here to bring some of these results together in a unified method of calculation in order to judge better the validity of this organizing power.

A truly relativistic quantum-mechanical theory today seems available only in the complexities of field theory with its many virtual states involving, for example, pairs, etc. It is so complex that no particular dynamic regularities among the resonances are expected of it, other than those resulting from symmetries of the original Hamiltonian. We have gone in a different direction, sacrificing theoretical adequacy for simplicity. We shall choose a relativistic theory which is naive and obviously wrong in its simplicity, but which is definite and in which we can calculate as many things as possible – not expecting the results to agree exactly with experiment, but to see how closely our "shadow of the truth" equation gives a partial reflection of reality. In our attempt to maintain simplicity, we shall evidently have to violate known principles of a complete relativistic field theory (for example, unitarity). We shall attempt to modify our calculated results in a general way to allow, in a vague way, for these errors.

This is, of course, quite dangerous – because if one allows too much latitude in modifying the results of the calculations, especially if empirical results are allowed to influence strongly the many arbitrary choices, the significance of later par-

Feynman’s original Ideas are

- Regge trajectories are degeneracies of three-dimensional harmonic-oscillators.

* Hadrons are relativistic extended particles and thus we have to construct Lorentz-covariant harmonic oscillators.

- But, Feynman’s wave functions do not satisfy boundary conditions. Feynman’s wave functions are “bad” wave functions. You cannot do physics with bad wave functions.

Solution to Feynman’s Problem

Construct a representation of the Lorentz group using harmonic oscillators, as we use spherical harmonics for the three-dimensional rotation group

How? Start from Feynman’s Lorentz-Invariant differential equation. It has 240 different boundary conditions. All are wrong from the physical point of view, except one

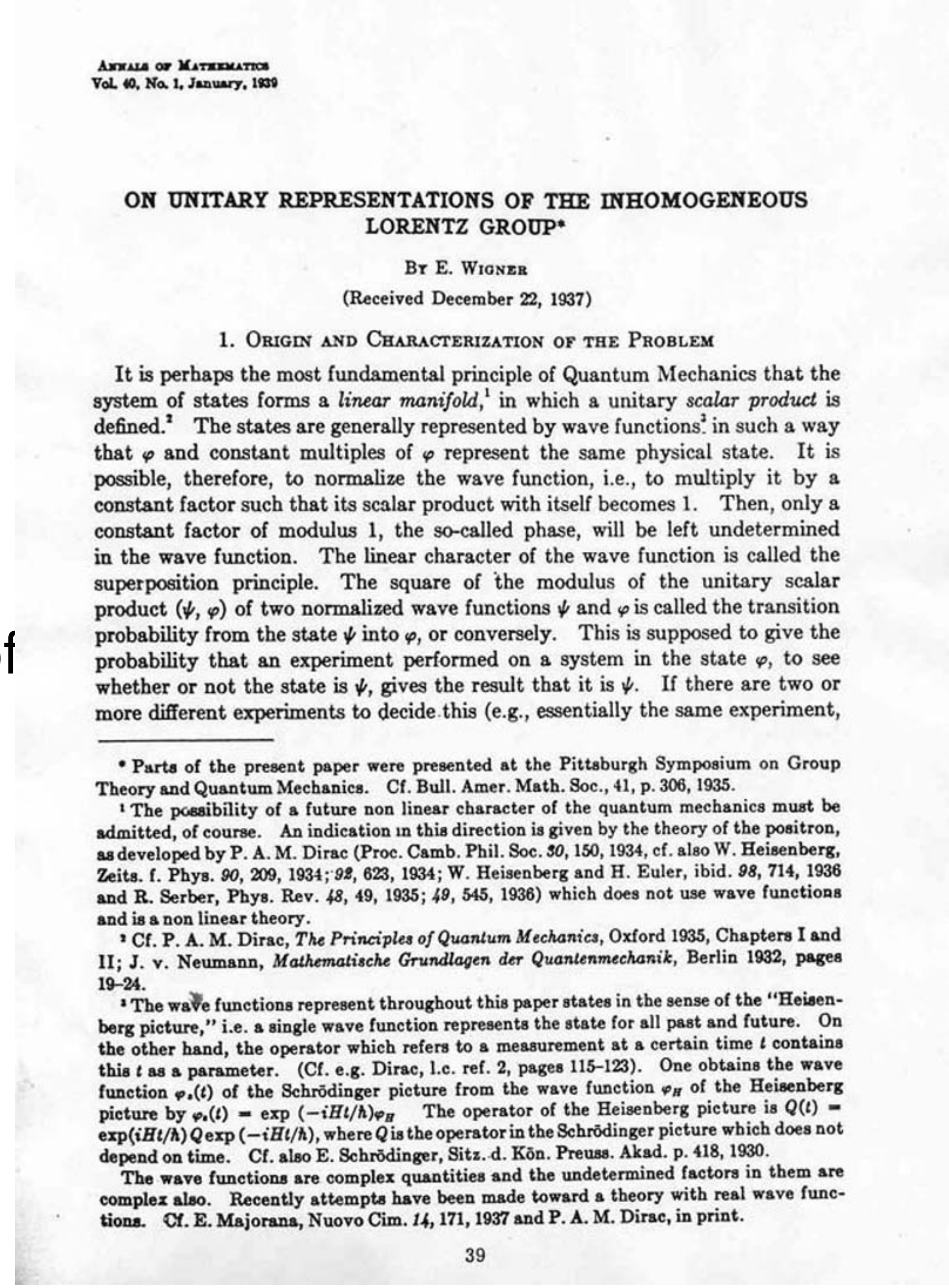
Fix up Ferynman. starting from his Lorentz-invariant diff. equation

- Paul A. M. Dirac
- Combine QM with SR.
- 1929.T-E Uncertainty
- 1946, Lorentz harmonics using oscillator wave functnions.
- 1949. Light-cone coordinate systrm.
- 1963. Coupled harmonic Oscillators..

- Eugene Paul Wigner
- Fundamental Internal Space-timte symmetrirs of Elementary Particles
- Representations of thre Poincare Group.
- Wigner 1939.

Wigner's Paper

- This paper was rejected by three journals, inculding the Physical Review
- John von Neuman was the editor of the Annals of Math.
- Nueman and Wigner went to the same High School in Budapest.
- Steven Weinberg spent four years on this paper. Feynman rules for arbitrary spin.



ANNALS OF MATHEMATICS
Vol. 40, No. 1, January, 1939

ON UNITARY REPRESENTATIONS OF THE INHOMOGENEOUS LORENTZ GROUP*

BY E. WIGNER

(Received December 22, 1937)

1. ORIGIN AND CHARACTERIZATION OF THE PROBLEM

It is perhaps the most fundamental principle of Quantum Mechanics that the system of states forms a *linear manifold*,[1] in which a unitary *scalar product* is defined.[2] The states are generally represented by wave functions[3] in such a way that φ and constant multiples of φ represent the same physical state. It is possible, therefore, to normalize the wave function, i.e., to multiply it by a constant factor such that its scalar product with itself becomes 1. Then, only a constant factor of modulus 1, the so-called phase, will be left undetermined in the wave function. The linear character of the wave function is called the superposition principle. The square of the modulus of the unitary scalar product (ψ, φ) of two normalized wave functions ψ and φ is called the transition probability from the state ψ into φ, or conversely. This is supposed to give the probability that an experiment performed on a system in the state φ, to see whether or not the state is ψ, gives the result that it is ψ. If there are two or more different experiments to decide this (e.g., essentially the same experiment,

* Parts of the present paper were presented at the Pittsburgh Symposium on Group Theory and Quantum Mechanics. Cf. Bull. Amer. Math. Soc., 41, p. 306, 1935.

[1] The possibility of a future non linear character of the quantum mechanics must be admitted, of course. An indication in this direction is given by the theory of the positron, as developed by P. A. M. Dirac (Proc. Camb. Phil. Soc. *30*, 150, 1934, cf. also W. Heisenberg, Zeits. f. Phys. *90*, 209, 1934; *92*, 623, 1934; W. Heisenberg and H. Euler, ibid. *98*, 714, 1936 and R. Serber, Phys. Rev. *48*, 49, 1935; *49*, 545, 1936) which does not use wave functions and is a non linear theory.

[2] Cf. P. A. M. Dirac, *The Principles of Quantum Mechanics*, Oxford 1935, Chapters I and II; J. v. Neumann, *Mathematische Grundlagen der Quantenmechanik*, Berlin 1932, pages 19–24.

[3] The wave functions represent throughout this paper states in the sense of the "Heisenberg picture," i.e. a single wave function represents the state for all past and future. On the other hand, the operator which refers to a measurement at a certain time t contains this t as a parameter. (Cf. e.g. Dirac, l.c. ref. 2, pages 115–123). One obtains the wave function $\varphi_s(t)$ of the Schrödinger picture from the wave function φ_H of the Heisenberg picture by $\varphi_s(t) = \exp(-iHt/\hbar)\varphi_H$ The operator of the Heisenberg picture is $Q(t) = \exp(iHt/\hbar)Q\exp(-iHt/\hbar)$, where Q is the operator in the Schrödinger picture which does not depend on time. Cf. also E. Schrödinger, Sitz. d. Kön. Preuss. Akad. p. 418, 1930.

The wave functions are complex quantities and the undetermined factors in them are complex also. Recently attempts have been made toward a theory with real wave functions. Cf. E. Majorana, Nuovo Cim. *14*, 171, 1937 and P. A. M. Dirac, in print.

Eugene Paul Wigner

A very diiificult person to approach.

Henryk Sienkiewicz wrote a book to approach God. Quo Vadis

Polish Wisdom!

Henryk Sienkiewicz in Warsaw

- I went there to worsdhip him

Moses wrote five books about God to talk to Him!

- I wrote book entitled “Theory and Aplications of the Poincare Group” using harmonic oscillators.

- Special relativity is the physics of the Lorentz group.
- Quantum mechnics is the physics of harmonic oscillators.
- Construct representations of the Lorentz/Poincare group using harmonic oscillators, to combine QM and SR.

With Marilyn E. Noz, since 1970

Last Lunch

Last Supper

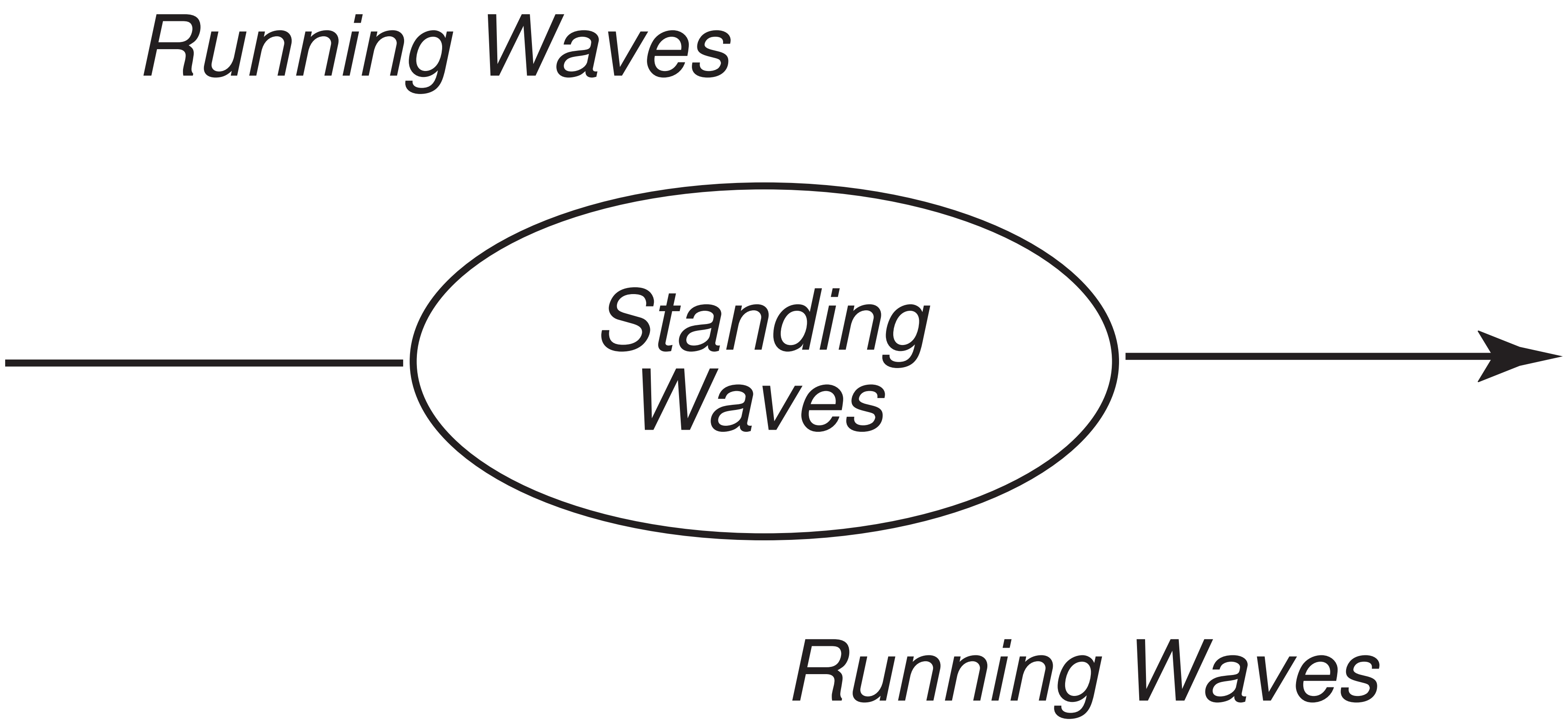
Waves in Relativistic World

- There are running waves and standing waves.

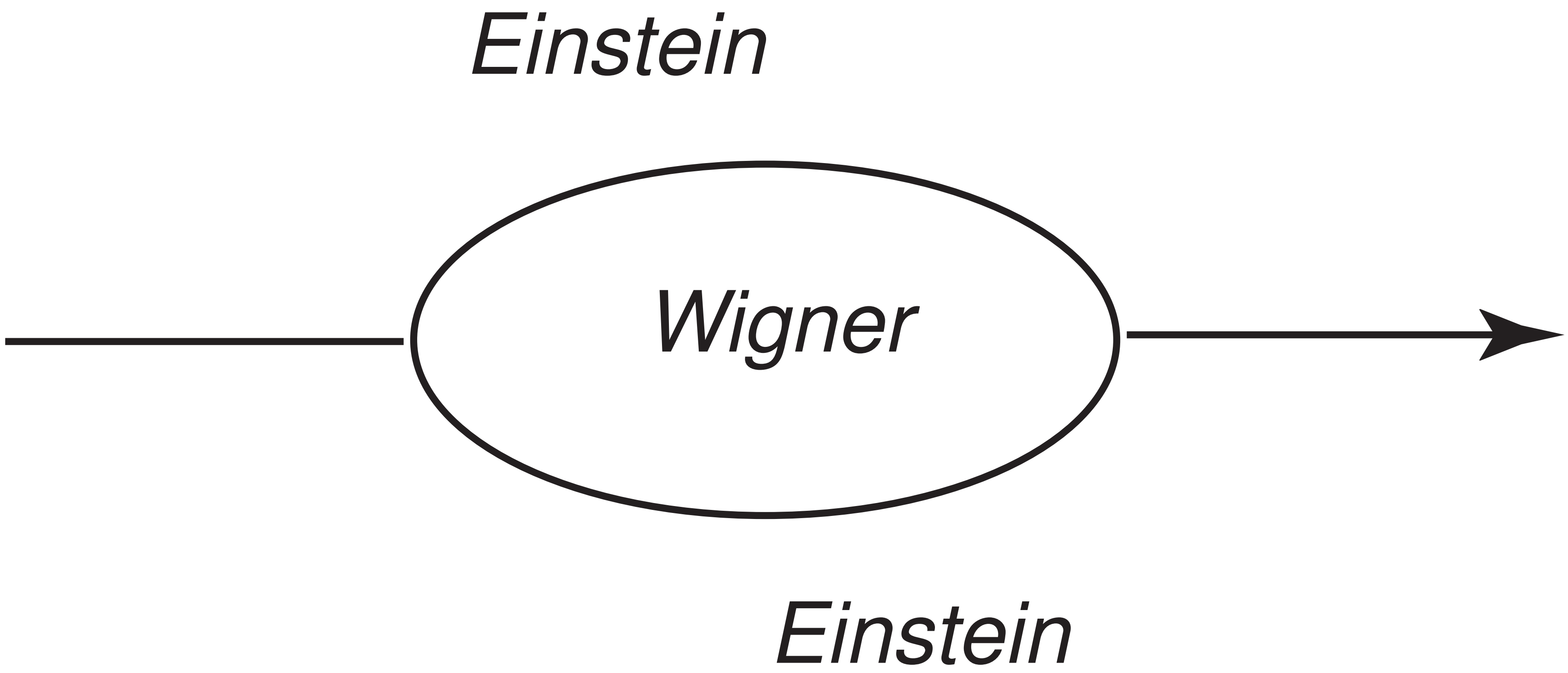
- Running waves can be approximated by plane waves. Running waves can be made Lorentz-covariant through the Klein-Gordon equation. Thus, the S-matrix and Feynman diagrams.

This is called Quantum Field Theory. Is the field theory capable of solving standing wave or bound-state problems?

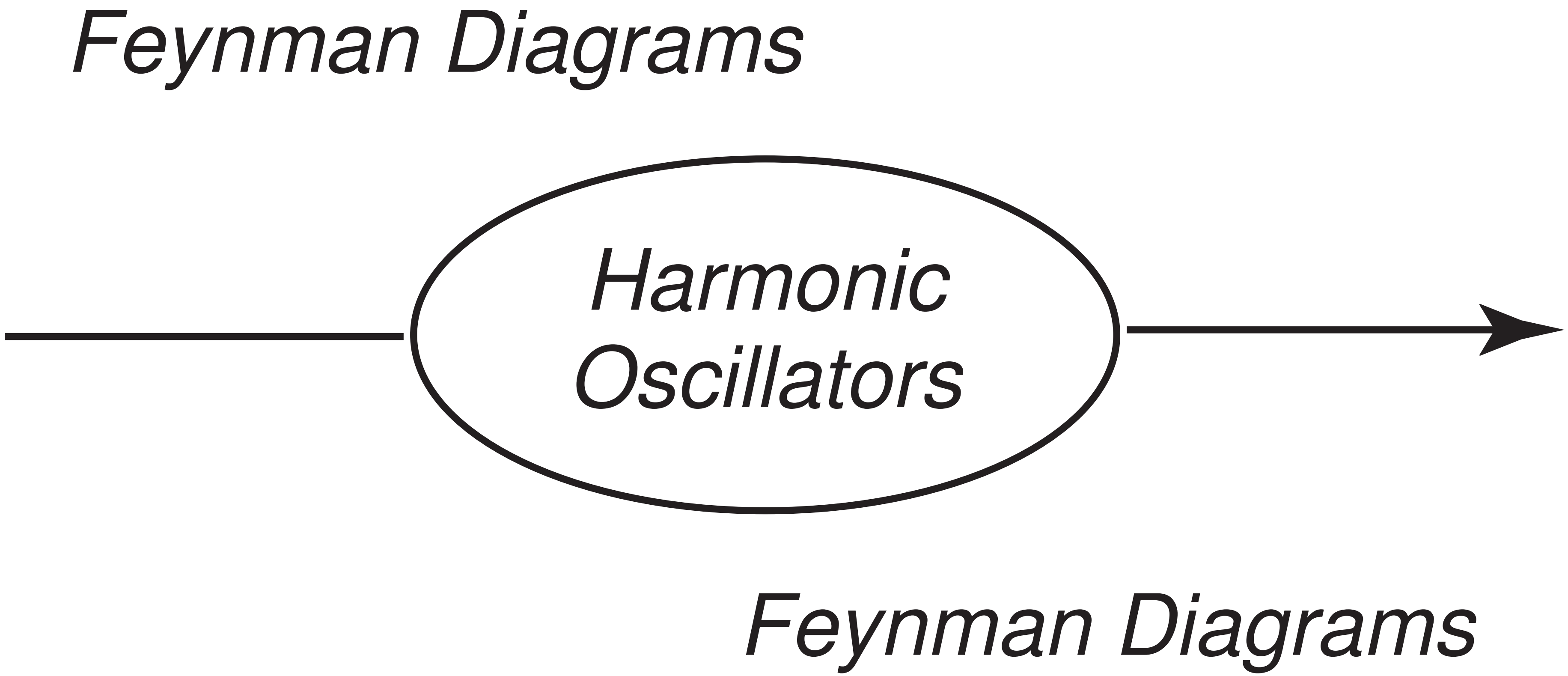
Running waves and Standing Waves



Symmetries. Wigner’s Little Groups



Feynman said



Three-D harmonic oscillators? How can they be relativistic or Lorentz-covariant?

- Wigner: Internal space-tim symmetry of massive particles is iomorphic to O(3), three-dimensional rotation group.

- For massless particles, the symmetry is isomorphic to E(2).
- Rotation corresponds to helicity.

- Translations to gauge transformation.

Paul A. M. Dirac
August 8, 1902 - October 20, 1984

MEMORIAL CONVOCATION
The Florida State University
Opperman Music Hall
November 19, 1984
11:00 a.m.

Paul A. M. Dirac
Professor of Physics, Florida State University
Nobel Laureate in Physics

Dirac and Feynman in Poland (1962)

- Dirac: construct a beautiful mathematics.
- Feynman: mathematical instrument that will produce numbers which compared with numbers observed in labs.
- In order to satisfy both, use two coupled oscillators.

In 1962, I had an audience with Paul A. M. Dirac

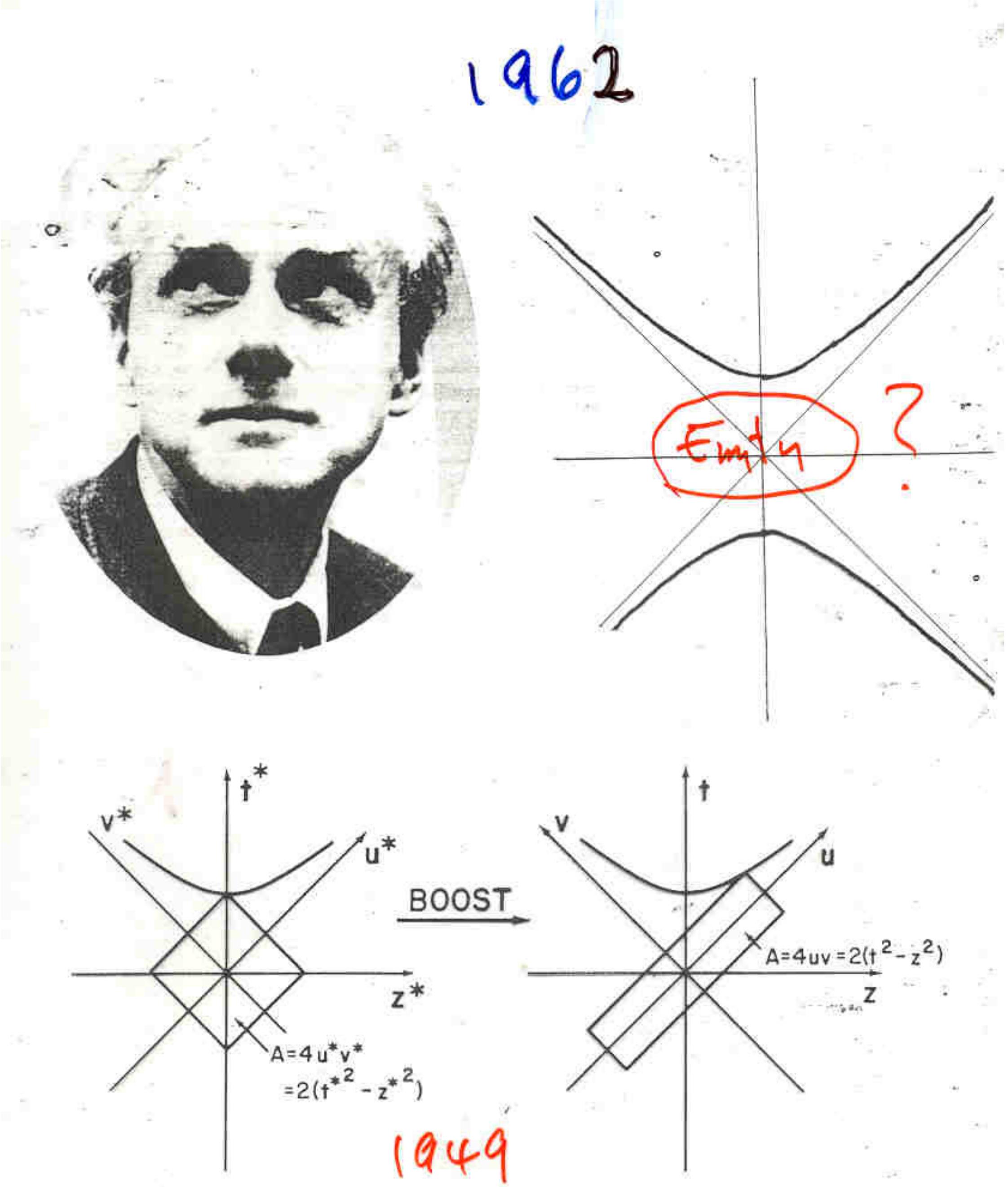
In 1962, everybody in USA was doing Regge poles and bootstraps. I did not like this environment, asked Dirac what was the most outstanding problem in American physics.

like Nikodemos asking Jesus

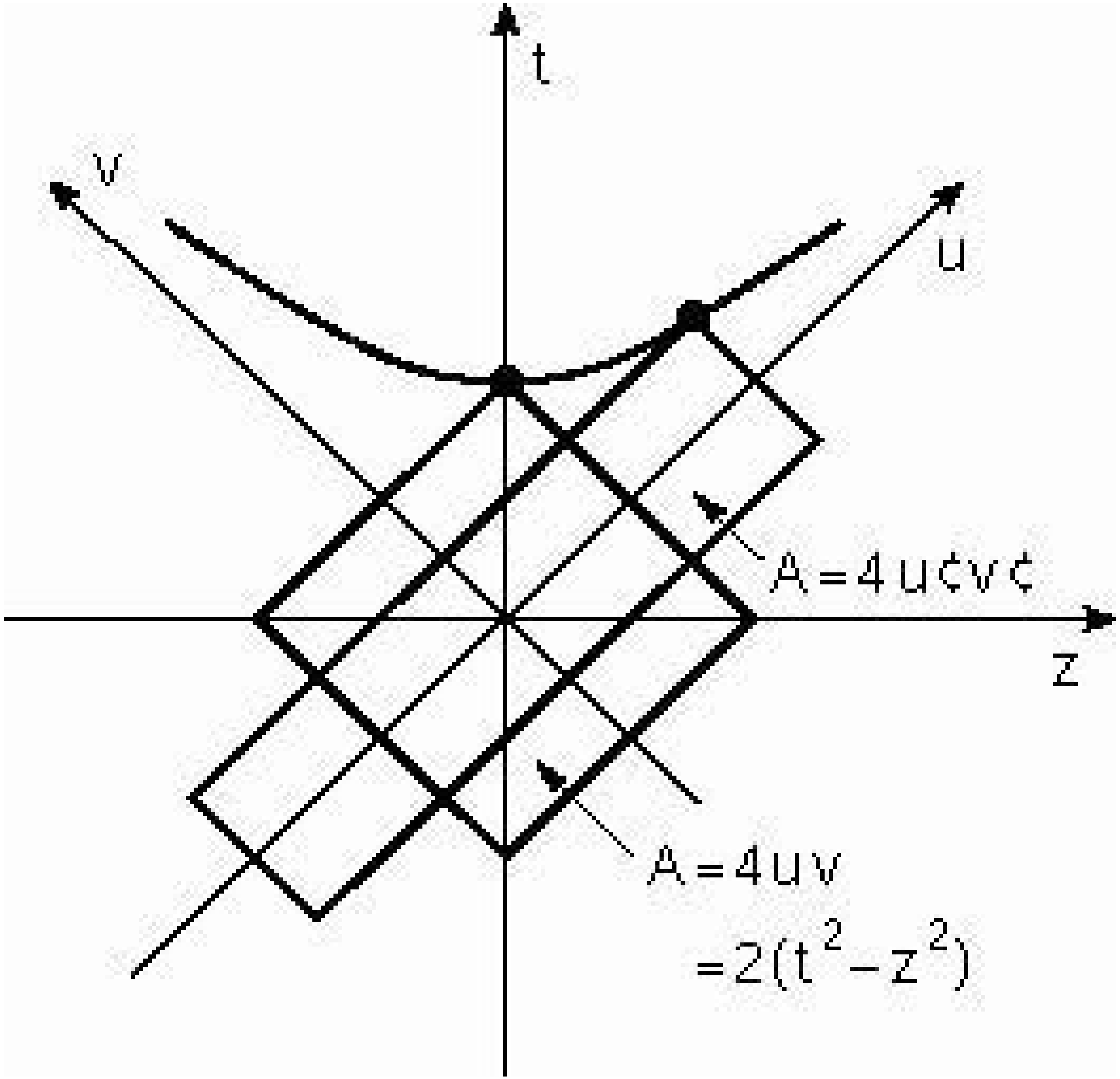
Dirac said “Young physicists should spend more time in understanding the difference between Lorenz covariance and Lorentz invariance.

Lesson from
Dirac, 1962

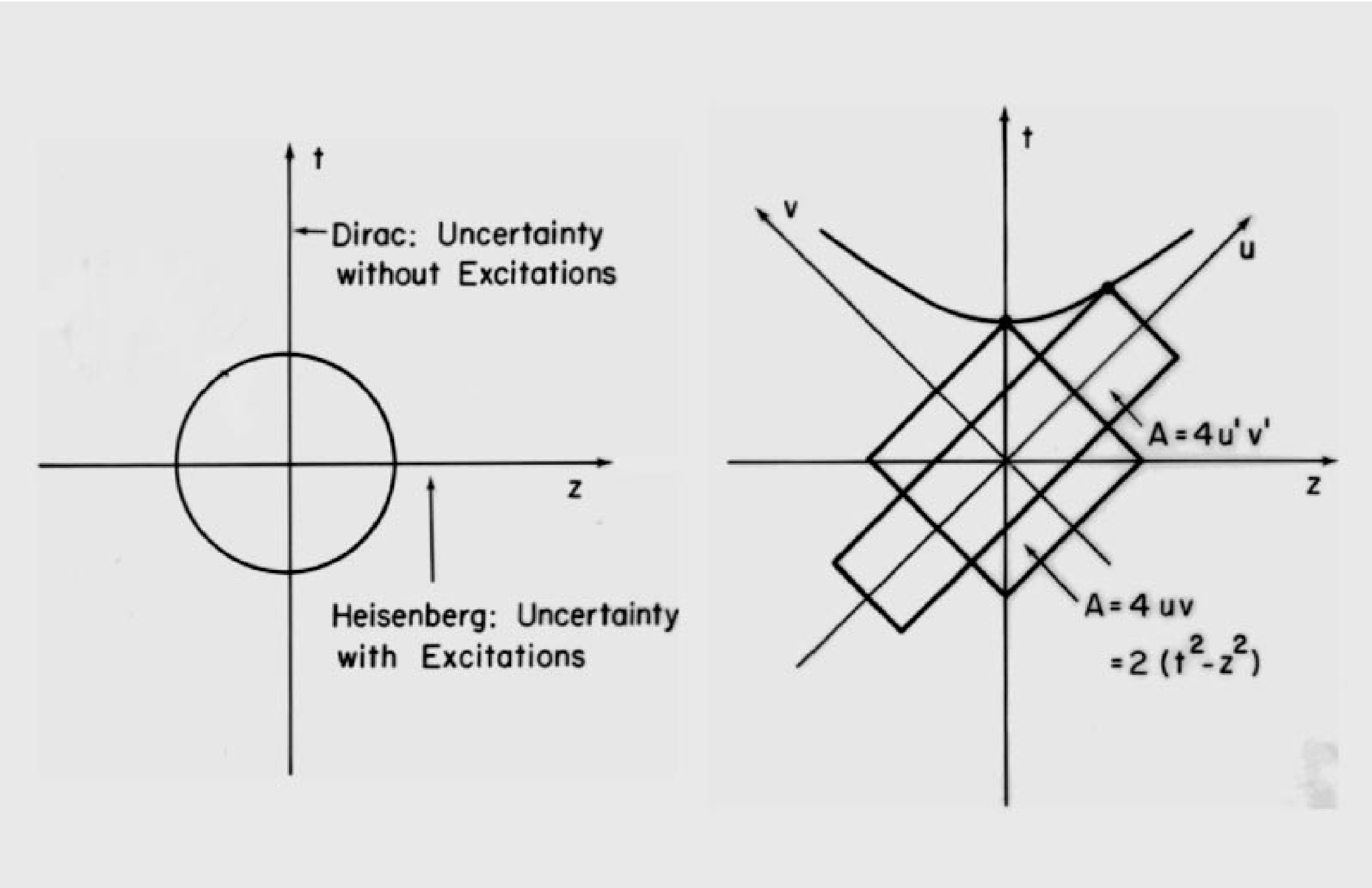
Fill in the empty
space with
quantum?



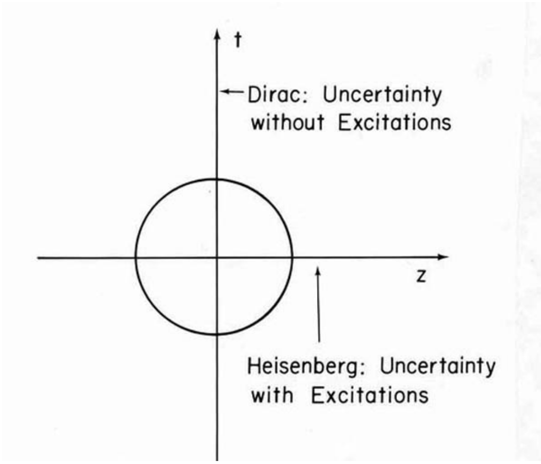
Dirac's Light-Cone System (1949)

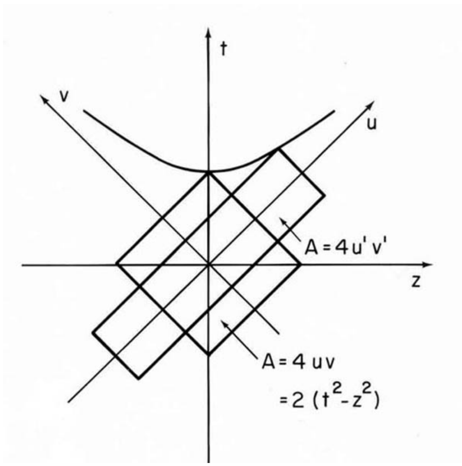


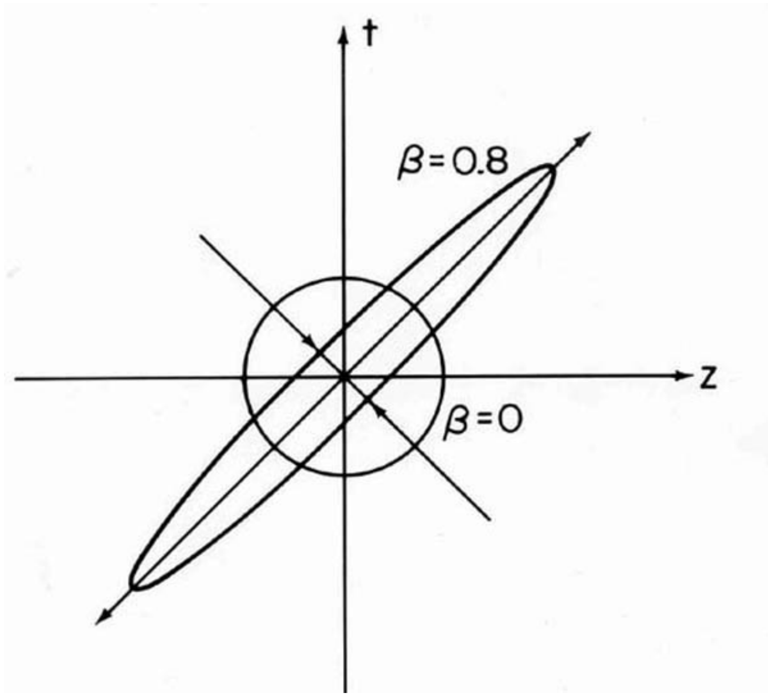
Dirac's Quantum Mech. and Relativity



Relativity and Quantum Mechanics





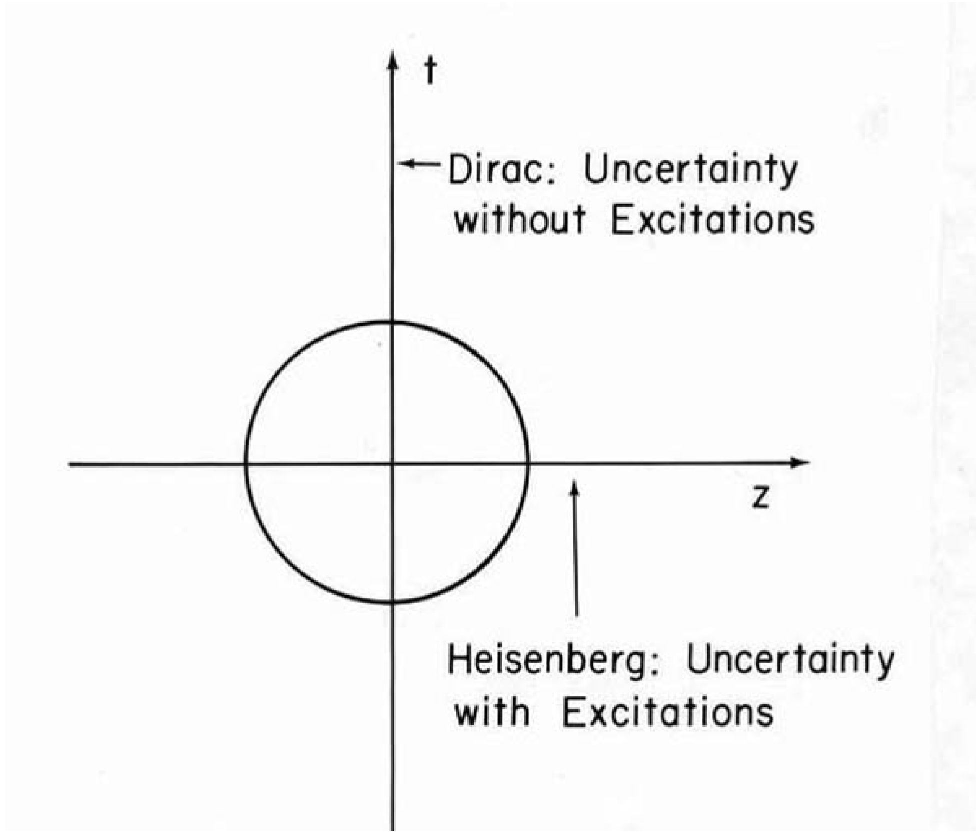


Dirac’s Quantum World

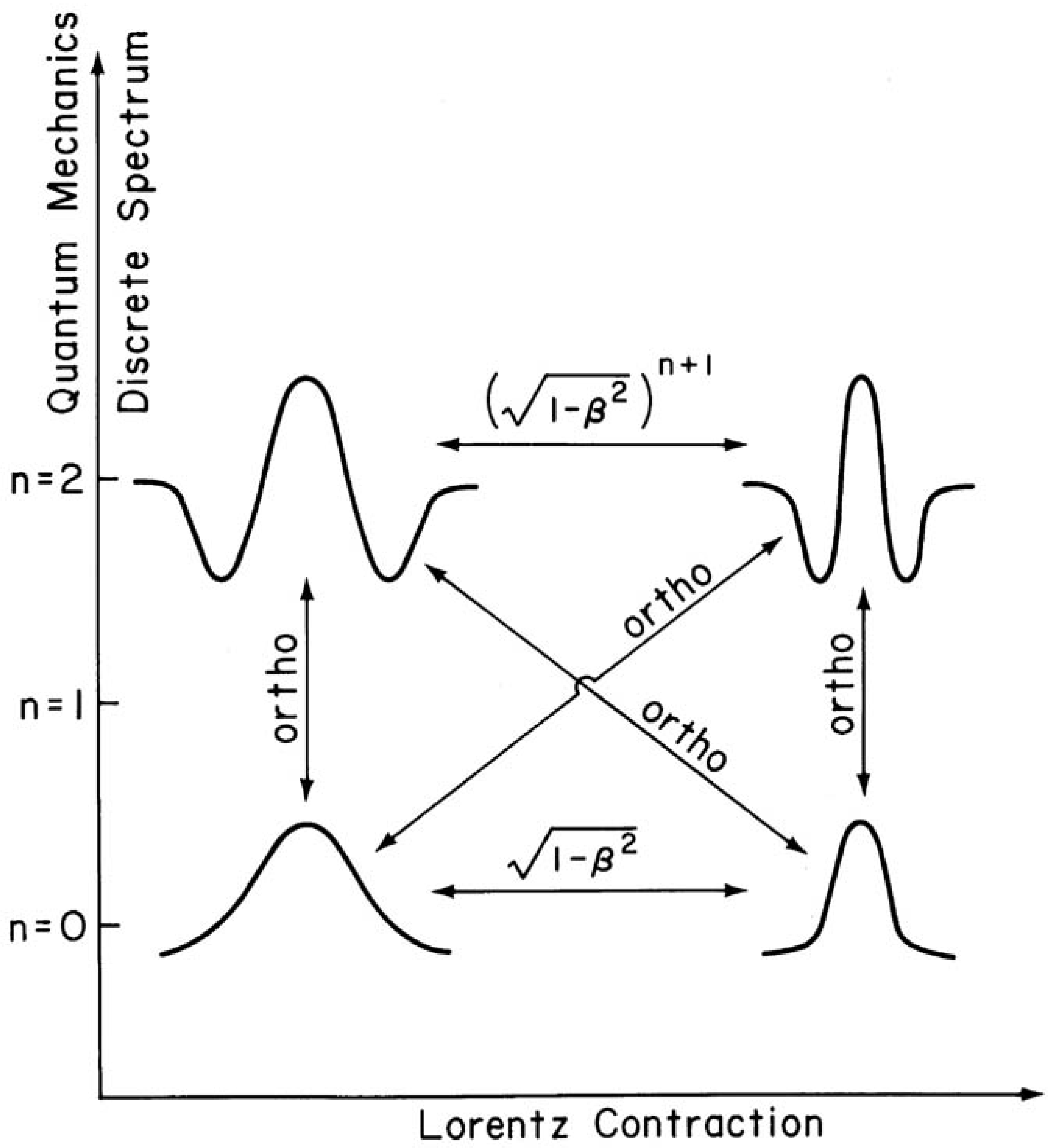
Time-energy uncertainty without excitations.

C-number T-E uncertainty.

Space-time asymmetry.



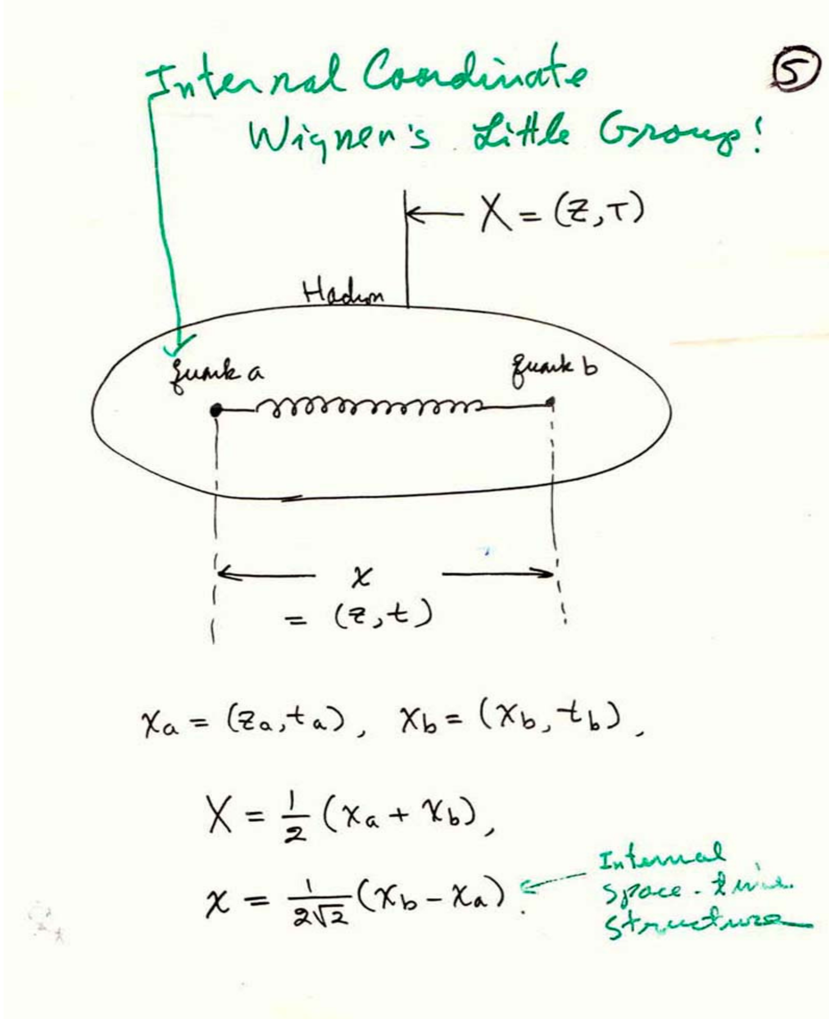
Orthogonality Relations



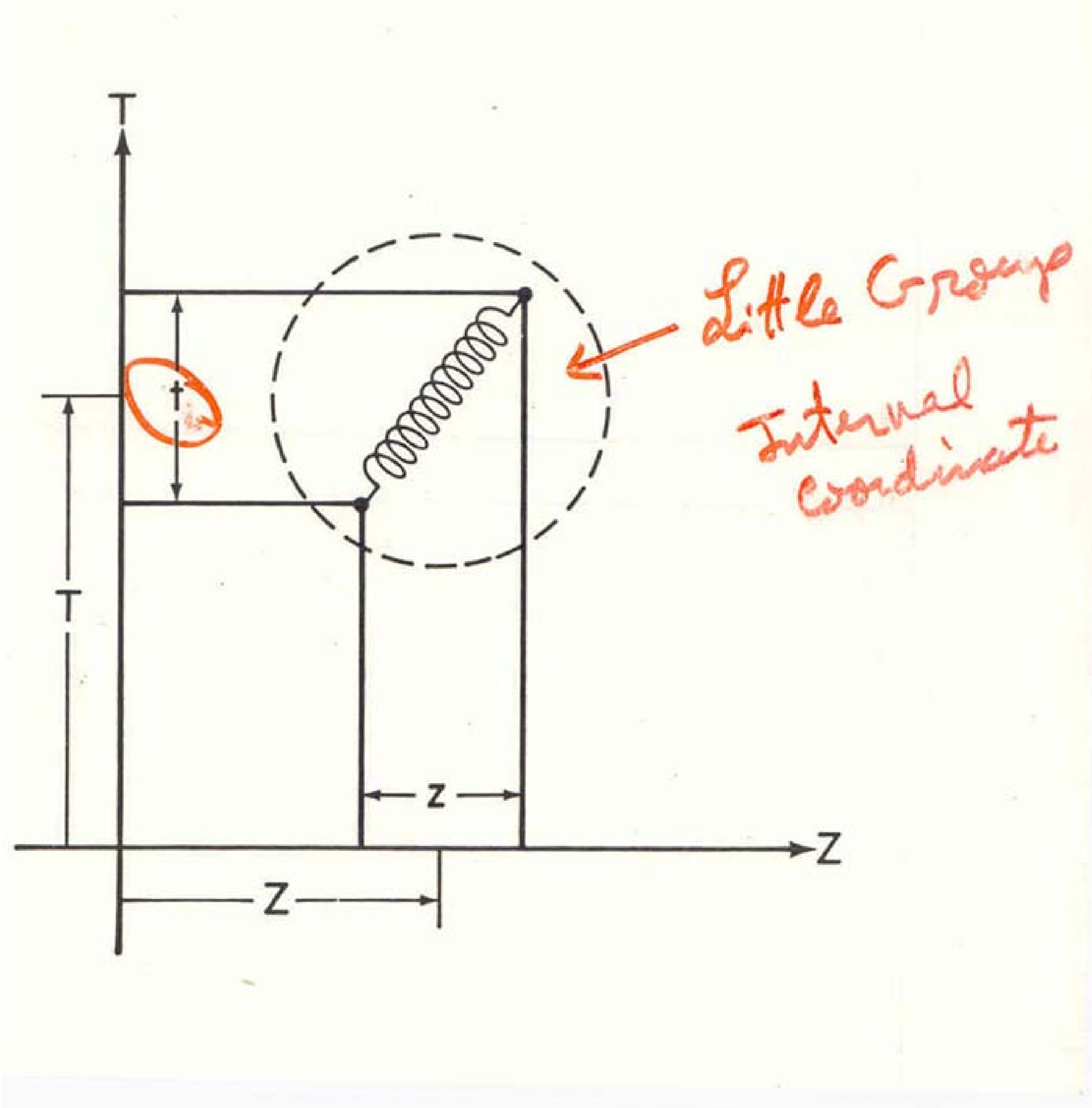
Covariant Bound States (Standing Waves)

- Bound States: Hydrogen Atom or

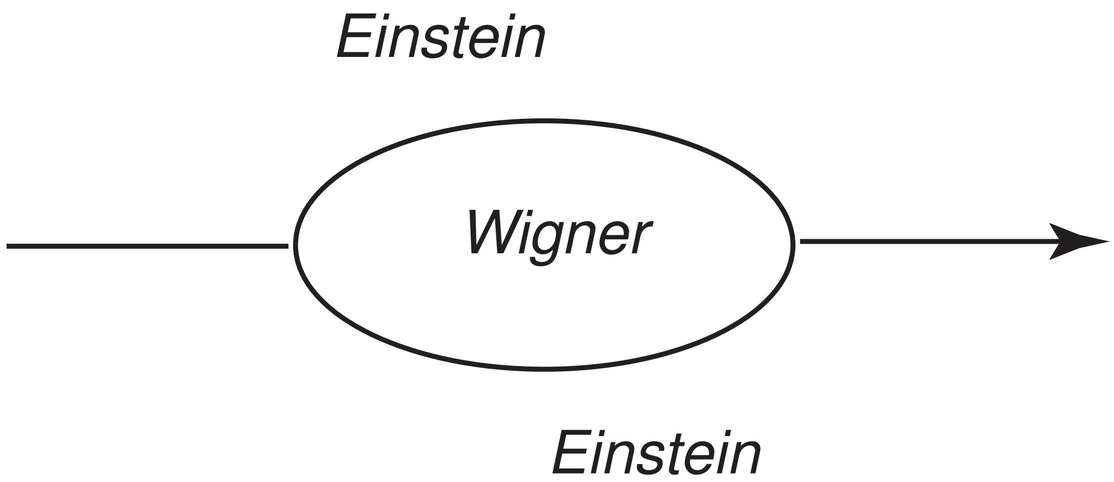
  Harmonic Oscillators.
- Feynman chooses osc. wave functions to understand the covariant world.
- Hadron consisting of two quarks. Overall coordinate and space-time separation.



Time separation

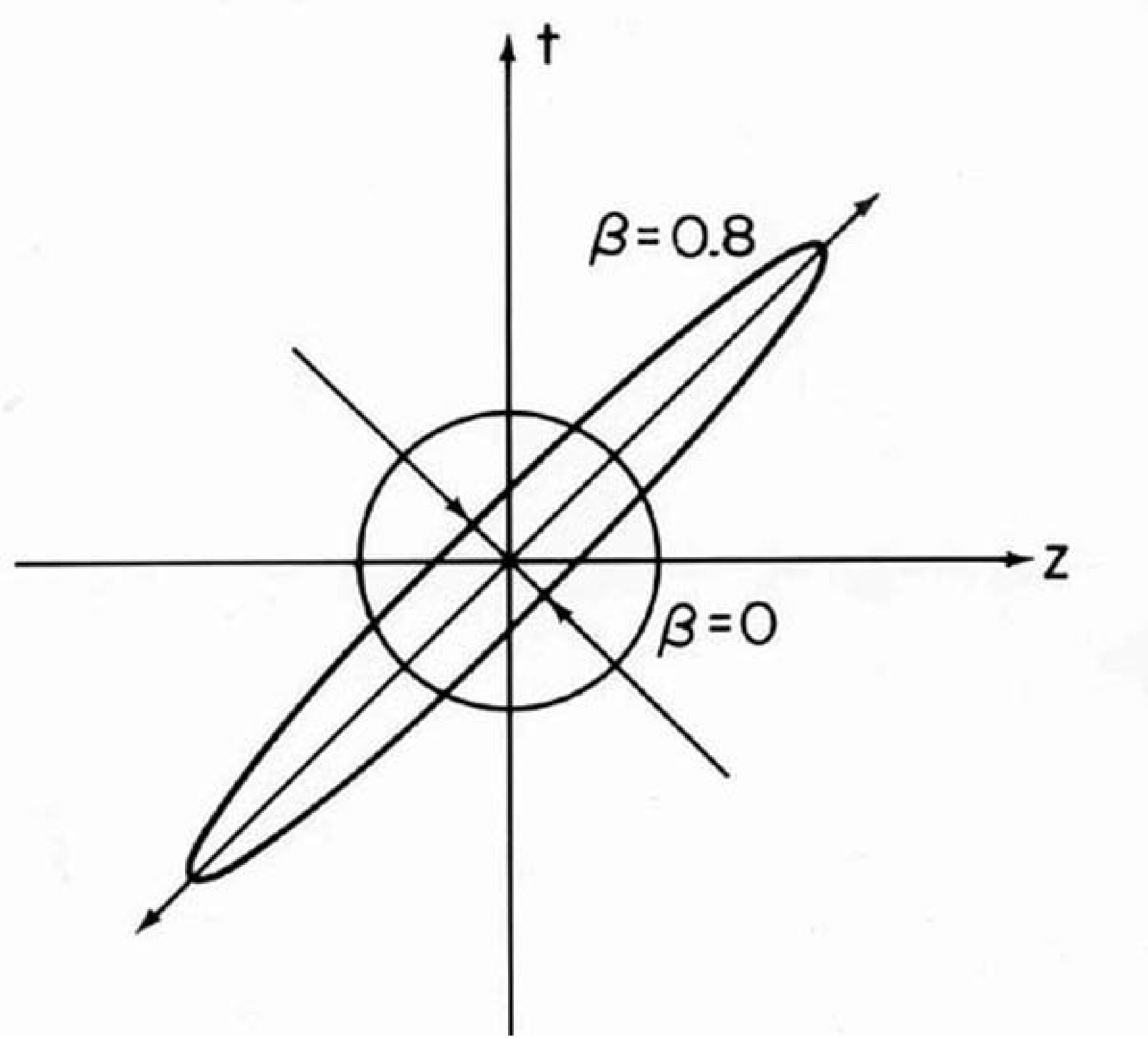


Wigner's Little Group



- The space-time symmetry inside the hadron is smaller than the Lorentz group. It is isomorphic to O(3), three-dimensional rotation group.

Lorentz-squeezed Hadron



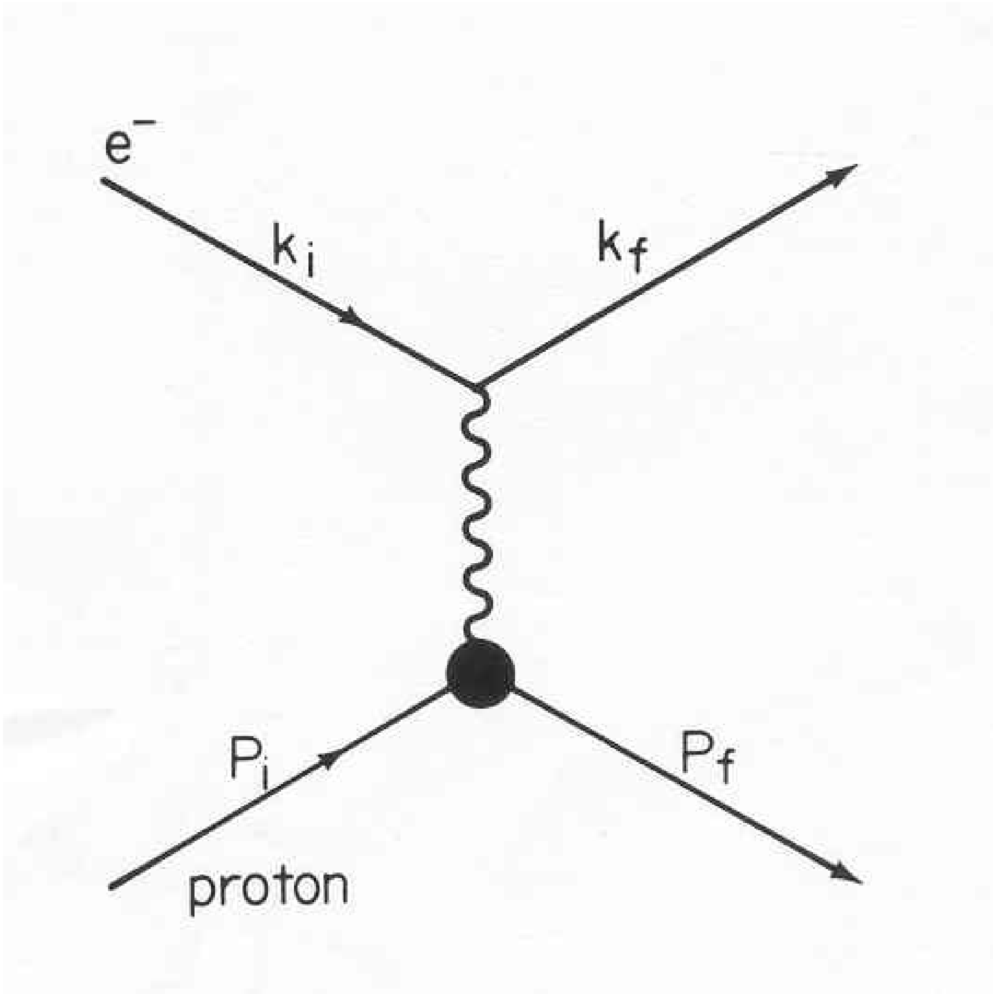
Feynman in Japan (1954)

In Japan, 1953 and 1970

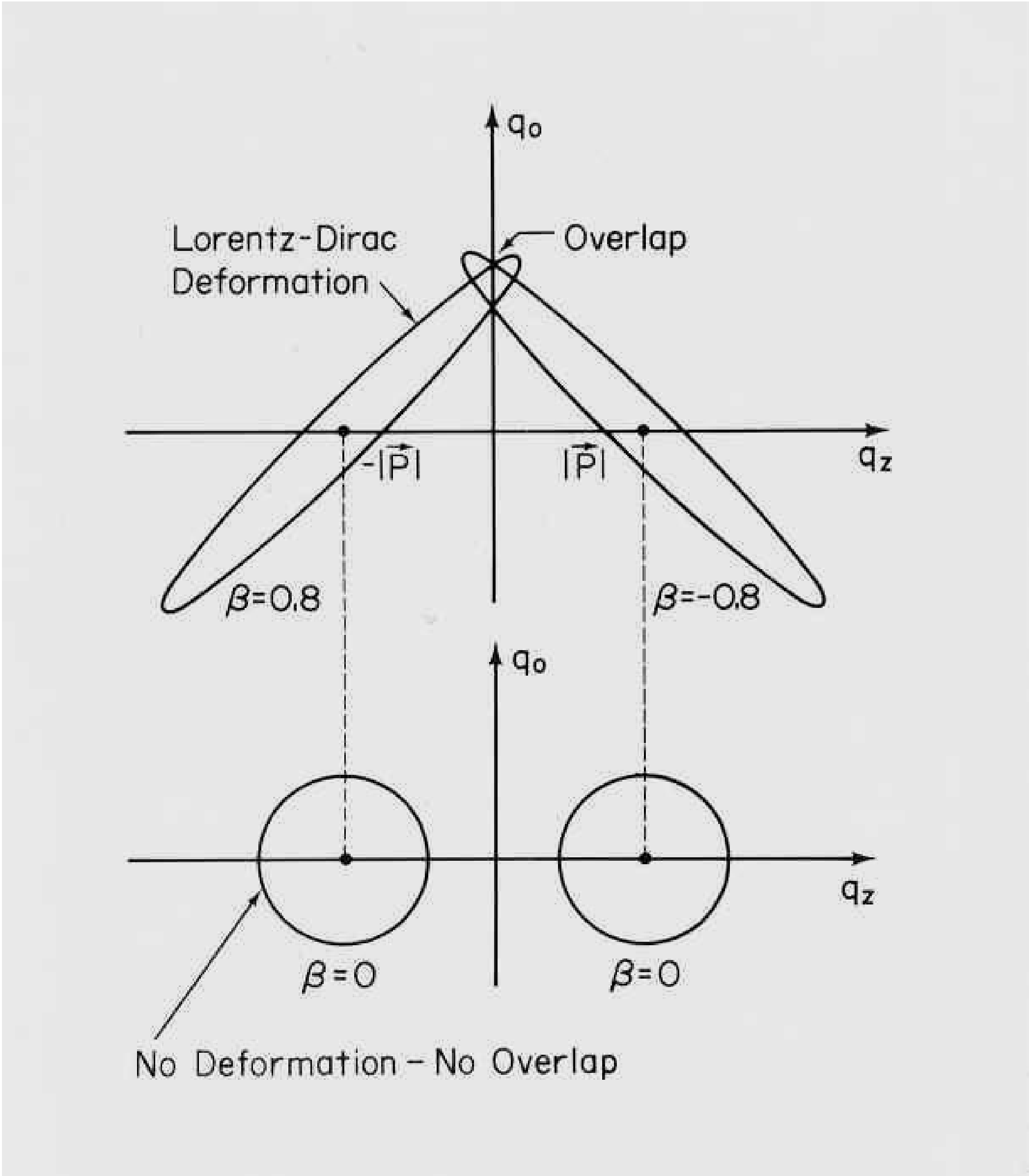
- In 1953, Yukawa was at Columbia University, and wrote a paper on extended particles based on a Lorentz-invariant differential equation and non-invariant but covariant solutions, essentially thee wave function I am using. But Japanese never looked at this paper

- In 1970, Fujimura, Kobayashi, and Namik calculated the proton form factor using Yukawa's \wave function, without quoting Yukawa.

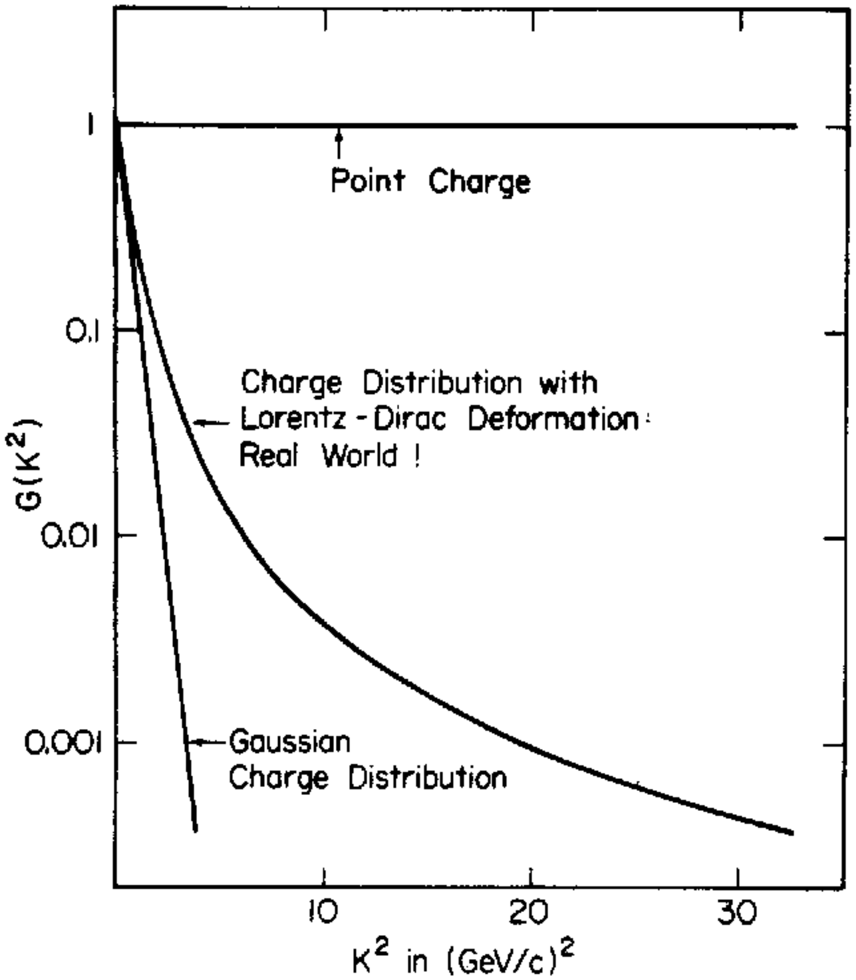
Electron-Proton Elastic Scattering Form Factor



Overlapping Wave Functions



Dipole Cut-off



Probability Interpretation?

- How can you give a Lorentz-covariant probability interpretation to non-relativistic oscillator wave functions?

- Feynman’s fifferential equation takes the same form as the schroedinger equation for coupled oscillators. We can then smuggle in the physics of coupled oscillators to Feynman’s oscillator.

Coupled Oscillators

$$H = \frac{1}{2}\left\{\frac{1}{m}p_1^2 + \frac{1}{m}p_2^2 + Ax_1^2 + Ax_2^2 + 2Cx_1x_2\right\}. \qquad (1)$$

If we choose coordinate variables

$$y_1 = \frac{1}{\sqrt{2}}(x_1 + x_2), \qquad y_2 = \frac{1}{\sqrt{2}}(x_1 - x_2), \qquad (2)$$

the Hamiltonian can be written as

$$H = \frac{1}{2m}\left\{p_1^2 + p_2^2\right\} + \frac{K}{2}\left\{e^{-2\eta}y_1^2 + e^{2\eta}y_2^2\right\}, \qquad (3)$$

where

$$K = \sqrt{A^2 - C^2}, \qquad \exp(2\eta) = \sqrt{\frac{A - C}{A + C}}, \qquad (4)$$

If y_1 and y_2 are measured in units of $(mK)^{1/4}$, the ground-state wave function of this oscillator system is

$$\psi_\eta(x_1, x_2) = \frac{1}{\sqrt{\pi}}\exp\left\{-\frac{1}{2}(e^{-\eta}y_1^2 + e^{\eta}y_2^2)\right\}, \qquad (5)$$

Let us write the wave function of Eq.(5) in terms of x_1 and x_2, then

$$\psi_\eta(x_1, x_2) = \frac{1}{\sqrt{\pi}} \exp\left\{-\frac{1}{4}\left[e^{-\eta}(x_1 + x_2)^2 + e^{\eta}(x_1 - x_2)^2\right]\right\}. \tag{6}$$

When the system is decoupled with $\eta = 0$, this wave function becomes

$$\psi_0(x_1, x_2) = \frac{1}{\sqrt{\pi}} \exp\left\{-\frac{1}{2}(x_1^2 + x_2^2)\right\}. \tag{7}$$

$$\psi_\eta(x_1, x_2) = \frac{1}{\cosh\eta} \sum_k \left(\tanh\frac{\eta}{2}\right)^k \phi_k(x_1)\phi_k(x_2), \tag{8}$$

where $\phi_k(x)$ is the harmonic oscillator wave function for the $k-th$ excited state. This expansion serves as the mathematical basis for squeezed states of light in quantum optics [3], among other applications.

Let us go back to Feynman!

Feynman's Oscillators

Let us use the simplest hadron consisting of two quarks bound together with an attractive force, and consider their space-time positions x_a and x_b, and use the variables

$$X = (x_a + x_b)/2, \qquad x = (x_a - x_b)/2\sqrt{2}. \tag{9}$$

The four-vector X specifies where the hadron is located in space and time, while the variable x measures the space-time separation between the quarks.

In their classic 1971 paper [9], Feynman, Kislinger and Ravndal start with Lorentz-invariant differential equation.

$$\frac{1}{2}\left\{x_\mu^2 - \frac{\partial^2}{\partial x_\mu^2}\right\}\psi(x) = \lambda\psi(x). \tag{10}$$

Although this paper contained the above mentioned original idea of Feynman, it contains some serious mathematical flaws. Feynman *et al.* start with a Lorentz-invariant differential equation for the harmonic oscillator for the quarks bound together inside a hadron. For the two-quark system, they write the wave function of the form

$$\exp\left\{-\frac{1}{2}\left(z^2 - t^2\right)\right\}, \tag{11}$$

where z and t are the longitudinal and time-like separations between the quarks. This form is invariant under the boost, but is not normalizable in the t variable. We do not know what physical interpretation to give to this the above expression.

Is it possible to fix Feynman's Oscillators?

If we adopt Dirac's picture which allows excitations along the longitudinal coordinate and forbid excitations along the time coordinate, the wave function takes the form

$$\psi_0^n(z,t) = C_n H_n(z) \exp\left\{-\frac{1}{2}\left(z^2 + t^2\right)\right\}, \tag{12}$$

If the system is boosted along the z direction, the z and t variables in the above wave function should be replaced by z' and t' respectively with

$$z' = (\cosh\eta)z - (\sinh\eta)t, \qquad t' = (\cosh\eta)t - (\sinh\eta)z. \tag{13}$$

The Lorentz-boosted wave function takes the form

$$\psi_\eta^n(z,t) = H_n(z') \exp\left\{-\frac{1}{2}\left(z'^2 + t'^2\right)\right\}, \tag{14}$$

We are interested in space-time localizations of the wave function dictated by the Gaussian factor. In Dirac's light-cone coordinate system,

$$u = \frac{z+t}{\sqrt{2}}, \qquad u = \frac{z-t}{\sqrt{2}}, \tag{15}$$

with the Lorentz boost of the form

$$u' = e^{\eta} u, \qquad v' = e^{-\eta} v, \tag{16}$$

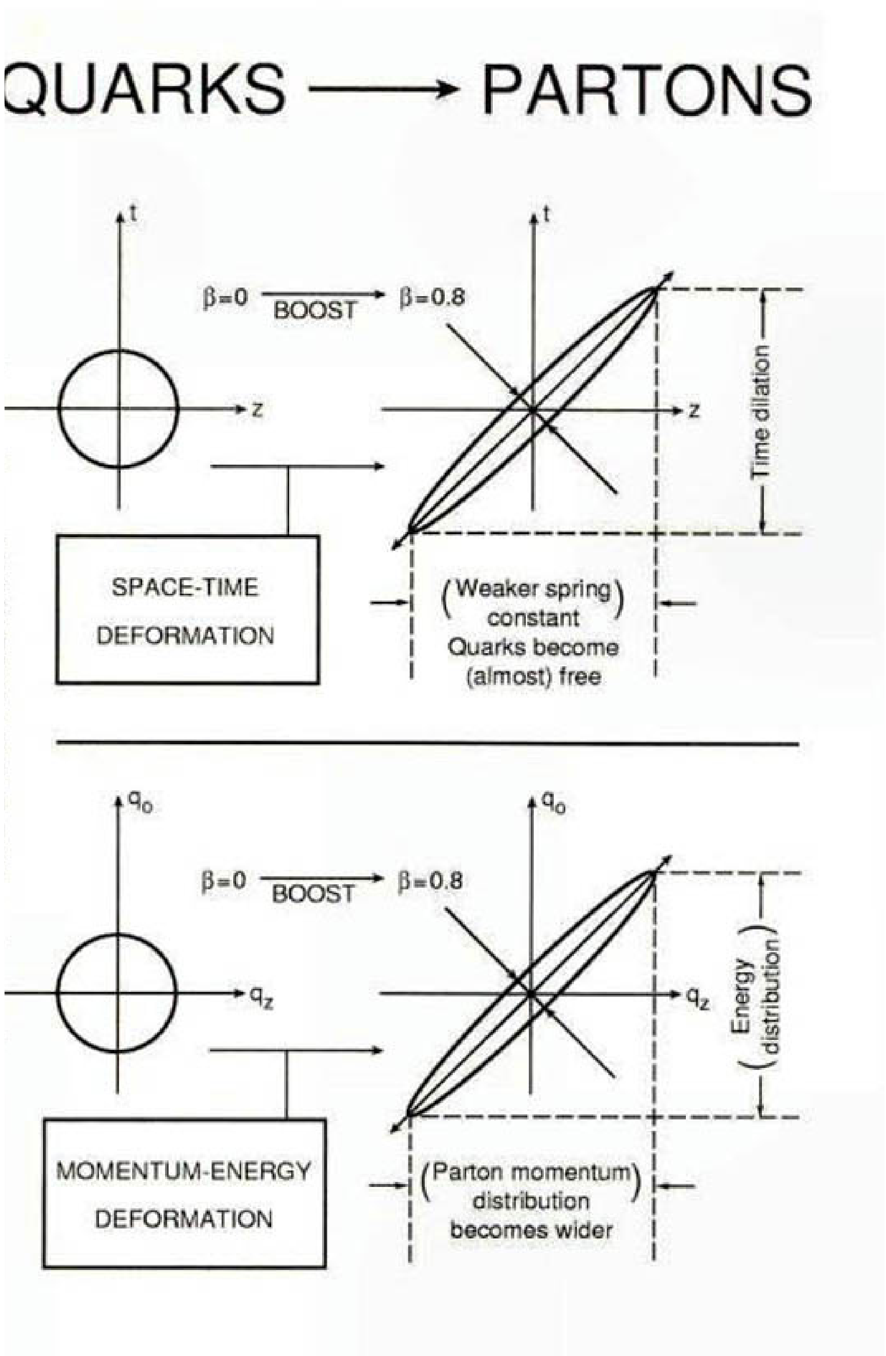
the Lorentz-boosted wave function becomes

$$\psi_\eta(z,t) = \left(\frac{1}{\pi}\right)^{1/2} \exp\left\{-\frac{1}{2}\left(e^{-2\eta} u^2 + e^{2\eta} v^2\right)\right\}, \tag{17}$$

This wave function can be written as

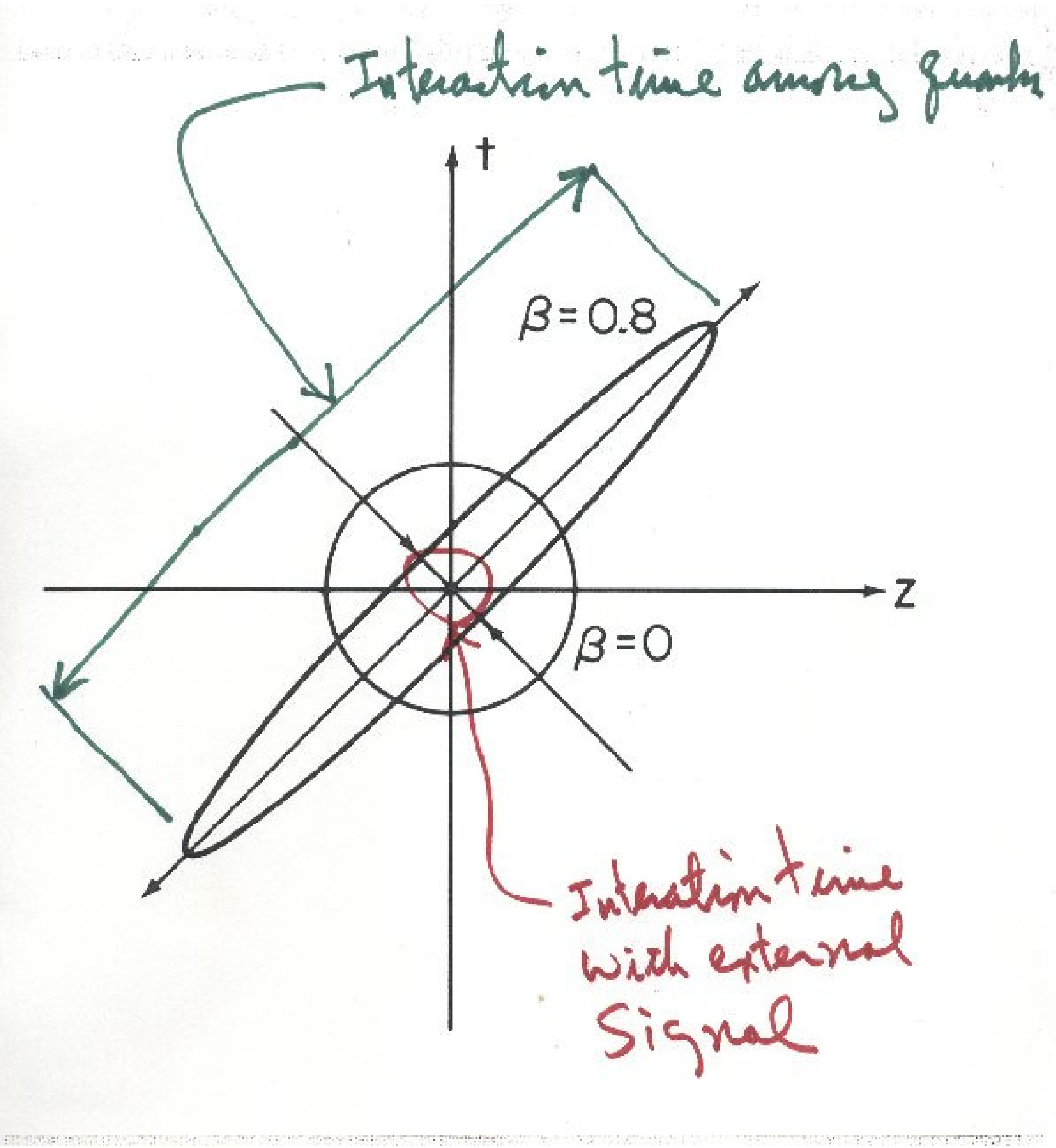
$$\psi_\eta(z,t) = \left(\frac{1}{\pi}\right)^{1/2} \exp\left\{-\frac{1}{4}\left[e^{-2\eta}(z+t)^2 + e^{2\eta}(z-t)^2\right]\right\}. \tag{18}$$

This form is mathematically identical to that of the coupled oscillators. The physics of x_1 and x_2 can be translated into that of x and t, namely the longitudinal and time-like separations.

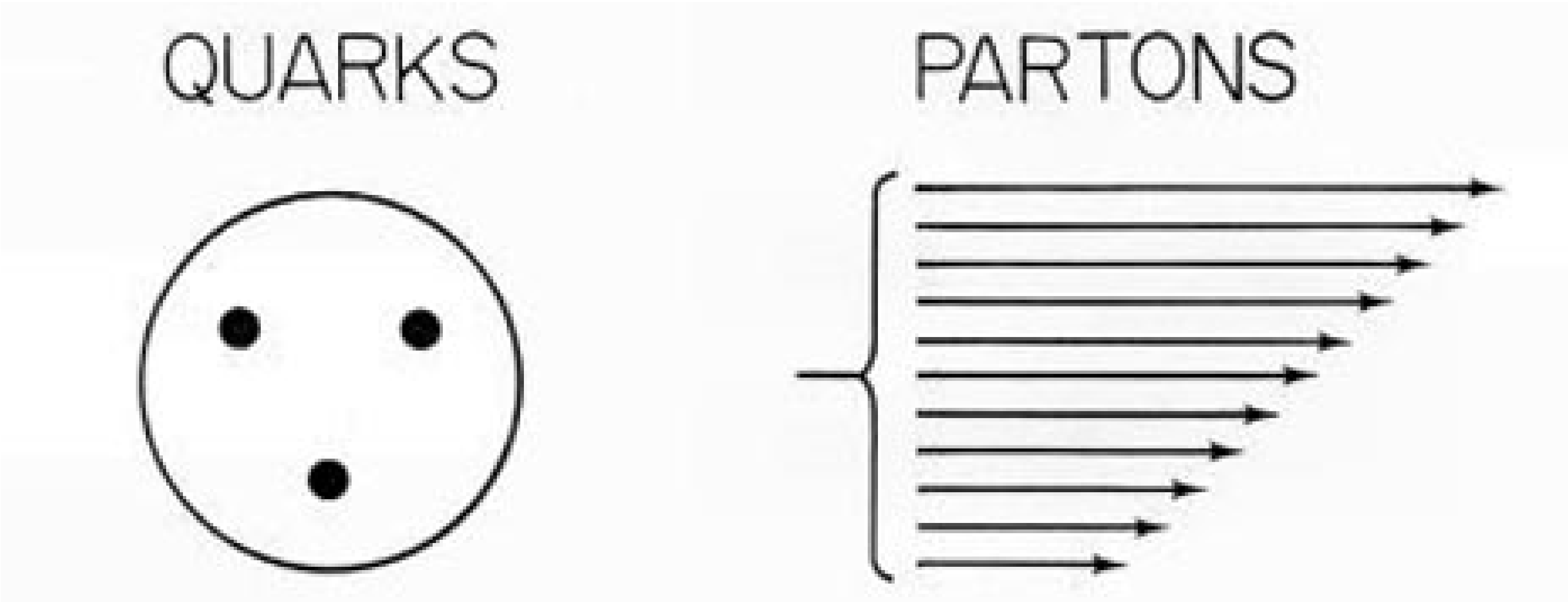


Lorentz-covariant Picture of Feynman's Parton Model

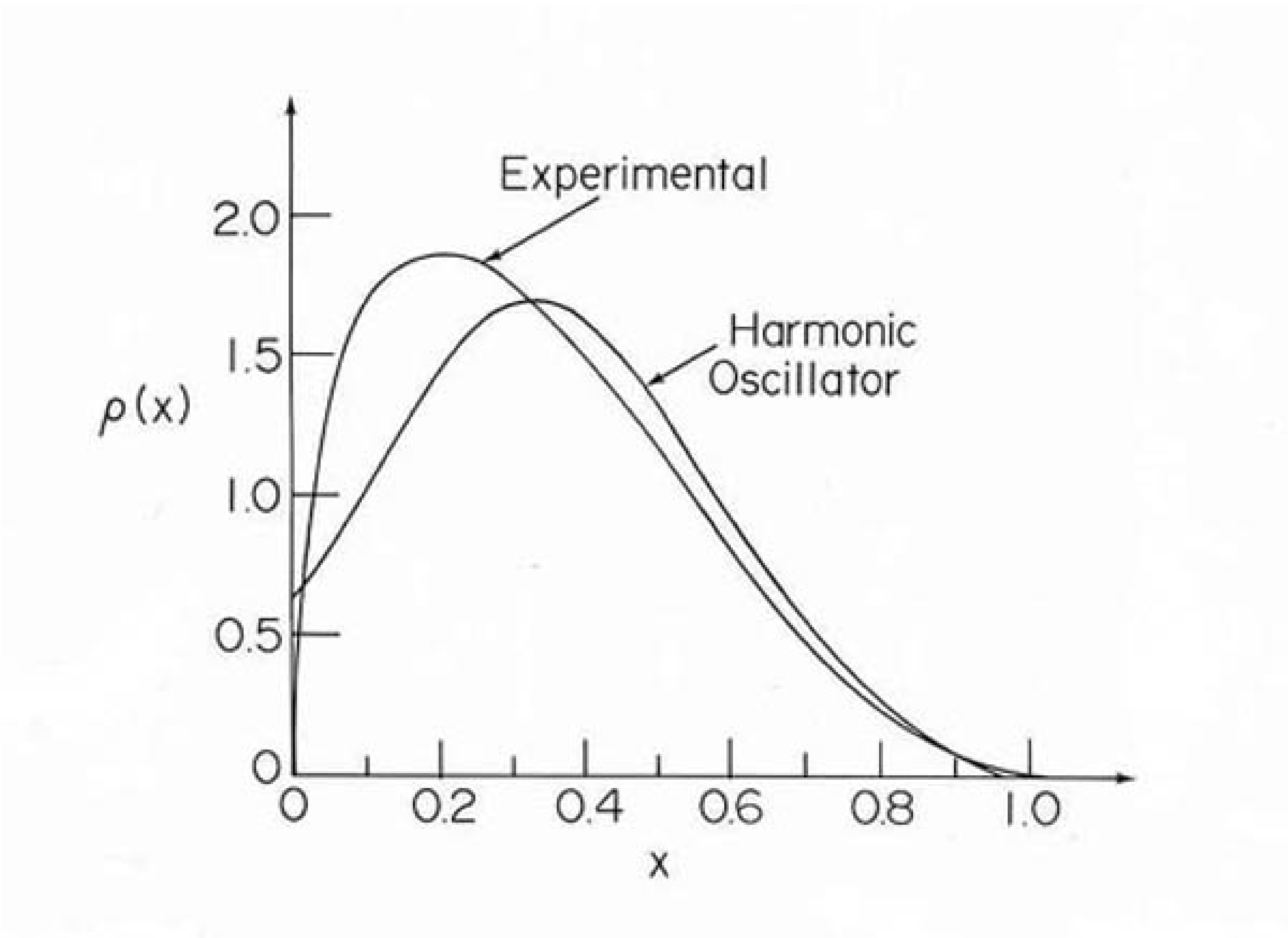
Time Dilation



Quarks and Partons



Parton Distribution Function, after removing Valon effects



Paul Hussar (1982, 1992)

- Paul Hussar was my PhD student at the University of Maryland., In 1992, he wrote a review article and presented an improved picture.

- http://ysfine.com/articles/hussar92.pdf

- We have to do some more work. R. Hwa's valon model is too simple. I came to this conference to find a new wisdom.

Since Einstein’s energy-momentum relation (1905)

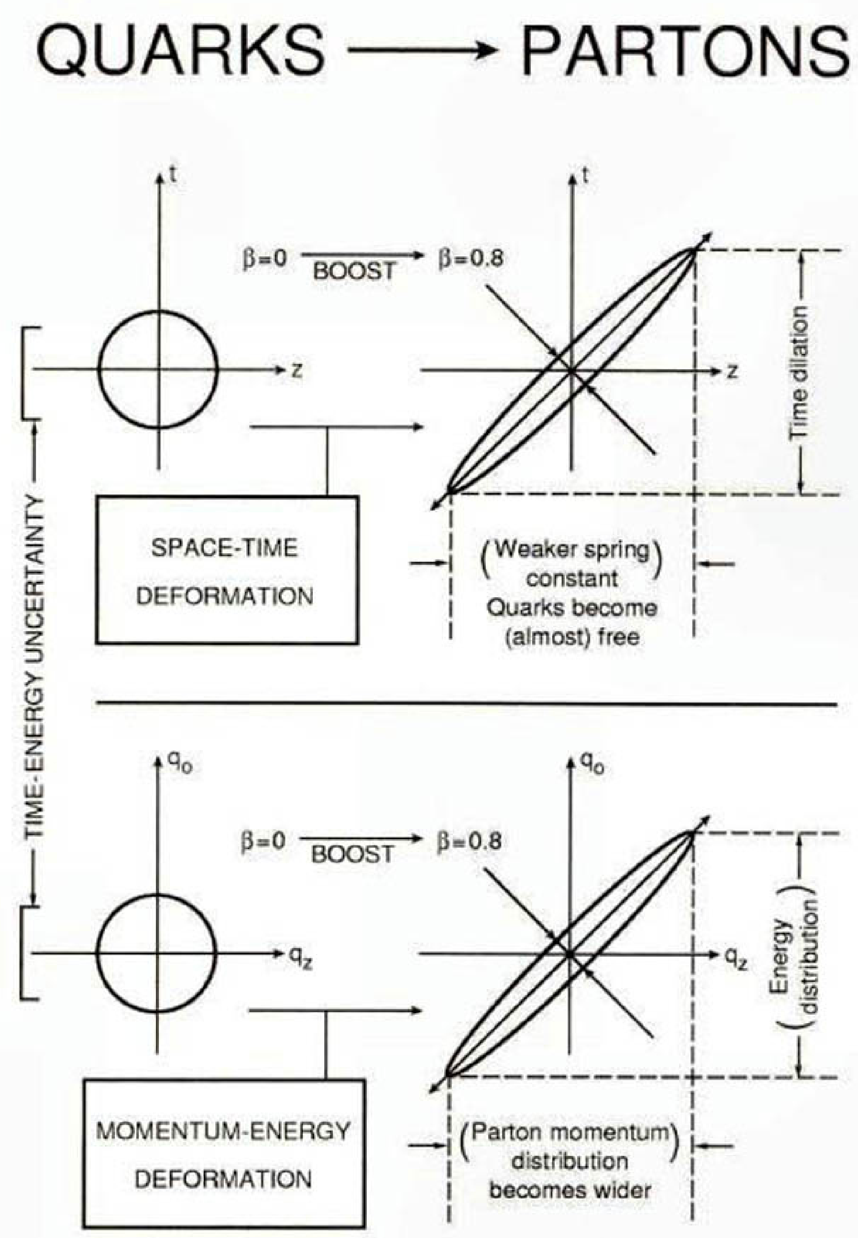
	Massive Slow	between	Massless Fast
Energy Momentum	$E = \frac{p^2}{2m}$	Einstein's $E = \sqrt{m^2 + p^2}$	$E = p$
Spin, Gauge Helicity	S_3 S_1 S_2	Wigner's Little Group	S_3 Gauge Trans.

BUILD YOUR OWN HOUSE

Quark Model and Parton Model are two different manisfestations

of one

Lorentz-covariant Entity.



You have to register your house ownership with the Authority

Observable Gauge Transformations in the Parton Picture

Y. S. Kim
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The internal space-time symmetry of rapidly moving composite particles is studied in terms of the little groups of the Poincaré group. It is shown that Feynman's x in his parton picture is a gauge-transformation parameter.

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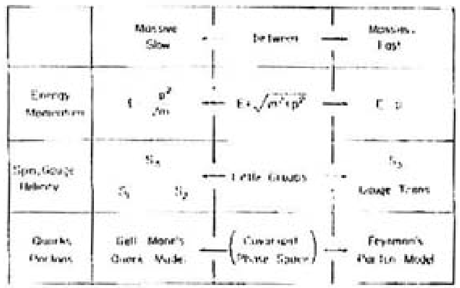
The internal space-time symmetries of elementary particles are governed by the little groups of the Poincaré group.[1] The little groups for massive and massless particles are locally isomorphic to O(3) and E(2) (two-dimensional Euclidean group), respectively. It was shown recently that the E(2)-like little group for massless particles is an infinite-momentum, zero-mass limit of the O(3)-like little group.[2,3] The role of the little groups is illustrated in the second row of Fig. 1.

The purpose of this Letter is to show that the internal space-time symmetry of composite particles can be formulated within the framework of Wigner's little groups, and that Feynman's x parameter in his parton picture[4] is a gauge-transformation parameter. This will unify the second and third rows of Fig. 1. This figure is from the recent paper by Kim and Wigner which deals primarily with the third row.[5]

Wigner's little group is the maximal subgroup of the Lorentz group whose transformations leave the four-momentum of a given particle invariant.[1] For a massive point particle, there is a Lorentz frame in which the particle is at rest. In this frame, the little group is the three-dimensional rotation group. This is the fundamental symmetry associated with the concept of spin.



The internal space-time symmetry of massless particles is governed by the cylindrical group which is locally isomorphic to E(2).[3] In this case, we can visualize a circular cylinder whose axis is parallel to the momentum. On the surface of this cylinder, we can rotate a point around the axis or translate along the direction of the axis. The rotational degree of freedom is associated with the helicity, while the translation corresponds to a gauge transformation in the case of photons.[1]

This translational degree of freedom is shared by all massless particles, including neutrinos and gravitons.[6] Indeed, the requirement of invariance under this symmetry leads to the polarization of neutrinos.[6,7] Since this translational degree of freedom is a gauge degree of freedom for photons, we can extend the concept of gauge transformations to all massless particles[8] and massive particles in the infinite-momentum limit.

It is not difficult to associate the symmetry of a point particle with that of a composite particle if they are massive and at rest, because both of them are governed by the three-dimensional rotation group.[9] The story is quite different for rapidly moving composite particles or hadrons. Does a rapidly moving hadron have the same set of space-time degrees of freedom as that of photons? We can study this problem by constructing a cylindrical symmetry for a hadron with infinite momentum. Then, is this symmetry consistent with Feynman's parton picture? This is the question we would like to address in the present paper.

The group of Lorentz transformations is generated by three rotation and three boost generators.[1,6,7] If we use the four-vector convention

Conclusion

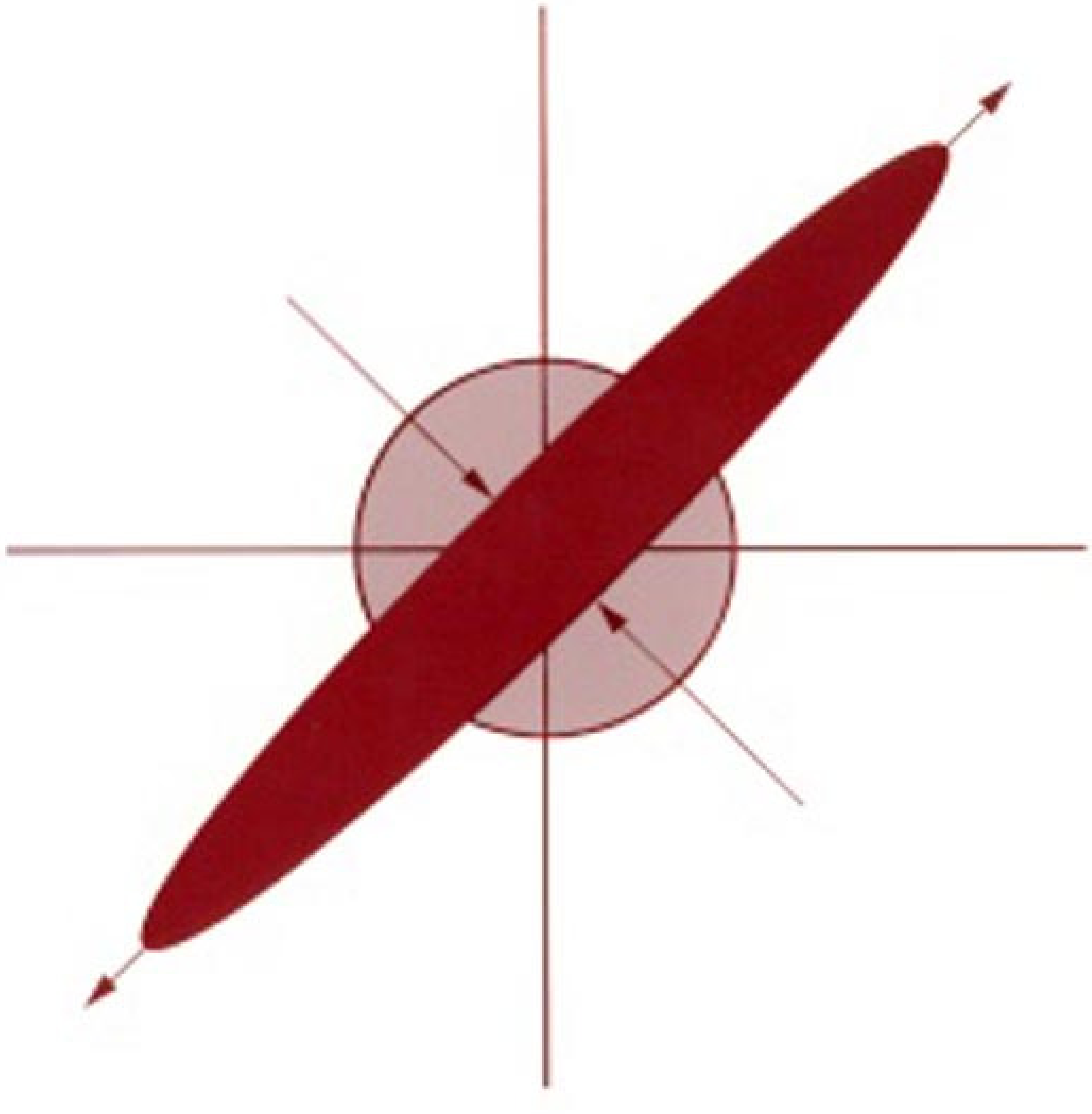
- Quantum mechanics and special relativity are two most important physical theories formulated during the past century. As far as we can see, both are right within their regions of applicability.

However, they cannot be complete theory unless they are consistent with each other.

We have to solve this problem before inventing a new quantum mechanics. There are too many people these days who claim to have invented new quantum mechanics. I don’t like them.

Feynman was a Kantianist, as Einstein was!

The adventure of our science of physics is a perpetual attempt to recognize that the different aspects of nature are really different aspects of the same thing.



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Covariant Harmonic Oscillators and the Quark Model*

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An attempt is made to give a physical interpretation to the phenomenological wave function of Yukawa, which gives a correct nucleon form factor in the symmetric quark model. This wave function is first compared with the Bethe-Salpeter wave function. It is shown that they have similar Lorentz-contraction properties in the high-momentum limit. A hyperplane harmonic oscillator is then introduced. It is shown that the Yukawa wave function, which is defined over the entire four-dimensional Euclidean space, can be interpreted in terms of the three-dimensional hyperplane oscillators. It is shown further that this wave function satisfies a Lorentz-invariant differential equation from which excited harmonic-oscillator states can be constructed, and from which a gauge-invariant electromagnetic interaction can be generated.

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tion.[15]

As we increase $|\vec{p}|$, this property holds for Eq. (2) until the kinetic energy becomes larger than the binding energy.[16] For $|\vec{p}|$ larger than the binding energy, the Bethe-Salpeter wave function is no longer normalizable in the above-mentioned four-dimensional Euclidean space. The harmonic-oscillator wave function of Eq. (1) does not suffer from this effect and remains normalizable for large values of $|\vec{p}|$. This is expected because particles bound by an oscillator potential have infinite binding energy.

Let us rewrite the oscillator wave function assuming that $\vec{p}$ is in the z direction. We use E for p_0 and p for p_z. Then

$$\Psi(x,p)=\exp[-\tfrac{1}{2}\omega(x^2+y^2)] \times\exp\{(-\omega/4m^2)[(E-p)^2(t+z)^2 +(E+p)^2(t-z)^2]\}. \quad (3)$$

For large p,

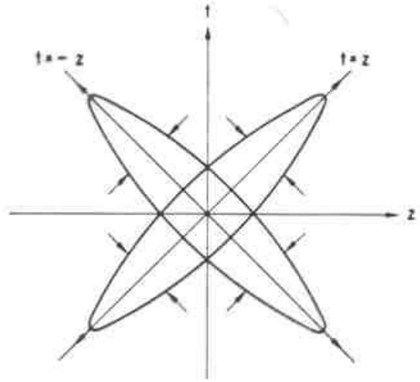

FIG. 1. Lorentz-contracted wave functions with two equal and opposite moments. The form-factor integral of Fujimura *et al.* receives contributions primarily from

portant analysis seems to be that of elastic form factors. The first objection to the use of the harmonic oscillator, that is the Gaussian, wave function is that the form factor decreases exponentially for large t values. This discrepancy with the real

days of quantum field theory.[17] This hyperplane technique was used recently by Fleming to understand the Newton-Wigner localization problem.[18,19] We shall use this hyperplane language in order to understand Lorentz-contracted Gaussian wave