Probing m_q from χSB insensitivity high in the spectrum

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Outline

1. Spontaneous chiral symmetry breaking
2. Example: model of Coulomb-QCD
3. Chiral doubling and quartets in the baryon spectrum
4. Sensitivity to m_q

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Chiral symmetry

On the classical fields

$$\psi \longrightarrow e^{i\alpha\gamma_5}\psi$$

Non-invariant terms

$$m\bar{\psi}\psi \qquad gA^{a\mu}\bar{\psi}\partial_\mu\psi$$

Chiral symmetry

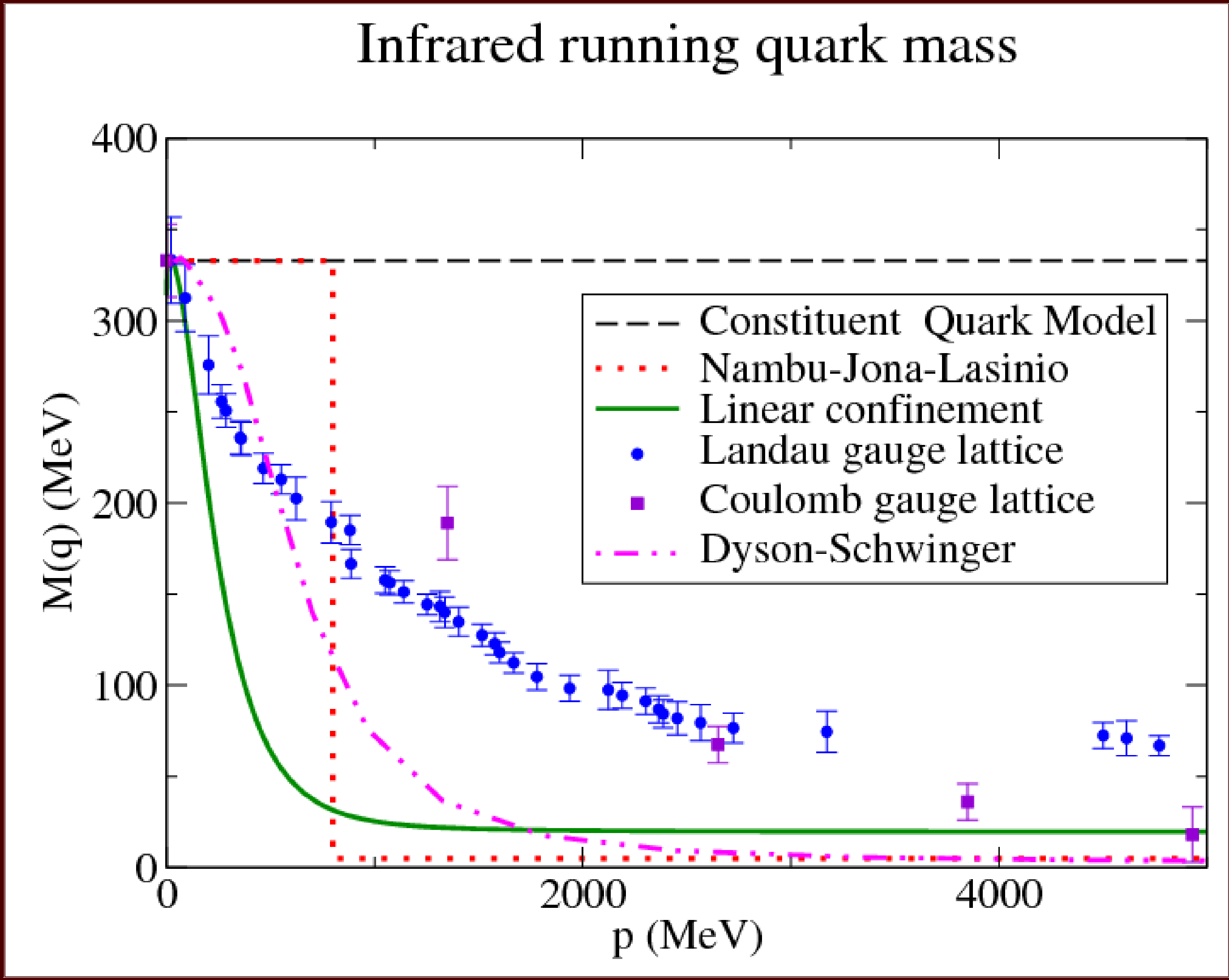
If they are absent, conserved chiral charge

$$Q_5^a = \int d\mathbf{x} \psi^\dagger(x)\gamma_5 \frac{\tau^a}{2}\psi(x)$$

Quantizing,

$$[Q_5^a, H] = 0$$

Spontaneously generated quark mass



How to distinguish the various running patterns experimentally?

BCS angle and spinors

$$\sin\phi(k) \equiv \frac{M(k)}{\sqrt{M(k)^2 + k^2}}$$

$$U_{\kappa\lambda} = \frac{1}{\sqrt{2}} \begin{bmatrix} \sqrt{1+\sin\phi_\kappa}\chi_\lambda \\ \sqrt{1-\sin\phi_\kappa}\vec{\sigma}\cdot\hat{\kappa}\chi_\lambda \end{bmatrix}$$

$$V_{-\kappa\lambda} = \frac{1}{\sqrt{2}} \begin{bmatrix} -\sqrt{1-\sin\phi_\kappa}\vec{\sigma}\cdot\hat{\kappa} i\sigma_2\chi_\lambda \\ \sqrt{1+\sin\phi_\kappa} i\sigma_2\chi_\lambda \end{bmatrix}$$

Goldstone bosons

$$\psi(x) = \int \frac{d\mathbf{k}}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{x}} \sum_{\lambda i} (B_{k\lambda i} U_{k\lambda i} + D^{\dagger}_{-k\lambda i} V_{-k\lambda})$$

$$Q_5^a = \int d\mathbf{x} \psi^{\dagger}(x) \gamma_5 \frac{\tau^a}{2} \psi(x)$$

$$Q_a^5 |\Omega\rangle_{BCS} = \int \frac{d^3k}{(2\pi)^3} \sum_{\lambda\lambda' ff' c} \left(\frac{\tau^a}{2}\right)_{ff'}$$

$$\sin\phi(k)(i\sigma_2)_{\lambda\lambda'} B^{\dagger}_{k\lambda fc} D^{\dagger}_{\lambda' f' c} |\Omega\rangle_{BCS}$$

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QCD in Coulomb gauge

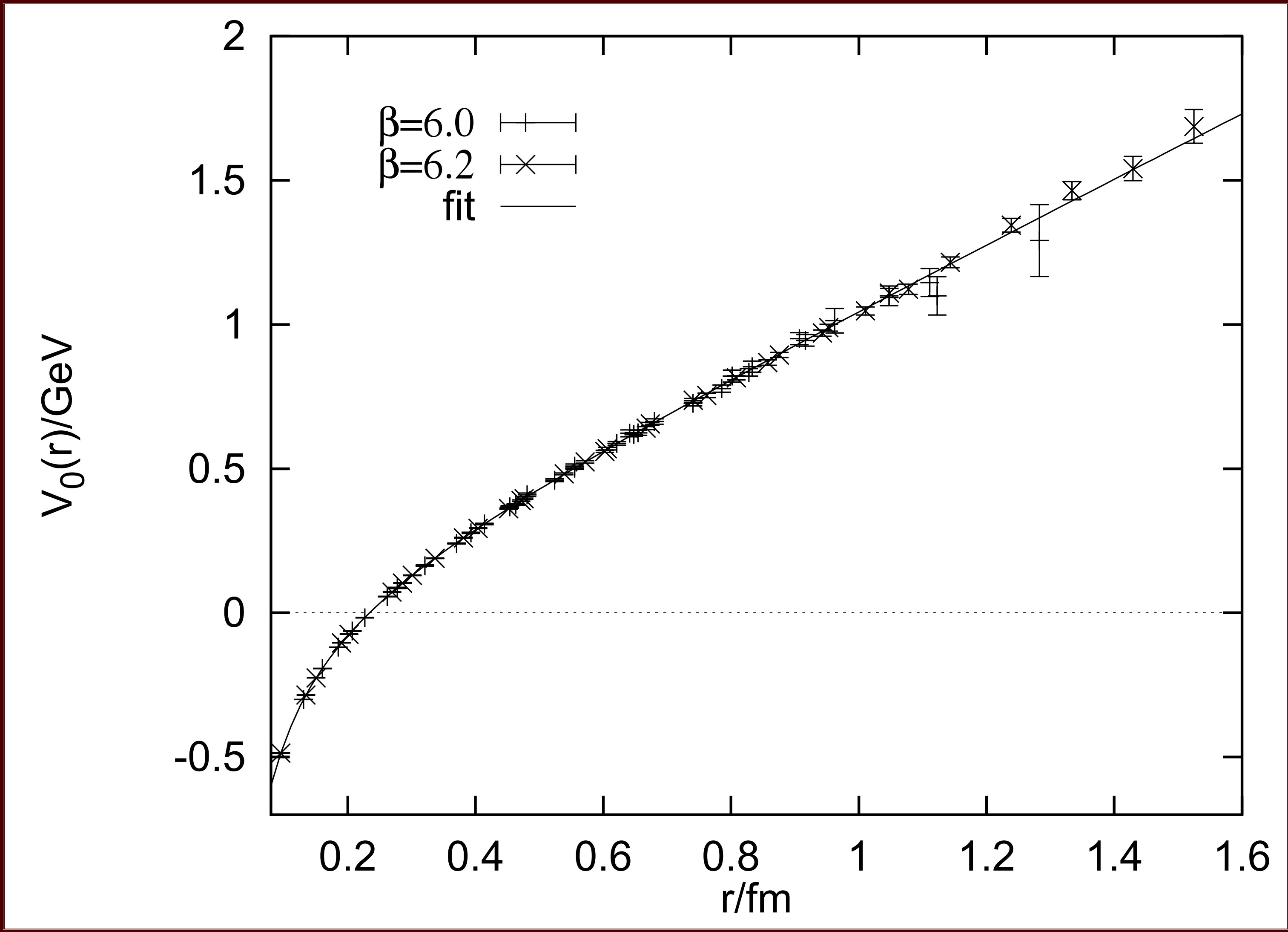
$$H = \int \left[\frac{1}{2} \mathcal{J}^{-1} \mathbf{E}_a^{tr} \mathcal{J} \mathbf{E}^{a\, tr} + \frac{1}{2} \mathbf{B}^a \mathbf{B}_a \right] + \int \overline{\Psi} [\gamma \cdot (\nabla - i g_0 T^a \mathbf{A}_a) + m] \Psi d\mathbf{x}$$

$$+ \frac{1}{2} g_0^2 \int \int \mathcal{J}^{-1} \rho^a(\mathbf{x}) v(|\mathbf{x} - \mathbf{y}|)_{aa'} \mathcal{J} \rho^{a'}(\mathbf{y}) d\mathbf{x} d\mathbf{y}$$

Color density and current:

$$\rho^a(\mathbf{x}) = \Psi^\dagger(\mathbf{x}) T^a \Psi(\mathbf{x}) + f^{abc} \mathbf{A}^b(\mathbf{x}) \cdot \mathbf{\Pi}^c(\mathbf{x})$$
$$\mathbf{J}^a = \Psi^\dagger(\mathbf{x}) \alpha T^a \Psi(\mathbf{x})$$

Lattice static potential



Bali and Schilling, PRD1997

Coulomb gauge model

Upgrade of the Cornell potential model to field theory

$$
\begin{aligned}
H = & -g_s \int d\mathbf{x} \Psi^\dagger(x) \alpha \cdot \mathbf{A}(x) \Psi(x) \\
& + Tr \int d\mathbf{x} (\mathbf{E} \cdot \mathbf{E} + \mathbf{B} \cdot \mathbf{B}) \\
+ & \int d\mathbf{x} \Psi_q^\dagger(\mathbf{x}) (-i\alpha \cdot \nabla + \beta m_q) \Psi_q(\mathbf{x}) \\
& - \frac{1}{2} \int d\mathbf{x} d\mathbf{y} \rho^a(\mathbf{x}) V_L(|\mathbf{x} - \mathbf{y}|) \rho^a(\mathbf{y})
\end{aligned}
$$

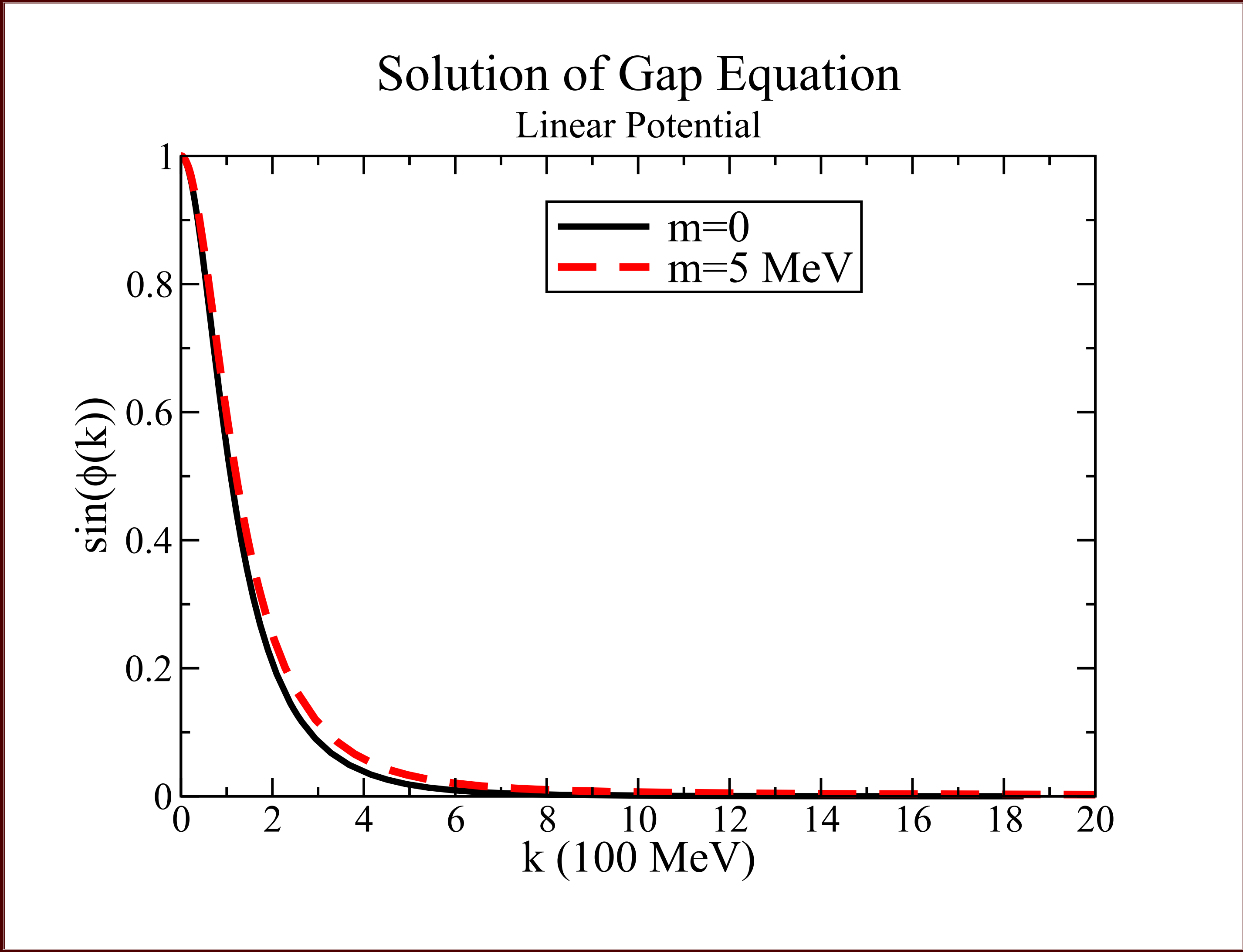
BCS gap equation

$$|\Omega\rangle = \exp\left(-\sum_{\lambda i}\int \frac{d\mathbf{k}}{(2\pi)^3}\lambda \tan\frac{\theta_k}{2} b^\dagger_{\mathbf{k}\lambda i} d^\dagger_{-\mathbf{k}\lambda i}\right)|0\rangle$$

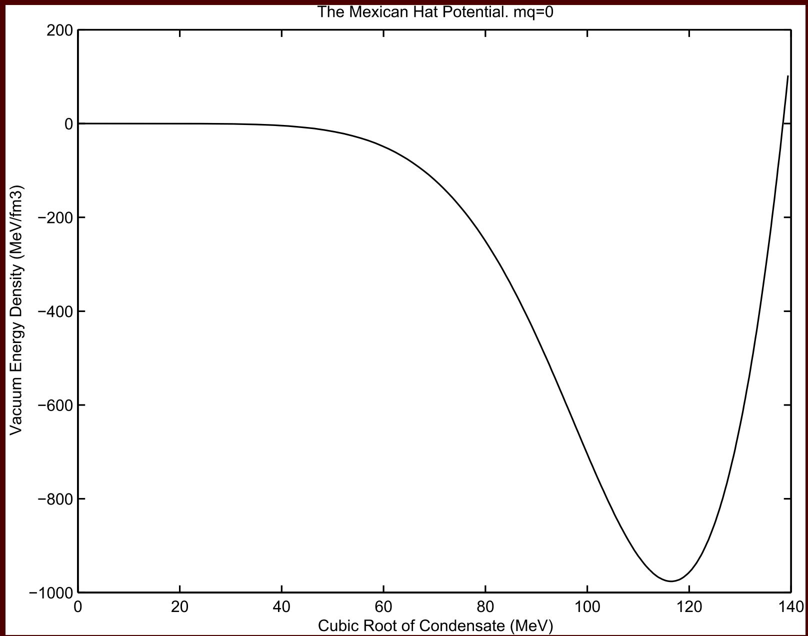
$$\begin{aligned} ks_k - mc_k = & \frac{2}{3}\int \frac{d\mathbf{q}}{(2\pi)^3}\left[(s_k c_q x - s_q c_k)V(|\mathbf{k}-\mathbf{q}|)\right. \\ - & \left. 2c_k s_q U(|\mathbf{k}-\mathbf{q}|) + 2c_q s_k W(|\mathbf{k}-\mathbf{q}|)\right] \end{aligned}$$

Adler and Davis NPB1984; Le Yaouanc et al PRD1986;
Bicudo and Ribeiro PRD1989

Mass gap generation

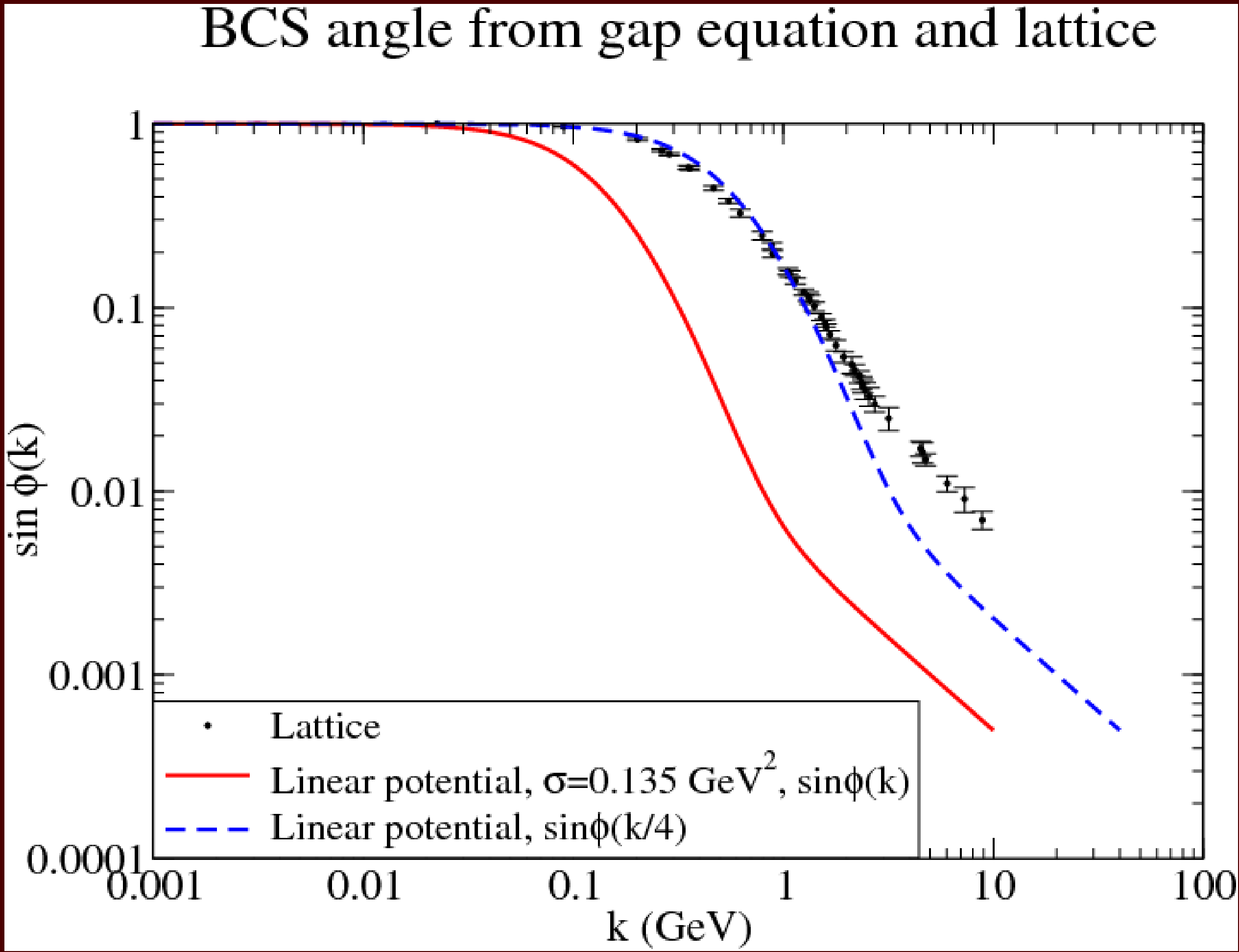


Mexican hat



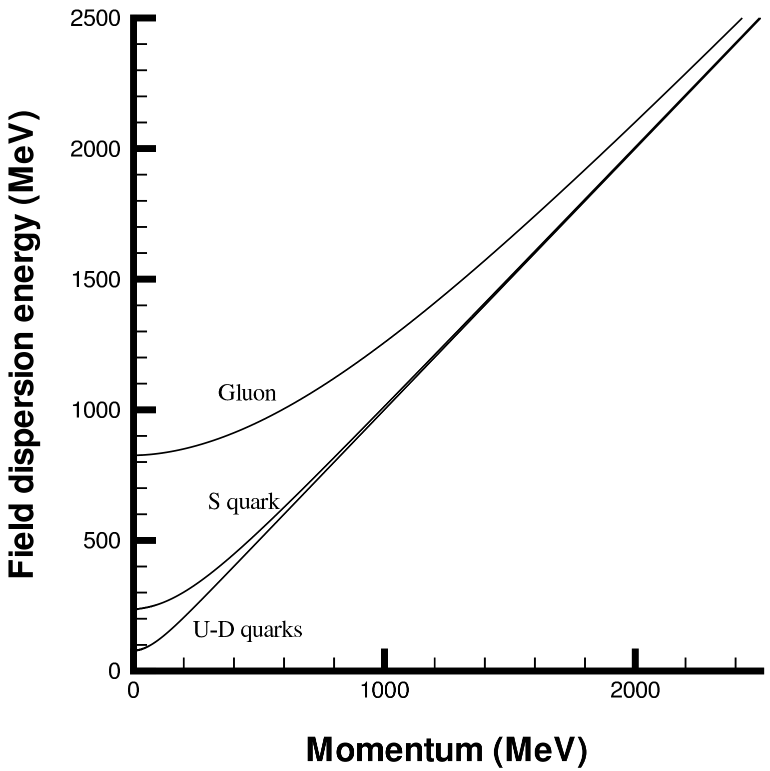
The Mexican Hat Potential. mq=0
Vacuum Energy Density (MeV/fm3)
200
0
−200
−400
−600
−800
−1000
0
20
40
60
80
100
120
140
Cubic Root of Condensate (MeV)

Sine of the gap angle



Lattice data (Landau gauge) courtesy of P. Bowman

Mass gap generation



Mesons in Tamm-Dancoff

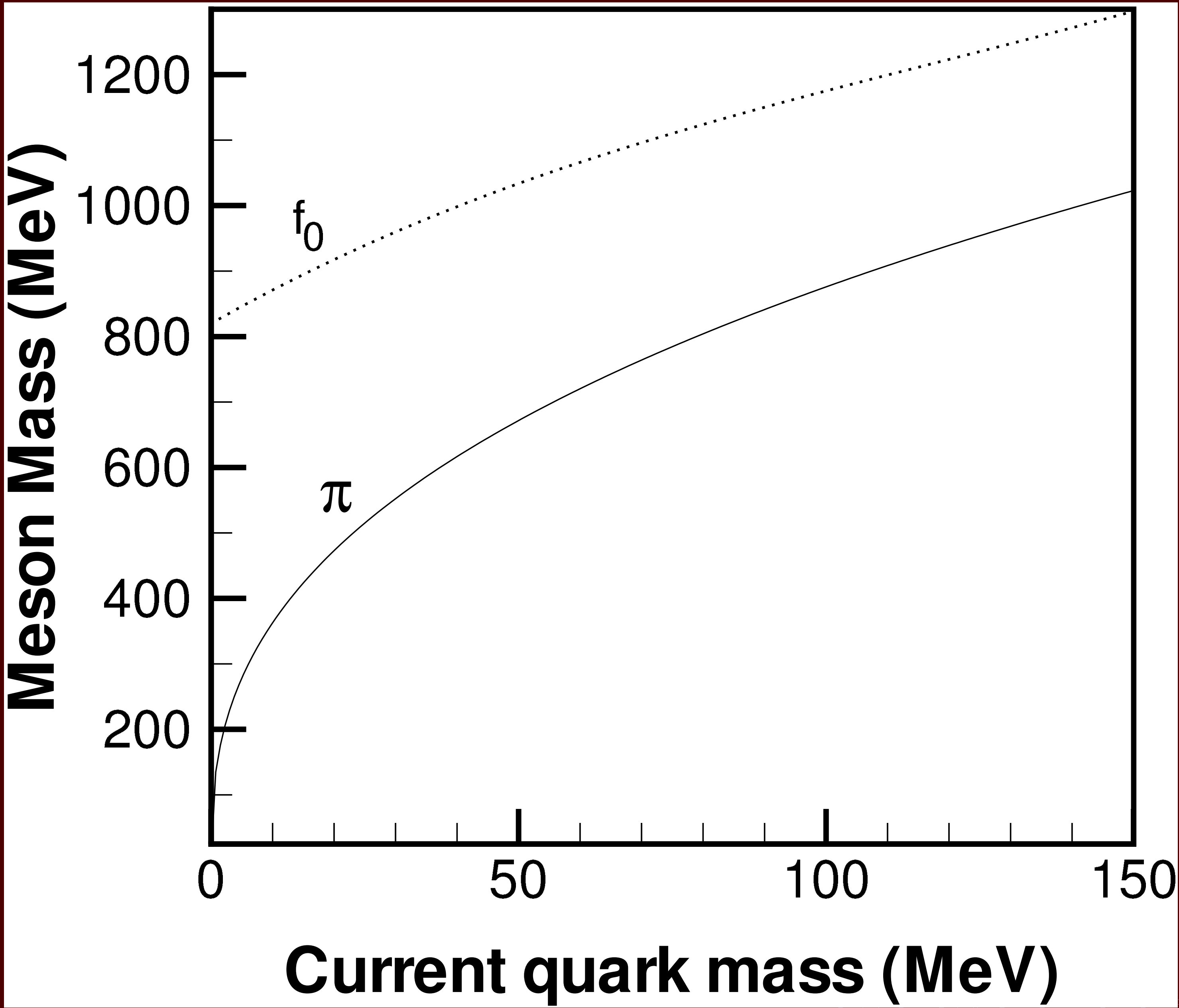
$$|G\rangle = \int \frac{d\mathbf{q}}{(2\pi)^3} \sum_{a\mu\nu} \phi^{(n)}_{\mu\nu}(\mathbf{q}) \alpha^{a\dagger}_{\mu}(\mathbf{q}) \alpha^{a\dagger}_{\nu}(-\mathbf{q}) |\Omega\rangle \ .$$

$$|Q\bar{Q}\rangle = \sum_{\gamma\delta} \Psi^{*n}_{\gamma\delta} B^{\dagger}_{\gamma} D^{\dagger}_{\delta} |\Omega\rangle_{BCS}$$

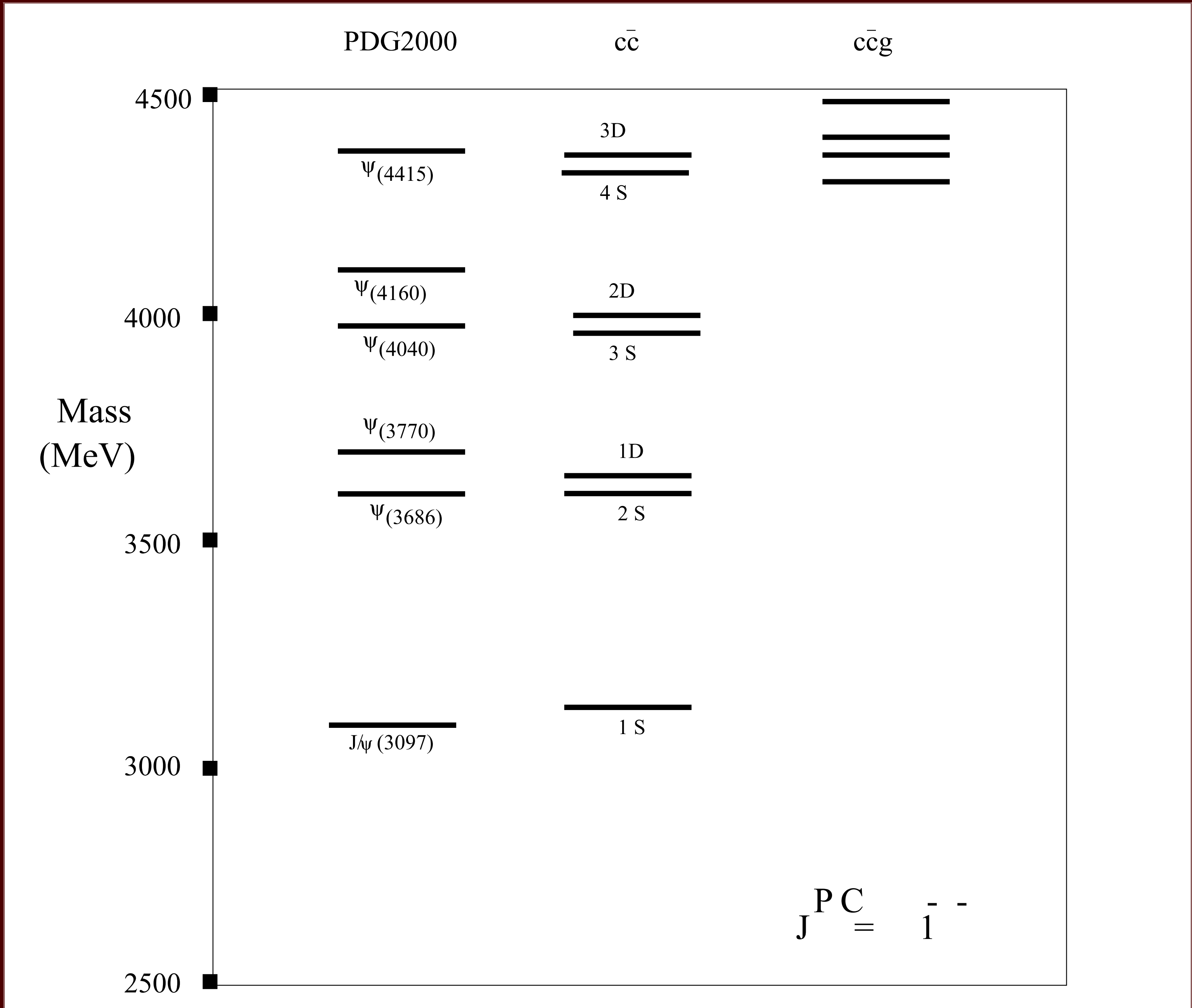
Cotanch and Ll-E NPA2002

RPA: Gell-Mann-Oakes-Renner

Llanes-Estrada and Cotanch, PRL 2000



Vector charmonium hybrids



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χS Wigner or Goldstone

$$[Q_5^a, N_i^+] = \Theta_{ij}^a N_j^-$$
$$[Q_5^a, N_i^-] = \Theta_{ij}^a N_j^+$$

$[Q_5^a, H] = 0$ implies $M^+ = M^-$

Alternatively

$$[Q_5^a, N_i^\pm] = v_0(\pi^2)\epsilon_{abc}\pi^c\Theta_{ij}^b N_j^\pm$$

Glozman (2001-2007): Wigner mode high in the spectrum

Q^5 in terms of $\sin\phi(k)$

$$Q^5_a|qqq\rangle = \int \frac{d^3k}{(2\pi)^3} \sum_{\lambda\lambda' ff' c} \left(\frac{\tau^a}{2}\right)_{ff'}$$
$$\left(\cos\phi(k)(\sigma\cdot\hat{\mathbf{k}})_{\lambda\lambda'} B^\dagger_{k\lambda fc} B_{\lambda' f' c} + \right.$$
$$\left. \sin\phi(k)(i\sigma_2)_{\lambda\lambda'} B^\dagger_{k\lambda fc} D^\dagger_{\lambda' f' c}\right)|qqq\rangle \ .$$

Nefediev, Ribeiro, Szczepaniak 08

Chiral multiplets

3-quark chiral quartet in baryons (for mesons, see Swanson 2003)

$$|N_1^P\rangle = \sum F^P_{ijk} B_i^\dagger B_j^\dagger B_k^\dagger |\Omega\rangle$$

$$|N_2^{-P}\rangle = \sum F^P_{ijk} \left(\sigma\cdot\hat{\mathbf{k}}_i B^\dagger\right)_i B_j^\dagger B_k^\dagger |\Omega\rangle$$

$$|N_3^P\rangle = \sum F^P_{ijk} \left(\sigma\cdot\hat{\mathbf{k}}_i B^\dagger\right)_i \left(\sigma\cdot\hat{\mathbf{k}}_j B^\dagger\right)_j B_k^\dagger |\Omega\rangle$$

$$|N_3^{-P}\rangle = \sum F^P_{ijk} \left(\sigma\cdot\hat{\mathbf{k}}_i B^\dagger\right)_i \left(\sigma\cdot\hat{\mathbf{k}}_j B^\dagger\right)_j \left(\sigma\cdot\hat{\mathbf{k}}_k B^\dagger\right)_k |\Omega\rangle$$

Diagonalize the chiral charge

$$Q_5(N_0 - N_2) = N_1 - N_3 \quad , \qquad Q_5(N_1 - N_3) = N_0 - N_2$$

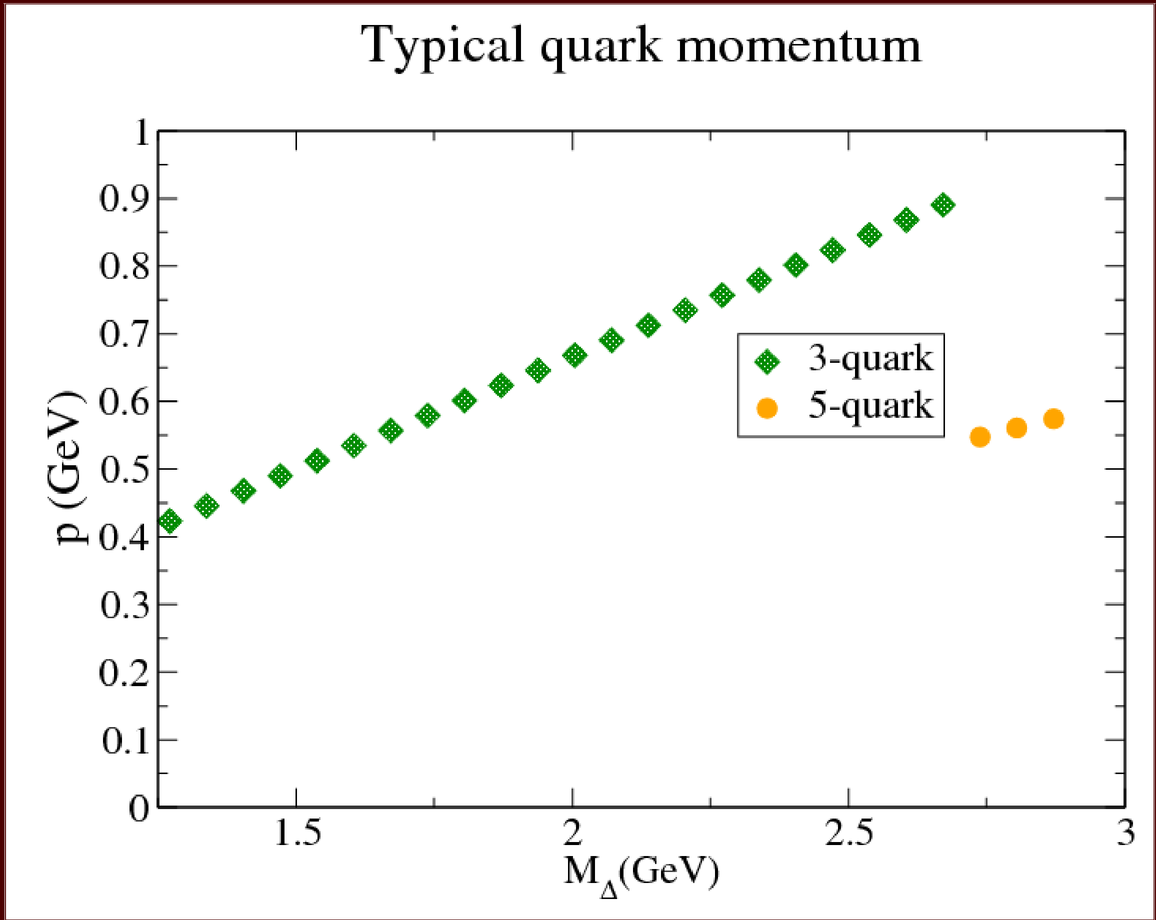
$$Q_5(N_0 + 3N_2) = 3(3N_1 + N_3) \quad , \quad Q_5(3N_1 + N_3) = 3(N_0 + 3N_2)$$

Two doublet-structure: $J^+ - J^-$, $J'^+ - J'^-$

Off to the spectrum...

- Fixed J, highly excited state?
- Ground state for increasing J?

Typical momentum



Phase space for 3,5... quarks

Even lower energy: dynamically generated resonances

Off to the spectrum...

- Fixed J^P, highly excited state?
- Ground state for increasing J?
- Molecules do not bind at large J

Regge trajectories of highly J-excited Δ baryons

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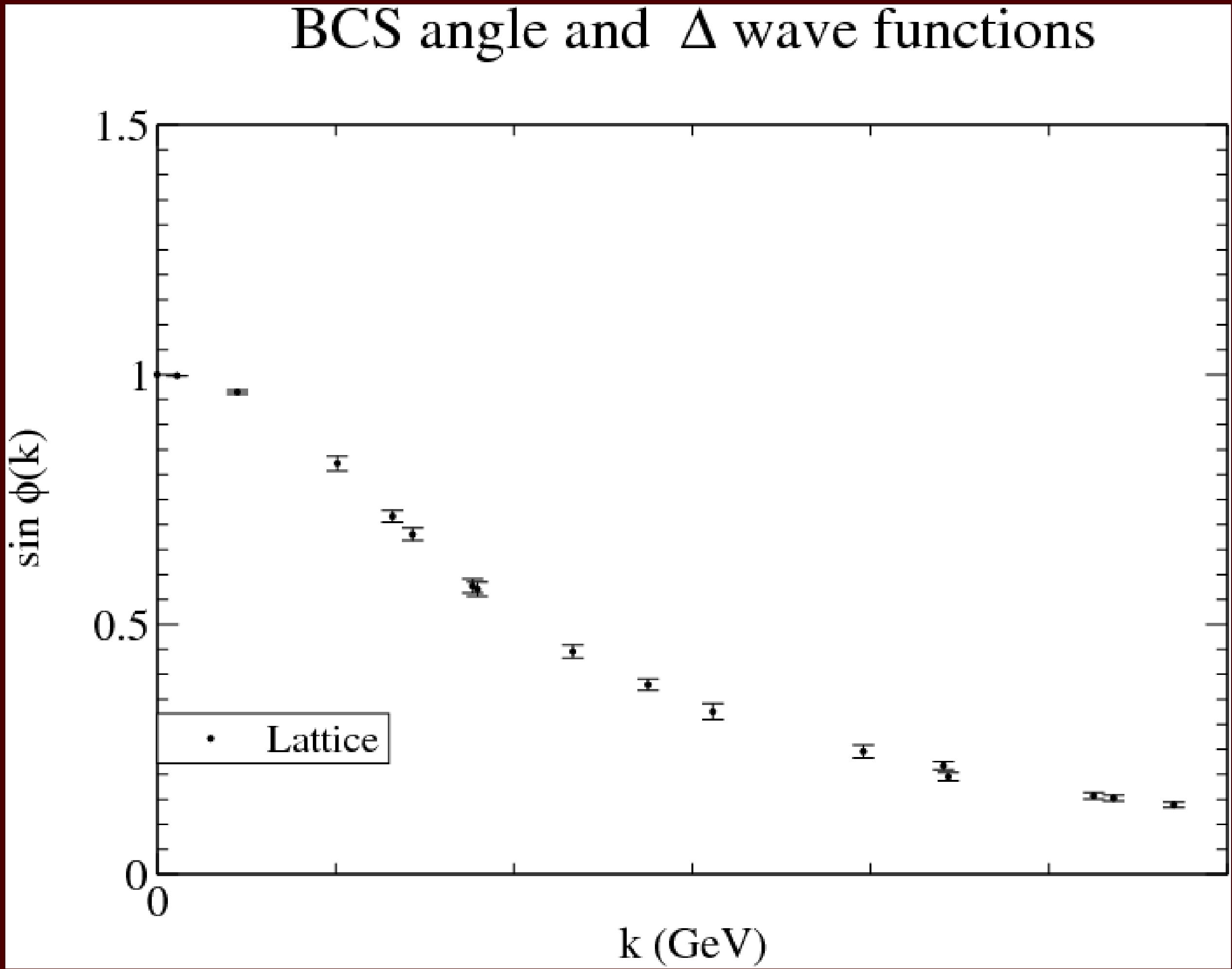
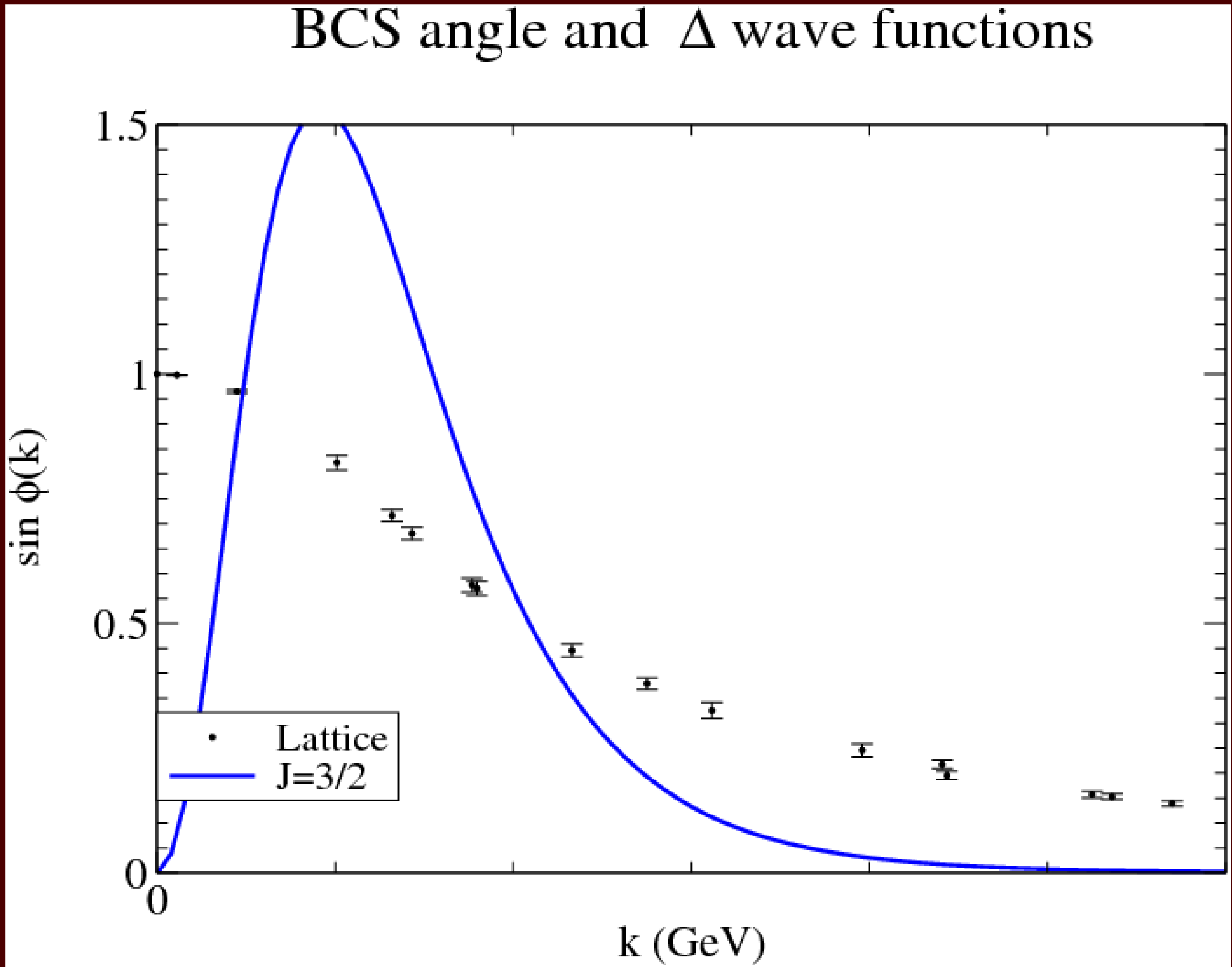
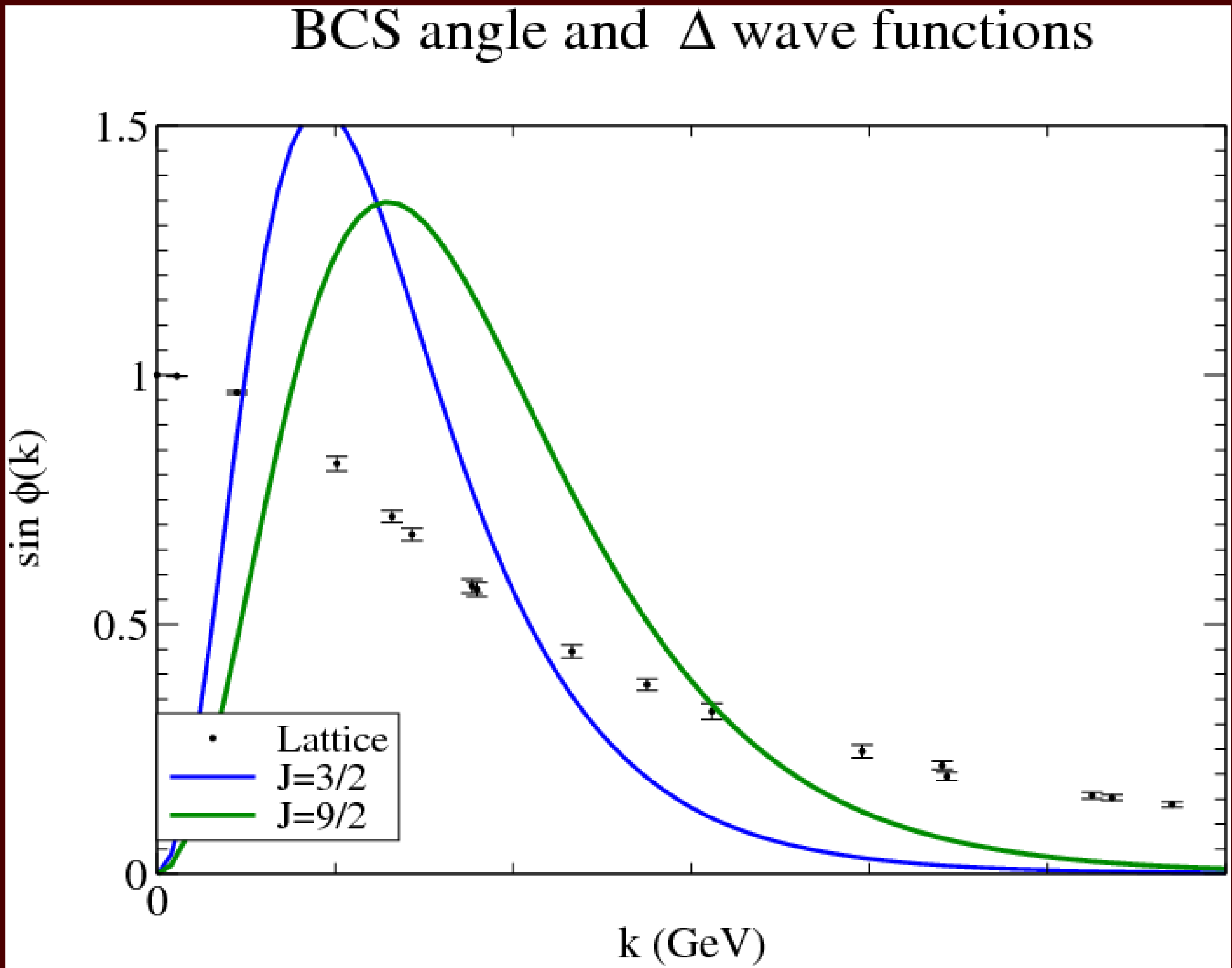


Lattice data (Landau gauge) courtesy of P. Bowman

Ground state Δ

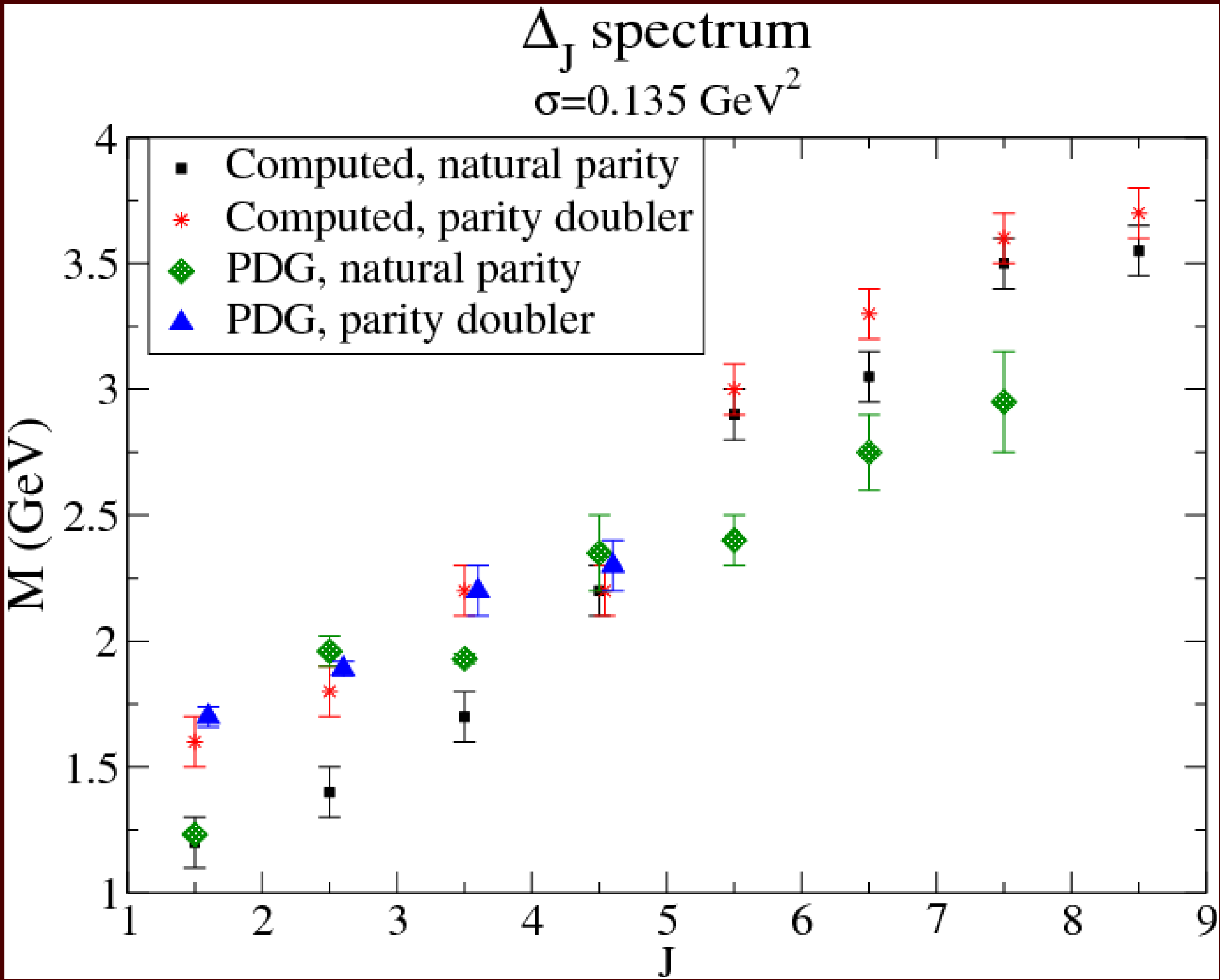


Excited Δ


BCS angle and Δ wave functions
1.5
1
0.5
0
0
sin φ(k)
k (GeV)
Lattice
J=3/2
J=9/2

Chiral Symmetry à la Wigner

Van Cauteren, Bicudo, Cardoso and LL-E
PRELIMINARY



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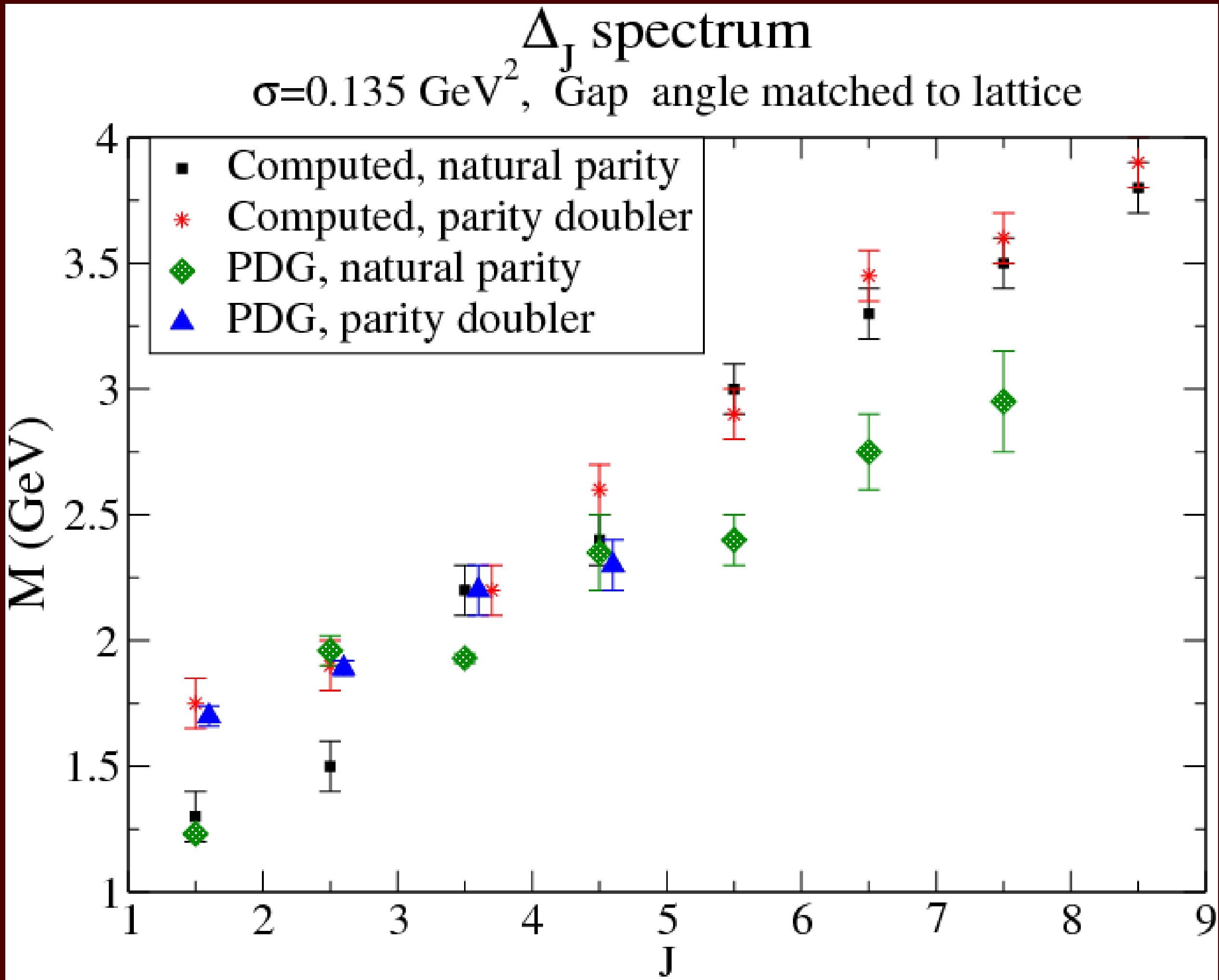
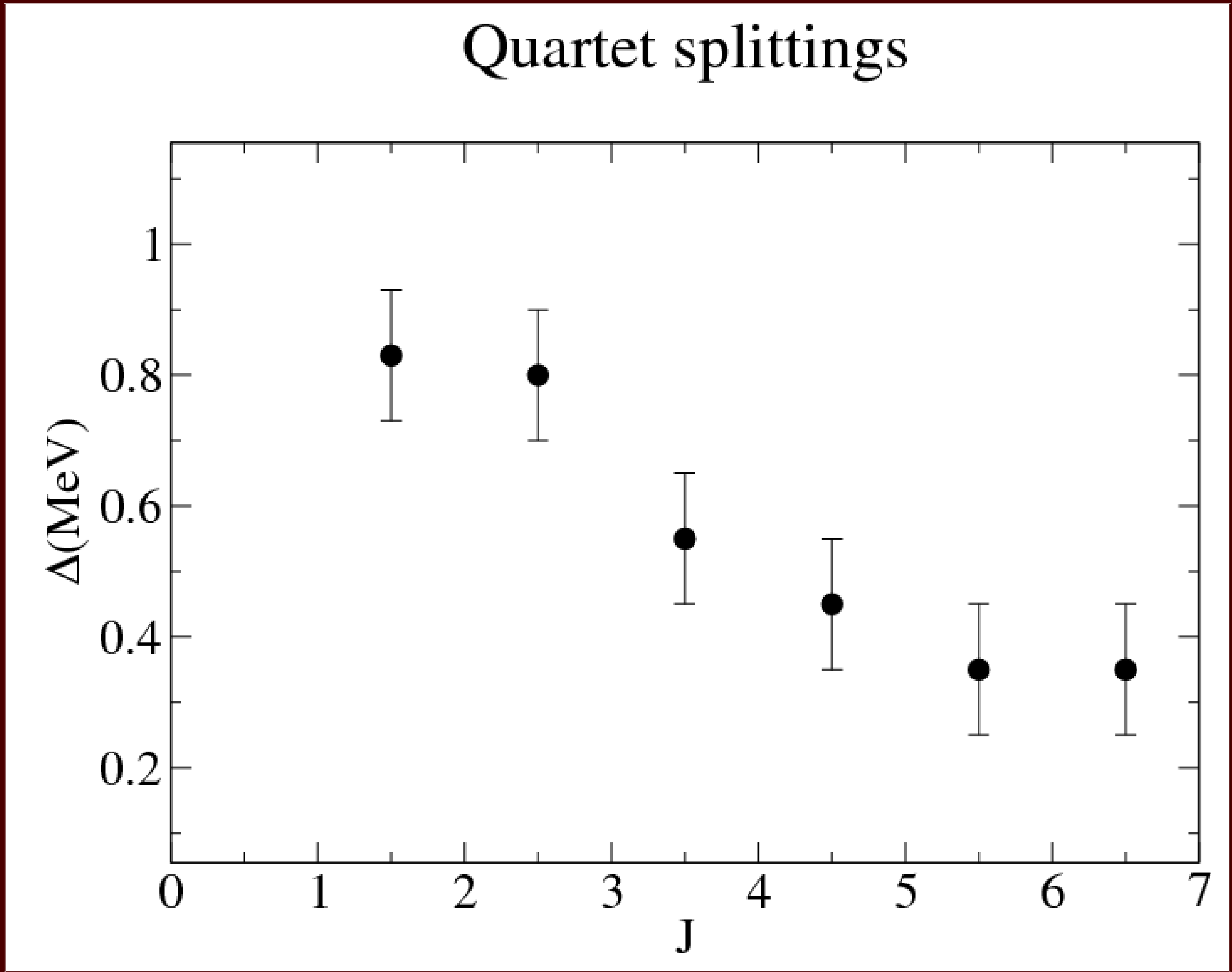


Van Cauteren, Bicudo, Cardoso and LL-E
PRELIMINARY

Decreasing parity splittings: first doublet



Van Cauteren, Bicudo, Cardoso and LL-E
PRELIMINARY

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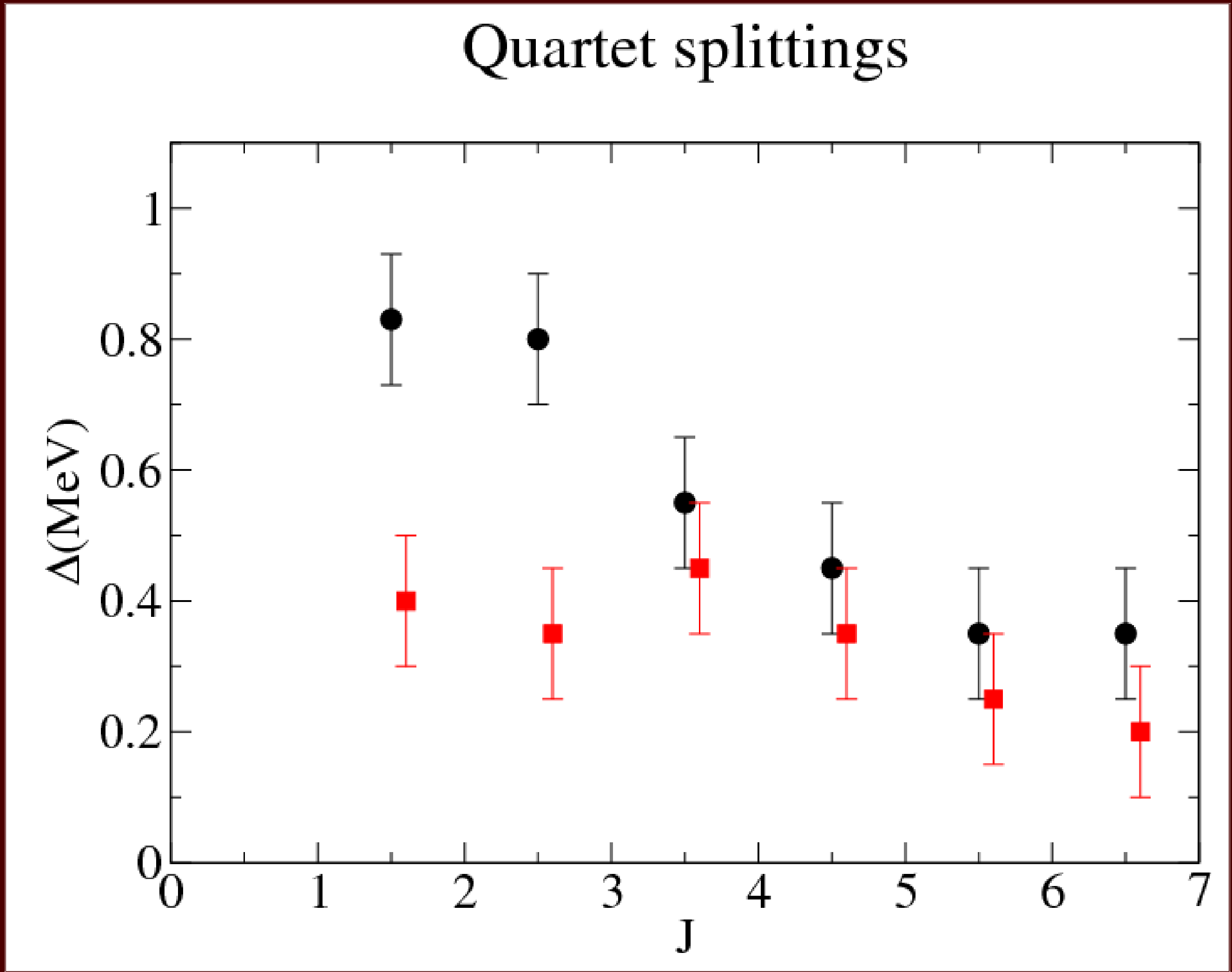
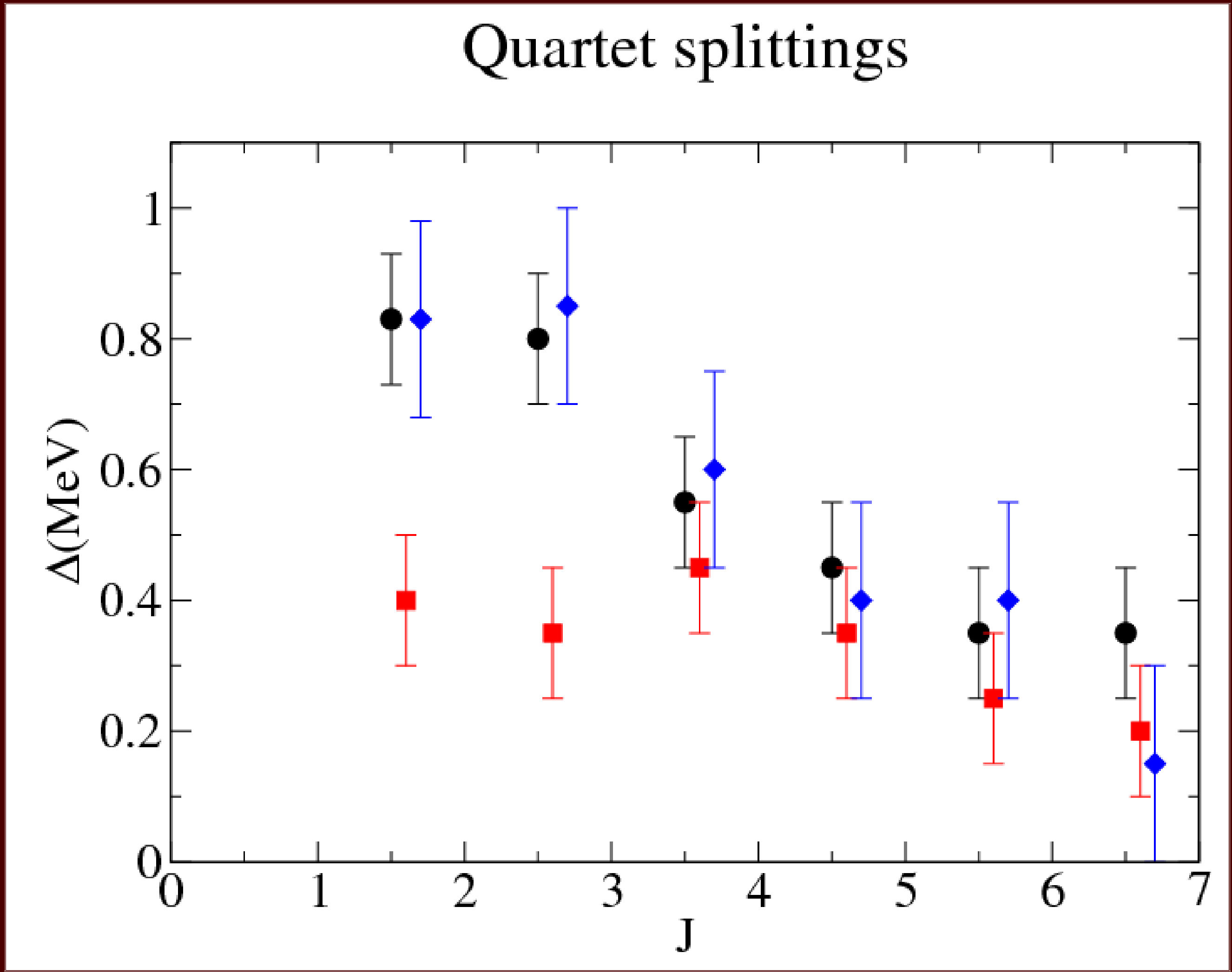


Van Cauteren, Bicudo, Cardoso and LL-E
PRELIMINARY

Decreasing splittings: interdoublet



Van Cauteren, Bicudo, Cardoso and LL-E
PRELIMINARY

Outline

Parity doubling and quark mass

$$M_+ - M_- =$$
$$3\int \frac{d^3k_1}{(2\pi)^3}\frac{d^3k_2}{(2\pi)^3}\left(\frac{2}{3}\right)\int\frac{d^3q}{(2\pi)^3}\hat{V}(q)\frac{1}{2}\left(\frac{m(|\mathbf{k}_1|)}{|\mathbf{k}_1|}+\frac{m(|\mathbf{k}_1+\mathbf{q}|)}{|\mathbf{k}_1+\mathbf{q}|}\right)\times$$
$$\left[F^{*\lambda_1\lambda_2\lambda_3}(\mathbf{k}_1,\mathbf{k}_2)\left(\mathbb{I}-\sigma\hat{\mathbf{k}}_1\sigma\widehat{\mathbf{k}_1+\mathbf{q}}\right)_{\lambda_1\mu_1}F^{\mu_1\lambda_2\lambda_3}(\mathbf{k}_1+\mathbf{q},\mathbf{k}_2-\mathbf{q})\right]$$

$$|M^+ - M^-| = \langle\frac{m(q)}{q}V_{\text{Tens}}\rangle$$
$$\rightarrow c_3\frac{m(\langle q\rangle)}{\langle q\rangle}\langle V_{\text{Tens}}\rangle$$

Compare with Gell-Mann-Oakes-Renner

$$M_\pi^2 = m_q\frac{\langle\bar{\Psi}\Psi\rangle}{f_\pi^2}$$

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Large-j scaling

- Regge trajectories: $M^{\pm} \propto \langle q \rangle \propto \sqrt{j}$
- Radial average: centrifugal barrier dominant $j^2 \langle V \rangle_r \propto M^{\pm}$
- Angular average: from spinors, $\langle V \rangle_\Omega \propto j$
- Together,tensor force scaling: $\langle V_{\rm tens} \rangle \propto \sqrt{\frac{1}{j}}$
- $|M^+ - M^-| \propto j^{-1} m(\langle q \rangle)$

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Experimental extraction

- Measure masses of (ground state) high j baryons of both parities
- Fit $|M^+ - M^-| \propto j^{-i}$
- $m(\Lambda \times \sqrt{j}) \propto j^{-i+1}$ or
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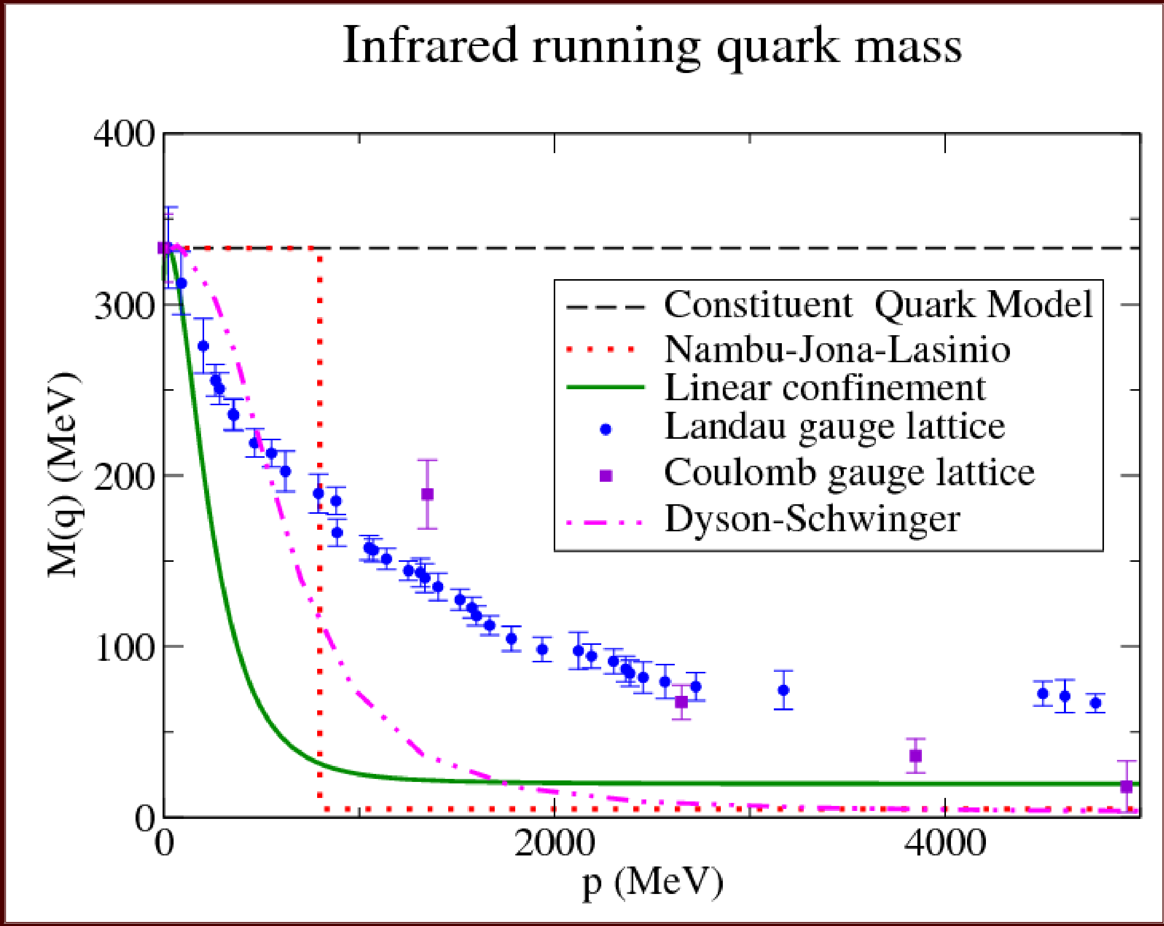
Note



$|M^+ - M^-| \to 0$ even in CQM (F. Fernández et al., 2008)

Need exponent i, good data: ELSA, JLAB

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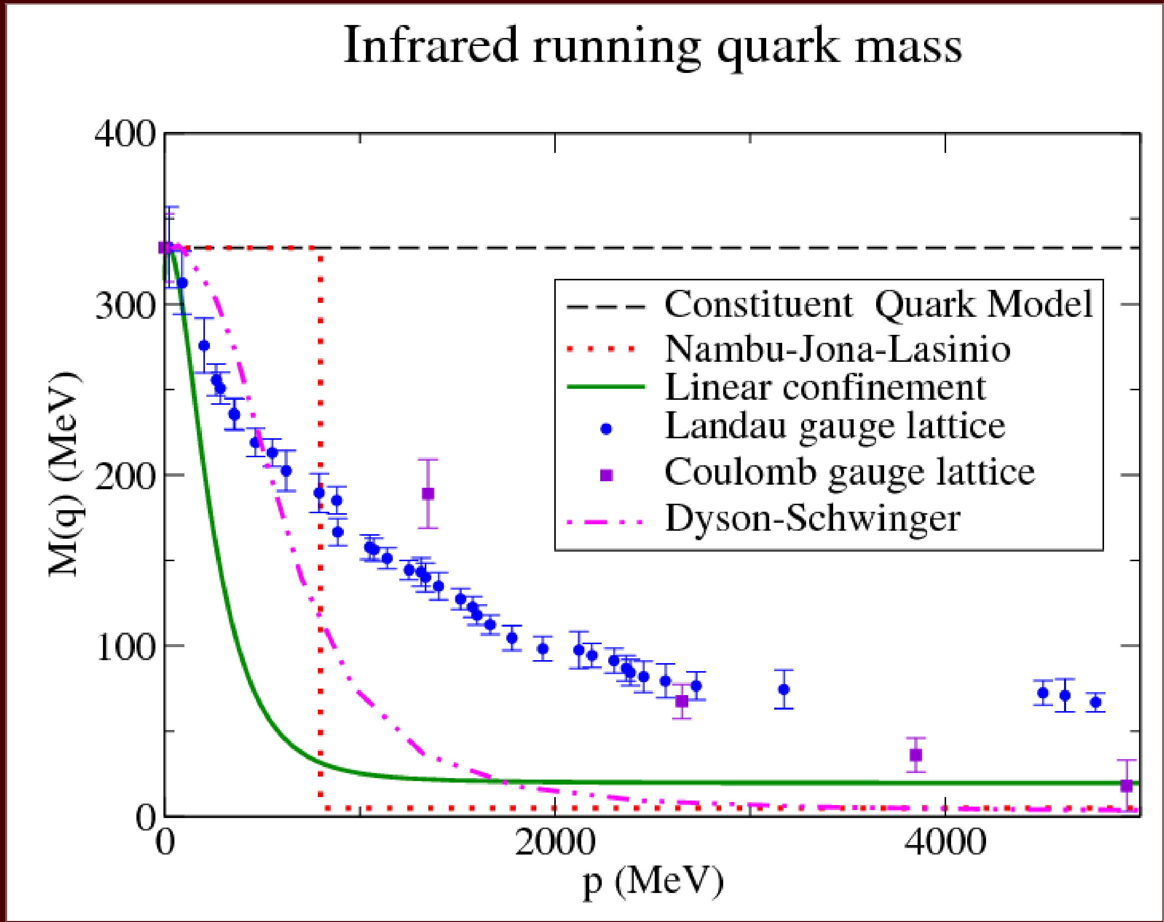

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Summary

- States high in the spectrum are insensitive to spontaneous Chiral Symmetry Breaking, hence parity doubling
- Baryons group in approximate quartets
- The decreasing splittings probe $m(k)$
- $\frac{m_q(\langle k \rangle)}{\langle k \rangle}$ a small quantity to expand in

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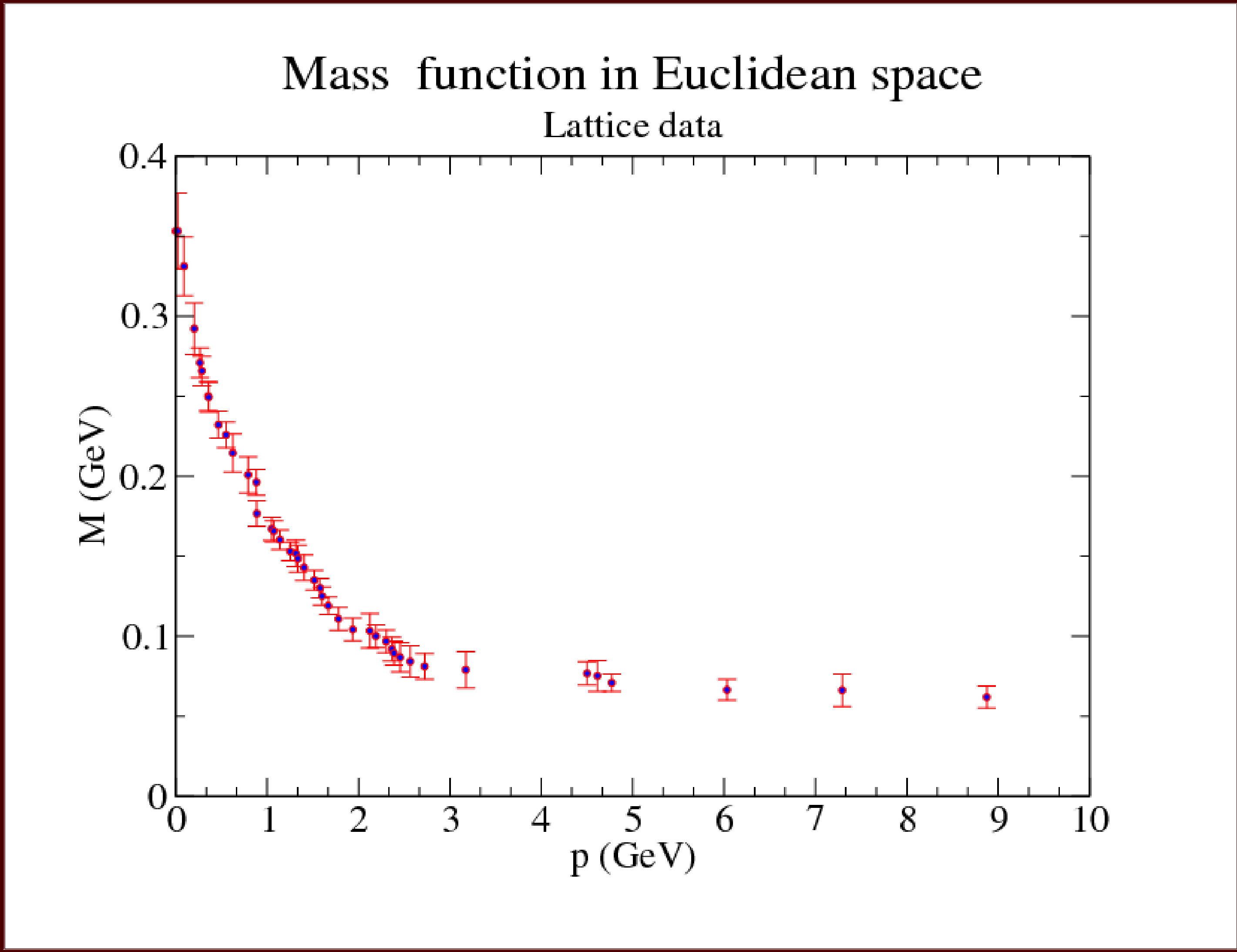
Quark mass in Landau gauge



Lattice data courtesy of P. Bowman et al

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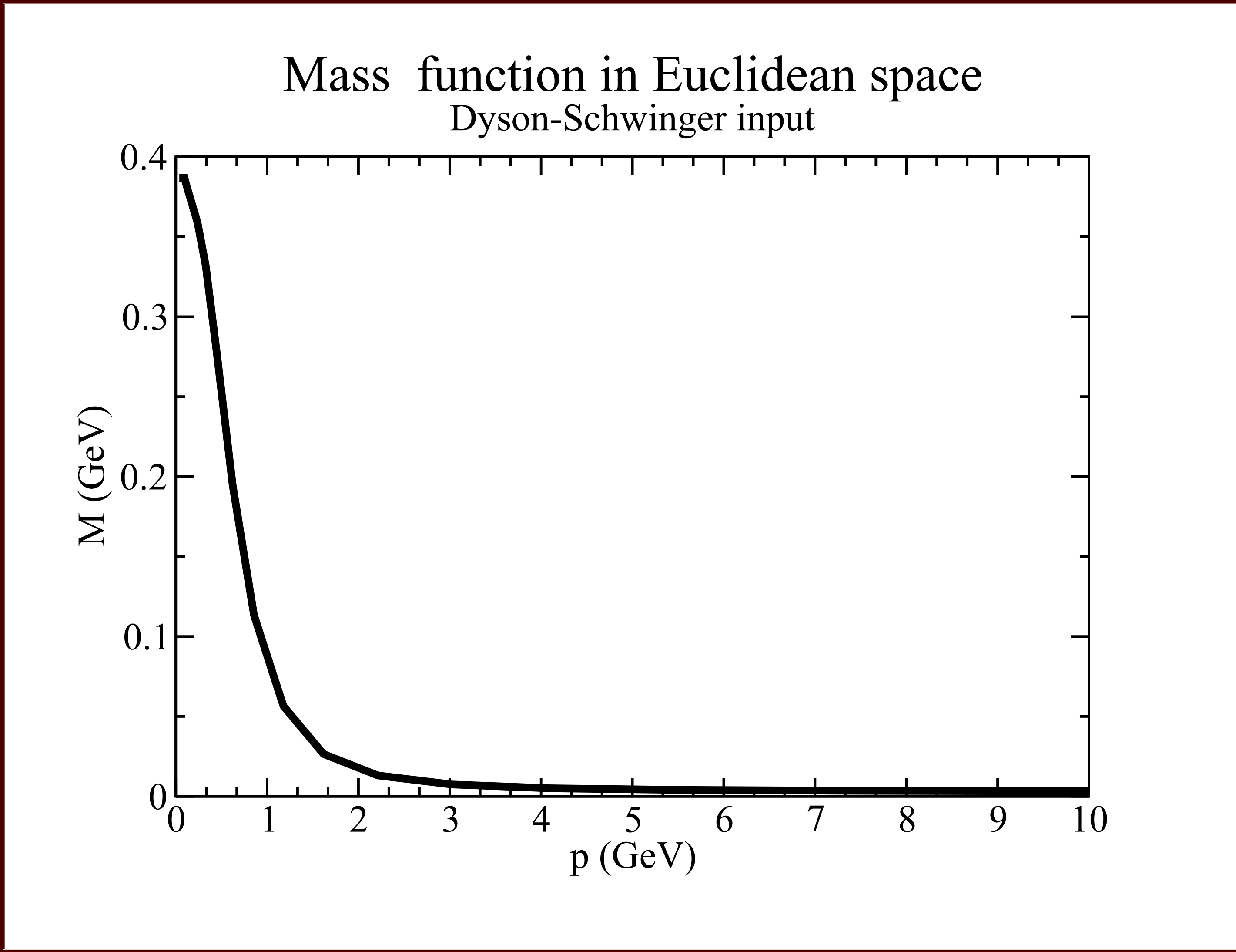
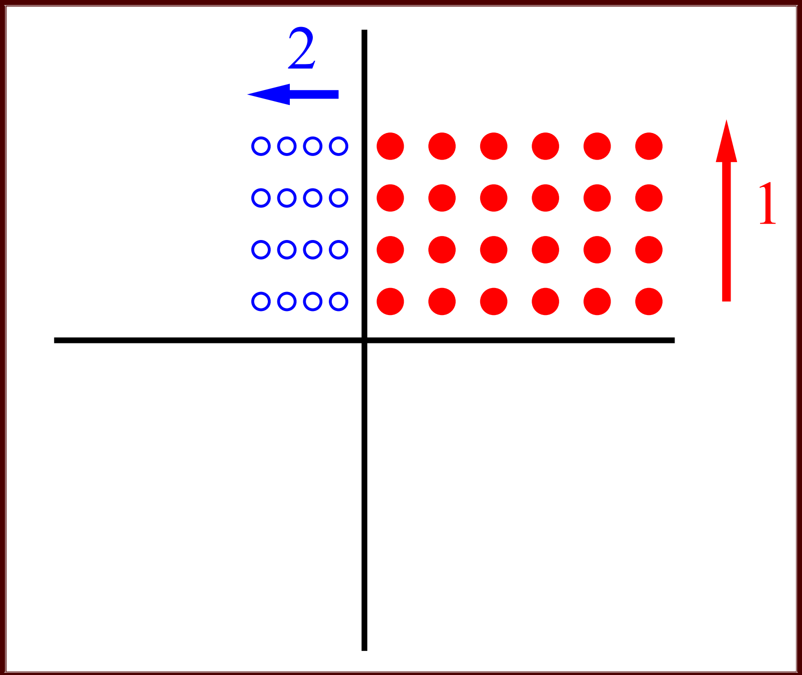


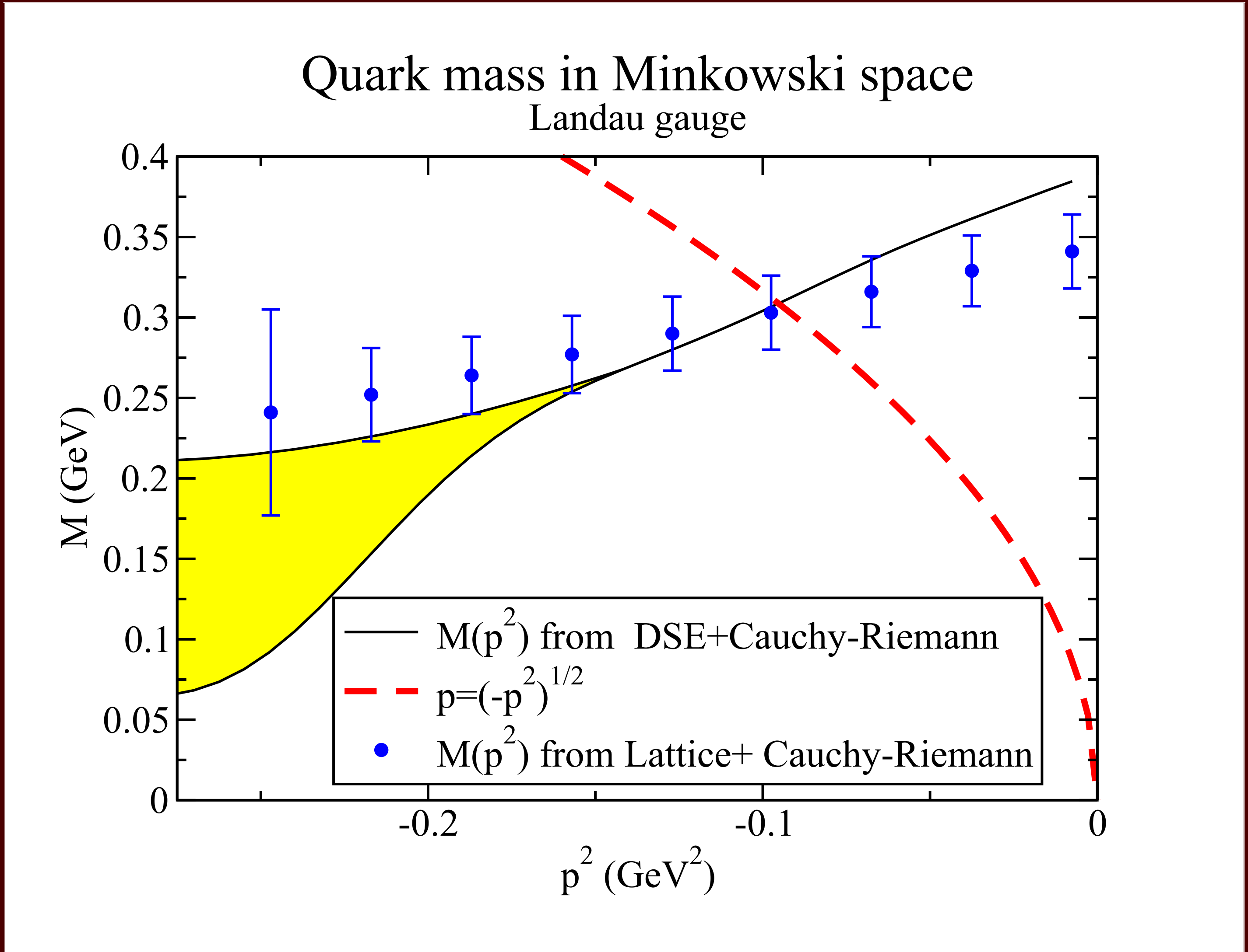
Cauchy-Riemann equations: local analyticity



$$\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \quad \frac{\partial v}{\partial y} = \frac{\partial u}{\partial x}$$

M. Gimeno-Segovia and F. LL-E, EPJC**56**, 557 (2008)

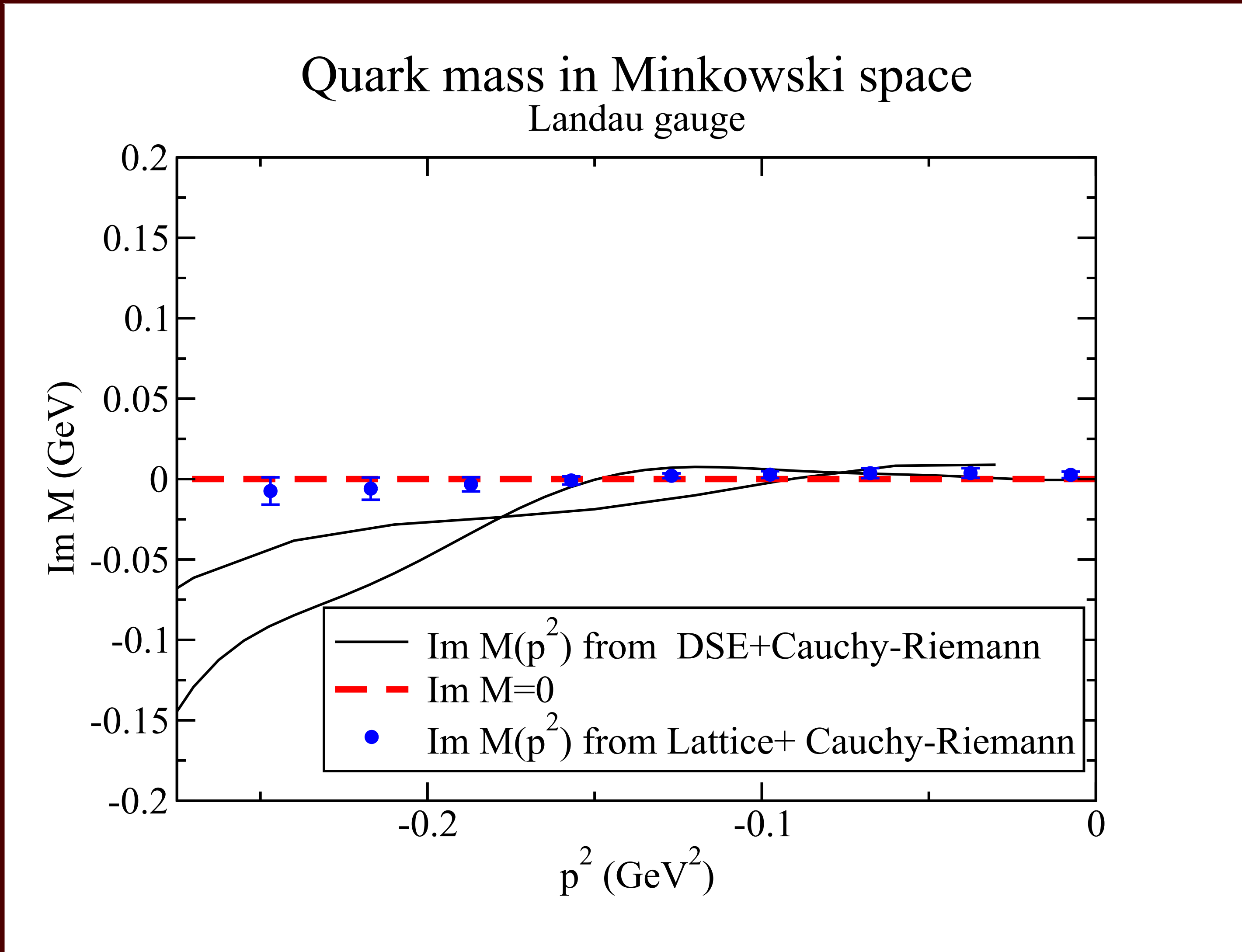
Wick rotation to Minkowski



Minimal assumption method: Cauchy-Riemann

Gimeno-Segovia and Ll-E EPJC2008

Wick rotation to Minkowski



Minimal assumption method: Cauchy-Riemann
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