A Linear Sigma Model with Vector Mesons and Global Chiral Invariance

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Motivation

- Sakurai: Vector Meson Dominance

  - EM processes mediated by vector mesons
- Linear Sigma Model: locally symmetric (up to vector mass terms)
- Local vs. Global Symmetry: Mesonic decay widths require global symmetry
- Nature of scalar mesons $f_0(600)$ and $a_0(980)$
- $q\bar{q}$ states under 1 GeV → $\sigma \equiv f_0(600)$, $a_0 \equiv a_0(980)$
- $q\bar{q}$ states above 1 GeV → $\sigma \equiv f_0(1370)$,

  $a_0 = a_0(1450)$ – additional scalar states under 1 GeV required (tetraquarks?)

Effective Theories of QCD and Linear Sigma Model

- Description of low-energy hadrons (mesons)
- Generalisation to T, μ ≠ 0

LSM:

- Treats chiral partners on the same footing
- Parameters fixed in $SU(2)$ and used in $SU(3)$
- Vacuum calculations → calculations at T≠0
- Degeneration of chiral partners above T_C

Lagrangian of a Locally Invariant Linear Sigma Model with Vector Mesons (N_f=2) (Gasiorowicz and Geffen, 1969)

$$\mathcal{L} = \mathrm{Tr}[(D^\mu\Phi)^\dagger(D^\mu\Phi)] + m_0^2\mathrm{Tr}(\Phi^\dagger\Phi) - \lambda_1[\mathrm{Tr}(\Phi^\dagger\Phi)]^2 - \lambda_2\mathrm{Tr}(\Phi^\dagger\Phi)^2$$

$$- \frac{1}{4}\mathrm{Tr}[(L^{\mu\nu})^2 + (R^{\mu\nu})^2] + \frac{m_1^2}{2}\mathrm{Tr}[(L^\mu)^2 + (R^\mu)^2]$$

$$+ \mathrm{Tr}[H(\Phi + \Phi^\dagger)] + c\,(\det\Phi + \det\Phi^\dagger)$$

scalars

$$\Phi = (\sigma + i\eta)\, t^0 + (\vec{a}_0 + i\vec{\pi}) \cdot \vec{t}$$

pseudoscalars

vectors

$$L^\mu = (\omega^\mu - f_1^\mu)\, t^0 + (\vec{\rho}^\mu - \vec{a}_1^\mu) \cdot \vec{t}$$

$$R^\mu = (\omega^\mu + f_1^\mu)\, t^0 + (\vec{\rho}^\mu + \vec{a}_1^\mu) \cdot \vec{t}$$

axialvectors

$$D^\mu\Phi = \partial^\mu\Phi + ig(\Phi L^\mu - R^\mu\Phi)$$

$$L^{\mu\nu} = \partial^\mu L^\nu - \partial^\nu L^\mu - ig[L^\mu, L^\nu]$$

$$R^{\mu\nu} = \partial^\mu R^\nu - \partial^\nu R^\mu - ig[R^\mu, R^\nu]$$

$\mathrm{Tr}H(\Phi + \Phi^\dagger) \equiv h\,\sigma$: ESB   "+": SSB $c\,(\det\Phi + \det\Phi^\dagger)$ Chiral Anomaly

Spontaneous Symmetry Breaking (SSB):

[S. Gasiorowicz and D. A. Geffen, *Rev. Mod. Phys.* 41, 531 (1969)]
[R. Pisarski, hep-ph/9503330 (1995)]

- **Shift:**

$$\sigma \longrightarrow \sigma + \phi$$

$$\Longrightarrow -g\partial_\mu \eta \phi f_1^\mu - g\partial_\mu \vec{\pi}\phi \cdot \vec{a}_1^\mu$$

non-diagonal!

- **Shift (Diagonalise):**

$$f_1^\mu \longrightarrow f_1^\mu + w\partial^\mu \eta$$
$$\vec{a}_1^\mu \longrightarrow \vec{a}_1^\mu + w\partial^\mu \vec{\pi}$$
$$w := \frac{g\phi}{m^2 + (g\phi)^2}$$

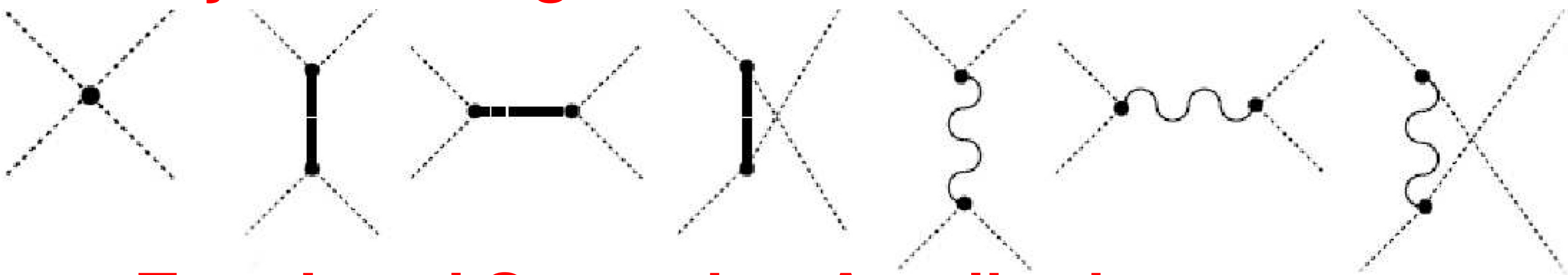
- **Renormalise Pseudoscalar Wave Functions:**

$$\frac{1}{Z}\eta \longrightarrow \eta \qquad \frac{1}{Z}\vec{\pi} \longrightarrow \vec{\pi}$$

$$Z = \frac{m_{a_1}}{m_\rho} \begin{cases} = 1.59 \ \text{ if } m_{a_1} = 1230\text{MeV}, m_\rho = 775.5\text{MeV (PDG)} \\ \overset{!}{=} \sqrt{2} \quad \text{KSFR Rule} \Longrightarrow m_{a_1} = 1097\text{MeV} \end{cases}$$

Calculation of Scattering Lengths

- **Feynman Diagrams At Tree Level:**



- **Tree-Level Scattering Amplitude**

$$\mathcal{M} = i\,\delta^{ab}\delta^{cd}\,A(s,t,u) + i\,\delta^{ac}\delta^{bd}\,A(t,s,u) + i\,\delta^{ad}\delta^{bc}A(u,t,s)$$

$$A(t,s,u):\ s \longleftrightarrow t$$

$$A(u,t,s):\ s \longleftrightarrow u$$

$\Longrightarrow$ **scattering lengths (on the threshold):**

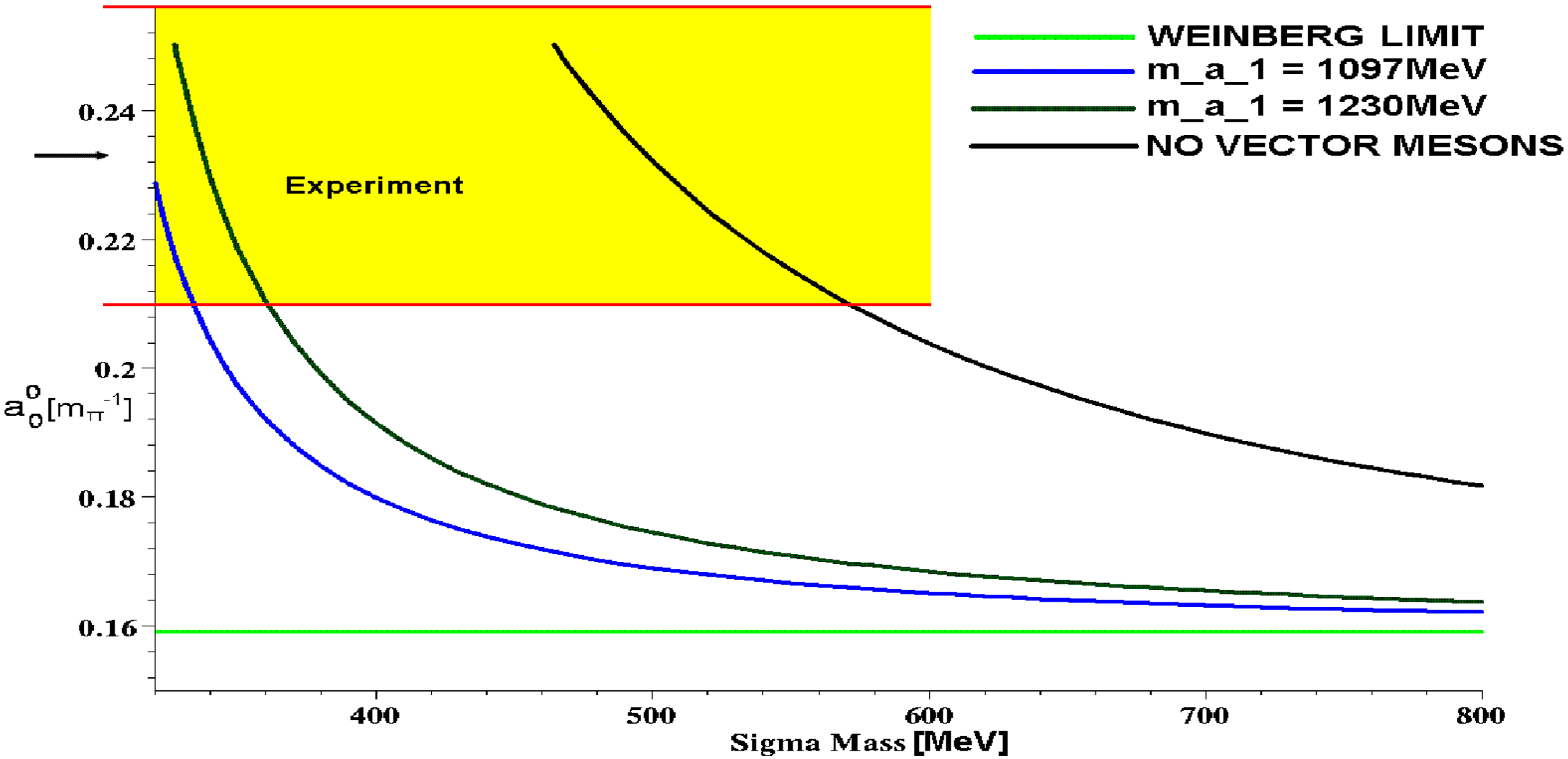
partial wave amplitude

$$\mathrm{Re}\, t_l^I(s) = q^{2l}\,[a_l^I + q^2 b_l^I + O(q^4)]$$

(l: angular momentum; q: momentum)

H. Leutwyler *et al.*, Phys. Rept. 353, 207 (2001)

Results I: a_0^0 as function of m_σ



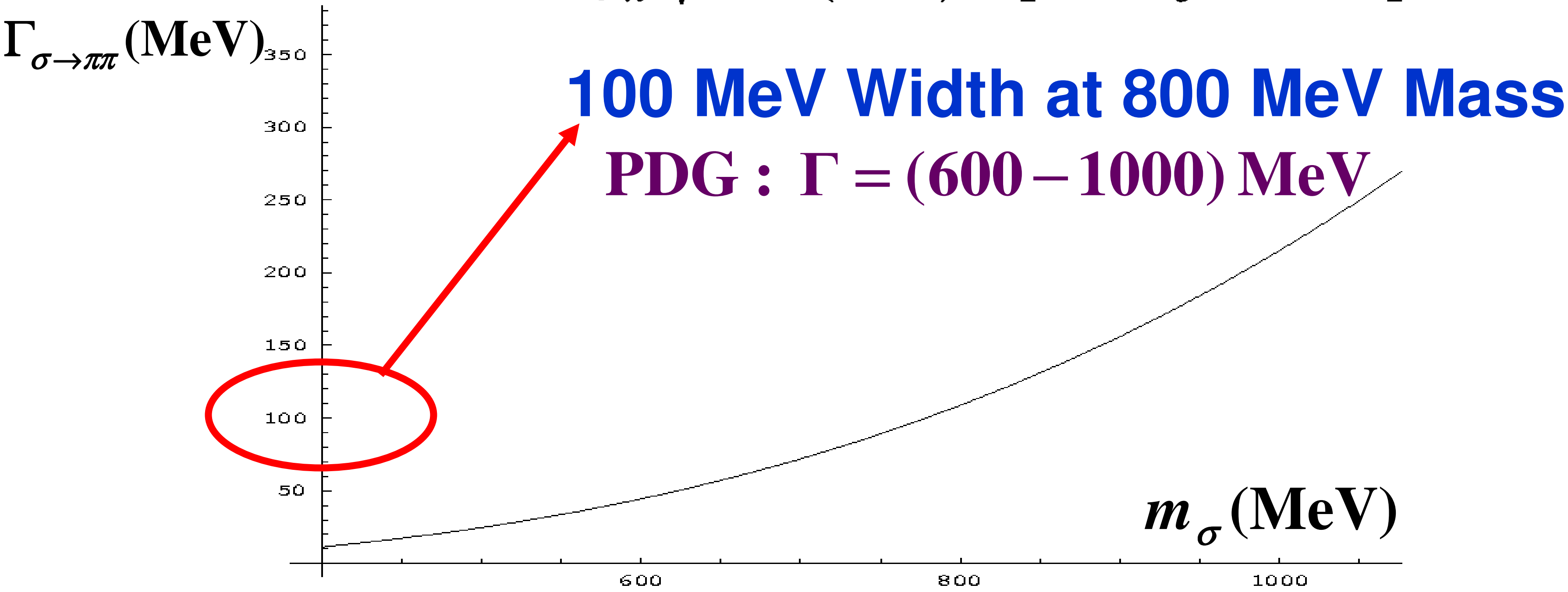
Experiment: a_0^0= 0.233 $\pm$ 0.023

J. R. Batley *et al.* [NA48/2 Collaboration], Eur. Phys. J. C 54, 411 (2008); Ke4 Data

Results II: Decay Widths

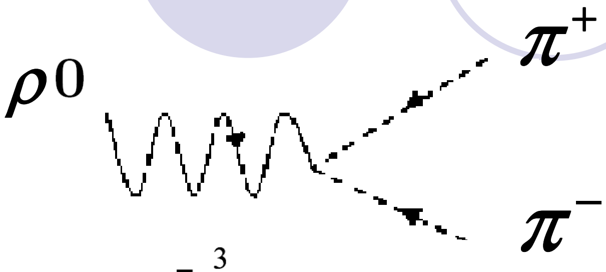
- **Sigma Decay** $\sigma \to \pi\pi$

$$\Gamma_{\sigma\to\pi\pi} = \frac{3}{32\pi}\frac{m_\sigma^3}{Z^6 f_\pi^2}\sqrt{1-\left(\frac{2m_\pi}{m_\sigma}\right)^2}\left[1+\frac{m_\pi^2}{m_\sigma^2}(Z^2-2)\right]^2$$



Results II: Decay Widths

- **Rho(770) Decay**



$$\Gamma_{\rho\to\pi\pi} = \frac{g^2}{192\pi} m_\rho \left[1 - \left(\frac{2m_\pi}{m_\rho}\right)^2\right]^{\frac{3}{2}} \left(1 + \frac{1}{Z^2}\right)^2$$

$$\Gamma_{\rho\to\pi\pi} = 86.5\text{MeV} \qquad \text{PDG}: (149.4 \pm 1.0)\text{MeV}$$

- **a_1(1260) Decay** $\quad a_1^0 \to \rho^\pm \pi^\mp$

$$\mathcal{L} = -\frac{gwZ}{2}\left[(\partial_\mu \vec{\rho}_\nu - \partial_\nu \vec{\rho}_\mu)\cdot(\partial_\mu \vec{\pi} \times \vec{a}_1^\nu + \vec{a}_1^\mu \times \partial_\nu \vec{\pi}) + (\partial_\mu \vec{a}_{1\,\nu} - \partial_\nu \vec{a}_{1\,\mu})\cdot(\vec{\rho}^\mu \times \partial_\nu \vec{\pi} + \partial_\mu \vec{\pi} \times \vec{\rho}^\nu)\right]$$
$$- g^2 \phi Z\, \vec{a}_{1_\mu} \cdot (\vec{\rho}^\mu \times \vec{\pi})$$

$$\Gamma_{a_1^0 \to \rho^\pm \pi^\mp} \approx 300\text{MeV}$$

$$\text{PDG}: (250 - 600)\text{MeV}$$

Results II: Decay Widths

- Decay Widths For Sigma And Rho(770) Do Not Match Experiments
- How To Improve Decay Widths?
- All Parameters Fixed Via Tree Level Masses and f_π $\longrightarrow$ No Free Parameters
- Need At Least One Additional Parameter To Try Adjusting Rho Width (149.4 $\pm$ 1.0) MeV

Improvements of the Lagrangian

[S. Gasiorowicz and D. A. Geffen (1969), U. G. Meissner (1988), P. Ko and S. Rudaz (1994)]

- **Strategy 1: Higher-dimension terms (local invariance)**
- **Strategy 2:**

Non-vanishing meson masses $\Longrightarrow$

$U(2)_R \times U(2)_L$ **becomes global**

QCD chiral symmetry: global $\longrightarrow$

LSM chiral symmetry global

[M. Urban, M. Buballa and J. Wambach, *Nucl. Phys. A* 697, 338 (2002)]
[D. Parganlija, F. Giacosa and D. Rischke, AIP Conf. Proc. 1030, 160 (2008)]
[D. Parganlija, F. Giacosa and D. Rischke, arXiv:0812.2183 [hep-ph]]

Globally Invariant Model

$$\mathcal{L} = \mathrm{Tr}[(D^\mu\Phi)^\dagger(D^\mu\Phi)] + m_0^2 \mathrm{Tr}(\Phi^\dagger\Phi) - \lambda_1[\mathrm{Tr}(\Phi^\dagger\Phi)]^2 - \lambda_2 \mathrm{Tr}(\Phi^\dagger\Phi)^2$$
$$- \frac{1}{4}\mathrm{Tr}[(L^{\mu\nu})^2 + (R^{\mu\nu})^2] + \frac{m_1^2}{2}\mathrm{Tr}[(L^\mu)^2 + (R^\mu)^2]$$
$$+ \mathrm{Tr}[H(\Phi + \Phi^\dagger)] + c(\det\Phi + \det\Phi^\dagger)$$
$$-2ig_2(\mathrm{Tr}\{L_{\mu\nu}[L^\mu, L^\nu]\} + \mathrm{Tr}\{R_{\mu\nu}[R^\mu, R^\nu]\})$$
$$-2g_3\{\mathrm{Tr}[(\partial_\mu L_\nu + \partial_\nu L_\mu)\{L^\mu, L^\nu\}] + \mathrm{Tr}[(\partial_\mu R_\nu + \partial_\nu R_\mu)\{R^\mu, R^\nu\}]\}$$
$$+\frac{h_1}{2}\mathrm{Tr}(\Phi^\dagger\Phi)\mathrm{Tr}[(L^\mu)^2 + (R^\mu)^2] + h_2 \mathrm{Tr}[(\Phi R^\mu)^2 + (L^\mu\Phi)^2]$$
$$+2h_3 \mathrm{Tr}(\Phi R_\mu \Phi^\dagger L^\mu) + \mathcal{L}_4$$

Z, m_σ

$$L^{\mu\nu} = \partial^\mu L^\nu - \partial^\nu L^\mu - (ieA^\mu[t^3, R^\nu] - ieA^\nu[t^3, R^\mu])$$
$$R^{\mu\nu} = \partial^\mu R^\nu - \partial^\nu R^\mu - (ieA^\mu[t^3, L^\nu] - ieA^\nu[t^3, L^\mu])$$
$$D^\mu\Phi = \partial^\mu\Phi + ig_1(\Phi L^\mu - R^\mu\Phi) - ieA^\mu t^3\Phi$$

→ photon

Parameter Determination

- **Masses:**

$$m_\pi, m_\eta, m_\sigma, m_{a_0}, m_\rho, m_{a_1}$$

- **Decays:**

$$\rho \to \pi\pi [Z, g_2] \quad f_1 \to a_0 \pi [Z, h_2] \quad a_1 \to \pi\gamma [Z]$$

$$a_0 \to \eta_N \pi [Z, h_2] \quad \sigma \to \pi\pi \; [Z, h_1, h_2]$$

$$a_1 \to \sigma\pi \; [Z, h_1, h_2] \quad a_1 \to \rho\pi \; [Z, g_2]$$

- **Scattering Lengths:** $Z, h_1, h_2, g_2, m_\sigma$

$$a_0^0(Z, m_\sigma, h_1, h_2), a_0^2(Z, m_\sigma, h_1, h_2)$$

Parameter Determination

- χ^2 **Method**

[D. V. Bugg *et al.*, Phys. Rev. D 50, 4412 (1994)]

$$\chi^2(Z,h_1,h_2,g_2,m_\sigma) := \sum_{\text{all decays}} \left(\frac{\Gamma^{(\text{th.})} - \Gamma^{(\text{exp.})}}{\Delta\Gamma^{(\text{exp.})}} \right)^2 + \left(\frac{A^{(\text{th.})}_{a_0 \to \eta_N \pi} - A^{(\text{exp.})}_{a_0 \to \eta_N \pi}}{\Delta A^{(\text{exp.})}_{a_0 \to \eta_N \pi}} \right)^2$$

- **Find global minimum of χ^2**
- **Look for $\chi^2 < 1$ per D.O.F. at minimum**
- **Parameter errors: diagonal elements of $\sqrt{\left(\frac{\partial^2}{\partial p_i \partial p_j}\chi^2\right)^{-1}}$ at global minimum for all parameters $p_{i,j} \in \{Z, h_1, h_2, g_2, m_\sigma\}$**

Results III

$\chi^2 = 0.7525$ per D.O.F.

Parameter / Observable	Our Value	Experiment
Z	1.5217	-
g_2	0.3365	-
h_1	- 100.7	-
h_2	106.0	-
m_σ	(330 ± 220) MeV	(400 – 1200) MeV
$\Gamma_{\rho\to\pi\pi}$	149.4 MeV	(149.4 ± 1.0) MeV
$\Gamma_{a_1\to\pi\gamma}$	0.473 MeV	(0.640 ± 0.246) MeV
$a_0^0[m_\pi^{-1}]$	0.233	0.233 ± 0.023
$a_0^2[m_\pi^{-1}]$	- 0.0454	- 0.0471 ± 0.015
$\Gamma_{f_1\to a_0\pi}$	9.356 MeV	(8.748 ± 2.097) MeV
$A_{a_0\to\eta\pi}$	4389 MeV	(4116 ± 185) MeV
$\Gamma_{\sigma\to\pi\pi}$	<10 MeV ± 84 MeV	(500 – 1200) MeV
$\Gamma_{a_1\to\sigma\pi}$	90 MeV	(250 – 600) MeV (total)
$\Gamma_{a_1\to\rho\pi}$	1413 MeV	(250 – 600) MeV (total)
m_1	758 MeV	-

$$m_\rho^2 = m_1^2 + \frac{\phi^2}{2}(h_1 + h_2 + h_3) = 775.49 \text{ MeV}$$

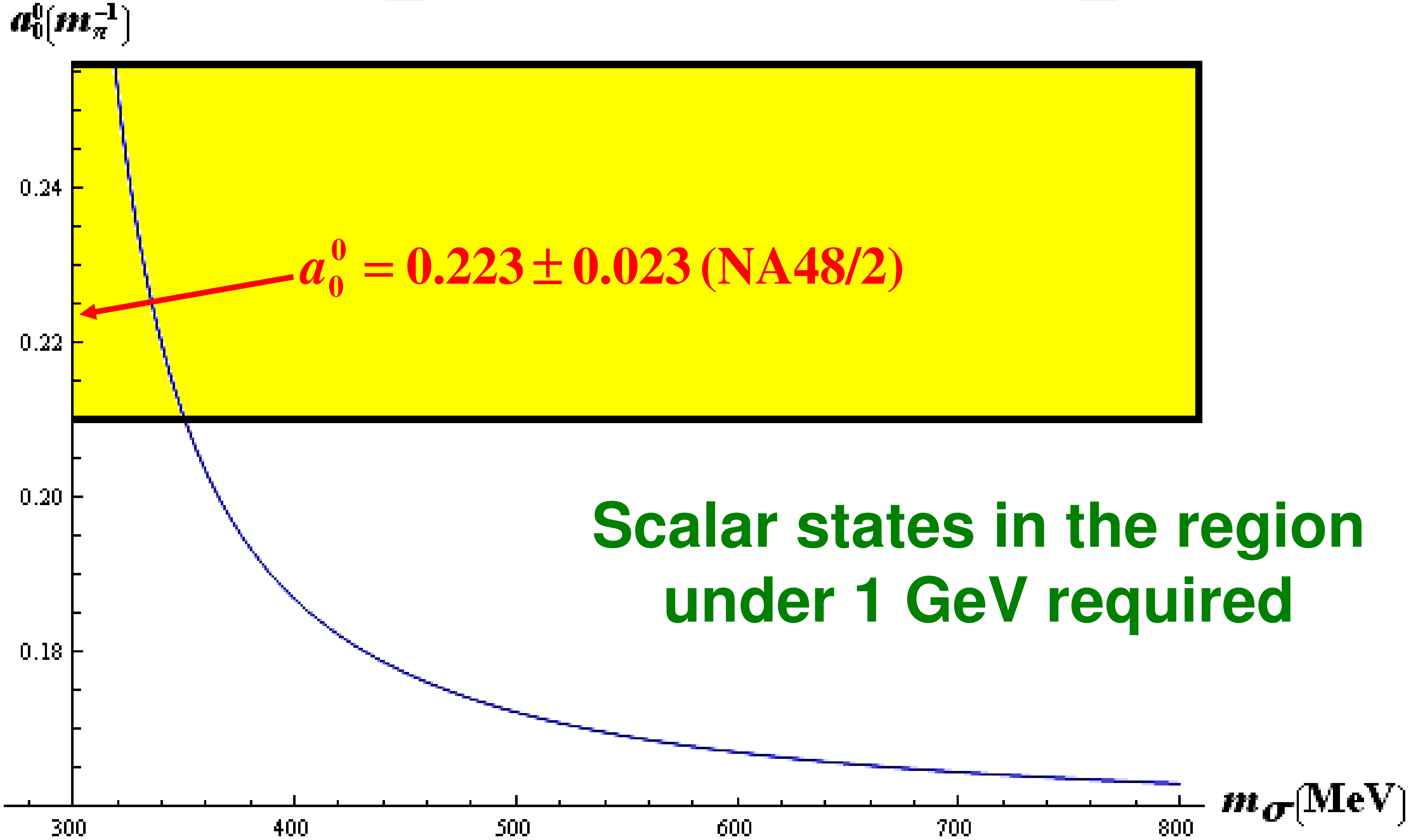
$$m_1 = 0$$

Results III: $m_1 = 0$ Limit

$\chi^2 = 0.7499$ per D.O.F.

Parameter / Observable	Our Value	Experiment
Z	1.5217	-
g_2	0.3365	-
h_1	- 45.2	-
h_2	106	-
m_σ	390 MeV	(400 – 1200) MeV
$\Gamma_{\rho\to\pi\pi}$	149.4 MeV	(149.4 ± 1.0) MeV
$\Gamma_{a_1\to\pi\gamma}$	0.473 MeV	(0.640 ± 0.246) MeV
$a_0^0[m_\pi^{-1}]$	0.233	0.233 ± 0.023
$a_0^2[m_\pi^{-1}]$	- 0.0470	- 0.0471 ± 0.015
$\Gamma_{f_1\to a_0\pi}$	9.336 MeV	(8.748 ± 2.097) MeV
$A_{a_0\to\eta\pi}$	4388 MeV	(4116 ± 185) MeV
$\Gamma_{\sigma\to\pi\pi}$	(45.7 ± 69.5) MeV	(500 – 1200) MeV
$\Gamma_{a_1\to\sigma\pi}$	≈ 0 MeV	(250 – 600) MeV (total)
$\Gamma_{a_1\to\rho\pi}$	1413 MeV	(250 – 600) MeV (total)

Sigma-mass dependence of a_0^0 - globally invariant case



Scalar states in the region under 1 GeV required

Summary

- LSM with global $U(2)_R \times U(2)_L$ invariance: scattering lengths, low-energy meson decay widths
- General phenomenology in agreement with experiment ($\rho \to \pi\pi$, $a_1 \to \pi\gamma$, $f_1 \to a_0\pi$, $a_0 \to \eta\pi$ decay, $\pi\pi$ scattering lengths)
- Two-pion sigma decay width fails to match experiment → quarkonium assignment for $\sigma \equiv f_0(600)$, $a_0 \equiv a_0(980)$ excluded
- Possible solution: interpret $f_0(1370)$ and $a_0(1450)$ as (dominantly) quarkonia
- Scalar states under 1 GeV required for correct description of pion-pion scattering lengths

Outlook

- **Parameters Errors and Consequences of Global Invariance up to 4th Order**
- **Low Energy Constants of QCD**
- ***p, d* Wave Scattering Lengths**
- **Chiral Models With Three Flavours**
- **Include Tensor, Pseudotensor Mesons, Baryons (*Nucleons*)**
- **Extension to Non-Zero Temperature: Study Chiral Symmetry Restoration**

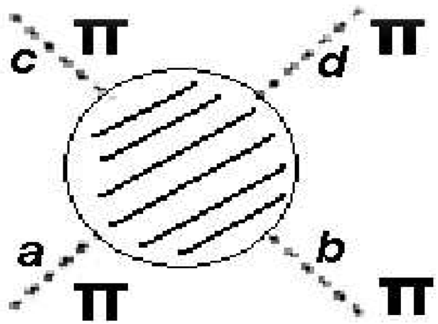
Spare Slides

Relevant Vertices for Pion-Pion Scattering

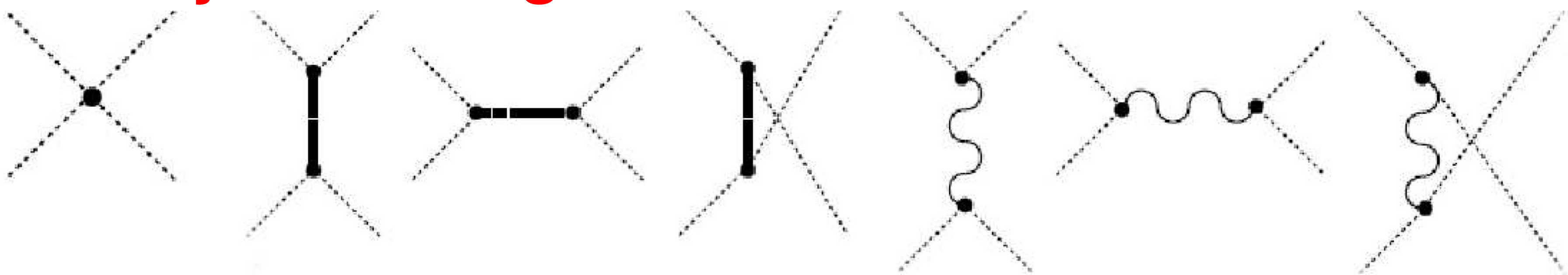
$$\mathcal{L}_{\pi/\pi} = -\frac{1}{4}(\lambda_1 + \frac{\lambda}{2})Z^4 \vec{\pi}^4 + \frac{1}{2} g^2 w^2 Z^4 (\partial^\mu \vec{\pi} \cdot \vec{\pi})^2 - \frac{1}{4} g^2 w^4 Z^4 [(\partial^\mu \vec{\pi}) \times (\partial^\nu \vec{\pi})]^2$$

$$\mathcal{L}_{\pi/\sigma} = -gwZ^2 \sigma \, \Box \, \vec{\pi} \cdot \vec{\pi} - g(1+Z^2) w\sigma (\partial^\mu \vec{\pi})^2 - (\lambda_1 + \frac{\lambda}{2}) \phi Z^2 \sigma \vec{\pi}^2$$

$$\mathcal{L}_{\pi/\rho} = -gw^2 Z^2 \partial_\mu \vec{\rho}_\nu \cdot [(\partial^\mu \vec{\pi}) \times (\partial^\nu \vec{\pi})] + g \, \partial_\mu \vec{\pi} \cdot (\vec{\rho}^\mu \times \vec{\pi})$$



- **Feynman Diagrams At Tree Level:**



Tree-Level Scattering Amplitude

$$\mathcal{M} = i\,\delta^{ab}\delta^{cd}\,A(s,t,u) + i\,\delta^{ac}\delta^{bd}\,A(t,s,u) + i\,\delta^{ad}\delta^{bc}A(u,t,s)$$

$$A(t,s,u):\ s \longleftrightarrow t$$

$$A(u,t,s):\ s \longleftrightarrow u$$

$\Rightarrow$ **s-channel isospin *I* components of the scattering amplitude:**

$$T^0(s,t) = 3A(s,t,u) + A(t,s,u) + A(u,t,s)$$

$$T^1(s,t) = A(t,s,u) - A(u,t,s)$$

$$T^2(s,t) = A(t,s,u) + A(u,t,s)$$

B. Ananthanarayan, G. Colangelo, J. Gasser and H. Leutwyler, arXiv:hepph/0005297 v1, 2000

partial wave amplitude

$\Rightarrow$ **scattering lengths (on the threshold):**

$$\mathrm{Re}\, t_l^I(s) = q^{2l}\left[a_l^I + q^2 b_l^I + O(q^4)\right]$$

(l: angular momentum; q: momentum)

Results I

Tree-level scattering lengths on the threshold ($s = 4m_\pi^2$):

$$a_0^0 = \frac{m_\pi^2}{32\pi f_\pi^2}\left[7 + \frac{2}{Z^2}\frac{m_\pi^2}{m_\sigma^2}\left(1-\frac{2}{Z^2}\right)^2 + \frac{3}{Z^2}\frac{m_\pi^2}{m_\sigma^2 - 4m_\pi^2}\left(1+\frac{2}{Z^2}\right)^2\right]; \quad Z = \frac{m_{a_1}}{m_\rho}$$

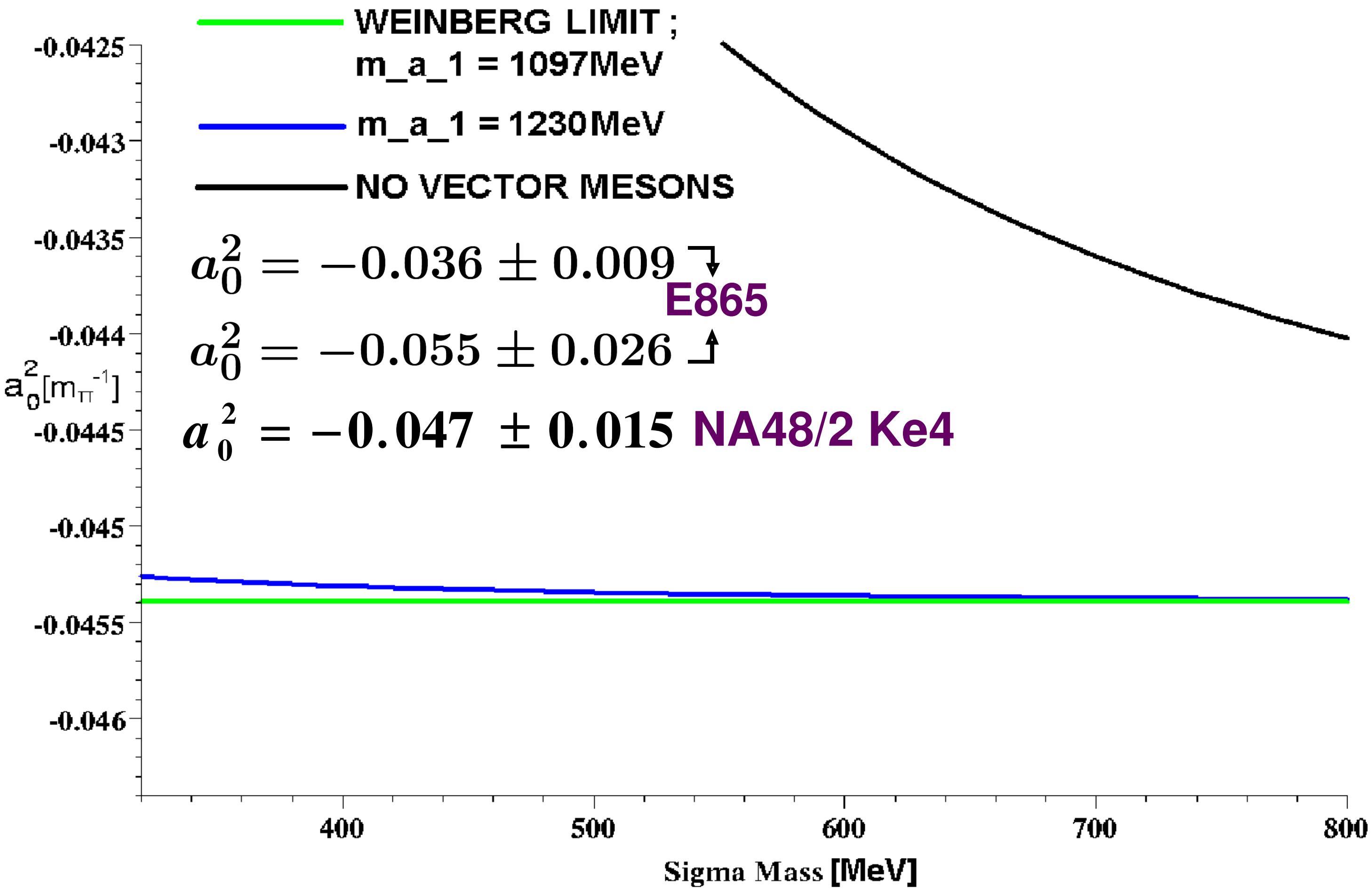
Weinberg Limit $m_\sigma \to \infty \; : \; a_0^0 = \frac{7m_\pi^2}{32\pi f_\pi^2}$ Chiral Limit $m_\pi \to 0 \; : \; a_0^0 = 0$

$$a_0^1 \equiv 0$$

$$a_0^2 = -\frac{m_\pi^2}{16\pi f_\pi^2}\left[1 - \frac{1}{Z^2}\frac{m_\pi^2}{m_\sigma^2}\left(1-\frac{2}{Z^2}\right)^2\right]$$

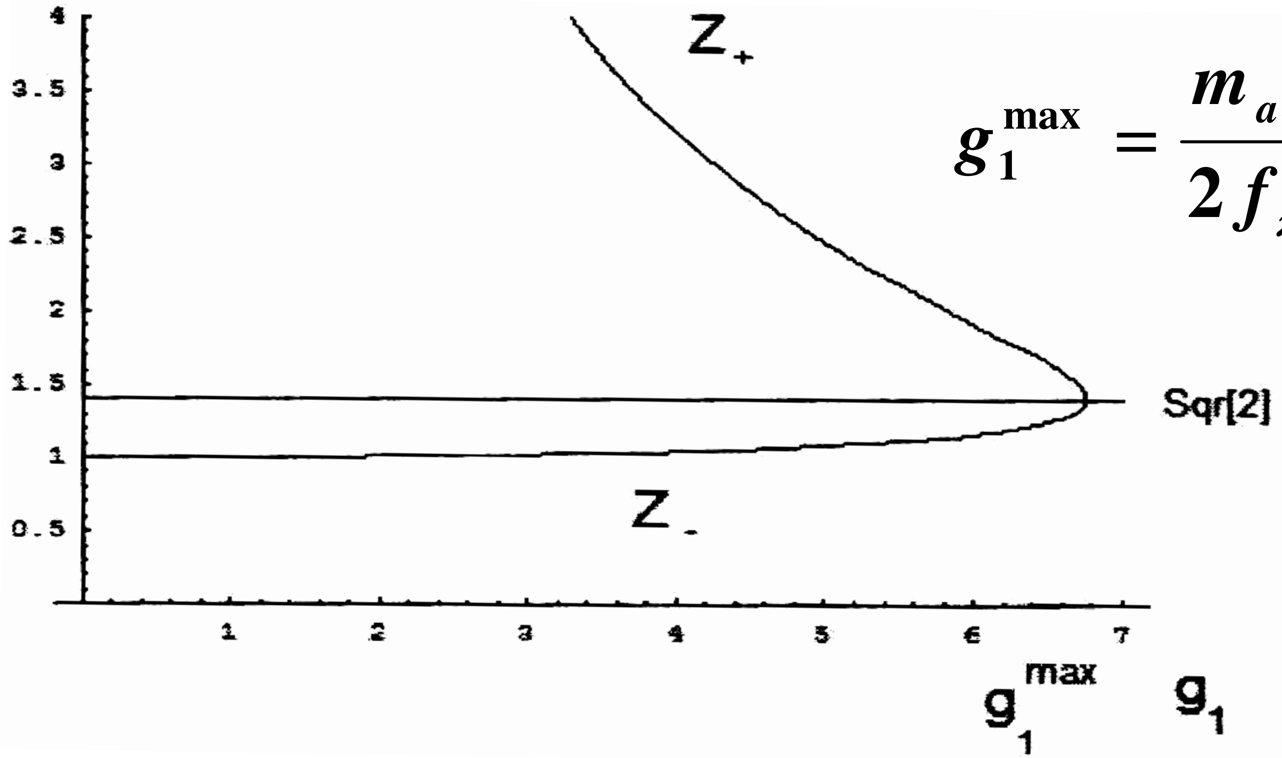
Weinberg Limit $m_\sigma \to \infty \; : \; a_0^2 = -\frac{m_\pi^2}{16\pi f_\pi^2}$ Chiral Limit $m_\pi \to 0 \; : \; a_0^2 = 0$

Results I: a_0^2 as function of m_σ



Renormalisation Constant Z

$$Z^2 = \frac{m_{a_1}^2}{m_\rho^2} = \frac{m_{a_1}^2}{m_{a_1}^2 - g_1^2 Z^2 f_\pi^2} \Rightarrow Z_\pm = \sqrt{\frac{m_{a_1}^2 \pm \sqrt{m_{a_1}^4 - 4 g_1^2 f_\pi^2 m_{a_1}^2}}{2 g_1^2 f_\pi^2}}$$



$$g_1^{\max} = \frac{m_{a_1}}{2 f_\pi} \approx 6.656$$

$\Gamma_{\sigma\rightarrow\pi\pi}$ as Function of m_σ – globally invariant model [f_0(600): quarkonium]