The running coupling from the four-gluon interaction in Yang-Mills theory

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Outline

1. Basics
2. The truncation scheme
3. Results
4. Summary/Outlook

QCD

QCD: Confinement/DχSB $\rightarrow$ non-perturbative methods mandatory

- method here: Dyson-Schwinger-Equations (DSE)
- advantage: DSEs provide equations for determining 1PI correlation functions
  - $\rightarrow$ no series expansion
  - $\rightarrow$ easy investigation of small momenta (in contrast to lattice QFT)
  - $\rightarrow$ unique gauge fixing possible (Gribov copies)
- disadvantage: DSEs form a coupled system of infinitely many equations $\rightarrow$ needs truncation

The running coupling of the four-gluon vertex

- The 4g-vertex is part of the QCD-Lagrangian
  $\longrightarrow$ has its own renormalisation constant
  $\longrightarrow$ allows for the definition of a running coupling via a Slavnov-Taylor-Identity (STI)
- does it make any difference?
- if so: are there consequences for gluon-confinement?

Yang-Mills theory and Landau gauge

- Yang-Mills theory $\Rightarrow$ No Quarks!
- gauge-fixing via Faddeev-Popov procedure introduces ghost fields and gauge-fixing parameter.

The Yang-Mills Lagrangian:

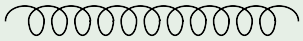
$$\mathcal{L}_{YM}[A,c,\bar{c}] = \frac{1}{4}F_{\mu\nu}^2 + \frac{(\partial_\mu A_\mu^a)^2}{2\xi} - i\bar{c}(\partial_\mu D_\mu^{ab})c$$

$$D_\mu^{ab} = \partial_\mu \delta^{ab} + g\, f^{abc} A_\mu^c$$

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g\, f^{abc} A_\mu^b A_\nu^c$$

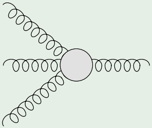
Landau gauge: $\xi \to 0$

Definitions



$$= Z(p^2) \cdot D^{\mu\nu}_{ab}(p^2)$$

$$= G(p^2) \cdot D^{G}_{ab}(p^2)$$



$$=^{4gl} \Gamma(p^2) \cdot \Gamma^{\kappa\lambda\mu\nu}_{abcd}$$

$$D^{\mu\nu}_{ab}(p^2) = \left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2} \right) \frac{1}{p^2} \delta_{ab}$$

$$D^{G}_{ab}(p^2) = -\frac{1}{p^2} \delta_{ab}$$

$$\Gamma^{\kappa\lambda\mu\nu}_{abcd} = ?$$

The tensor-basis

$$C^{(1)}_{abcd} = \delta_{ab}\delta_{cd}\,, \quad C^{(2)}_{abcd} = \delta_{ac}\delta_{bd}\,, \quad C^{(3)}_{abcd} = \delta_{ad}\delta_{bc}\,,$$
$$C^{(4)}_{abcd} = f_{abn}f_{cdn}\,, \quad C^{(5)}_{abcd} = f_{acn}f_{dbn}$$

$$L^{\kappa\lambda\mu\nu}_{(1)} = \delta^{\kappa\lambda}\delta^{\mu\nu} \quad L^{\kappa\lambda\mu\nu}_{(2)} = \delta^{\kappa\mu}\delta^{\lambda\nu} \quad L^{\kappa\lambda\mu\nu}_{(3)} = \delta^{\kappa\nu}\delta^{\lambda\mu}$$

- contains the perturbative vertex
- does not create new structures in lower DSEs or Bethe-Salpeter equations
- Bose-symmetrisation!

parametrisation:

$$^{4Gl}\Gamma^{\kappa\lambda\mu\nu}_{abcd} = \sum_{j=1}^{3} \Gamma_j(p_1,p_2,p_3)\, T^{\kappa\lambda\mu\nu}_{abcd,j}$$

The definition of the running coupling from the STI

The STI for the four-gluon vertex:

$$Z_4 = Z_g^2 Z_3^2$$

Renormalisation $\alpha(\Lambda^2) \to Z_g^2\, \alpha(\mu^2)$:

$$\alpha(\Lambda^2) = \frac{Z_4(\mu^2, \Lambda^2)}{Z_3^2(\mu^2, \Lambda^2)} \alpha(\mu^2)$$

$$\begin{aligned} Z_0(p^2, \Lambda^2) &= Z_3(\mu^2, \Lambda^2)\, Z(p^2, \mu^2) \\ {}^{4gl}\Gamma(p^2, \mu^2) &= Z_4(\mu^2, \Lambda^2)\, {}^{4gl}\Gamma_0(p^2, \Lambda^2). \end{aligned}$$

The defintion of the running coupling from the STI

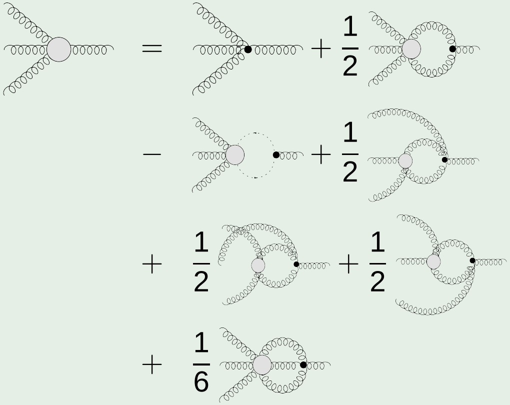
Trading the renormalisation constants for 1PI dressing-functions one obtains

$$\alpha_{4gl}(\Lambda^2)\,{}^{4gl}\Gamma_0(p^2,\Lambda^2)\,Z_0^2(p^2,\Lambda^2) = \alpha(\mu^2)\,{}^{4gl}\Gamma(p^2,\mu^2)\,Z^2(p^2,\mu^2).$$

Renormalising at $\mu^2 = p^2$ with ${}^{4gl}\Gamma(p^2,p^2)\,Z^2(p^2,p^2) = 1$ leads to

$$\alpha(p^2) = \alpha(\mu^2)\,{}^{4gl}\Gamma(p^2,\mu^2)\,Z^2(p^2,\mu^2)$$

The four-gluon-DSE

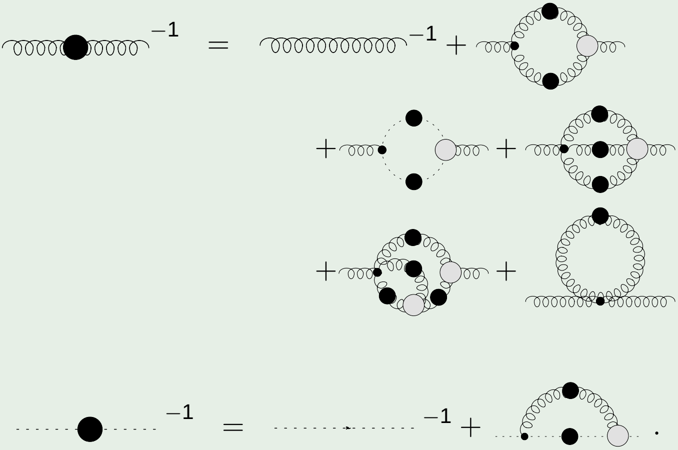


Driesen, Stingl, Eur.Phys.J.A4:401-419,1999.

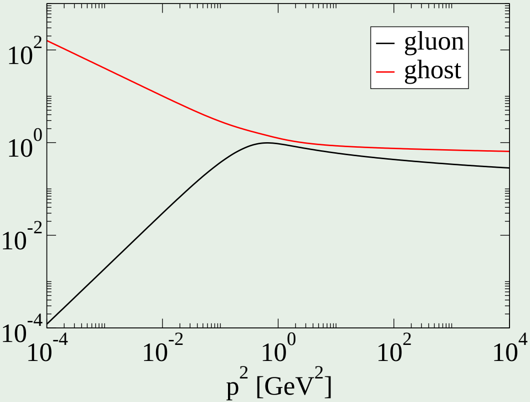
The truncation scheme

- Landau-gauge, Yang-Mills-theory (no quarks)
- no self-consistency $\rightarrow$ neglect all 4-point- and higher correlations
- influence of higher correlations via an effective three-gluon vertex-model (anomalous dimension in the UV)
- bare ghost-gluon vertex
  (Maas, Cucchieri, Mendes
  Braz.J.Phys.37N1B:219-225,2007
  Schleifenbaum, Maas, Wambach, Alkofer
  Phys.Rev.D72:014017,2005)
- identification of the leading IR/UV diagrams via dressed skeleton-expansion

The ghost-gluon-system in Landau gauge



Numerical solution of the coupled ghost-gluon DSEs



Fischer, Alkofer Phys.Lett.B536:177-184,2002

IR behaviour of Landau-gauge YM-theory

A general IR-solution for the whole tower of coupled DSEs has been found:

$$\Gamma^{n,m}(p^2 \to 0) = C \left(p^2\right)^{\left(\frac{n}{2}-m\right)\kappa}, \qquad \kappa \approx 0.595$$

n ghost-legs, m gluon-legs

Alkofer, Fischer, Llanes-Estrada *Phys.Lett*. **B611** (2005) 279-288

For completeness: the quark-gluon vertex:

$$\Gamma^{\mu}_{q-g}\left(p_1^2 \sim p_2^2 = p \to 0\right) = \tilde{C}\left(p^2\right)^{-\frac{1}{2}-\kappa}$$

Alkofer, Fischer, Llanes-Estrada, Schwenzer (2008)
to appear in Annals of Physics (arXiv: 0804.3042)

The effective 3-gluon-vertex

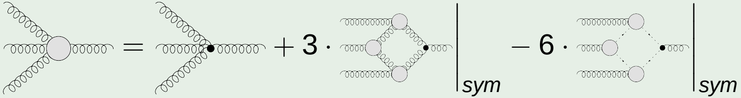
$$ {}^{3g}\Gamma^{ab}_{\lambda\mu\nu}(q,p) = \frac{G(q^2)^{(-\frac{1}{6}-\delta)}}{Z(q^2)^{(\frac{5+3\delta}{6})}} \frac{G(p^2)^{(-\frac{1}{6}-\delta)}}{Z(p^2)^{(\frac{5+3\delta}{6})}} \, {}^{3g}_{(0)}\Gamma^{ab}_{\lambda\mu\nu}(q,p) $$

$\delta = -9/44$: ghost one-loop anomalous dimension

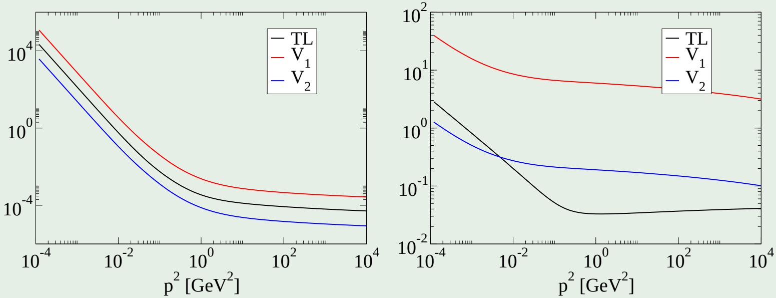
- preserves correct UV behaviour known from resummed perturbation theory $\rightarrow$ asymptotic freedom
- preserves consistent IR power-law behaviour

The approximate equation

The truncations scheme leads to the following approximate equation for the DSE:



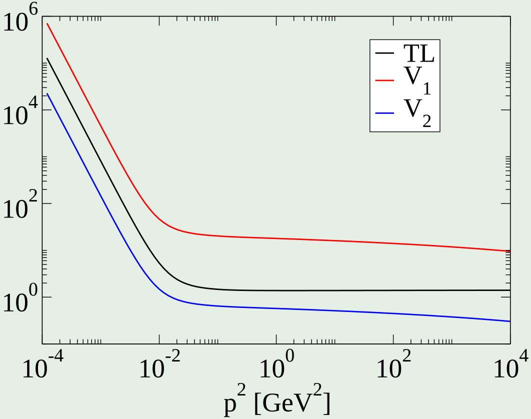
The dressing functions ($p_1 = p_2 = p_3 = p$):



(a) GhostBox (b) GluonBox

Results

The dressing-functions ($p_1 = p_2 = p_3 = p$):



Results

- IR: all tensor-structures show predicted power-law behaviour

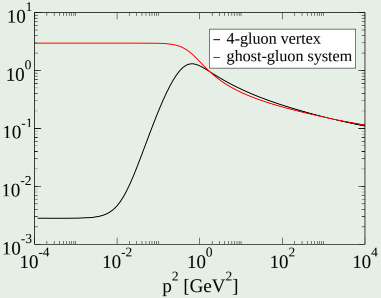
$$\Gamma(p^2 \to 0) = A\,(p^2)^{4\kappa}, \qquad \kappa = 0.595353\ldots$$

(Alkofer,Fischer,Llanes-Estrada Phys. Lett. B611 (2005) 279-288)

- UV: correct anomalous dimension for the perturbative structure
- UV: the other dressing functions vanish (slowly) $\rightarrow$ perturbative structure leads for very large momenta

Results

The running coupling



$$\alpha_{4gl}(p^2 \to 0) = 0.00285$$

$$\alpha_{g/gl}(p^2 \to 0) = 2.97$$

Fischer, Alkofer
Phys. Lett. B536 (2002) 177-184

$$\alpha_{3gl}(p^2 \to 0) = 0.00134$$

K. Schwenzer
priv.comm.

The coupling from the ghost-gluon-vertex is three orders of magnitude larger than the couplings from the other superficially divergent vertices!

Summary/Outlook

- non-trivial IR-fixed-point
- three orders of magnitude smaller than α_{g-gl}
- future: improvement of the truncation (self-consistence!)

Thank you!

The derivation of the non-perturbative coupling

For the running coupling one usually defines

$$\alpha = \frac{g^2}{4\pi}.$$

Renormalisation of the theory:

$$\begin{aligned} A^a_\mu &\rightarrow \sqrt{Z_3}\, A^a_\mu \\ \bar{c}^a\, c^b &\rightarrow \tilde{Z}_3\, \bar{c}^a\, c^b \\ g &\rightarrow Z_g\, g \end{aligned}$$

The derivation of the non-perturbative coupling

The renormalisation constants are not independent. Their relation is described with the Slavnov-Taylor-Identities (STI).

The STI of interest here is

$$Z_4 = Z_g^2 Z_3^2.$$

Putting all together one finds

$$\alpha(\Lambda^2) = \frac{Z_4(\mu^2, \Lambda^2)}{Z_3^2(\mu^2, \Lambda^2)} \alpha(\mu^2).$$

The derivation of the non-perturbative coupling

The renormalisation of the gluon-propagator and the four-gluon vertex is done via:

$$\begin{aligned} Z_0(p^2,\Lambda^2) &= Z_3(\mu^2,\Lambda^2)\,Z(p^2,\mu^2) \\ {}^{4gl}\Gamma(p^2,\mu^2) &= Z_4(\mu^2,\Lambda^2)\,{}^{4gl}\Gamma_0(p^2,\Lambda^2). \end{aligned}$$

$$\alpha(\Lambda^2)\,{}^{4gl}\Gamma_0(p^2,\Lambda^2)Z_0^2(p^2,\Lambda^2) = \alpha(\mu^2)\,{}^{4gl}\Gamma(p^2,\mu^2)Z^2(p^2,\mu^2)$$

Renormalisation condition:

$${}^{4gl}\Gamma(p^2,p^2)\,Z^2(p^2,p^2) = 1$$

$$L_{(1)}^{\kappa\lambda\mu\nu} = \delta^{\kappa\lambda}\delta^{\mu\nu}, \quad L_{(2)}^{\kappa\lambda\mu\nu} = \delta^{\kappa\mu}\delta^{\lambda\nu}, \quad L_{(3)}^{\kappa\lambda\mu\nu} = \delta^{\kappa\nu}\delta^{\lambda\mu}$$

$$C_{abcd}^{(1)} = \delta_{ab}\delta_{cd}, \quad C_{abcd}^{(2)} = \delta_{ac}\delta_{bd}, \quad C_{abcd}^{(3)} = \delta_{ad}\delta_{bc},$$
$$C_{abcd}^{(4)} = f_{abn}f_{cdn}, \quad C_{abcd}^{(5)} = f_{acn}f_{bdn}.$$

$$\begin{array}{llll} B_1 = L^{(1)}C_{(1)} & B_2 = L^{(1)}C_{(2)} & B_3 = L^{(1)}C_{(3)} & B_4 = L^{(1)}C_{(4)} \\ B_5 = L^{(1)}C_{(5)} & B_6 = L^{(2)}C_{(1)} & B_7 = L^{(2)}C_{(2)} & B_8 = L^{(2)}C_{(3)} \\ B_9 = L^{(2)}C_{(4)} & B_{10} = L^{(2)}C_{(5)} & B_{11} = L^{(3)}C_{(1)} & B_{12} = L^{(3)}C_{(2)} \\ B_{13} = L^{(3)}C_{(3)} & B_{14} = L^{(3)}C_{(4)} & B_{15} = L^{(3)}C_{(5)}, & \end{array}$$

$$V_1 = \frac{1}{108N_c^2\left(N_c^2-1\right)}\Big(-B_4+2B_5+2B_9+-B_{10}-B_{14}-B_{15}\Big)$$

$$V_2 = \frac{1}{48N_c^4-120N_c^2+72}\Big(B_1+\frac{2}{3N_c}B_4-\frac{4}{3N_c}B_5+B_7-\frac{4}{3N_c}B_9 +\frac{2}{3N_c}B_{10}+B_{13}+\frac{2}{3N_c}B_{14}+\frac{2}{3N_c}B_{15}\Big)$$

$$V_3 = \frac{1}{216\left(N_c^6-4N_c^4+N_c^2+4\right)} \Big(\frac{N_c^2+6}{3-2N_c^2}B_1+B_2+B_3+\frac{2(N_c^2+1)}{3N_c-2N_c^3}B_4+\frac{4(N_c^2-1)}{N_c(2N_c^2-3)}B_5 +B6+\frac{N_c^2+6}{3-2N_c^2}B_7+B_8+\frac{4(N_c^2+1)}{N_c(2N_c^2-3)}B_9+\frac{2(N_c^2-1)}{3N_c-2N_c^3}B_{10} +B_{11}+B_{12}+\frac{N_c^2+6}{3-2N_c^2}B_{13}+\frac{2(N_c^2+1)}{3N_c-2N_c^3}B_{14}+\frac{2(N_c^2+1)}{3N_c-2N_c^3}B_{15}\Big).$$