

# The running coupling from the four-gluon interaction in Yang-Mills theory

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1 Basics

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4 Summary/Outlook



QCD: Confinement/ $D\chi$ SB  $\rightarrow$  non-perturbative methods mandatory

- method here: Dyson-Schwinger-Equations (DSE)
- advantage: DSEs provide equations for determining 1PI correlation functions
  - $\rightarrow$  no series expansion
  - $\rightarrow$  easy investigation of small momenta (in contrast to lattice QFT)
  - $\rightarrow$  unique gauge fixing possible (Gribov copies)
- disadvantage: DSEs form a coupled system of infinitely many equations  $\rightarrow$  needs truncation



# The running coupling of the four-gluon vertex

- The 4g-vertex is part of the QCD-Lagrangian
  - has its own renormalisation constant
  - allows for the definition of a running coupling via a Slavnov-Taylor-Identity (STI)
- does it make any difference?
- if so: are there consequences for gluon-confinement?



# Yang-Mills theory and Landau gauge

- Yang-Mills theory  $\Rightarrow$  No Quarks!
- gauge-fixing via Faddeev-Popov procedure introduces ghost fields and gauge-fixing parameter.

The Yang-Mills Lagrangian:

$$\mathcal{L}_{YM}[A, c, \bar{c}] = \frac{1}{4} F_{\mu\nu}^2 + \frac{(\partial_\mu A_\mu^a)^2}{2\xi} - i\bar{c}(\partial_\mu D_\mu^{ab})c$$

$$D_\mu^{ab} = \partial_\mu \delta^{ab} + g f^{abc} A_\mu^c$$

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g f^{abc} A_\mu^b A_\nu^c$$

Landau gauge:  $\xi \rightarrow 0$



# Definitions

$$\text{wavy line} = Z(p^2) \cdot D_{ab}^{\mu\nu}(p^2)$$

$$\text{dashed line} = G(p^2) \cdot D_{ab}^G(p^2)$$

$$\text{four-gluon vertex} = 4g \Gamma(p^2) \cdot \Gamma_{abcd}^{\kappa\lambda\mu\nu}$$

$$D_{ab}^{\mu\nu}(p^2) = \left( g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2} \right) \frac{1}{p^2} \delta_{ab}$$

$$D_{ab}^G(p^2) = -\frac{1}{p^2} \delta_{ab}$$

$$\Gamma_{abcd}^{\kappa\lambda\mu\nu} = ?$$

# The tensor-basis

$$C_{abcd}^{(1)} = \delta_{ab}\delta_{cd}, \quad C_{abcd}^{(2)} = \delta_{ac}\delta_{bd}, \quad C_{abcd}^{(3)} = \delta_{ad}\delta_{bc},$$
$$C_{abcd}^{(4)} = f_{abn}f_{cdn}, \quad C_{abcd}^{(5)} = f_{acn}f_{dbn}$$

$$L_{(1)}^{\kappa\lambda\mu\nu} = \delta^{\kappa\lambda}\delta^{\mu\nu} \quad L_{(2)}^{\kappa\lambda\mu\nu} = \delta^{\kappa\mu}\delta^{\lambda\nu} \quad L_{(3)}^{\kappa\lambda\mu\nu} = \delta^{\kappa\nu}\delta^{\lambda\mu}$$

- contains the perturbative vertex
- does not create new structures in lower DSEs or Bethe-Salpeter equations
- Bose-symmetrisation!

parametrisation:

$$4G\Gamma_{abcd}^{\kappa\lambda\mu\nu} = \sum_{j=1}^3 \Gamma_j(p_1, p_2, p_3) T_{abcd,j}^{\kappa\lambda\mu\nu}$$

# The definition of the running coupling from the STI

The STI for the four-gluon vertex:

$$Z_4 = Z_g^2 Z_3^2$$

Renormalisation  $\alpha(\Lambda^2) \rightarrow Z_g^2 \alpha(\mu^2)$ :

$$\alpha(\Lambda^2) = \frac{Z_4(\mu^2, \Lambda^2)}{Z_3^2(\mu^2, \Lambda^2)} \alpha(\mu^2)$$

$$\begin{aligned} Z_0(p^2, \Lambda^2) &= Z_3(\mu^2, \Lambda^2) Z(p^2, \mu^2) \\ {}^{4g} \Gamma(p^2, \mu^2) &= Z_4(\mu^2, \Lambda^2) {}^{4g} \Gamma_0(p^2, \Lambda^2). \end{aligned}$$



# The definition of the running coupling from the STI

Trading the renormalisation constants for 1PI dressing-functions one obtains

$$\alpha_{4gl}(\Lambda^2) {}^{4gl}\Gamma_0(p^2, \Lambda^2) Z_0^2(p^2, \Lambda^2) = \alpha(\mu^2) {}^{4gl}\Gamma(p^2, \mu^2) Z^2(p^2, \mu^2).$$

Renormalising at  $\mu^2 = p^2$  with  ${}^{4gl}\Gamma(p^2, p^2) Z^2(p^2, p^2) = 1$  leads to

$$\alpha(p^2) = \alpha(\mu^2) {}^{4gl}\Gamma(p^2, \mu^2) Z^2(p^2, \mu^2)$$



# The four-gluon-DSE

$$\begin{aligned} &= \text{Diagram 1} + \frac{1}{2} \text{Diagram 2} \\ &- \text{Diagram 3} + \frac{1}{2} \text{Diagram 4} \\ &+ \frac{1}{2} \text{Diagram 5} + \frac{1}{2} \text{Diagram 6} \\ &+ \frac{1}{6} \text{Diagram 7} \end{aligned}$$

Driesen, Stingl, Eur.Phys.J.A4:401-419,1999.

# The truncation scheme

- Landau-gauge, Yang-Mills-theory (no quarks)
- no self-consistency  $\rightarrow$  neglect all 4-point- and higher correlations
- influence of higher correlations via an effective three-gluon vertex-model (anomalous dimension in the UV)
- bare ghost-gluon vertex  
(Maas, Cucchieri, Mendes  
Braz.J.Phys.37N1B:219-225,2007  
Schleifenbaum, Maas, Wambach, Alkofer  
Phys.Rev.D72:014017,2005)
- identification of the leading IR/UV diagrams via dressed skeleton-expansion

# The ghost-gluon-system in Landau gauge

The image displays two Dyson-Schwinger equations for the ghost-gluon system in Landau gauge. The first equation is for the ghost-gluon vertex, and the second is for the ghost self-energy.

**Ghost-gluon vertex equation:**

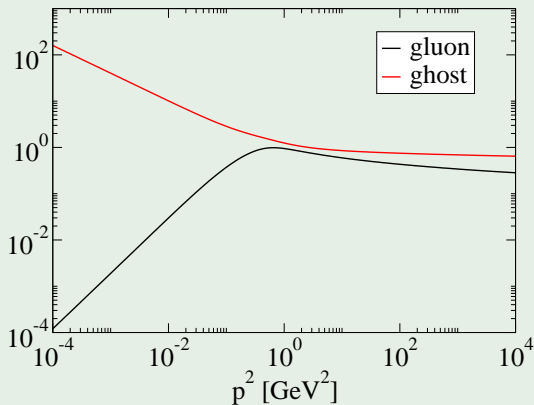
$$\text{Wavy line with black dot}^{-1} = \text{Wavy line}^{-1} + \text{Loop with black and white dots} + \text{Triangle with black and white dots} + \text{Box with black and white dots} + \text{Triangle with black and white dots} + \text{Loop with wavy lines}$$

**Ghost self-energy equation:**

$$\text{Dashed line with black dot}^{-1} = \text{Dashed line}^{-1} + \text{Loop with wavy lines and black dots}$$

The diagrams use the following conventions: wavy lines for gluons, dashed lines for ghosts, black dots for ghost-gluon vertices, and white dots for ghost-ghost vertices. The superscript  $-1$  indicates the inverse of the corresponding propagator.

# Numerical solution of the coupled ghost-gluon DSEs



Fischer, Alkofer Phys.Lett.B536:177-184,2002

# IR behaviour of Landau-gauge YM-theory

A general IR-solution for the whole tower of coupled DSEs has been found:

$$\Gamma^{n,m}(p^2 \rightarrow 0) = C (p^2)^{\left(\frac{n}{2}-m\right)\kappa}, \quad \kappa \approx 0.595$$

$n$  ghost-legs,  $m$  gluon-legs

Alkofer, Fischer, Llanes-Estrada *Phys.Lett.* **B611** (2005) 279-288

For completeness: the quark-gluon vertex:

$$\Gamma_{q-g}^{\mu}(p_1^2 \sim p_2^2 = p \rightarrow 0) = \tilde{C} (p^2)^{-\frac{1}{2}-\kappa}$$

Alkofer, Fischer, Llanes-Estrada, Schwenzer (2008)  
to appear in *Annals of Physics* (arXiv: 0804.3042)

# The effective 3-gluon-vertex

$${}^3g\Gamma_{\lambda\mu\nu}^{ab}(q, p) = \frac{G(q^2)^{(-\frac{1}{6}-\delta)}}{Z(q^2)^{(\frac{5+3\delta}{6})}} \frac{G(p^2)^{(-\frac{1}{6}-\delta)}}{Z(p^2)^{(\frac{5+3\delta}{6})}} {}^3g\Gamma_{\lambda\mu\nu}^{ab(0)}(q, p)$$

$\delta = -9/44$ : ghost one-loop anomalous dimension

- preserves correct UV behaviour known from resummed perturbation theory  $\rightarrow$  asymptotic freedom
- preserves consistent IR power-law behaviour



# The approximate equation

The truncations scheme leads to the following approximate equation for the DSE:

The diagram shows an equation for the Dyson-Schwinger Equation (DSE) for a quark propagator. On the left is the full quark propagator, represented by a wavy line entering a circle vertex from the left and another wavy line exiting to the right. This is equal to the sum of three terms:

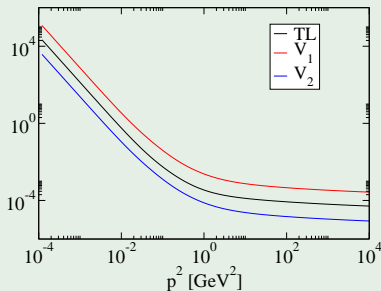
- A tree-level term: a wavy line entering a black dot vertex from the left and another wavy line exiting to the right.
- A term with a coefficient of 3: a wavy line entering a vertex from the left, which is connected to a loop of three gluons (represented by wavy lines and circles), and another wavy line exiting to the right.
- A term with a coefficient of -6: a wavy line entering a vertex from the left, which is connected to a loop of two gluons (represented by wavy lines and circles), and another wavy line exiting to the right.

Each of the loop diagrams is followed by a vertical line and the label "sym", indicating that the terms are multiplied by their respective symmetry factors.

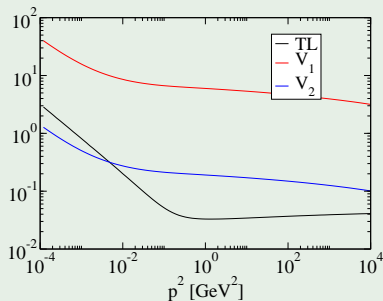




The dressing functions ( $p_1 = p_2 = p_3 = p$ ):



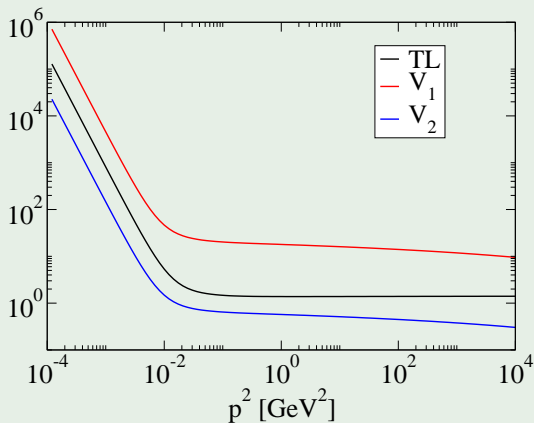
(a) GhostBox



(b) GluonBox



The dressing-functions ( $p_1 = p_2 = p_3 = p$ ):



- IR: all tensor-structures show predicted power-law behaviour

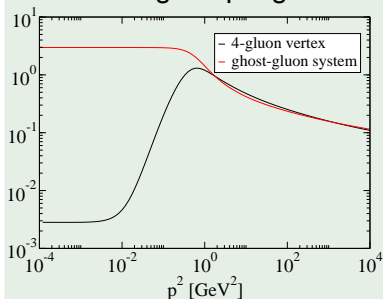
$$\Gamma(p^2 \rightarrow 0) = A(p^2)^{4\kappa}, \quad \kappa = 0.595353\dots$$

(Alkofer,Fischer,Llanes-Estrada Phys. Lett. B611 (2005) 279-288)

- UV: correct anomalous dimension for the perturbative structure
- UV: the other dressing functions vanish (slowly)  $\rightarrow$  perturbative structure leads for very large momenta



## The running coupling



$$\alpha_{4gl}(p^2 \rightarrow 0) = 0.00285$$

$$\alpha_{g/gl}(p^2 \rightarrow 0) = 2.97$$

Fischer, Alkofer

Phys. Lett. B536 (2002) 177-184

$$\alpha_{3gl}(p^2 \rightarrow 0) = 0.00134$$

K. Schwenzer

*priv.comm.*

The coupling from the ghost-gluon-vertex is three orders of magnitude larger than the couplings from the other superficially divergent vertices!

- non-trivial IR-fixed-point
- three orders of magnitude smaller than  $\alpha_{g-g}$
- future: improvement of the truncation (self-consistence!)



# Thank you!



# The derivation of the non-perturbative coupling

For the running coupling one usually defines

$$\alpha = \frac{g^2}{4\pi}.$$

Renormalisation of the theory:

$$\begin{aligned} A_\mu^a &\rightarrow \sqrt{Z_3} A_\mu^a \\ \bar{c}^a c^b &\rightarrow \tilde{Z}_3 \bar{c}^a c^b \\ g &\rightarrow Z_g g \end{aligned}$$



# The derivation of the non-perturbative coupling

The renormalisation constants are not independent. Their relation is described with the Slavnov-Taylor-Identities (STI).

The STI of interest here is

$$Z_4 = Z_g^2 Z_3^2.$$

Putting all together one finds

$$\alpha(\Lambda^2) = \frac{Z_4(\mu^2, \Lambda^2)}{Z_3^2(\mu^2, \Lambda^2)} \alpha(\mu^2).$$





# The derivation of the non-perturbative coupling

The renormalisation of the gluon-propagator and the four-gluon vertex is done via:

$$\begin{aligned}Z_0(p^2, \Lambda^2) &= Z_3(\mu^2, \Lambda^2) Z(p^2, \mu^2) \\ {}^{4g} \Gamma(p^2, \mu^2) &= Z_4(\mu^2, \Lambda^2) {}^{4g} \Gamma_0(p^2, \Lambda^2).\end{aligned}$$

$$\alpha(\Lambda^2) {}^{4g} \Gamma_0(p^2, \Lambda^2) Z_0^2(p^2, \Lambda^2) = \alpha(\mu^2) {}^{4g} \Gamma(p^2, \mu^2) Z^2(p^2, \mu^2)$$

Renormalisation condition:

$${}^{4g} \Gamma(p^2, p^2) Z^2(p^2, p^2) = 1$$

$$L_{(1)}^{\kappa\lambda\mu\nu} = \delta^{\kappa\lambda}\delta^{\mu\nu}, \quad L_{(2)}^{\kappa\lambda\mu\nu} = \delta^{\kappa\mu}\delta^{\lambda\nu}, \quad L_{(3)}^{\kappa\lambda\mu\nu} = \delta^{\kappa\nu}\delta^{\lambda\mu}$$

$$C_{abcd}^{(1)} = \delta_{ab}\delta_{cd}, \quad C_{abcd}^{(2)} = \delta_{ac}\delta_{bd}, \quad C_{abcd}^{(3)} = \delta_{ad}\delta_{bc},$$

$$C_{abcd}^{(4)} = f_{abn}f_{cdn}, \quad C_{abcd}^{(5)} = f_{acn}f_{bdn}.$$

$$B_1 = L^{(1)}C_{(1)} \quad B_2 = L^{(1)}C_{(2)} \quad B_3 = L^{(1)}C_{(3)} \quad B_4 = L^{(1)}C_{(4)}$$

$$B_5 = L^{(1)}C_{(5)} \quad B_6 = L^{(2)}C_{(1)} \quad B_7 = L^{(2)}C_{(2)} \quad B_8 = L^{(2)}C_{(3)}$$

$$B_9 = L^{(2)}C_{(4)} \quad B_{10} = L^{(2)}C_{(5)} \quad B_{11} = L^{(3)}C_{(1)} \quad B_{12} = L^{(3)}C_{(2)}$$

$$B_{13} = L^{(3)}C_{(3)} \quad B_{14} = L^{(3)}C_{(4)} \quad B_{15} = L^{(3)}C_{(5)},$$



$$V_1 = \frac{1}{108N_c^2(N_c^2 - 1)} \left( -B_4 + 2B_5 + 2B_9 + -B_{10} - B_{14} - B_{15} \right)$$

$$V_2 = \frac{1}{48N_c^4 - 120N_c^2 + 72} \left( B_1 + \frac{2}{3N_c}B_4 - \frac{4}{3N_c}B_5 + B_7 - \frac{4}{3N_c}B_9 \right. \\ \left. + \frac{2}{3N_c}B_{10} + B_{13} + \frac{2}{3N_c}B_{14} + \frac{2}{3N_c}B_{15} \right)$$

$$V_3 = \frac{1}{216(N_c^6 - 4N_c^4 + N_c^2 + 4)} \\ \left( \frac{N_c^2 + 6}{3 - 2N_c^2}B_1 + B_2 + B_3 + \frac{2(N_c^2 + 1)}{3N_c - 2N_c^3}B_4 + \frac{4(N_c^2 - 1)}{N_c(2N_c^2 - 3)}B_5 \right. \\ \left. + B_6 + \frac{N_c^2 + 6}{3 - 2N_c^2}B_7 + B_8 + \frac{4(N_c^2 + 1)}{N_c(2N_c^2 - 3)}B_9 + \frac{2(N_c^2 - 1)}{3N_c - 2N_c^3}B_{10} \right. \\ \left. + B_{11} + B_{12} + \frac{N_c^2 + 6}{3 - 2N_c^2}B_{13} + \frac{2(N_c^2 + 1)}{3N_c - 2N_c^3}B_{14} + \frac{2(N_c^2 + 1)}{3N_c - 2N_c^3}B_{15} \right).$$