Energy Losses in a Hot Plasma

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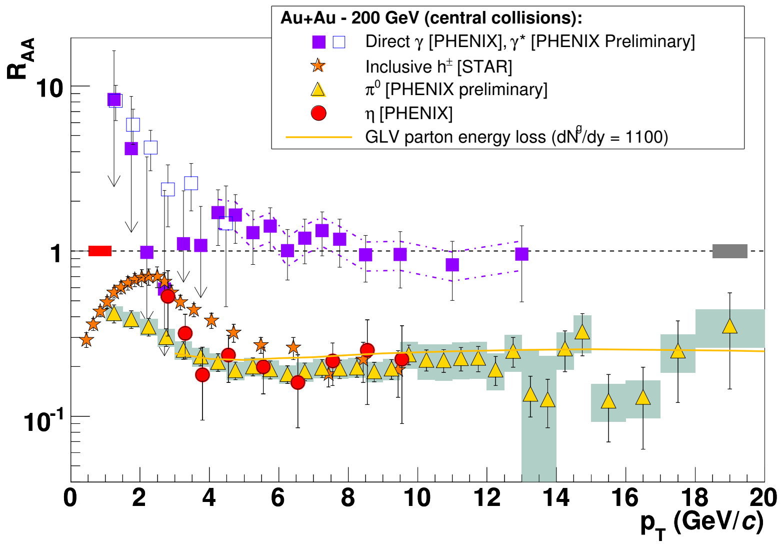
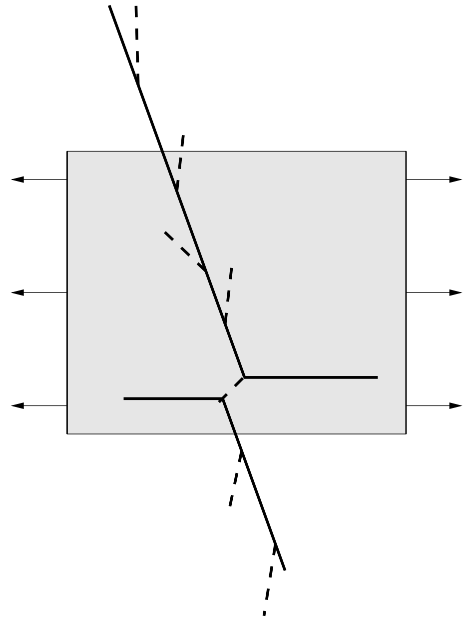
ΔE of fast particle crossing hot plasma





- Physics of ΔE_{parton} quite rich:

- transverse momentum broadening in multiple scattering

- LPM effect

- dead cone effect

- $\Delta E_{\rm parton}$ studied by many groups

Bjorken
Thoma, Gyulassy
Braaten, Thoma
Peshier
Peigné, Peshier
...

Baier et al
Zakharov
Gyulassy, Levai, Vitev
Arnold, Moore, Yaffe
Dokshitzer, Kharzeev
...

with different assumptions and methods

- complicated formalisms, but model-dependent results

$$\Delta E_{\rm rad}(q) = c_1 \alpha_s \hat{q} L^2 \qquad \hat{q} = \frac{\mu^2}{\lambda}$$
$$\Delta E_{\rm rad}(q) = c_2 \alpha_s^3 T^3 L^2 \qquad c_2 = ?$$

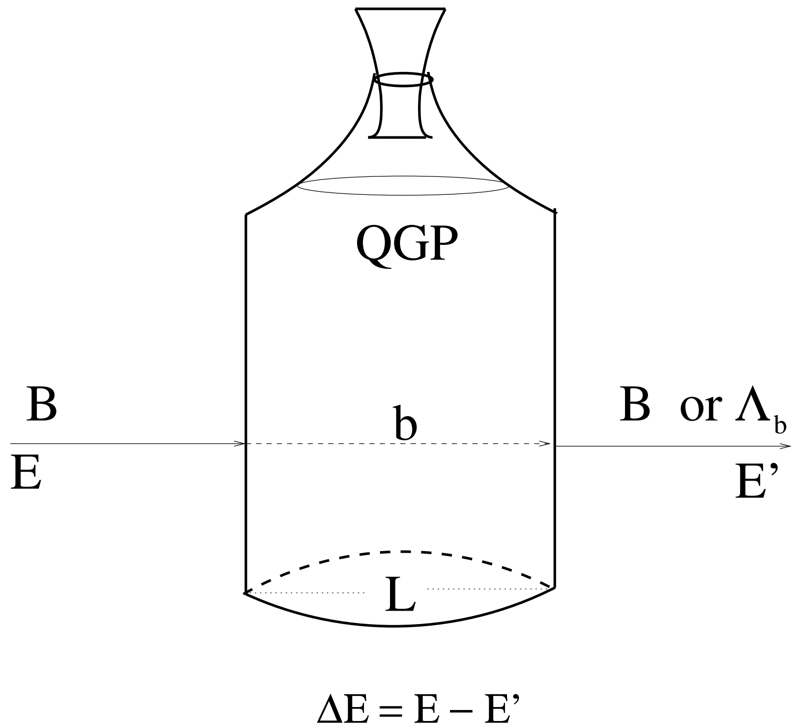
- in this context, using simple heuristic arguments to find $\Delta E(E, M, T, \alpha, L)$ may be as good

S. P., A. Smilga 0810.5702 [hep-ph]

- QED, natural situation: asymptotic particle ($t_{\text{prod}} = -\infty$) entering a plasma
- QCD: no asymptotic color charge $\Rightarrow$ $t_{\text{prod}} = 0$ more natural

Thought experiment

to measure ΔE
of "asymptotic parton"



We can study in parallel:

QED / QCD ; $M = 0$ / $M \neq 0$; $t_{\rm prod} = -\infty$ / $t_{\rm prod} = 0$

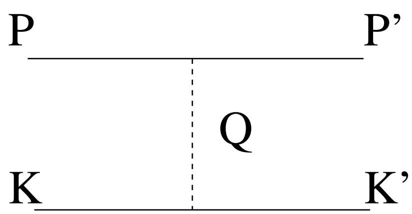
all known results (and a few new ones)
from simple arguments

OUTLINE

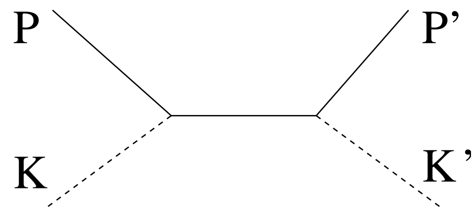
- Collisional loss
- $\Delta E_{\rm rad}(e^-, t_{\rm prod} = -\infty)$
- $\Delta E_{\rm rad}(e^-, t_{\rm prod} = 0)$
- $\Delta E_{\rm rad}(Q, t_{\rm prod} = 0)$

Collisional energy loss

Muon crossing e^+e^- plasma $\qquad E \gg M \gg T$



(a) Coulomb



(b) Compton

(a) Coulomb: $\frac{d\sigma}{dt} \sim \frac{\alpha^2}{(t-\mu^2)^2}$ (Debye mass $\mu \sim eT$)

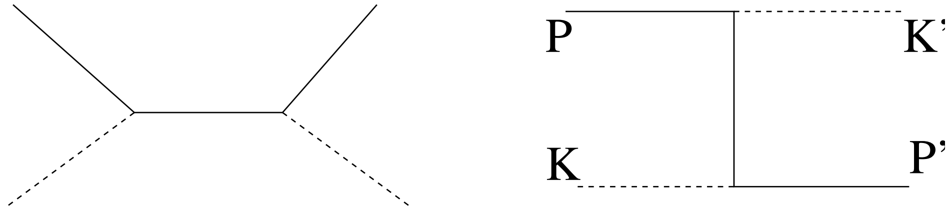
$$\frac{dE}{dx} = \frac{1}{\lambda} \cdot \langle \Delta E \rangle_{1\,\mathrm{scat.}} \sim n\sigma \cdot \frac{1}{\sigma} \int dt \, \frac{d\sigma}{dt} \, Q_0$$

$$t = -2K \cdot Q = -2(K_0 Q_0 - \vec{K} \cdot \vec{Q}) \Rightarrow Q_0 \sim |t|/K_0 \sim |t|/T$$

$$\frac{dE}{dx} \sim n \int dt \, \frac{\alpha^2}{t^2} \frac{|t|}{T} \sim \alpha^2 T^2 \int \frac{dt}{t} \sim \alpha^2 T^2 \ln \frac{ET}{\mu^2}$$

- E-dependence is smooth
- $|t| \ll |t|_{\max} \sim ET \Leftrightarrow Q_0 \ll E$ final muon is leading

(b) Compton



$$M^2 \ll |u| \ll s \sim ET \Rightarrow \frac{d\sigma}{dt} \sim \frac{\alpha^2}{su} \qquad (u\text{-channel})$$

$$\frac{dE}{dx} \sim n \int dt \, \frac{\alpha^2}{su} \frac{|t|}{T} \sim \alpha^2 T^2 \int \frac{du}{u} \sim \alpha^2 T^2 \ln \frac{ET}{M^2}$$

- $|t| \simeq |t|_{\mathrm{max}} \Leftrightarrow Q_0 \simeq E$ full stopping of final muon

(see: Compton backscattering of laser beams)

$$\left.\frac{dE}{dx}\right|_{\mathrm{coll}}^{\mu^-} \sim \alpha^2 T^2 \left[\mathrm{cst} \ln \frac{ET}{\mu^2} + \mathrm{cst}' \ln \frac{ET}{M^2} \right]$$

Electron crossing e^+e^- plasma $E \gg T \gg m \sim m_{\rm th}$

$$\left.\frac{dE}{dx}\right|_{\rm coll}^{e^-} = \left.\frac{dE}{dx}\right|_{\rm coll}^{\mu^-} (M \to m_{\rm th} \sim \mu) \text{ ???} \qquad \text{NO!}$$

in u-channel contribution, fully stopped electron cannot be distinguished from thermal electrons

$\Rightarrow$ this part of $\frac{dE}{dx}$ is not observable

For light particle, observable ΔE requires

$$E' > E/2 \quad \Rightarrow \quad \left.\frac{dE}{dx}\right|_{\rm coll}^{e^-} \sim \alpha^2 T^2 \ln \frac{ET}{\mu^2}$$

- QCD: similar discussion (except running of α_s)

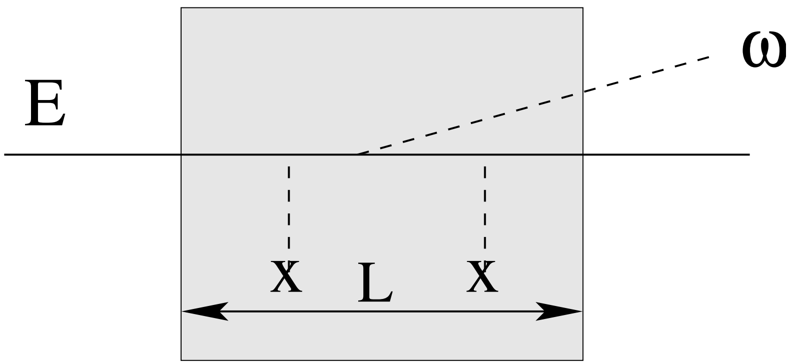
Heavy Quark (tagged): $\Delta E \equiv E'_Q - E_Q \to$ Coul. + Comp.

Light quark: $\Delta E \equiv E'_{\rm leading} - E_q \to$ Coulomb

Radiative energy loss

two cases: $\Delta E_{\rm rad}(t_{\rm prod}=-\infty)$ and $\Delta E_{\rm rad}(t_{\rm prod}=0)$

$t_{\rm prod}=-\infty$

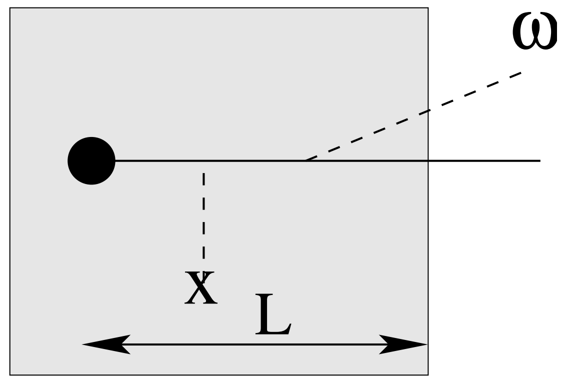


$\Delta E_{\rm rad} = \Delta E_{\rm rad}^{\rm tot}$

$\Delta E_{\rm rad}(L=0) = 0$

(on-shell particle travelling in vacuum does not radiate)

$t_{\rm prod}=0$



$\Delta E_{\rm rad}^{\rm tot}(L=0) \neq 0$

(newly created particle suffers DGLAP radiation)

$\Delta E_{\rm rad}^{\rm med} - \Delta E_{\rm rad}^{\rm vac} \equiv \Delta E_{\rm induced}$

$\Delta E_{\rm rad}$ obtained from essential quantum features:

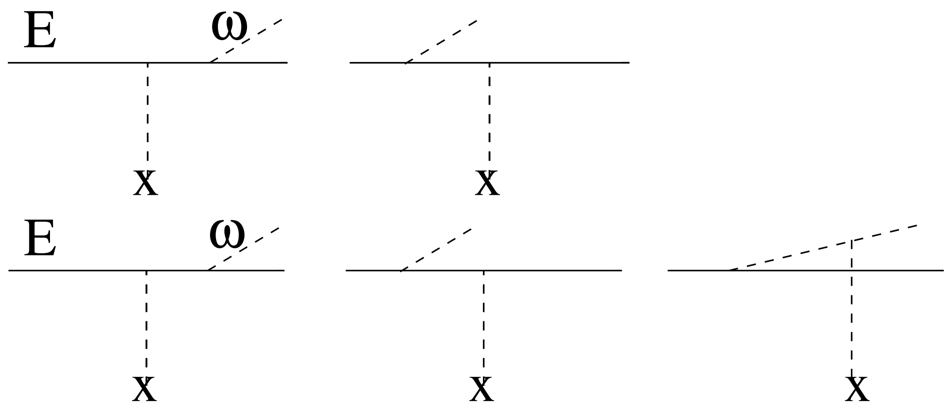
- formation time $\quad t_f(\omega,\theta) \sim \frac{1}{\omega\theta^2}$

$$\omega \frac{dI}{d\omega d\theta}\Big)_L \sim \frac{L}{t_f} \cdot \omega \frac{dI}{d\omega d\theta}\Big)_{1,\mathrm{BH}}$$

- Bethe-Heitler spectrum $\Rightarrow$ 2 radiation cones

QED

QCD



$$\theta_{\rm cone}^{\rm QED} \sim \frac{\mu_{\rm eff}}{E}$$

$$\theta_{\rm cone}^{\rm QCD} \sim \frac{\mu_{\rm eff}}{\omega}$$

$\Delta E_{\rm rad}(e^-, t_{\rm prod} = -\infty)$

- $L \ll \lambda$

$$\Delta E_{\rm rad} \sim \frac{L}{\lambda} \cdot \Delta E_{1,\rm BH} \sim \frac{L}{\lambda} \int d^2\vec{\theta}\, d\omega\, \alpha\, \frac{\theta_s^2}{\theta^2(\vec{\theta}-\vec{\theta}_s)^2} \quad \left(\theta_s = \frac{q_\perp}{E} \sim \frac{\mu}{E}\right)$$

$$\Delta E_{\rm rad} \sim \alpha E \frac{L}{\lambda} \ln \frac{\theta_s^2}{\theta_{\rm min}^2} \sim L \cdot \alpha^2 ET \gg \Delta E_{\rm coll}$$

typical formation time $\quad t_f \sim \frac{1}{\omega\theta^2} \sim \frac{1}{E\theta_s^2} \sim \frac{E}{\mu^2} \gg \lambda \gg L$

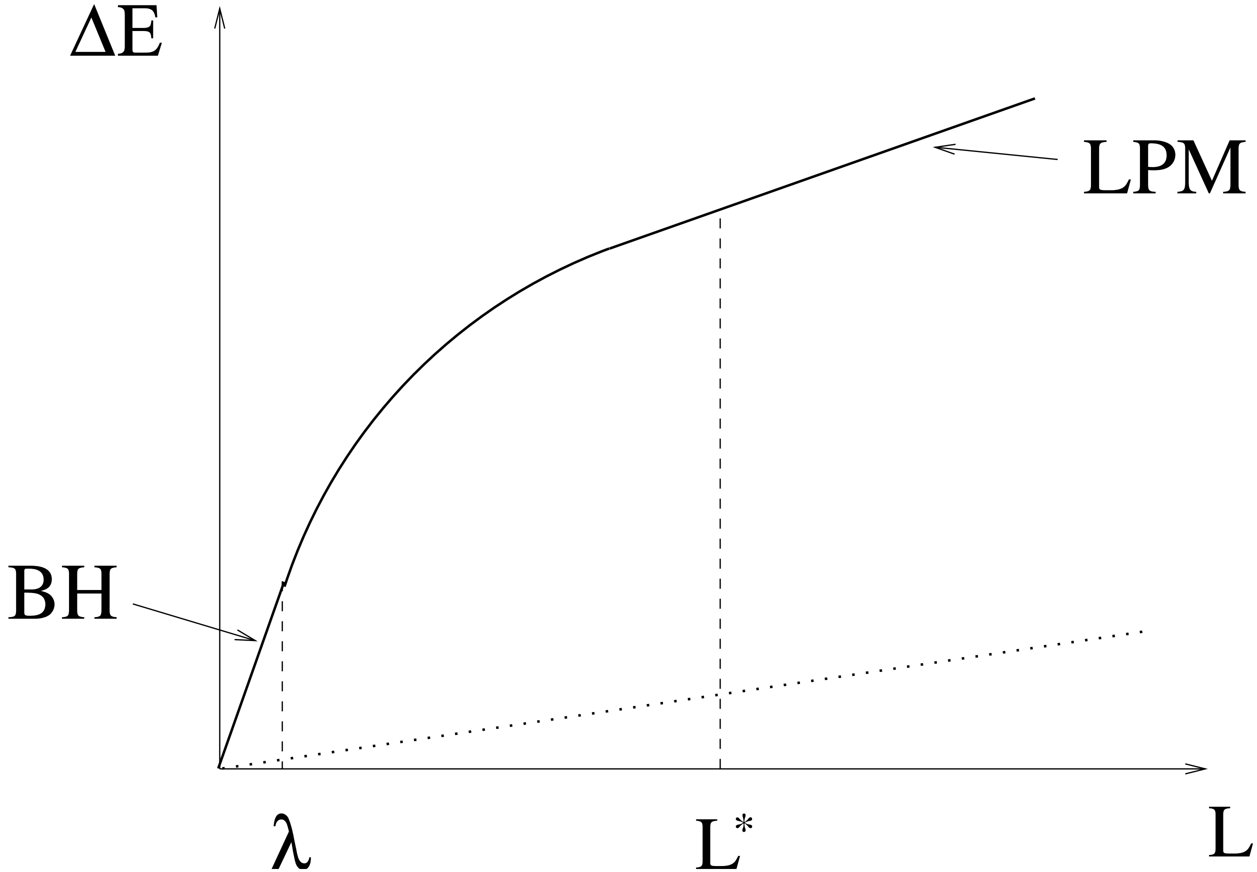
- $L \to \infty$

$$\Delta E_{\rm rad} = \frac{L}{t_f} \cdot \Delta E_{1,\rm BH} \ll \frac{L}{\lambda} \cdot \Delta E_{1,\rm BH}$$

LPM effect

$$t_f \sim \frac{E}{\mu_{\rm eff}^2} \sim \frac{E}{\mu^2 t_f/\lambda} \Rightarrow t_f \sim \sqrt{\frac{\lambda E}{\mu^2}} \equiv L^*$$

$$L \gg L^* \Rightarrow \Delta E_{\rm rad} \sim \frac{L}{L^*}\,\alpha E \sim L \cdot \alpha^2 \sqrt{ET^3} \gg \Delta E_{\rm coll}$$



ΔE of "asymptotic light parton" is similar

$\Delta E_{\rm induced}(e^-, t_{\rm prod} = 0)$

radiation associated with $t_f > L$ cancels in $\Delta E_{\rm induced}$

$$\Delta E_{\rm induced}(L) \sim \int d\theta\, d\omega\, \underbrace{\left.\omega \frac{dI}{d\omega d\theta}\right)_L}_{\rm as.\ particle} \Theta(t_f \le L)$$

- $L \ll \lambda$ recall $\Delta E(t_{\rm prod} = -\infty)$ arises from $t_f \gg L \Rightarrow$

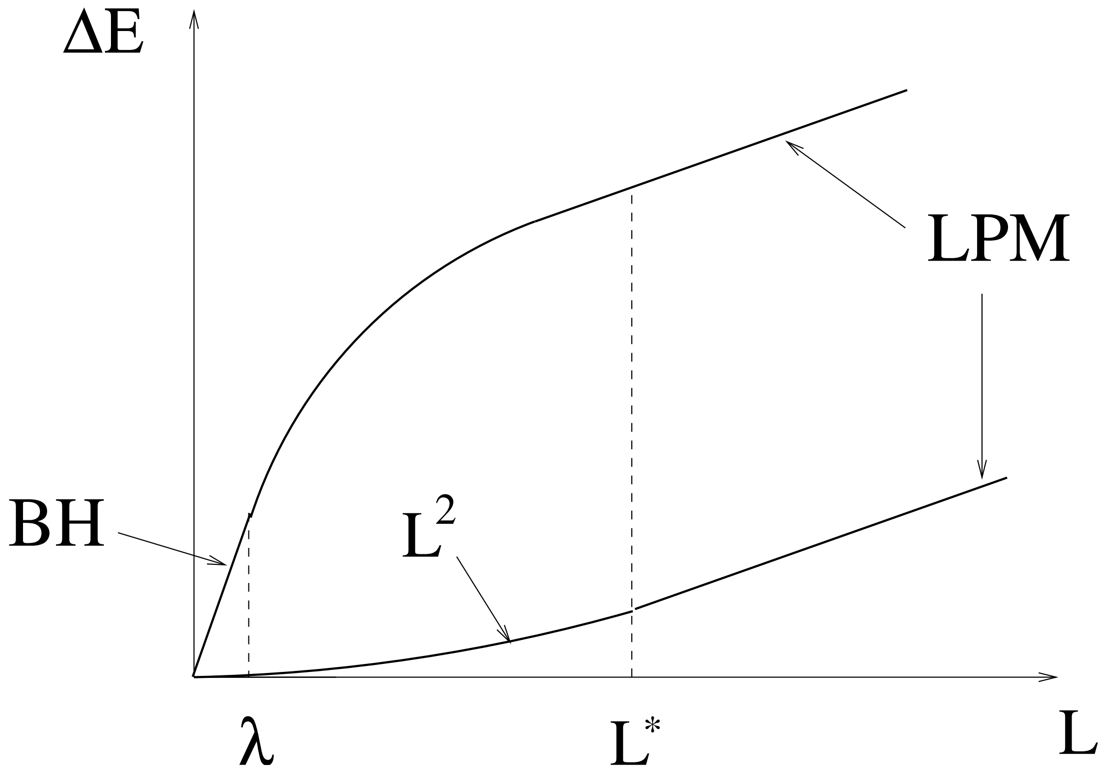
constraint $\Theta(t_f \le L)$ will strongly suppress $\Delta E_{\rm induced}$

$$\Delta E_{\rm induced} \sim \int d\theta^2\, d\omega\, \Theta(\tfrac{1}{\omega\theta^2} \le L)\, \frac{L}{\lambda}\, \alpha\, \frac{\theta_s^2}{\theta^2(\vec{\theta}-\vec{\theta}_s)^2} \sim \alpha \frac{\mu^2}{\lambda} L^2$$

$\Delta E_{\rm induced} \sim \alpha\, \hat{q} L^2$ valid up to $L \sim L^*$

- $L \gg L^*$ $\Delta E(t_{\rm prod} = -\infty)$ arises from $t_f \sim L^* \Rightarrow$

$$\Delta E_{\rm induced}(t_{\rm prod} = 0) \sim \Delta E(t_{\rm prod} = -\infty) \sim \alpha E L / L^*$$



similar result for $\Delta E_{\text{induced}}$ in QCD

news: $\Delta E_{\text{induced}}(\text{small } L) \sim L^2$ not specific to QCD

$\Delta E_{\text{induced}}(Q, t_{\text{prod}} = 0)$

mass dependence from

- $t_f \sim \frac{1}{\omega(\theta^2+\theta_M^2)}$ $\quad (\theta_M \equiv M/E)$
- mass dependence of BH spectrum

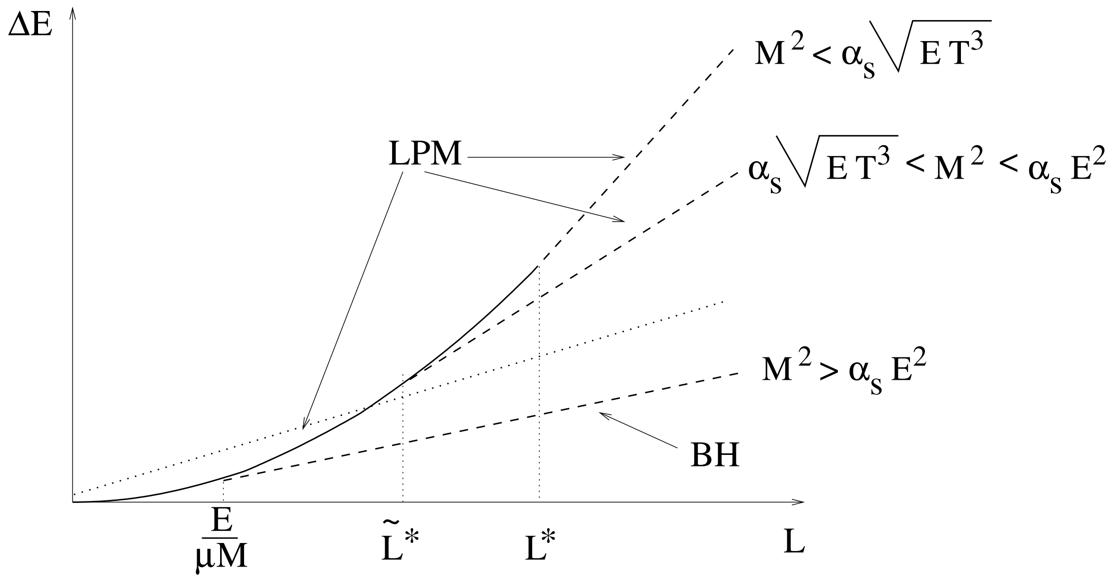
$$\omega \frac{dI}{d\omega}\Big)_{1,\text{BH}} \sim \alpha_s \ln\left(1 + \frac{\mu^2/\omega^2}{M^2/E^2}\right)$$

$$\rightarrow \omega_{\text{char}} \sim E\frac{\mu}{M} \ll E$$

$$\theta_{\text{char}} \sim \frac{M}{E} \sim \frac{\mu}{\omega_{\text{char}}}$$

typical formation time

$$t_f \sim \frac{1}{\omega_{\text{char}}\theta_{\text{char}}^2} \sim \frac{E}{\mu M}$$



$$\Delta E \sim \alpha_s\, \hat{q}\, L\, \mathrm{Min}(L, L_{\rm cr}) \qquad L_{\rm cr} = \mathrm{Min}(L^*, \tilde{L}^*, \frac{E}{\mu M})$$

$\tilde{L}^* =$??

$L \to \infty$: effectively one gluon radiated every t_f

$$t_f \sim \frac{E}{\mu_{\rm eff} M} \Rightarrow t_f^2 \sim \frac{E^2}{\mu_{\rm eff}^2 M^2} \sim \frac{E^2}{\mu^2 \frac{t_f}{\lambda} M^2} \Rightarrow t_f \sim \left(\frac{\lambda E^2}{\mu^2 M^2}\right)^{1/3} = \tilde{L}^*$$

Summary

- ΔE_{coll} : $\Delta E_{\text{coll}}(q) \neq \Delta E_{\text{coll}}(Q)|_{M \to 0}$

- ΔE_{rad} : all $\Delta E_{\text{rad}}(E, M, T, \alpha, L)$ obtained assuming

  single effective scattering on time t_f

  - QED/QCD very similar for light particles
  - $t_{\text{prod}} = 0$:

  L^2-law for $\Delta E_{\text{induced}}(\text{small } L)$ is universal