Confinement of electrons in QED2+1 and quarks in QCD3+1 in Temporal Euclidean space

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Outline

- Motivation and introduction
- Schwinger-Dyson equations in Minkowski, spacelike Euclidean and Temporal Euclidean spaces
- Confinement of electrons in QED2+1
- Equivalence between Temporal Euclidean and Minkowski ladder QED2+1
- Confinement of light quarks in QCD in E_T space
- future application, Walking Technicolor, ...

Motivation and Introduction

- Confinement and chiral symmetry breaking as nonperturbative phenomena in QCD.
  - Confinement - there are no free colored (long time living) states.
  - Chsb - Dynamical mass generation; the masses appear without its explicit presence in the classical action. For spin $1/2$- chiral symmetry is broken because of "strong" interaction in QCD, 2+1 QED with small N_f, Technicolors ...

If there are no singularities in the Green's functions associated with colored objects, the associated objects cannot propagate freely in spacetime, they must be confined.

QFT: classical action-> Green's function->knowledge of the poles S/matrix-> $d\sigma$

Instead of some general proof of mass gap in QCD or a general proof of confinement (Witten at all) I expect step by step approximate evidence of confinement of simple colored object that we can imagine.

free moving particles or bound states- Singularities are expected in GFs in timelike regime of momenta

permanently confined objects- Singularities are NOT expected in GFs in timelike regime of momenta

Technique providing the results for GFs in timelike axis of momenta is welcome!

In practise- Wick rotation is done. All lattice and most of the other nonperturbative first principle QCD calculations are performed in the Euclidean space. The property of GFs at timelike momenta are estimated by continuation (within a doubts).

Direct Minkowski approach:

- Perturbation theory (any standard text-books)
- spectral SDE solutions $C - R_+$ analytical assumption V.S. JHEP2001,FBS2005 works only for subcritical coupling V.S. , J. Adam, P. Bicudo; PRD2007
- Gauge (Pinch) Technique- Salam, Delbourgho (QED), Cornwall (QCD)

conclusion: Solution for timelike momenta based on the weaker assumption is necessary

Schwinger-Dyson equations

Quantum equation of motion- $\delta\Gamma/\delta\phi$ system of Eqs. for Greens functions (see Itzykson-Zuber)

S/quark, G/gluon, g-ghost

$$S = F[S, G, \Gamma]$$
$$G = F[G, g, \Gamma_{GGG}, \Gamma_{GGGG}, \Gamma_{Ggg}, S, \Gamma_{SGS}]$$
$$g = F[G, g, \Gamma_{GGG}, \Gamma_{GGGG}, \Gamma_{Ggg}]$$
$$\Gamma_3 = F[S, G, g, \Gamma_3, \Gamma_4]$$

... reviews, applications, methods of solution:

Euclidean space: R. Alkofer, L. Smekal, An. Phys. 1999. P. Maris, C.D. Roberts, Int.J.Mod.Phys. E12 (2003) 297-365, +1000 of other papers

Minkowski space by spectral representations: V. Sauli. Few-Body Systems, 39, 45 (2006)

Temporal Euclidean space: V. Sauli and Z. Batiz, arXiv:0901.0110, Confinement and Chiral Symmetry Breaking in QED2+1

Sauli,Batiz, J.Phys.G36 (2009)

Sauli, arXiv: last week ...

Standard Euclidean space formulation

Wick Rotations in 4dim:

$t \to i\tau \ , p_0 \to -ip_4$

The Wick rotation is a calculational trick in quantum theory in which we assume that the energy or the time are pure imaginary. We do the calculations given these assumptions, which are often more well-defined, and then analytically continue the results back the usual real values of time and/or energy. It works usually in perturbation theory

in momentum space, quark gap:

$$S^{-1}(p) = \not{p} - m - \Sigma(p)$$

$$\Sigma(p) = -i \int \frac{d^4q}{(2\pi)^4} \Gamma_\mu(p,q) S(q) \gamma_\nu G^{\mu\nu}(p-q)$$

singularity structure $S \simeq 1/(p^2 - m^2)$

$p^2 = p_o^2 - p_x^2 - p_y^2 - p_z^2.$

E. GFs are regular and smooth in E space

SDE in E space after WR $q_0, p_0 \to -iq_4, -ip_4$

$$\Sigma(p_E) = \int \frac{d^4 q_E}{(2\pi)^4} \Gamma_\mu(p_E, q_E) S(q_E) \gamma_\nu G^{\mu\nu}(p_E - q_E)$$

feasible since no sing. tree level: $S \simeq 1/(p^2 + m^2)$ $p_E^2 = -p^2 = p_1^2 + p_2^2 + p_3^2 + p_4^4$

E GF.s are real functions note- Euclidean means Minkowskian spacelike (assuming validity of assumptions)

Advantages:

- no singularities
- convenient usage of spherical coordinates

Disadvantages:

- unknown physics at timelike domain if back WR is not performed

SDE in Temporal Euclidean space

The idea: deforming contour for space components of fourmomentum P to arrive to the Euclidean space again

multidimensional analogue of WR

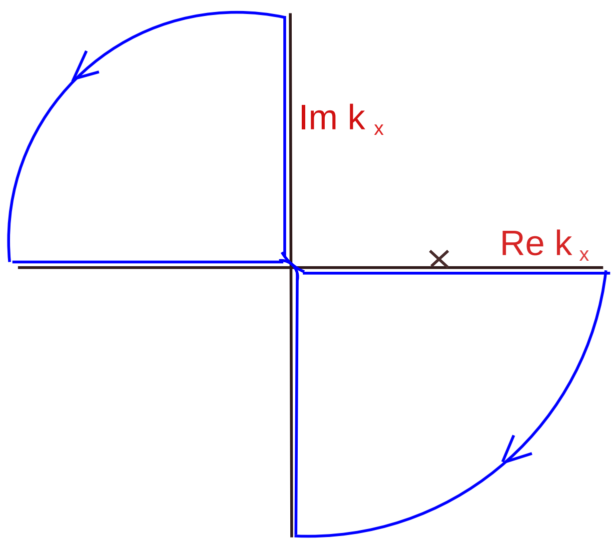

Figure 1: Deformation of the contour of space component of P in complex plane

Assumptions: the GFs are holomorphic in the fourth and the second quadrants of complex plane of $P_{x,y,z}$

$$p_{ET}^2 = \sum_{i=1}^4 p_i^2$$

$$p = (p_1, p_2, p_3, p_4)^{[ET]} = (p_0, -ip_1, -ip_2, -ip_3)^{[M]} ,$$

measure $d^4p_M = id^4_{ET}$ with real $\pm\infty$ boundaries in a momentum loop integrals.

Results:

Equations and loop expressions simplify, perturbative singularity persist.

tree level propag.:

$\simeq \frac{1}{p^2-m^2}$

ambiguity free assumptions for confining theory-

- There are no poles at real axis of p^2 for Greens functions describing permanently confined object (quarks, gluons)

because of "selfenergy property"-Σ **becomes complex dynamically bellow naive perturbative threshold**

Advantages of E_t:

- (technical) SDEs for confined objects are smooth functions even for any p^2

- (technical) convenient usage of spherical coordinates

  The hyperbolic angle of Minkowskian "spherical" coordinates:

  $k_0 = k \cosh\theta \, , \theta \in (0, \infty)$

  turns to be standard angle

  $k_4 = k \cos\theta \, , \theta \in (-\pi/2, \pi/2)$

- (physical) knowledge about timelike behaviour of GFs

Disadvantage (or perhaps warning):

- some "weak" singularities persist

Example: Propagators of SM W, Z, H fields have predominantly poles close to the real axis (we must define how to work with) and their GFs are convoluted with GFs of confined quarks. However their contribution to Σ is substantial.–> no problem

E_t SDE are formulated

Suited for study of confinement

QED2+1 and QED3

Minkowski metric $g_{\mu\nu} = diag(1, -1, -1)$, four dimensional Dirac matrices $\{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu}$ Minkowski formulation:

$$S^{-1}(p) = \not{p} - m - ie^2 \int \frac{d^3k}{(2\pi)^3} G_{\mu\nu}(k-p)\Gamma^\mu(k,p)S(k)\gamma^\nu$$

Ladder approximation of SDE=bare vertex+ free photon propagator

$$G_{\mu\nu} = \frac{-g_{\mu\nu} + (1-\xi)\frac{k_\mu k_\nu}{k^2}}{k^2}$$

Standard Euclidean treatment:

T. Appelquist,PRL. 60,(1988).

N.E. Mavromatos, J. Papavassiliou, cond-mat/0311421.

A. Bashir, A. Raya, I. C. Cloet, C. D. Roberts, arXiv:0806.3305.

Temporal Euclidean space for 2+1 dimensional Quantum Electrodynamics

V. Sauli and Z. Batiz, arXiv:0901.0110,

2dim WR leaves i in measure

$$k_{x,y} \to i k_{2,3}$$

$$i \int d^3k \to -i \int d^3k_{E_T}$$

Integrating SDE over the angels gives

$$B(p) = m + i(2+\xi)\frac{e^2}{4\pi^2}\int_0^\infty dk \frac{k}{p} \ln\left|\frac{k+p}{k-p}\right| S_s(k)$$

ξ - gauge parameter

complexification:

$$
\begin{aligned}
& S_s(x) = \frac{B(k)}{A^2(k)k^2 - B^2(k)} \\
= \quad & \frac{R_B\left[(R_A^2 - \Gamma_A^2)k^2 - R_B^2 - \Gamma_B^2\right] + 2R_A\Gamma_B\Gamma_A\, k^2}{D} \\
+ \quad & i\,\frac{\Gamma_B\left[(R_A^2 - \Gamma_A^2)k^2 + R_B^2 + \Gamma_B^2\right] - 2R_BR_A\Gamma_A\, k^2}{D} \\
& S_v(k) = \frac{A(k)}{A^2(k)k^2 - B^2(k)} \\
= \quad & \frac{R_A\left[(R_A^2 + \Gamma_A^2)k^2 - R_B^2 + \Gamma_B^2\right] - 2R_B\Gamma_A\Gamma_B}{D} \\
+ \quad & i\,\frac{\Gamma_A\left[-(R_A^2 + \Gamma_A^2)k^2 - R_B^2 + \Gamma_B^2\right] + 2R_AR_B\Gamma_B}{D}
\end{aligned}
$$

where R_A, R_B (Γ_A, Γ_B) are real (imaginary) parts of the functions A, B and the denominator D reads

$$D = ([R_A^2 - \Gamma_A^2]k^2 - [R_B^2 - \Gamma_B^2])^2 + 4(\Gamma_A R_A - \Gamma_B B)^2 \,.$$

Confinement in QED2+1

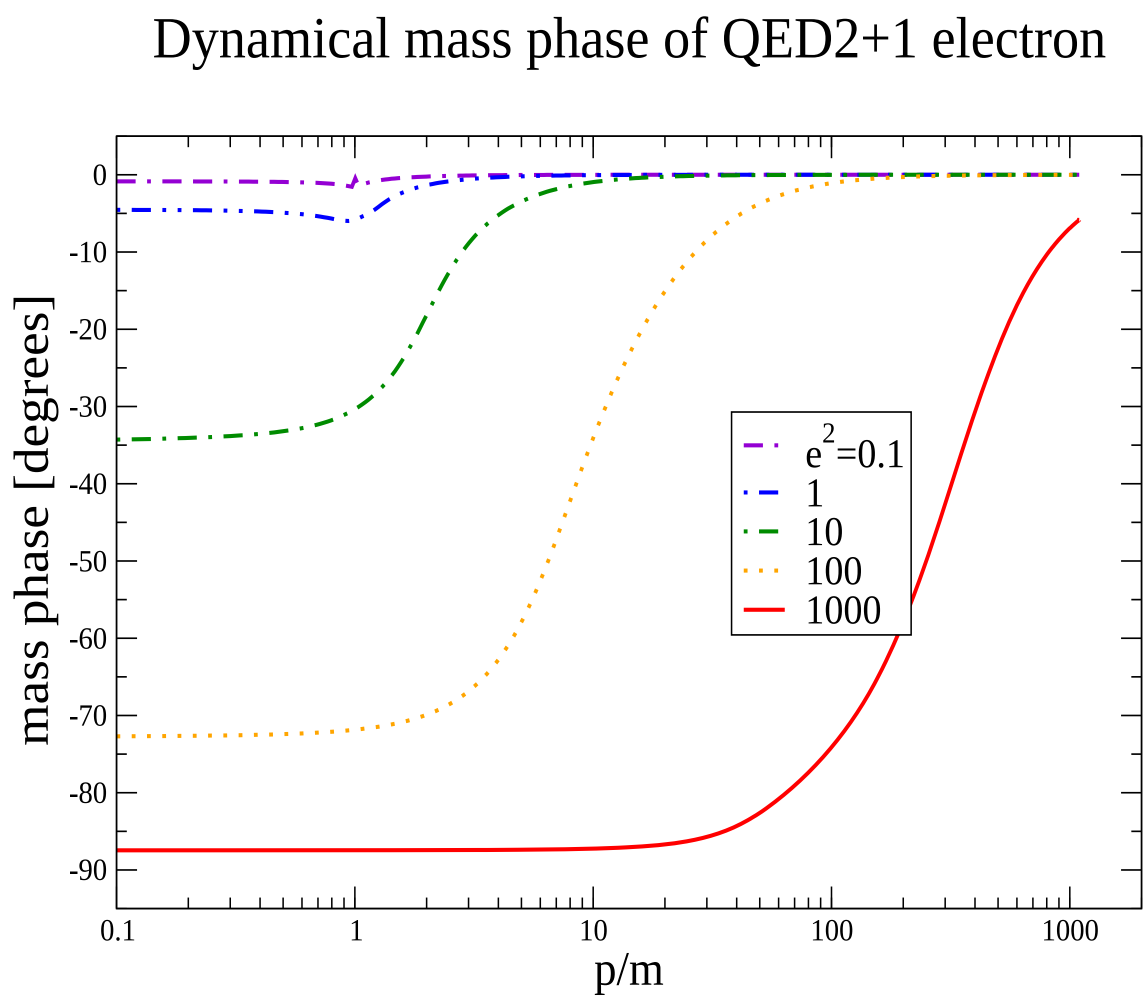

Figure 2: Phase ϕ of mass a dynamical mass function $M = |M|e^{i\phi}$ of electron living in 2+1 dimensions with different $\mathcal{C}$, $m = 1$. The real pole is absent! for larger driving parameter :$\mathcal{C} = \frac{e^2}{m}$. For $\mathcal{C} > 0.0191 \pm 001$ there are no free moving electrons.

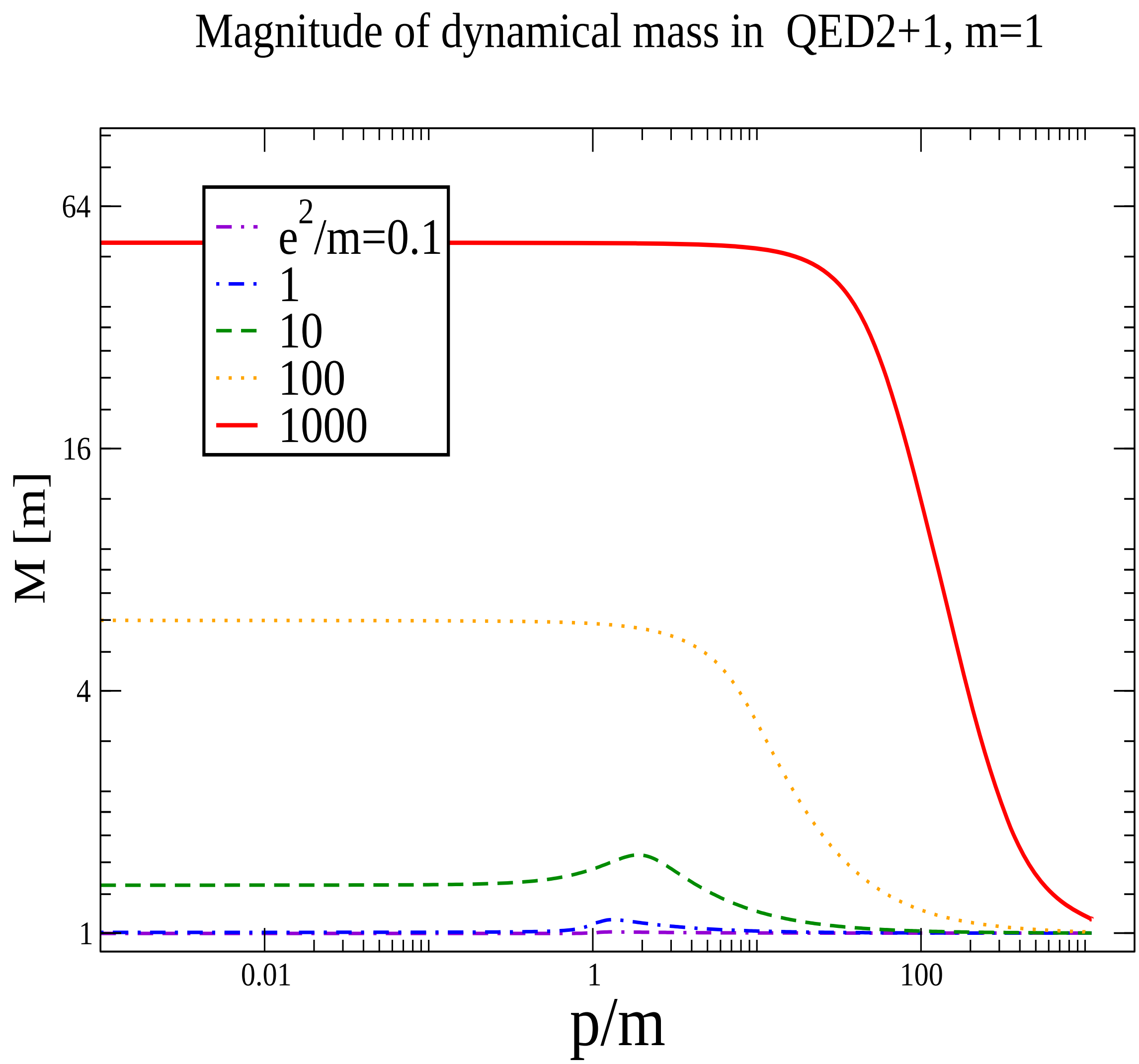

Figure 3: Magnitude of the running mass $M = |M|e^{i\phi}$ of electron living in 2+1 dimensions with different $\mathcal{C}$

Direct Minkowski space calculation. Proof of equivalence between $M2+1$ and E_T3 QED

$$\int d^3k K(k,p) = \int_0^\infty dr r^2 \int_0^{2\pi} d\theta \int_0^\infty d\alpha K(k,p)$$

$$\left\{ sh\alpha \left| \begin{array}{c} k_o = -r\, ch\alpha \\ k_x = -r\, sh\alpha\, \sin\theta \\ k_y = -r\, sh\alpha\, \cos\theta \end{array} \right| + sh\alpha \left| \begin{array}{c} k_o = r\, ch\alpha \\ k_x = r\, sh\alpha\, \sin\theta \\ k_y = r\, sh\alpha\, \cos\theta \end{array} \right| \right.$$

$$\left. + ch\alpha \left| \begin{array}{c} k_o = -r sh\alpha \\ k_x = -r ch\alpha\, \sin\theta \\ k_y = -r ch\alpha\, \cos\theta \end{array} \right| + ch\alpha \left| \begin{array}{c} k_o = r sh\alpha \\ k_x = r ch\alpha\, \sin\theta \\ k_y = r ch\alpha\, \cos\theta \end{array} \right| \right\}$$

For ladder QED, for timelike momenta

$$p_\mu = (p, 0, 0)$$

$$B(p_T) = m + i(2+\xi)\frac{e^2}{4\pi^2}\int_0^\infty dk \frac{k}{p} \ln\left|\frac{k+p}{k-p}\right| S_s(k)$$

Discovery:

- spacelike part of Minkowski subspace$= 0$
- QED2+1 $=$ QED3 in $E_T 3$

for spacelike external

$$p_\mu = (0, p, 0)$$

$$B(p_S) = \mathcal{F} B(p_T)$$

It is not a gap equation but twodim integrals

but QED2+1 is inequivalent with standard QED3 "WR si wrong". ——————————————————————————————

The Simple Models for QCD CSB in ET space

Model 0

Ladder Rainbow Landau gauge quark gap equation with "analyticized" gluon form factor

Sauli,Batiz, J.Phys.G36 (2009)

$$G^{\mu\nu}(k) = \left[-g^{\mu\nu} + \frac{k^\mu k^\nu}{k^2}\right] G(k^2) - \xi \frac{k^\mu k^\nu}{(k^2)^2}$$

$$\frac{g^2}{4\pi} G(q^2, \Lambda_{QCD}) = \int_0^\infty d\nu \, \frac{\rho_g(\nu, \Lambda_{QCD})}{q^2 - \nu + i\varepsilon}$$

Thus, contrary to studied quark propagator, the standard analyticity for gluon propagator is still assumed.

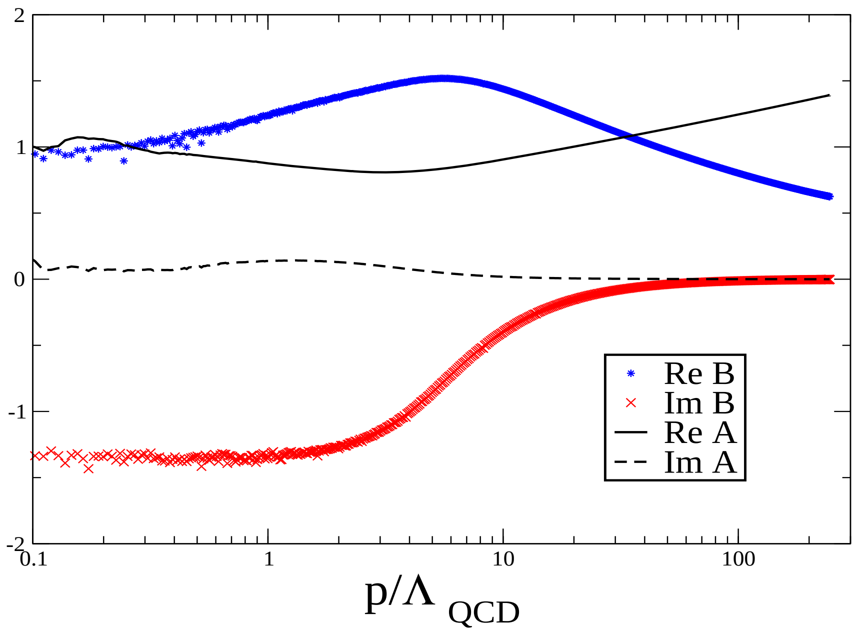

Figure 4: The s-quark propagator function A, B. The upper (down) curves represent the real (imaginary) parts. The dimensionfull quantities are rescaled by QCD scale Λ_{QCD}. Parameters $m_r(0.2\Lambda \simeq 0) \simeq \Lambda$

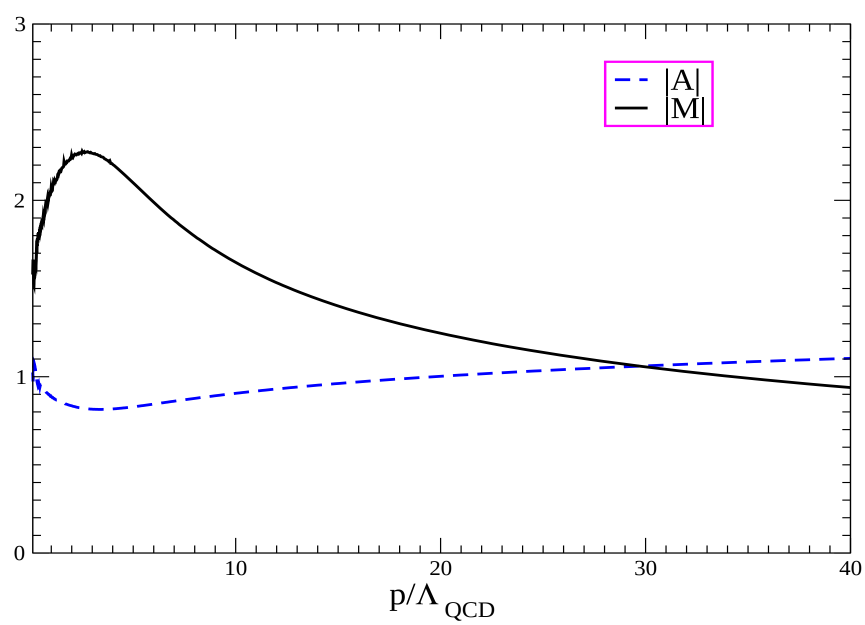

Figure 5: The absolute value of quark mass function $M = |M|e^{i\phi_M}$ and the inverse of renormalization function- the function $||A||$ is displayed.

Confinement of light quarks,

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Rainbow, ladder gap quark SDE with two different modeled kernel

Renormalization at timelike ultraviolet, for explicit CSB renormalized value $M(250 - 350GeV) \approx 2 - 3MeV$

Model I.:

$$\frac{g^2}{4\pi}G(q^2) = \frac{4\pi/\beta}{\frac{1}{2}ln\left[e + \left(\frac{q^2}{\Lambda^2}\right)^2\right]} \tag{1}$$

Model II.:

=Model 0 , with softened gluon form factor by factor $1/2$ Feynman epsilon ε is dropped out.

for the first time we get CSB and confinement for light and massless quarks in ET space

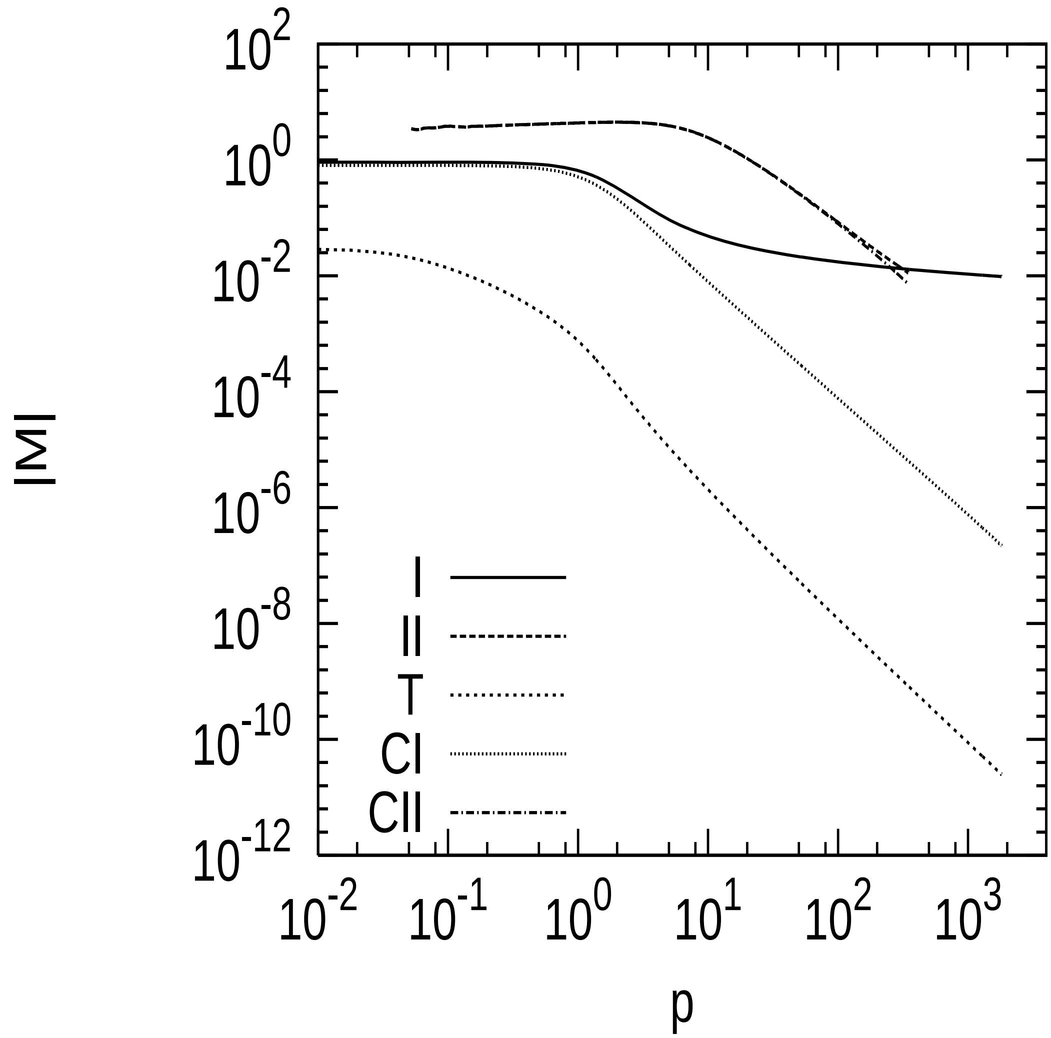

Figure 6: Magnitude $|M|$ of the running quark mass function $M = |M|e^{i\phi}$ for modeled QCD I,II, its chiral limit CI,CII. The $SU(3)$ "Walking Technicolor" T solution is added for for the comparison, scale is $\Lambda_{QCD} = 1$ (and $\Lambda_{Tech.} = 1$ is set up in this case as well)

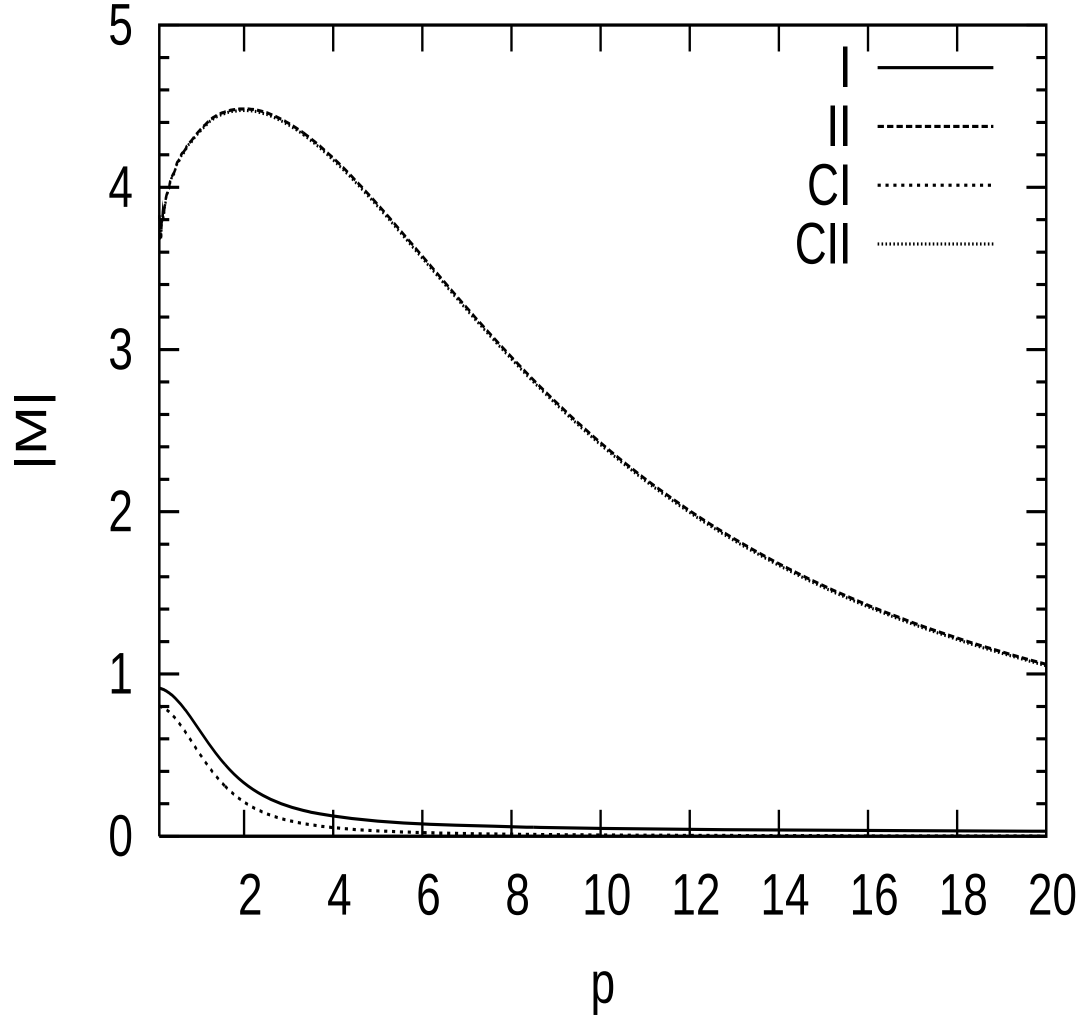

Figure 7: I infrared behaviour of the functions M as they are in the Fig. 1, but in with linear axis. The solution for large N_f is omitted here.

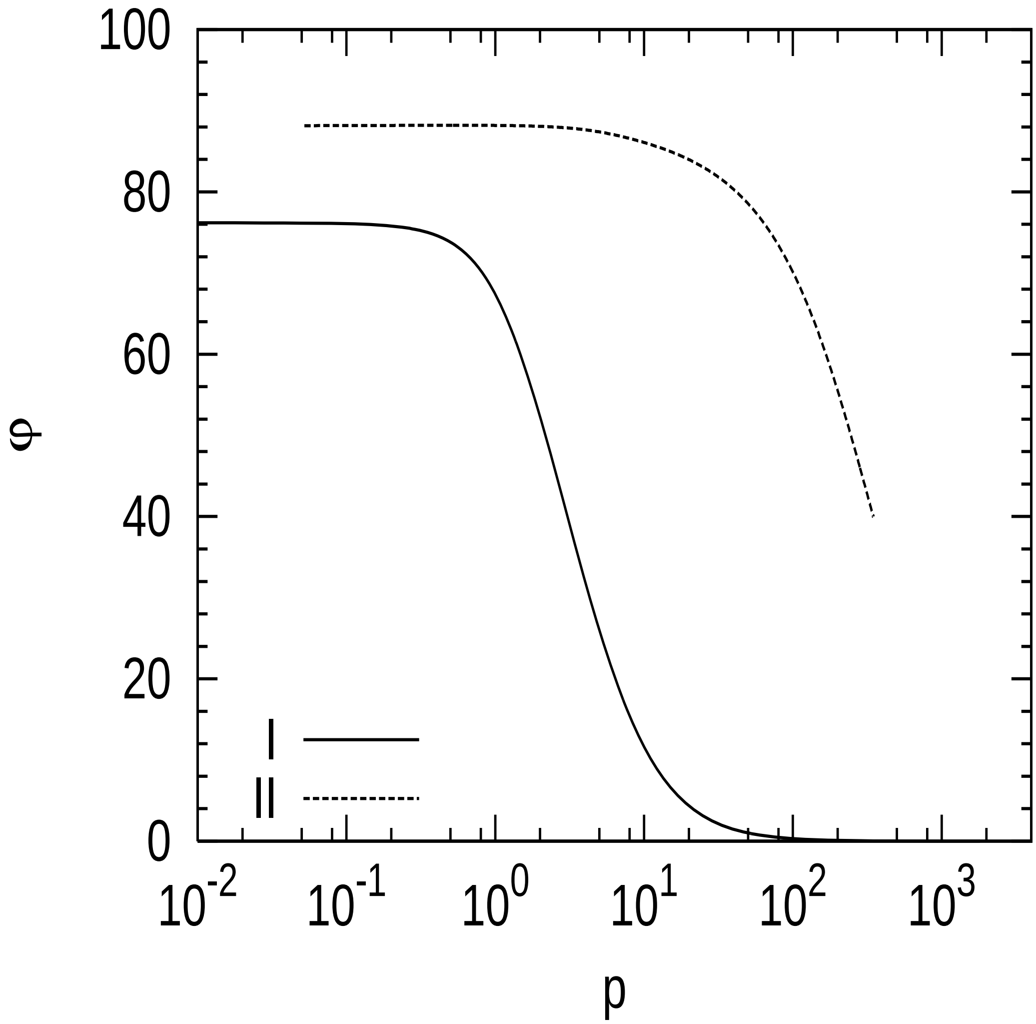

Figure 8: Phase ϕ of the running quark mass function $M = |M|e^{i\phi}$ for the models I and II respectively, axis momentum is in the units of Λ_{QCD}.

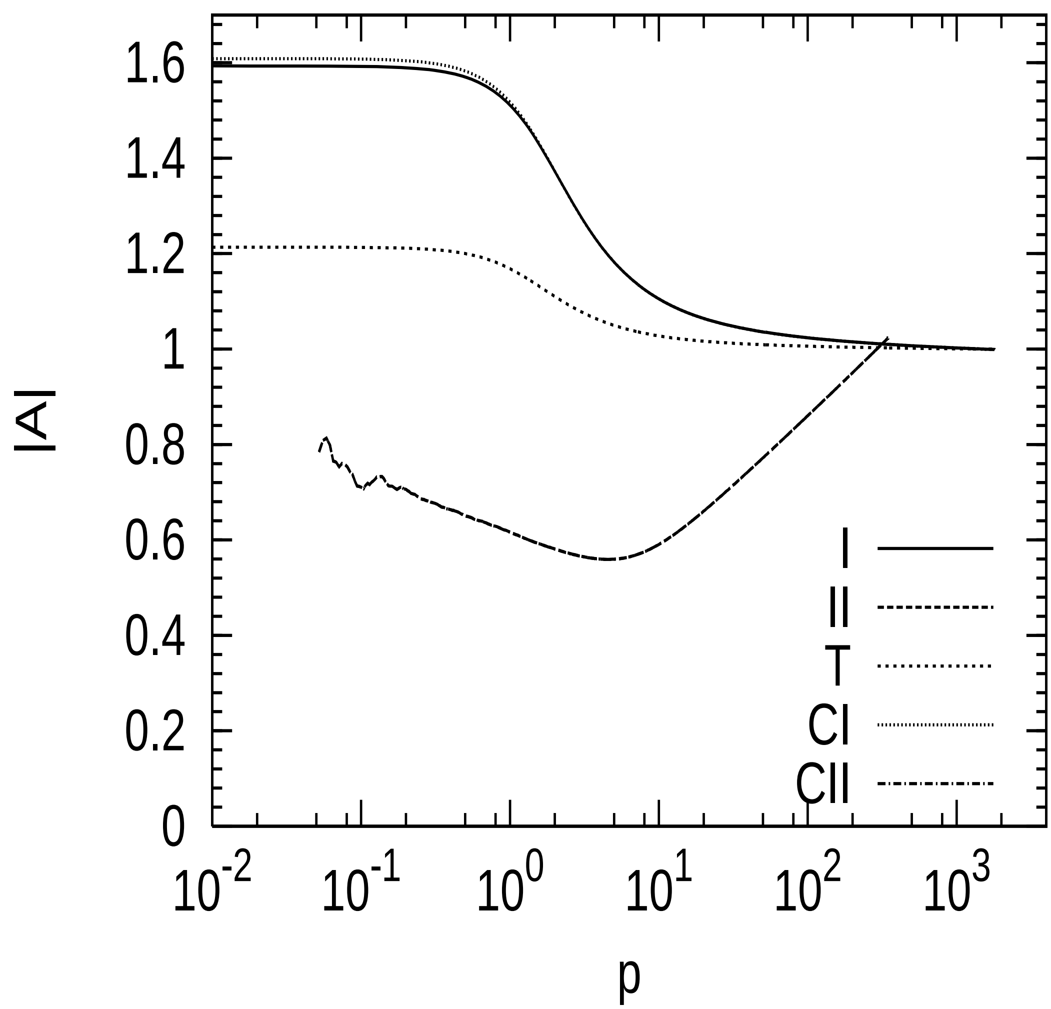

Figure 9: Renormalization wave functions for the model I and II. The same is shown for the chiral limit (which are indistinguishable in one case). The "Technicolor" T solution is added for for the comparison, the momentum axis is scaled in Λ_{QCD} and Λ_T respectively.

Alternative derivation , relation between E spaces

Do I have time?

Another possibility to arrive to the same equation is to start with standard E_s formulation and make the continuation to E_t at level of SDE

Assumption of analyticity :

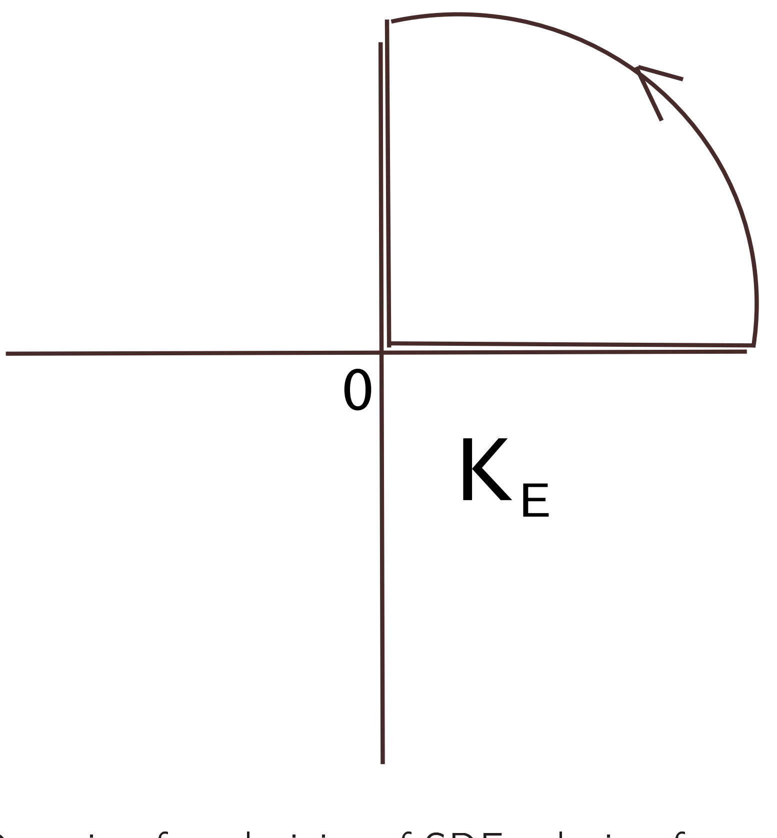

Figure 10: Domain of analyticity of SDE solution for complex norm of Euclidean momentum

Alternative derivation, 4-dim QCD example

$$S^{-1} = \not{p} - m - \Sigma(p)$$
$$\Sigma = \Sigma_A \ \not{p} + \Sigma_B; \ S = S_V \ \not{p} + S_S$$

$$\begin{aligned}
\Sigma_A(p) &= \int \frac{dk^2}{2\pi} S_V(k^2) I_A(p,k) \\
\Sigma_B(p) &= \int \frac{dk^2}{2\pi} S_S(k^2) I_B(p,k)
\end{aligned}$$

I_A, I_B are kernels obtained from E_s by $k \longrightarrow ik$,

Lorentz invariance

$$I_A(p,k) = I_A^{E_s}(p^2 \rightarrow -p^2, k^2 \rightarrow -k^2)$$

$$S_{S,V}(x) = \frac{A, B(x)}{xA^2(x) - B^2(x)}$$

Conclusion

First analysis of the quark gap equation in the temporal Euclidean space has been presented. Using 2d and 3d Wick rotated kernel the analysis of the QED2+1 electron and QCD quark gap equation in the temporal Euclidean space has been presented.

For ladder QED2+1 in $A = 1$ the exact equivalence between ET and M space is proved.

The mass function of light quarks is predominantly imaginary, providing the confining solution for the quark propagator; such quark propagator has no pole nor branch point at real p^2 axis.

At last but not at least the dynamical CSB has been studied near the critical coupling. The observed numerical solutions obtained do not suggests separation of the dynamical CSB from confinement. These phenomena go hand by hand in hypothetical Walking Technicolors models.

Future study:

1. gluon propagator and confinement of gluons.

2. Clarification and further justification of of ET space (the best way is through the study of M.)

3. Hadron phenomenology- BSE, form factors.