Heavy multiquarks

available at *http://lpsc.in2p3.fr/theorie/Richard/SemConf/Talks.html*

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Excited QCD, Zakopane (Poland), February, 2009

GRENOBLE 1

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Dodecatoplet ($t^6\bar{t}^6$)

- Higgs exchange

$$-\alpha_H \frac{\exp(-\mu_H r)}{r}\,, \qquad \text{with} \quad \alpha_H = g_t^2/(4\pi) \quad \text{and} \quad g_t \sim 1\,.$$

- Does it bind $t^n\bar{t}^m$? (Frogatt, H.B. Nielsen)
- Up to $n \leq 6$ and $m \leq 6$, behave as bosons.
- Optimistic estimate by Nielsen and Frogatt, who neglected the Debye factor!
- Corrected by a Hartree (self consistent effective one-particle potential) by Shuryak et al. $\rightarrow$ upper variational bound on ground state energy
- In fact, the calculation of the self-Yukawian boson system already available in the literature (Pacheco et al.) and a lower bound is also possible.

Dodecatoplet ($t^6\bar{t}^6$)

- If Higgs exchange alone, by scaling, the only parameter is $G = m_t \alpha_H / \mu_H$
- for 2-body, no binding for $G \leq 1.68$ (Blatt and Jackson, 1949), i.e., $\mu_H \leq 8.2$ GeV for $\alpha_H = 1/(4\pi)$.
- For ($t^6\bar{t}^6$), estimate $\mu_H \lesssim 29$ GeV By Shuriak, and $\mu_H \lesssim 31$ GeV from Pacheco et al.
- Perhaps slightly heavier with a better variational calculation,
- From the lower-bound on the ground-state energy, the critical mass [again for $\alpha_H = 1/(4\pi)$], cannot exceed $\mu_H^{(c)} = 49$ GeV.
- If α_H bigger, $\mu_H^{(c)}$ inversely proportional.

The chromomagnetic scenario

- In the 70s, it was realised that a model based on

$$H_{SS} = -\sum_{i<j} \frac{C}{m_i m_j} \boldsymbol{\sigma}_i.\boldsymbol{\sigma}_j\, \tilde{\lambda}_i.\tilde{\lambda}_j\ ,$$

  inspired by the Breit–Fermi term in QED, reproduces the observed patterns, $N < \Delta$, $\eta_c < J/\Psi$, etc.
- An astute treatment of the colour-spin algebra exhibits interesting coherences, e.g., for the H, suggesting its stability

$$\langle \left[\sum \boldsymbol{\sigma}_i.\boldsymbol{\sigma}_j\, \tilde{\lambda}_i.\tilde{\lambda}_j\right] (uuddss)\rangle < \langle[...](uds)\rangle + \langle[...](uds)\rangle\ ,$$

- and its analogue for the 1987-vintage pentaquark P

$$\langle \left[\sum \boldsymbol{\sigma}_i.\boldsymbol{\sigma}_j\, \tilde{\lambda}_i.\tilde{\lambda}_j\right] (\overline{Q}qqqq)\rangle < \langle[...](\overline{Q}q)\rangle + \langle[...](qqq)\rangle$$

- However no stability with $SU(3)_F$ breaking and an estimate of short-range correlations in multiquarks, $C \propto \langle \delta^{(3)}(\boldsymbol{r}_{ij})\rangle$,
- H not found (in many experiments), nor the P (in 1 exp.)

Flavour independence & symmetry breaking

- If chromomagnetism fails, why not chromo-electricity and its properties under symmetry breaking ?
- Consider

$$H = H_0(even) + \lambda H_1(odd).$$

Then for the ground state, with $\psi_0(H_0)$ as trial w.f, $\langle\psi_0|H_1|\psi_0\rangle = 0$

$$E(H) \leq E(H_0),$$

i.e., *H benefits* of symmetry breaking.
For instance $E(p^2 + x^2 + \lambda x) = 1 - \lambda^2/4$.

- This is very general.
- Starting, e.g., from a symmetrical four-body system $(a, a, \bar{a}, \bar{a})$ breaking particle identity or charge conjugation lowers the ground state, but has different consequences on stability.

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Breaking particle identity

$H(M, m, M, m)$, where V <u>does not change</u> if M or m is modified, can be rewritten as

$$H = \underbrace{\left(\frac{1}{4M} + \frac{1}{4m}\right)\left[\boldsymbol{p}_1^2 + \cdots + \boldsymbol{p}_4^2\right] + V}_{H_0} + \underbrace{\left(\frac{1}{4M} - \frac{1}{4m}\right)\left[\boldsymbol{p}_1^2 - \boldsymbol{p}_2^2 + \boldsymbol{p}_3^2 - \boldsymbol{p}_4^2\right]}_{H_1}$$

Thus $E(H) \leq E(H_0)$. But in general, the threshold *also* benefits from this symmetry breaking, and actually benefits more, so that four-body binding deteriorates.
For instance, in atomic physics (e^+, e^+, e^-, e^-) and any equal-mass $(\mu^+, \mu^+, \mu^-, \mu^-)$ weakly bound below the atom–atom threshold, but (M^+, m^+, M^-, m^-) unstable for $M/m \gtrsim 2.2$, see Bressanini, Varga...
Then: breaking the symmetry of identical particles does not help

Breaking charge conjugation

$H(M, M, m, m)$ written as

$$H = \underbrace{\left(\frac{1}{4M} + \frac{1}{4m}\right)\left[\boldsymbol{p}_1^2 + \cdots + \boldsymbol{p}_4^2\right] + V}_{H_0} + \underbrace{\left(\frac{1}{4M} - \frac{1}{4m}\right)\left[\boldsymbol{p}_1^2 + \boldsymbol{p}_2^2 - \boldsymbol{p}_3^2 - \boldsymbol{p}_4^2\right]}_{H_1}$$

still benefits to the four-body system, $E(H) \leq E(H_0)$, but H and H_0 have the same threshold $(M^+, m^-) + (M^+, m^-)$. Hence binding improves. Indeed, H_2 more bound than Ps_2 and has even a rich spectrum of excitations.

Quark model analogs

For a central, flavour-independent, confining interaction V,

- Equal mass case $(q, q, \bar{q}, \bar{q})$ hardly bound
- Hidden-flavour case $(Q, q, \bar{Q}, \bar{q})$ even farther from binding,
- $(QQ\bar{q}\bar{q})$ with flavour = 2 bound if M/m large enough
  See Ader et al. (then at CERN), Heller et al. (Los Alamos), Zouzou et al. (Grenoble), D. Brink et al. (Oxford), Rosina et al. (Slovenia), Lipkin, Nussinov, Semay et al., Vijande et al., etc.

$(QQ\bar{q}\bar{q})$ expected at least in the limit of large or very large M/m.
As compared to the "colour-chemistry" (late 70's and early 80's), the $(QQ\bar{q}\bar{q})$ with very large M/m seems on safe grounds

- no exotic colour configuration
- for large M/m, almost pure $3 \to \bar{3}$ for (QQ) as in every baryon,
- and then $\bar{3} \times \bar{3} \times \bar{3} \to 1$ for $[(QQ) - \bar{q}\bar{q}]$ as in every antibaryon: well probed colour structure.

Early phenomenology of $(QQ\bar{q}\bar{q})$

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Early phenomenology of $(QQ\bar{q}\bar{q})$

- Very difficult 4-body problem
- Strong competition between the collective mode $(QQ\bar{q}\bar{q})$ and the splitting into two mesons, $(Q\bar{q}) + (Q\bar{q})$
- Usually $(cc\bar{n}\bar{n})$ $(n = u, d)$ found marginally unbound or bound, see Rosina (FBS, 2001)
- $(bb\bar{n}\bar{n})$, or perhaps $(bc\bar{n}\bar{n})$ usually stable
- However, questionable assumptions about confinement
- Hence: effect of a better treatment of confinement?

The additive model of tetraquark confinement-1

- Questions:
  - What is V for tetraquarks?
  - Even earlier: what is the link from mesons to baryons?
- The additive model
  By analogy with QED,

$$V(1,2,\ldots) = -\frac{3}{16}\sum_{i<j}\tilde{\lambda}_i^{(c)}.\tilde{\lambda}_j^{(c)}v(r_{ij}) \ ,$$

  - $\lambda^{(c)}$ is the non-abelian colour operator
  - $v(r)$ is the quarkonium potential fitted to mesons,
  - For baryons, this ansatz gives the "1/2 rule"

$$V = [v(r_{12}) + v(r_{23} + v(r_{31})]/2$$

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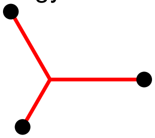
The Steiner-tree model of baryons

Y-shape potential:

- Artru, Dosch, Merkuriev, etc., proposed a better ansatz, often verified and rediscovered (strong coupling, adiabatic bag model (Kuti et al.), flux tube (Kogut et al.), lattice QCD, etc.)
- The linear $q - \bar{q}$ potential of mesons  interpreted as minimising the gluon energy in the flux tube limit
- The $q - q - q$ potential of baryons is  with the junction optimised, i.e., fulfilling the conditions of the well-known Fermat-Torricelli problem.

This potential was used for baryons (Taxil et al., Semay et al., Carlson et al.), but it does not make much difference as compared with the additive ansatz $V = (r_{12} + r_{23} + r_{31})/2$.

The Steiner tree model of tetraquarks

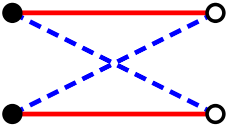
Generalisation to tetraquarks [e.g., Sugunama et al., Lattice QCD]

$$V_4 = \min(V_f, V_S)$$

combination of

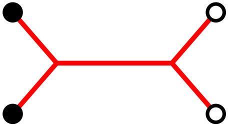
- flip-flop V_f (already used in its quadratic version by Lenz et al.)

$$V_f = \lambda \min(r_{13} + r_{24}, r_{23} + r_{14})$$



- and Steiner-tree V_S

$$V_S = \lambda \min_{k,\ell}(r_{1k} + r_{2k} + r_{k\ell} + r_{\ell 3} + r_{\ell 4}) .$$



- This QCD-inspired potential is more favourable

The Steiner tree model of tetraquarks-2

- As an illustration, we consider two variants of a purely linear potential
  1. the additive model

$$H_1 = \sum_i \frac{\boldsymbol{p}^2}{2m_i} - \frac{3}{16}\sum_{i<j} \tilde{\lambda}_i^{(c)}.\tilde{\lambda}_j^{(c)}\, r_{ij}$$

  2. The Steiner-tree model

$$H_2 = \sum_i \frac{\boldsymbol{p}^2}{2m_i} + V_4$$

- H_1 does not bind for masses (m, m, m, m) but for masses (M, M, m, m), if $M/m \gtrsim 5$
- J. Carlson and V.R. Pandharipande concluded that H_2 does not bind, but

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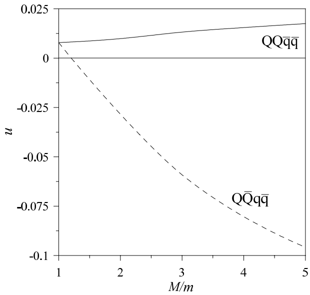
- J. Carlson and V.R. Pandharipande concluded that H_2 does not bind, but
- they used too simple trial wave functions for the 4-body problem, and did not consider unequal masses.

Tetraquarks in the minimal-path model-1

Vijande, Valcarce and R. revisited the calculation of Carlson at al. with a basis of correlated Gaussians (matrix elements painfully calculated numerically), and obtained stability for ($QQ\bar{q}\bar{q}$) even for $M/m = 1$, but better stability for $M/m \gg 1$.



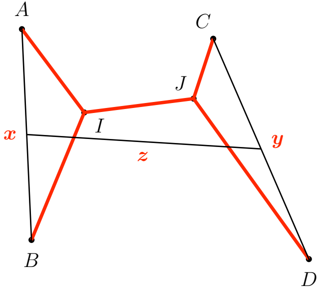
$$u = (E_{th} - E_4)/E_{th}$$

Tetraquarks in the minimal-path model-2

More recently, Cafer Ay, Hyam Rubinstein (Melbourne) and R.: rigorous proof of stability within the minimal-path model if $M \gg m$. Obviously,

$V_4 \leq V_S \leq |\boldsymbol{x}| + |\boldsymbol{y}| + |\boldsymbol{z}|$

where $\boldsymbol{x} = \overrightarrow{AB}$, $\quad \boldsymbol{y} = \overrightarrow{CD}$,
and $\boldsymbol{z}$ links the middles.



Then

$$H \leq \left[\frac{\boldsymbol{p}_x^2}{M} + |\boldsymbol{x}|\right] + \left[\frac{\boldsymbol{p}_y^2}{m} + |\boldsymbol{y}|\right] + \left[\frac{\boldsymbol{p}_z^2}{2\mu} + |\boldsymbol{z}|\right]$$

exactly solvable, but not does not demonstrate binding of $(QQ\bar{q}\bar{q})$

Better bound

- A better bound demonstrates stability for large M/m:

$$H \leq \left[\frac{\boldsymbol{p}_x^2}{M} + \frac{\sqrt{3}}{2}|\boldsymbol{x}|\right] + \left[\frac{\boldsymbol{p}_y^2}{m} + \frac{\sqrt{3}}{2}|\boldsymbol{y}|\right] + \left[\frac{\boldsymbol{p}_z^2}{2\mu} + |\boldsymbol{z}|\right]$$

- $p^2 + |\boldsymbol{x}| \quad \Longrightarrow \quad e_0 = 2.3381\ldots$ (Airy function)
- by scaling $\boldsymbol{p}^2/m + \lambda|\boldsymbol{x}| \quad \Longrightarrow \quad e_0\,\lambda^{2/3}\,m^{-1/3}$.
- Threshold $2(Q\bar{q})Q\bar{q})$ at $E_{\rm th} = 2e_0\mu^{-1/3}$, $\mu = Mm/(M+m)$.
- The tetraquark energy has a upper bound

$$E_4 \leq E_4^{\rm up} = e_0 \left\{ \left(\frac{3}{4}\right)^{1/3} \left[M^{-1/3} + m^{-1/3}\right] + (2\mu)^{-1/3} \right\}$$

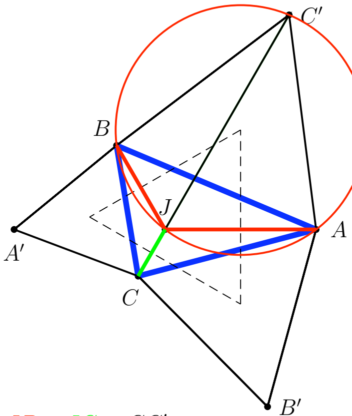
- Straightforward to check that $E_4^{\rm up} < E_{\rm th}$ for $M/m < 6403$
- Thus $\boxed{E_4 < E_{\rm th}}$ at large M/m demonstrated rigorously
- Actually $\forall\, M/m$ from solving numerically the 4-body pb.

Proof-1

A flavour of the proof. In the 3-body case, Steiner tree linked to Napoleon's theorem.

$JA + JB + JC = CC'$ where C' makes an external equilateral triangle associated to the side AB.

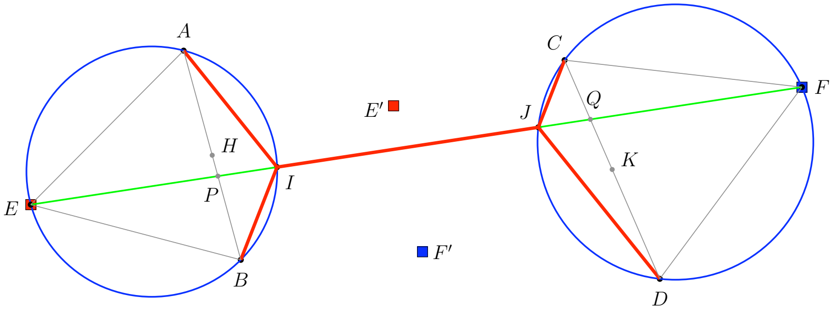
Well-known property of the Fermat-Torricelli problem. (C' belongs to the torroïdal domain associated to AB)



$JA + JB + JC = CC'$

Proof-2

The analogue for the planar tetraquark is



$$V_S = JA + JB + JK + KC + KD = EF$$

The minimal network linking (A, B, C, D) is the maximal distance beween $\{E, E'\}$ and $\{F, F'\}$, which are the torroïdal domains associated to (A, B) and (C, D) (= points completing an equilateral tr.)

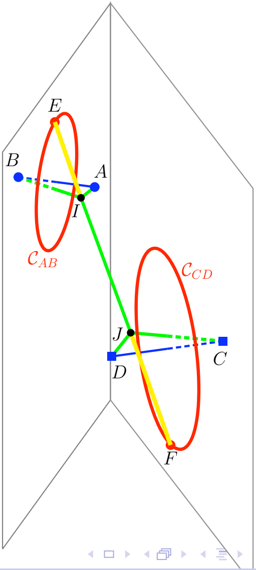
Proof-3

In space, still

$$V_S = JA + JB + JK + KC + KD = EF$$

where

- $E \in \mathcal{C}_{AB}$= torroïdal domain of quarks AB, (equilateral circle)
- $F \in \mathcal{C}_{CD}$= torroïdal domain of antiquarks CD,
- V_S is the maximal distance between the circles $\mathcal{C}_{AB}$ and $\mathcal{C}_{CD}$, which is less than the distance between the centres and the sum of radii.



Conclusions : the four-body problem

- Drastic revision of the four-body spectrum within this model
- Analogous to the Wheeler (1945) – Ore (1946) – Hyllerras & Ore (1947) views on the Ps_2 molecule.

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Binding Energy of Polyelectrons

AADNE ORE
Sloane Physics Laboratory, Yale University, New Haven, Connecticut
June 10, 1946

THE question as to the existence of groups of electrons and positrons having temporary stability has recently been raised by J. A. Wheeler,[1] who shows that clusters of

Although the evidence here presented against the stability of the polyelectron composed of two electrons and two positrons is not conclusive in a strict mathematical sense, it counsels against the assumption that clusters of this (or even of higher) complexity can be formed.

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Binding Energy of the Positronium Molecule

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Institute of Theoretical Physics, University of Oslo, Oslo, Norway
AND
AADNE ORE*
Sloane Physics Laboratory, Yale University, New Haven, Connecticut
(Received December 26, 1946)

A system of two electrons and two positrons is shown to possess dynamic stability. The variational calculation performed leads to a binding energy of at least 0.11 ev for this cluster. The approximate wave function which yields this value depends on the four electron-positron distances only. Neglect of the two distances between particles of the same kind permits an essential mathematical simplification which might be of interest in other problems.

Conclusions : exotic hadrons

- ($t^6\bar{t}^6$) probably unbound,
- Better models of confinement beyond naive additive models,
- Steiner-tree model, $\Rightarrow$ $(QQ\bar{q}\bar{q})$ bound $\forall$ M/m (numerical)
- Stability rigorously proved for large M/m
- One should further study short-range corrections, and other refinements,
- States with large M/m, e.g., $(cc\bar{q}\bar{q})$ likely to survive,
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- The hadron family already rich, but likely to welcome new members,

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- One should perhaps wait
- The positronium molecule predicted in 1945-47
- Found in 2007,
- $(QQ\bar{q}\bar{q})$ predicted in 1982
- Found in ???

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