S wave interactions between charmed mesons and Goldstone bosons from chiral dynamics

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Based on the following two papers:
1) F.-K. Guo, C. Hanhart, S. Krewald and Ulf-G. Meißner, Phys. Lett. B **666** (2008) 251
2) F.-K. Guo, C. Hanhart and Ulf-G. Meißner, arXiv:0901.1597 [hep-ph]

What and how do we know about the pseudoscalar Goldstone boson–charmed meson interactions?

- Experimental fact: a scalar meson $D_{s0}^*(2317)$ with a mass of 2317.8 MeV.

  — DK bound state? $\Rightarrow$ test it!

  $\Rightarrow$ Experimental side: $\Gamma(D_{s0}^*(2317) \to D_s\pi^0)$

  $\Rightarrow$ Lattice side: S wave scattering lengths !

  some channels were reported recently

  L.Liu, H.W. Lin and K. Orginos, arXiv:0810.5412[hep-lat]

- Theoretical tools: heavy quark symmetry + chiral symmetry

  G.Burdman, J.F.Donoghue, M.B.Wise, T.M.Yan, H.Y.Cheng,...

I. Width of the $D_{s0}^*(2317)$ in the hadronic molecular picture

Based on

Subleading contributions to the width of the $D_{s0}^*(2317)$

F.-K. Guo, C. Hanhart, S. Krewald and Ulf-G. Meißner, Phys. Lett. B **666** (2008) 251

$D^*_{s0}(2317)$

- First observed in the $D_s\pi$ final state
(B.Aubert, *et al.* [BABAR Collaboration], Phys. Rev. Lett. 90 (2003) 242001)

$$M(D^*_{s0}(2317)) = 2317.8 \pm 0.6 \text{ MeV}, \quad \Gamma < 3.8 \text{ MeV}$$

- $I(J^P) = 0(0^+)$
- Much lower than the predictions of the lowest 0^+ $c\bar{s}$ state in most quark models
- No accurate measurement on the branching ratios
- $D^*_{s0}(2317) \to D_s\pi^0$ violates isospin symmetry

The nature of the $D^*_{s0}(2317)$ is still in debate:

$c\bar{s}$? tetraquark? DK molecule?

A consistent treatment of the mass and width is required.
Mass alone is not so useful to tell us the nature!

(see E.van Beveren,G.Rupp(2003), D.S.Hwang,D.W.Kim(2004),Yu.A.Simonov,J.A.Tjon(2004), Guo,Krewald,Meißner(2008)...)

Isospin violation

- Isospin violation has two sources (QCD+QED):

  QCD: $m_u - m_d$

  QED: virtual photons, $\propto e^2$

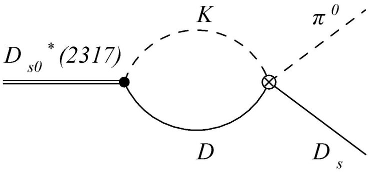
- Both are small and of the same order
  (e.g. $m_p - m_n \sim -2.1\ \mathrm{MeV(QCD)} + 0.8\ \mathrm{MeV(QED)}$ see Gasser, Leutwyler, Phys. Rept. 87 (1982) 77)

  $\Rightarrow$ A systematic study must include both these simultaneously

- Chiral perturbation theory with virtual photons is the tool to analyze structures of the spontaneously and explicitly broken chiral symmetry of QCD

  Weinberg, Gasser, Leutwyler, Urech, Neufeld, Knecht, Meißner, Müller, Steininger, ...

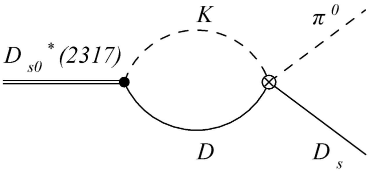
Isospin violation — width of the $D^*_{s0}(2317)$



Assuming the $D^*_{s0}(2317)$ to be an $I = 0$ hadronic molecule (mainly DK), sources for isospin violating decay $D^*_{s0}(2317) \to D_s\pi^0$:

- π^0–η mixing $\Rightarrow \Gamma \sim 10$ keV (Guo et al(2006)) ~width assuming $c\bar{s}$ (S.Godfrey(2003))
- Meson mass differences $M_{K^+} - M_{K^0}$ and $m_{D^+} - m_{D^0}$
  $\Gamma = 79.3 \pm 32.6$ keV A.Faessler et al., Phys. Rev. D 76 (2007) 014005
  $\Gamma \approx 140$ keV M.Lutz,M.Soyeur, Nucl. Phys. A 813 (2008) 14

Isospin violation — width of the $D_{s0}^*(2317)$



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  $\Gamma \approx 140$ keV M.Lutz,M.Soyeur, Nucl. Phys. A 813 (2008) 14
- Electromagnetic corrections!

LO Lagrangian

The leading order Lagrangian:

$$\mathcal{L}^{(1)} = \mathcal{D}_\mu D \mathcal{D}^\mu D^\dagger - m_D^2 D D^\dagger$$

with $D = (D^0, D^+, D_s^+)$ denoting the D–mesons, and the covariant derivative being

$$\begin{aligned} \mathcal{D}_\mu &= \partial_\mu + \Gamma_\mu, \\ \Gamma_\mu &= \frac{1}{2}\left(u^\dagger \partial_\mu u + u \partial_\mu u^\dagger\right), \end{aligned}$$

where

$$U = \exp\left(\frac{\sqrt{2} i \phi}{F}\right), u^2 = U, \phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}.$$

G. Burdman and J. F. Donoghue, Phys. Lett. B 280 (1992) 287; M. B. Wise, Phys. Rev. D 45 (1992) 2188; T. M. Yan, H. Y. Cheng, C. Y. Cheung, G. L. Lin, Y. C. Lin and H. L. Yu, Phys. Rev. D 46, 1148 (1992) [Erratum-ibid. D 55, 5851 (1997)].

NLO Lagrangians: Strong + em.

The strong part is

$$\begin{aligned}\mathcal{L}^{(2)}_{\rm str.} &= D\left(-h_0\langle\chi_+\rangle - h_1\tilde{\chi}_+ + h_2\langle u_\mu u^\mu\rangle - h_3 u_\mu u^\mu\right)\bar{D} \\ &\quad + \mathcal{D}_\mu D\left(h_4\langle u^\mu u^\nu\rangle - h_5\{u^\mu, u^\nu\} - h_6[u^\mu, u^\nu]\right)\mathcal{D}_\nu\bar{D},\end{aligned}$$

and the electromagnetic part is

$$\mathcal{L}^{(2)}_{\rm e.m.} = F^2 D\left[g_0\left(Q_+^2 - Q_-^2\right) + g_1\left\langle Q_+^2 - Q_-^2\right\rangle + g_2 Q_+\langle Q_+\rangle + g_3\langle Q_+\rangle^2\right]\bar{D},$$

where

$$\begin{aligned}\chi &= 2B\cdot{\rm diag}\{m_u, m_d, m_s\}, \quad Q = e\cdot{\rm diag}\{0,1,1\}, \\ \chi_+ &= u^\dagger\chi u^\dagger + u\chi u, \quad \tilde{\chi}_+ = \chi_+ - \frac{1}{3}\langle\chi_+\rangle, \\ u_\mu &= iu^\dagger\mathcal{D}_\mu U u^\dagger, \quad Q_\pm = \frac{1}{2}\left(u^\dagger Q u \pm u Q u^\dagger\right).\end{aligned}$$

The h_0, h_1 terms have been introduced in H.Y.Cheng et al.(1994) & M.Lutz et al.(2004), h_2, h_3 terms introduced in M.Lutz et al. (2004).

Low energy constants

$$\mathcal{L}^{(2)}_{\text{str.}} = D\left(-h_0\langle\chi_+\rangle - h_1\tilde{\chi}_+ + h_2\langle u_\mu u^\mu\rangle - h_3 u_\mu u^\mu\right)\bar{D} + \mathcal{D}_\mu D\left(h_4\langle u^\mu u^\nu\rangle - h_5\{u^\mu, u^\nu\} - h_6[u^\mu, u^\nu]\right)\mathcal{D}_\nu\bar{D},$$

$$\mathcal{L}^{(2)}_{\text{e.m.}} = F^2 D\left[g_0\left(Q_+^2 - Q_-^2\right) + g_1\left\langle Q_+^2 - Q_-^2\right\rangle + g_2 Q_+\langle Q_+\rangle + g_3\langle Q_+\rangle^2\right]\bar{D}.$$

h_0, h_2, h_4: large N_c suppressed (M.Lutz et al. (2007))

h_6: suppressed due to the commutator

g_3: an overall em mass shift of the D–mesons

g_1: isospin symmetric em interaction, irrelevant

h_1 and $g_0 + 2g_2$: fixed by $m_{D^+} - m_{D^0}$, $m_{D_s^+} - m_{D^+}$

$$h_1 = 0.42 \pm 0.00, \quad g_0 + 2g_2 = 11 \pm 3.$$

Unitarization of the scattering amplitudes

In particle basis, there are four channels:

$$D^0 K^+, D^+ K^0, D_s^+ \eta, \text{ and } D_s^+ \pi^0$$

Unitarization of the scattering amplitudes to NLO:

$$T(s) = V(s) \left[1 - G(s) \cdot V(s)\right]^{-1}$$

with $V(s) = V_{\rm LO}(s) + V_{\rm NLO}(s)$ the sum of the S–wave scattering amplitudes of the LO and NLO orders, $G(s)$ the two–meson loop functions containing one parameter, a subtraction constant.

(J.A. Oller, Ulf-G. Meißner, Phys. Lett. B 500 (2001) 263)

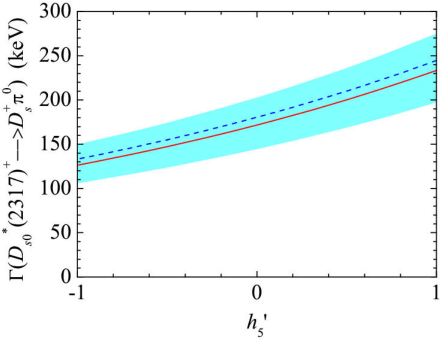
Pole of $T(s)$ in unphysical Riemann sheet:

$$m(D_{s0}^*) - i\Gamma(D_{s0}^* \to D_s \pi^0)/2$$

Regulator $a(1\ \mathrm{GeV}) = -1.846$ is fixed to reproduce $M_{D_{s0}^*} = 2317.8$ MeV at LO

Width of the $D^*_{s0}(2317)$ as a criterion for testing its nature

$h'_5 \equiv h_5 m^2_{D^0} \in [-1, 1]$: naturalness
h_3: determined from the mass of the $D^*_{s0}(2317)$ for each h'_5.



$\Gamma(D^{*+}_{s0} \to D^+_s \pi^0) = 180 \pm 40(\mathrm{exp}) \pm 100(\mathrm{th})$ keV

Consistent with

- A.Faessler et al. (2007): 79.3 ± 32.6 keV
- M.Lutz et al. (2008): 140 keV

Solid curve: result with central values of all parameters
Dashed curve: result without virtual photons

Compare with $c\bar{s}$, tetraquark: $\sim$ 10 keV (S.Godfrey(2003), M.Nielsen(2006),...)

$\Gamma(D^*_{s0}(2317)^+ \to D^+_s \pi^0)$ could be a criterion for testing the nature of the $D^*_{s0}(2317)$

II. *S* wave charmed-meson–Goldstone-boson scattering lengths

Based on

Interactions between heavy mesons and Goldstone bosons from chiral dynamics

F.-K. Guo, C. Hanhart and Ulf-G. Meißner, arXiv:0901.1597 [hep-ph]

CM–GB scattering lengths

- The S-wave scattering length parameterizes the scattering amplitude at threshold

$$a_0 = -\frac{1}{8\pi(M_1 + M_2)} T(s)\Big|_{s=(M_1+M_2)^2}$$

- Exp: No experimental data
- Lattice:

  1) Direct simulations for $I = 1\ D\bar{K}$, $I = \frac{3}{2}\ D\pi$, $D_s\pi$, D_sK

  L.Liu, H.W. Lin and K. Orginos, arXiv:0810.5412[hep-lat]

  2) Indirect extraction of $a^{D\pi}_{0,I=1/2}$ & $a^{DK}_{0,I=0}$ from lattice calculations of the heavy-light form factors in semileptonic decays:

  J.M. Flynn and J. Nieves, Phys. Rev. D **75** (2007) 074024

Isospin symmetric scattering amplitudes

- Lagrangians

$$\mathcal{L}^{(1)} = \mathcal{D}_\mu D \mathcal{D}^\mu D^\dagger - m_D^2 D D^\dagger$$
$$\mathcal{L}^{(2)}_{\text{str.}} = D\left(-h_0\langle\chi_+\rangle - h_1\tilde{\chi}_+ + h_2\langle u_\mu u^\mu\rangle - h_3 u_\mu u^\mu\right)\bar{D} + \mathcal{D}_\mu D\left(h_4\langle u^\mu u^\nu\rangle - h_5\{u^\mu, u^\nu\} - h_6[u^\mu, u^\nu]\right)\mathcal{D}_\nu \bar{D}$$

h_0, h_2, h_4, h_6 terms suppressed

- Scattering amplitudes at threshold to NLO

$$T_{\text{thr.}} = \frac{1}{F^2}\left[C_0 M_1 M_2 + \frac{2C_1}{3}h_1 + 2C_{35}\left(h_3 M_2^2 + 2h_5 M_1^2 M_2^2\right)\right]$$

M_1 : mass of charmed meson, M_2 : mass of GB; C_0, C_1, C_{35}: coefficients

★The most attractive interaction happens in the $(S, I) = (1, 0)$ DK channel!

$C_0 = -2$ for this channel

- Unitarized amplitude:

$$T(s) = V(s)[1 - G(s)\cdot V(s)]^{-1}$$

Predictions

- The S wave scattering length for the $(S, I) = (1, 0)$ DK channel:

$(1,0)$ DK	LO	NLO	UChPT	CUChPT
a (fm)	0.72	0.67 ± 0.04	-1.47 ± 0.20	-0.93 ± 0.05

UChPT: One–channel unitarized ChPT   CUChPT: Coupled–channel unitarized ChPT

⋆ Scattering length changes sign ⇒ presence of a bound state ⇒ $D_{s0}^*(2317)$

⋆ Weinberg's model-independent formula in the limit of small binding energy:

$$a_0 = -2\frac{1-Z}{2-Z}\frac{1}{\sqrt{2\mu\epsilon}} \Rightarrow a_{0,I=0}^{DK} = -1.05 \text{ fm} \leftarrow \text{Assuming } DK \text{ bound state}$$

Z: wave function renormalization constant; μ: reduced mass; ϵ: binding energy

$Z = 0$ for a pure bound state and $Z = 1$ for a pure elementary state

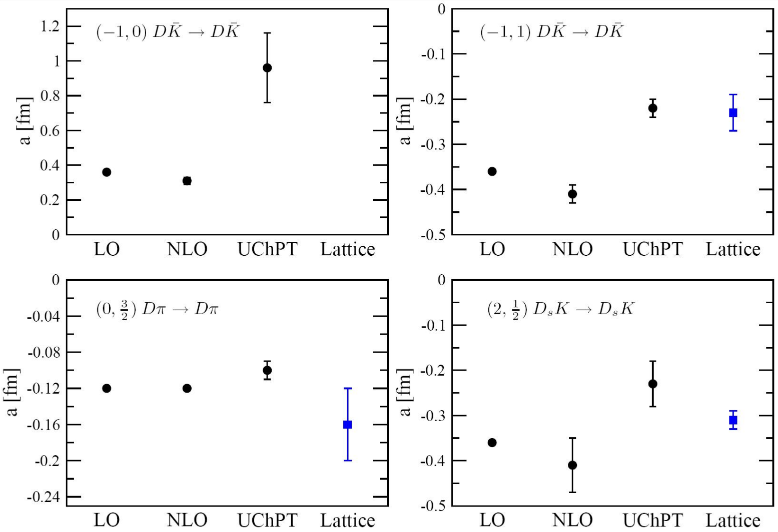
S.Weinberg, Phys. Rev. 137 (1965) B672; V.Baru *et al.* Phys. Lett. B 586 (2004) 53

- The S wave scattering length for the $(S, I) = (0, \frac{1}{2})$ $D\pi$ channel:

$(0, \frac{1}{2})$ $D\pi$	LO	NLO	UChPT	CUChPT
a (fm)	0.24	0.23 ± 0.00	0.36 ± 0.01	0.35 ± 0.01

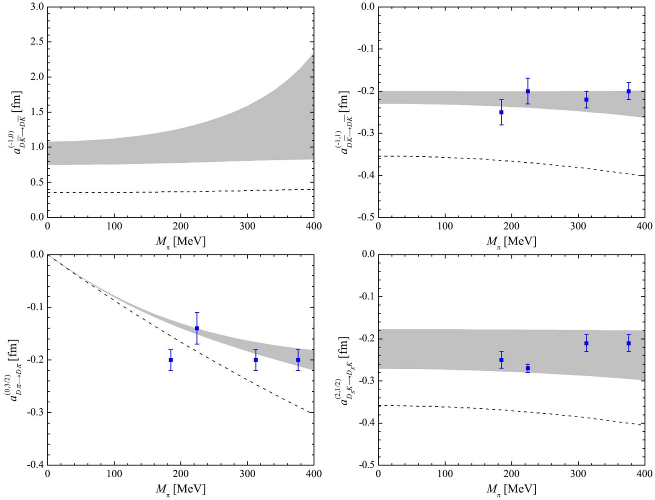
Cf. $a_0^{I=1/2\ D\pi} = 0.41 \pm 0.06$ fm   J.M. Flynn and J. Nieves, Phys. Rev. D **75** (2007) 074024

Comparison with the lattice data



Lattice data: L.Liu, H.W. Lin and K. Orginos, arXiv:0810.5412[hep-lat]

Chiral extrapolation using UChPT



The bands reflect the remaining freedom in the choice of the parameters. The dashed lines show the LO results.

Summary

- Width of the $D_{s0}^*(2317)$
  - ⋆ chiral effective Lagrangians at NLO for both str. and em interactions
  - ⋆ the $D_{s0}^*(2317)$ as a pole of the unitarized scattering amplitude
  - ⋆ width of the $D_{s0}^*(2317)$ as a hadronic molecule is 180 ± 110 keV, much larger than that obtained assuming the $c\bar{s}$ or 4-q structure
- S wave Goldstone boson–D-meson scattering lengths
  - ⋆ systematically using ChPT to NLO and a unitarized version of it
  - ⋆ in the (1,0) DK channel, consistent with Weinberg's formula for a bound state $\Rightarrow$ $D_{s0}^*(2317)$ as a bound state of DK
  - ⋆ chiral extrapolation
- Suggestions towards identifying the nature of the $D_{s0}^*(2317)$
  - ⋆ To experimental community: to measure $\Gamma(D_{s0}^*(2317) \to D_s\pi^0)$
  - ⋆ To lattice community: to calculate the $I = 0$ DK scattering length

Isospin violation — Goldstone bosons

Weinberg, Gasser, Leutwyler, Urech, Neufeld, Knecht, Meißner, Müller, Steininger, ...

- SU(3) ChPT Lagrangian for Goldstone bosons with virtual photons to LO ($\mathcal{O}(p^2)$):

$$\mathcal{L}^{(2)} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{F^2}{4}\left\langle D_\mu U D^\mu U^\dagger + \chi U^\dagger + \chi^\dagger U \right\rangle + C\left\langle QUQU^\dagger \right\rangle$$

♡ $Q =$ light quark charge matrix, $\chi \sim$ quark masses

♡ Last term generates the pion mass difference (to LO, e.m. solely), low energy constant (LEC) C fixed from this

♡ For $m_u = m_d$, $\Rightarrow M_{K^+}^2 - M_{K^0}^2 = M_{\pi^+}^2 - M_{\pi^0}^2 = 2Ce^2/F^2$ (Dashen's theorem)

♡ For $m_u \neq m_d$, LO kaon mass difference

$$\left(M_{K^+}^2 - M_{K^0}^2\right)^{\text{strong}} = B_0(m_d - m_u) + \mathcal{O}(m_q^2)$$

where $B_0 = |\langle 0|\bar{q}q|0\rangle|/F^2$

$$
\begin{aligned}
G(s)_{ii} &= i\int \frac{d^4q}{(2\pi)^4}\frac{1}{\left(q^2-m_1^2+i\epsilon\right)\left[(P-q)^2-m_2^2+i\epsilon\right]} \\
&= \frac{1}{16\pi^2}\left\{a(\mu)+\ln\frac{m_2^2}{\mu^2}+\frac{m_1^2-m_2^2+s}{2s}\ln\frac{m_1^2}{m_2^2}+\frac{\sigma}{2s}\left[\ln(s-m_1^2+m_2^2+\sigma)\right.\right. \\
&\quad \left.\left.-\ln(-s+m_1^2-m_2^2+\sigma)+\ln(s+m_1^2-m_2^2+\sigma)-\ln(-s-m_1^2+m_2^2+\sigma)\right]\right\}
\end{aligned}
$$

with $\sigma=[s-(m_1+m_2)^2][s-(m_1-m_2)^2]^{1/2}$. $a(\mu)$ fixed by reproducing the mass of the $D^*_{s0}(2317)$.

Results at LO

$$F_\pi = 92.42 \text{ MeV}, \quad M_{\pi^0} = 134.98 \text{ MeV}, \quad M_{\pi^+} = 139.57 \text{ MeV},$$
$$M_{K^0} = 497.65 \pm 0.02 \text{ MeV}, \quad M_{K^+} = 493.68 \pm 0.02 \text{ MeV},$$
$$m_{D^0} = 1864.84 \pm 0.17 \text{ MeV}, \quad m_{D^+} = 1869.62 \pm 0.20 \text{ MeV},$$
$$m_{D_s^+} = 1968.49 \pm 0.34 \text{ MeV}, \quad M_\eta = 547.51 \pm 0.18 \text{ MeV},$$
$$\epsilon_{\pi^0\eta} = 0.010 \pm 0.001.$$

Regulator $a(1 \text{ GeV}) = -1.846$ is fixed to reproduce $M_{D^*_{s0}} = 2317.8$ MeV

	LO	π^0–η mixing	Mass differences
Decay width	149.4 keV	15.0 keV	69.7 keV

- Meson mass differences give a larger contribution than the π^0–η mixing. Consistent with A.Faessler et al. (2007) & M.Lutz et al. (2007)
  $\epsilon_{\pi^0\eta} = 1/\sqrt{3}B(m_d - m_u)/\left(M^2_\eta - M^2_{\pi^0}\right)$ suppressed in SU(2) chiral perturbation theory

Predictions

(S, I)	Channel	LO	NLO	UChPT	CUChPT	Lattice [5]
$(-1,0)$	$D\bar{K} \to D\bar{K}$	0.36	$0.31(2)$	$0.96(20)$		
$(-1,1)$	$D\bar{K} \to D\bar{K}$	-0.36	$-0.41(2)$	$-0.22(2)$		$-0.23(4)$
$(0,\frac{1}{2})$	$D\pi \to D\pi$	0.24	$0.23(0)$	$0.36(1)$	$0.35(1)$	
	$D\eta \to D\eta$	0	$-0.09(1)$	$-0.08(1)$	$0.19(9) + i0.02(2)$	
	$D_s\bar{K} \to D_s\bar{K}$	0.36	$0.31(6)$	$1.10(57)$	$-0.60(53) + i0.77(15)$	
$(0,\frac{3}{2})$	$D\pi \to D\pi$	-0.12	$-0.12(0)$	$-0.10(1)$		$-0.16(4)$
$(1,0)$	$DK \to DK$	0.72	$0.67(4)$	$-1.47(20)$	$-0.93(5)$	
	$D_s\eta \to D_s\eta$	0	$0.00(10)$	$0.02(10)$	$-0.33(4) + i0.05(1)$	
$(1,1)$	$D_s\pi \to D_s\pi$	0	-0.005	-0.005	$-0.0003(4)$	$0.00(1)$
	$DK \to DK$	0	-0.054	-0.049	$-0.04(6) + i0.29(11)$	
$(2,\frac{1}{2})$	$D_sK \to D_sK$	-0.36	$-0.41(6)$	$-0.23(5)$		$-0.31(2)$

UChPT: One–channel unitarized ChPT   CUChPT: Coupled–channel unitarized ChPT

Lattice data: L.Liu, H.W. Lin and K. Orginos, arXiv:0810.5412[hep-lat]

- ⋆ Consistent with the lattice data
- ⋆ Cf. $a_0^{I=1/2\ D\pi} = 0.41 \pm 0.06$ fm   J.M. Flynn and J. Nieves, Phys. Rev. D **75** (2007) 074024
- ⋆ Scattering length changes sign ⇒ presence of a bound state ⇒ $D_{s0}^*(2317)$
- ⋆ Uncertainty comes from lack of knowledge of the LECs

Understanding the numbers from S-matrix poles

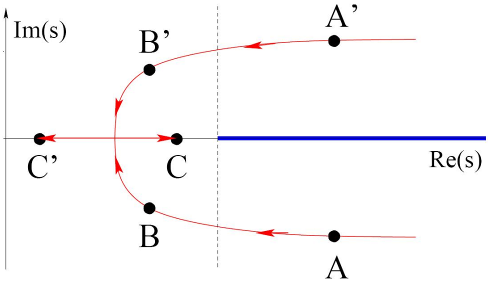
Poles in one-channel unitarized amplitudes (units are MeV):

(S, I)	Channel	Thr	$h'_5 = +1$			$h'_5 = -1$		
			Re	Im	RS	Re	Im	RS
$(-1, 0)$	$D\bar{K} \to D\bar{K}$	2363	2354	± 56	II	2301	± 97	II
$\left(0, \frac{1}{2}\right)$	$D\pi \to D\pi$	2005	2098	± 124	II	2102	± 106	II
	$D_s\bar{K} \to D_s\bar{K}$	2464	2286	± 54	II	2354	0	II
						2431	0	II
$(1, 0)$	$DK \to DK$	2363	2343	0	I	2337	0	I

Poles in coupled-channel unitarized amplitudes (units are MeV):

(S, I)	$h'_5 = +1$			$h'_5 = -1$		
	Re	Im	RS	Re	Im	RS
$\left(0, \frac{1}{2}\right)$	2107	± 123	II	2107	± 105	II
	2452	± 17	III	2519	± 69	III
$\left(0, \frac{1}{2}\right)$ $(V_{ii} = 0)$	2466	± 24	III	2388	± 49	III
$(1, 0)$	2318	0	I	2318	0	I

Pole movement



C.Hanhart, J.R.Pelaez and G.Rios, Phys. Rev. Lett. **100** (2008) 152001

Chiral extrapolation

- Physical light quark masses [using $\overline{\rm MS}$ at $\mu = 2$ GeV]

  $m_u = 1.5 - 3.3$ MeV, $m_d = 3.5 - 6.0$ MeV, $m_s = 104^{+26}_{-34}$ MeV

- Light quark masses used in lattice calculations

  $m_s = 80$ MeV — physical

  $m_{u,d} = 11, 16, 32, 48$ MeV — unphysical $\Rightarrow$ chiral extrapolation needed

- ChPT provides one nice approach to do chiral extrapolation

  ♡ $M_\pi^2 = 2B_0\hat{m} + \mathcal{O}(m_q^2)$ where $\hat{m} = \frac{1}{2}(m_u + m_d)$

  ♡ $M_K = \mathring{M}_K + \frac{M_\pi^2}{4\mathring{M}_K} + \mathcal{O}(M_\pi^4)$, $M_D = \mathring{M}_D + h_1\frac{M_\pi^2}{\mathring{M}_D} + \mathcal{O}(M_\pi^4)$

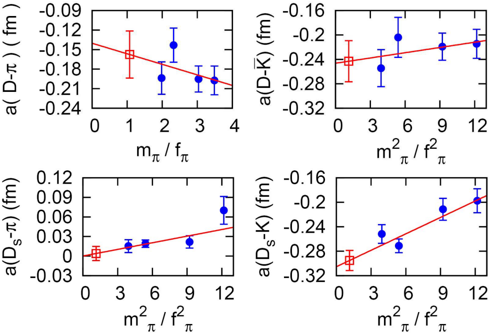
  $\mathring{M}_K$ ($\mathring{M}_D$): kaon (D) mass in SU(2) chiral limit

  ♡ To $\mathcal{O}(M_\pi^2)$, M_{D_s} is independent of M_π $\Rightarrow$ using the physical value

  ♡ LECs (F, h_i) are quark mass independent ay definition

Chiral extrapolation in the lattice paper

L.Liu, H.W. Lin and K. Orginos, arXiv:0810.5412[hep-lat]



⋆ Extrapolation functions:

$a = c_1 + c_2 M_\pi / F_\pi$ for $I = 3/2$ $D\pi$ $\quad a = c_1 + c_2 M_\pi^2 / F_\pi^2$ for $D_s\pi(K)$, $I = 1$ $D\bar{K}$

⋆ Results in the physical world:

(S, I) Channel	$(-1, 1)$ $D\bar{K}$	$\left(0, \frac{3}{2}\right)$ $D\pi$	$(1, 1)$ $D_s\pi$	$\left(2, \frac{1}{2}\right)$ D_sK
a_0 (fm)	$-0.23(4)$	$-0.16(4)$	$0.00(1)$	$-0.31(2)$