The chiral partner of the nucleon in the mirror assignment

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Excited QCD

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JOHANN WOLFGANG GOETHE
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Talk 09.02.09 by D. Parganlija:

$$L_{mes} = Tr[(D^{\mu}\Phi)(D_{\mu}\Phi)] - m^2 Tr[\Phi^{+}\Phi] - \lambda_1 [Tr(\Phi^{+}\Phi)]^2 - \lambda_2 Tr(\Phi^{+}\Phi)^2 +$$

$$c[\det(\Phi^{+}) + \det(\Phi)] + Tr[H(\Phi^{+} + \Phi)] -$$

$$\frac{1}{4} Tr[(L^{\mu\nu})^2 + (R^{\mu\nu})^2] + \frac{m_1^2}{2} Tr[(L^{\mu})^2 + (R^{\mu})^2]$$

+ Baryons:

$$\overline{\Psi}_L i\gamma_\mu \partial^\mu \Psi_L + \overline{g\Psi}_L \gamma_\mu L^\mu \Psi_L + \overline{\Psi}_R i\gamma_\mu \partial^\mu \Psi_R + g\overline{\Psi}_R \gamma_\mu R^\mu \Psi_R$$

$$- \hat{g}\ (\overline{\Psi}_L \Phi \Psi_R + \overline{\Psi}_R \Phi^{+} \Psi_L)$$

Motivation

Introduction of N* for chiral realization of the nucleonic Lagrangian
→ chiral invariant mass term / m_0

Generation of the nucleonic mass:
how is m_N generated?
Quark condensate vs. m_0

Correct description of the axial coupling constant
gA(N)=1.26
gA(N*)=?

Study of pion-nucleon scattering

The chiral partner:

what is N*?

The chiral partner of the nucleon in the mirror assignment

Mirror assignment vs naive assignment: chiral transformation

$$U_L \times U_R \in SU(2)_L \times SU(2)_R$$

Naive

$$\Psi_{1L} \to U_L \Psi_{1L}$$
$$\Psi_{2L} \to U_L \Psi_{2L}$$

$$\Psi_{1R} \to U_R \Psi_{1R}$$
$$\Psi_{2R} \to U_R \Psi_{2R}$$
$$\Downarrow$$

No chiral invariant mass term

2 independent Linear Sigma models

$$\Psi_1 = N$$
$$\Psi_2 = N^*$$
$$\hat{M} = 1_{2x2}$$

Mirror

$$\Psi_{1L} \to U_L \Psi_{1L}$$
$$\Psi_{2L} \to U_R \Psi_{2L}$$

$$\Psi_{1R} \to U_R \Psi_{1R}$$
$$\Psi_{2R} \to U_L \Psi_{2R}$$
$$\Downarrow$$

$$-m_0(\overline{\Psi}_{1L}\Psi_{2R} - \overline{\Psi}_{1R}\Psi_{2L} - \overline{\Psi}_{2R}\Psi_{1L} + \overline{\Psi}_{2L}\Psi_{1R})$$

Nucleon mass not only dynamically generated

$$\hat{M} \neq 1_{2x2}$$

$$\boxed{\begin{pmatrix} N \\ N^* \end{pmatrix} = \hat{M} \begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix}}$$

The chiral partner of the nucleon in the mirror assignment

Candidates with the correct quantum numbers from PDG:

N*(1535) lowest nucleonic resonance
N*(1650)

$$J^P = \frac{1}{2}^-$$

Other possibility (speculative):

N*(1200) (D. Zschiesche, L. Tolos, J. Schaffner-Bielich, R. Pisarski, Phys.Rev.C75:055202,2007)

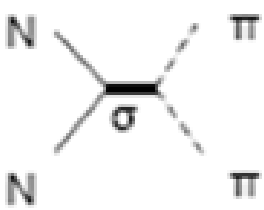
Very broad resonance? Not yet measured?

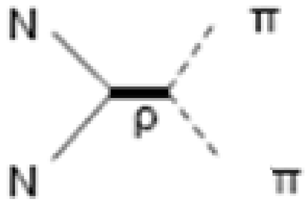
A little bit of scattering theory

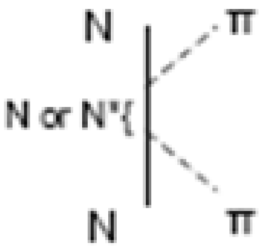
A little bit of scattering theory

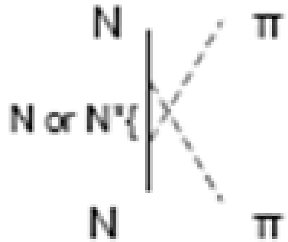
General form of the Lorentz-invariant scattering amplitude: $\bar{u}(p_2)T_{ba}u(p_1)$

$$T_{ba} = \left[A^{(+)} + \frac{1}{2}(q_1^\mu + q_2^\mu)\gamma_\mu B^{(+)} \right]\delta_{ab} + \left[A^{(-)} + \frac{1}{2}(q_1^\mu + q_2^\mu)\gamma_\mu B^{(-)} \right] i\varepsilon_{bac}\tau_c$$









s-wave scattering lengths: $a_0^{(\pm)} = \eta(A_0^{(\pm)} + m_\pi B_0^{(\pm)})$

$$\eta = \frac{1}{4\pi(1+\frac{m_\pi}{m_N})}$$

$$t = 0, s = (m_N + m_\pi)^2, u = (m_N - m_\pi)^2$$

Some important data

H. Schroder: Measured pion nucleon scattering lengths from pionic hydrogen and deuterium at PSI (Eur.Phys.J.C21:473-488,2001)

$$a_0^{(+)} = (-8.85783 \pm 7.165) 10^{-6} MeV^{-1}$$

$$a_0^{(-)} = (6.41256 \pm 0.1146) 10^{-4} MeV^{-1}$$

T. Takahashi and T. Kunihiro: Axial charge of N(1535) in lattice QCD (e-Print: arXiv:0801.4707)

$$g_A^{N^*} = 0.2 \pm 0.3$$

L. Ya. Glozman: Phys.Rept.444:1-49,2007

$$g_A^{N^*} \approx 0 \rightarrow N^* \text{ is not the partner}$$

$$\text{if } N^* \text{ is the partner} \rightarrow g_A^{N^*} \approx -1$$

we shall present a different result

The model

Linear sigma model
with vector and axial vector mesons,
as well as the nucleonic fields
in the mirror assignment
local symmetric case

Spontaneus symmetry breaking

$$\sigma \rightarrow \sigma + \varphi$$

leads to mixing terms in the Lagrangian: $-g\partial_{\mu}\eta\varphi f_1^{\mu} - g\partial_{\mu}\vec{\pi}\varphi \cdot \vec{a}_1^{\mu}$

we need to perform the shifts of the axial fields:

$$\vec{a}_1^{\mu} \rightarrow \vec{a}_1^{\mu} + w\partial^{\mu}\vec{\pi}$$

$$f_1^{\mu} \rightarrow f_1^{\mu} + w\partial^{\mu}\eta$$

$$w = \frac{g\varphi}{m_{a_1} + (g\varphi)^2}$$

and to renormalize the pseudoscalar fields:

$$\frac{1}{Z}\eta \rightarrow \eta \qquad \frac{1}{Z}\vec{\pi} \rightarrow \vec{\pi}$$

$Z = \sqrt{2}$ as predicted by KSFR - relation

$Z = \frac{m_a}{m_\rho}$

S. Gasiorowicz and D.A.Geffen , **Rev.Mod.Phys.41:531-573,1969**.

Mesonic sector of the lagrangian

$$L_{mes} = Tr[(D^{\mu}\Phi)(D_{\mu}\Phi)] - m^2 Tr[\Phi^+\Phi] - \lambda_1[Tr(\Phi^+\Phi)]^2 - \lambda_2 Tr(\Phi^+\Phi)^2 +$$
$$c[\det(\Phi^+) + \det(\Phi)] + Tr[H(\Phi^+ + \Phi)] -$$
$$\frac{1}{4}Tr[(L^{\mu\nu})^2 + (R^{\mu\nu})^2] + \frac{m_1^2}{2}Tr[(L^{\mu})^2 + (R^{\mu})^2]$$

Fields

$$\Phi = (\sigma + i\eta)t_0 + (\vec{a}_0 + i\vec{\pi})\vec{t}$$
$$L^{\mu} = (\omega^{\mu} + f_1^{\mu})t_0 + (\vec{\rho}^{\mu} + \vec{a}_1^{\mu})\vec{t}$$
$$R^{\mu} = (\omega^{\mu} - f_1^{\mu})t_0 + (\vec{\rho}^{\mu} - \vec{a}_1^{\mu})\vec{t}$$

Field strength tensor

$$L^{\mu\nu} = \partial^{\mu}L^{\nu} - \partial^{\nu}L^{\mu} - ig[L^{\mu}, L^{\nu}]$$
$$R^{\mu\nu} = \partial^{\mu}R^{\nu} - \partial^{\nu}R^{\mu} - ig[R^{\mu}, R^{\nu}]$$

$$D^{\mu}\Phi = \partial^{\mu}\Phi + ig(\Phi R^{\mu} - L^{\mu}\Phi)$$

$$+ L_{nucleon}$$

Introducing the chiral partner

$$
\begin{aligned}
L_{nucl} = {} & \overline{\Psi}_{1L} i\gamma_\mu \partial^\mu \Psi_{1L} + g\overline{\Psi}_{1L}\gamma_\mu L^\mu \Psi_{1L} + \overline{\Psi}_{1R} i\gamma_\mu \partial^\mu \Psi_{1R} + g\overline{\Psi}_{1R}\gamma_\mu R^\mu \Psi_{1R} \\
& + \overline{\Psi}_{2L} i\gamma_\mu \partial^\mu \Psi_{2L} + g\overline{\Psi}_{2L}\gamma_\mu R^\mu \Psi_{2L} + \overline{\Psi}_{2R} i\gamma_\mu \partial^\mu \Psi_{2R} + g\overline{\Psi}_{2R}\gamma_\mu L^\mu \Psi_{2R} \\
& - \hat{g}_1(\overline{\Psi}_{1L}\Phi\Psi_{1R} + \overline{\Psi}_{1R}\Phi^+\Psi_{1L}) - \hat{g}_2(\overline{\Psi}_{2L}\Phi^+\Psi_{2R} + \overline{\Psi}_{2R}\Phi\Psi_{2L}) \\
& - m_0(\overline{\Psi}_{1L}\Psi_{2R} - \overline{\Psi}_{1R}\Psi_{2L} - \overline{\Psi}_{2R}\Psi_{1L} + \overline{\Psi}_{2L}\Psi_{1R})
\end{aligned}
$$

$$m_N \sim \langle \overline{q} q \rangle + m_0 \quad ?$$

Mixing angle and mass m_0

$$\begin{pmatrix} N \\ N^* \end{pmatrix} = \hat{M} \begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix}$$

with

$$\hat{M} = \frac{1}{\sqrt{2\cosh\delta}} \begin{pmatrix} e^{\delta/2} & \gamma_5 e^{-\delta/2} \\ \gamma_5 e^{-\delta/2} & -e^{\delta/2} \end{pmatrix}$$

diagonalize $\hat{M} \Rightarrow$ nucleon masses:

$$m_{N,N^*} = \frac{1}{2}\left(\sqrt{(\hat{g}_1 + \hat{g}_2)^2 \varphi^2 + 4m_0^2} \pm (\hat{g}_1 - \hat{g}_2)\frac{\varphi}{2} \right)$$

Mixing angle:

$$\boxed{\delta = ar\cosh\left[\frac{m_N + m_{N^*}}{2m_0}\right]}$$

$$\delta \to \infty \Rightarrow \Psi_1 = N, \Psi_2 = N^*$$

$$\delta \to -\infty \Rightarrow \Psi_1 = N^*, \Psi_2 = N$$

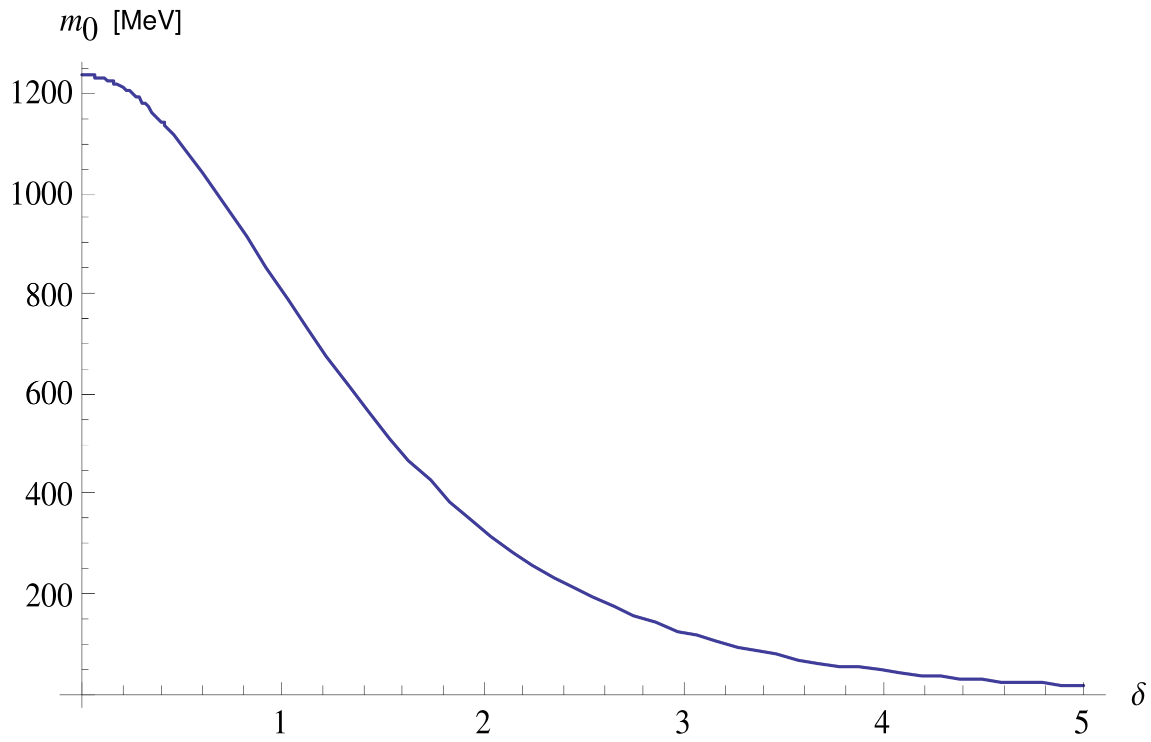
no mixing

$$\delta \to 0 \Rightarrow \Psi_{1,2} = \frac{N \pm N^*}{\sqrt{2}}$$

maximal mixing

Mixing angle and mass m_0

$$m_0 = \frac{\hat{g}_1(m_N, m_{N^*}, \delta) + \hat{g}_2(m_N, m_{N^*}, \delta)}{4\sinh\delta}$$



Linear sigma model
with vector and axial vector
mesons,
as well as the nucleonic
fields
in the mirror assignment
local symmetric case:

Results

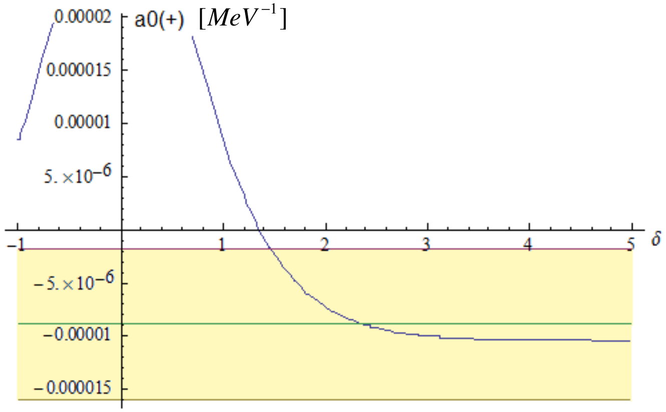
Results: locally symmetric model

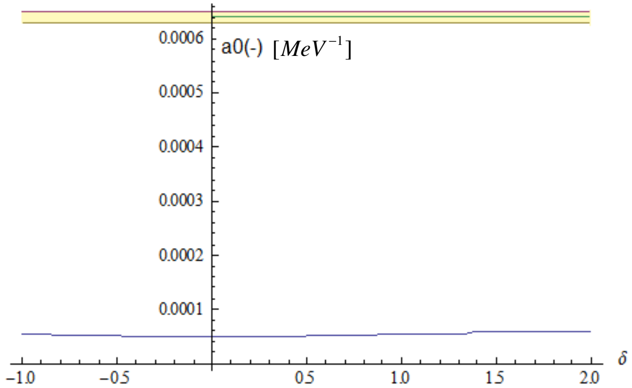
The s-wave scattering lengths

$$a_{0,\exp}^{(+)} = (-8.85783 \pm 7.165) 10^{-6} MeV^{-1}$$

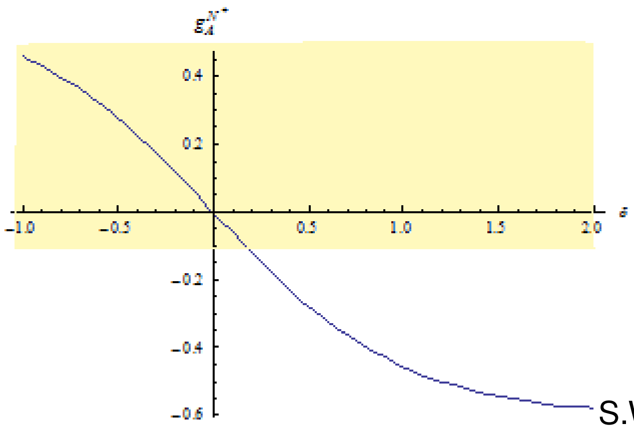
$$a_{0,\exp}^{(-)} = (6.41256 \pm 0.1146) 10^{-4} MeV^{-1}$$

$$Z = \sqrt{2}$$





Axial coupling constant of N*(1535)



$$g_{A,Lattice}^{N^*} = 0.2 \pm 0.3$$

$$g_{A,Glozman}^{N^*} = -1.26$$

S.W.(S.G.), F. Giacosa, D.Rischke,e-Print: **nucl-th/0702076**

Conclusions: locally symmetric model

- Scattering length $a_0^{(+)}$ in agreement with experimental values for $\delta > 1.5$
- Scattering length $a_0^{(-)}$ is not in agreement with experimental values
- $g_A^N = \frac{\tanh(\delta)}{Z^2} < 1$ but experimental value is 1.26
- $g_A^{N^*} = 0.2 \pm 0.3 \rightarrow \delta < 0$ or very small (not compatible with $a_0^{(+)}$)

$\Rightarrow$ Results are not consistent

$\Rightarrow$ we need to improve our model

Global chiral symmetry in agreement with mesonic decays

Model with global symmetry

$$L = L_{mes} + \frac{h_1}{2} Tr[\Phi^+\Phi] Tr[(L^\mu)^2 + (R^\mu)^2]$$
$$+ h_2 Tr[\Phi^+ L_\mu L^\mu \Phi + \Phi R_\mu R^\mu \Phi^+] + 2h_3 Tr[\Phi R_\mu \Phi^+ L^\mu] +$$
$$\overline{\Psi}_{1L} i\gamma_\mu \partial^\mu \Psi_{1L} + c_1 \overline{\Psi}_{1L} \gamma_\mu L^\mu \Psi_{1L} + \overline{\Psi}_{1R} i\gamma_\mu \partial^\mu \Psi_{1R} + c_1 \overline{\Psi}_{1R} \gamma_\mu R^\mu \Psi_{1R}$$
$$+ \overline{\Psi}_{2L} i\gamma_\mu \partial^\mu \Psi_{2L} + c_2 \overline{\Psi}_{2L} \gamma_\mu R^\mu \Psi_{2L} + \overline{\Psi}_{2R} i\gamma_\mu \partial^\mu \Psi_{2R} + c_2 \overline{\Psi}_{2R} \gamma_\mu L^\mu \Psi_{2R}$$
$$- \hat{g}_1 (\overline{\Psi}_{1L} \Phi \Psi_{1R} + \overline{\Psi}_{1R} \Phi^+ \Psi_{1L}) - \hat{g}_2 (\overline{\Psi}_{2L} \Phi^+ \Psi_{2R} + \overline{\Psi}_{2R} \Phi \Psi_{2L})$$
$$- m_0 (\overline{\Psi}_{1L} \Psi_{2R} - \overline{\Psi}_{1R} \Psi_{2L} - \overline{\Psi}_{2R} \Psi_{1L} + \overline{\Psi}_{2L} \Psi_{1R})$$

Changes in L_{mes} :

$$D^\mu \Phi = \partial^\mu \Phi + i g_1 (\Phi R^\mu - L^\mu \Phi)$$

$$L^{\mu\nu} = \partial^\mu L^\nu - \partial^\nu L^\mu - i g_2 \left[L^\mu, L^\nu\right]$$
$$R^{\mu\nu} = \partial^\mu R^\nu - \partial^\nu R^\mu - i g_2 \left[R^\mu, R^\nu\right]$$

Model with global symmetry

We determine the free parameters of the model m_0, c_1, and c_2 from $g_A^N, g_A^{N*}, \Gamma_{N^* \to \pi N}$

N*(1200): speculative candidate, decay width $\Gamma_{N^* \to \pi N}$ > 800 MeV

N*(1500) : PDG , decay width $\Gamma_{N^* \to \pi N}$ = 67.5 MeV

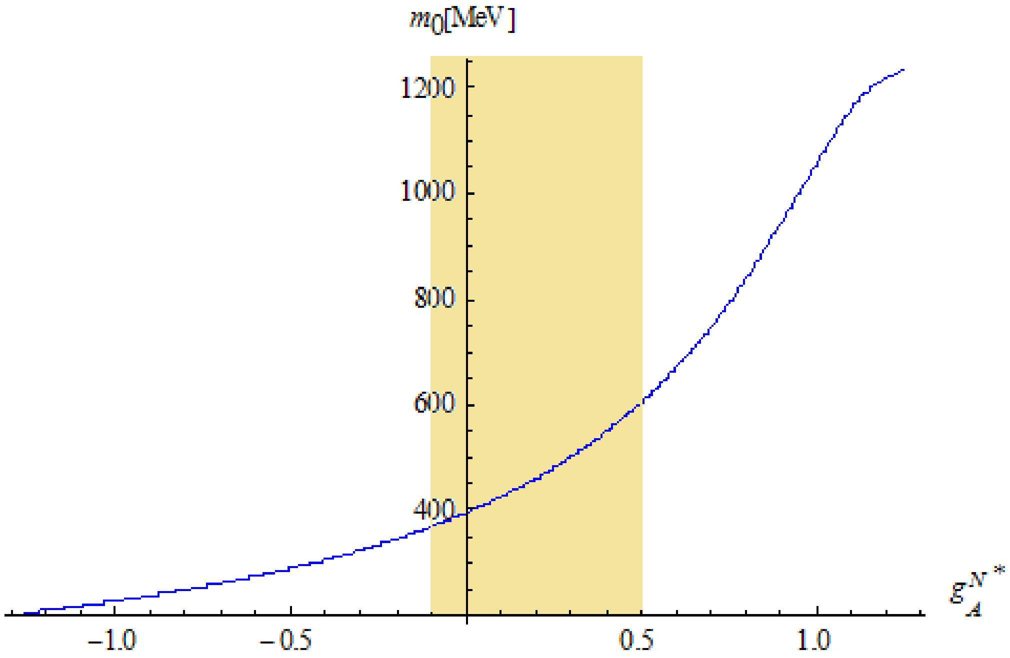
N*(1650): PDG, decay width $\Gamma_{N^* \to \pi N}$ =92.5+/-37.5 MeV

Global chiral symmetry in agreement with mesonic decays

Results

Results: globally symmetric model

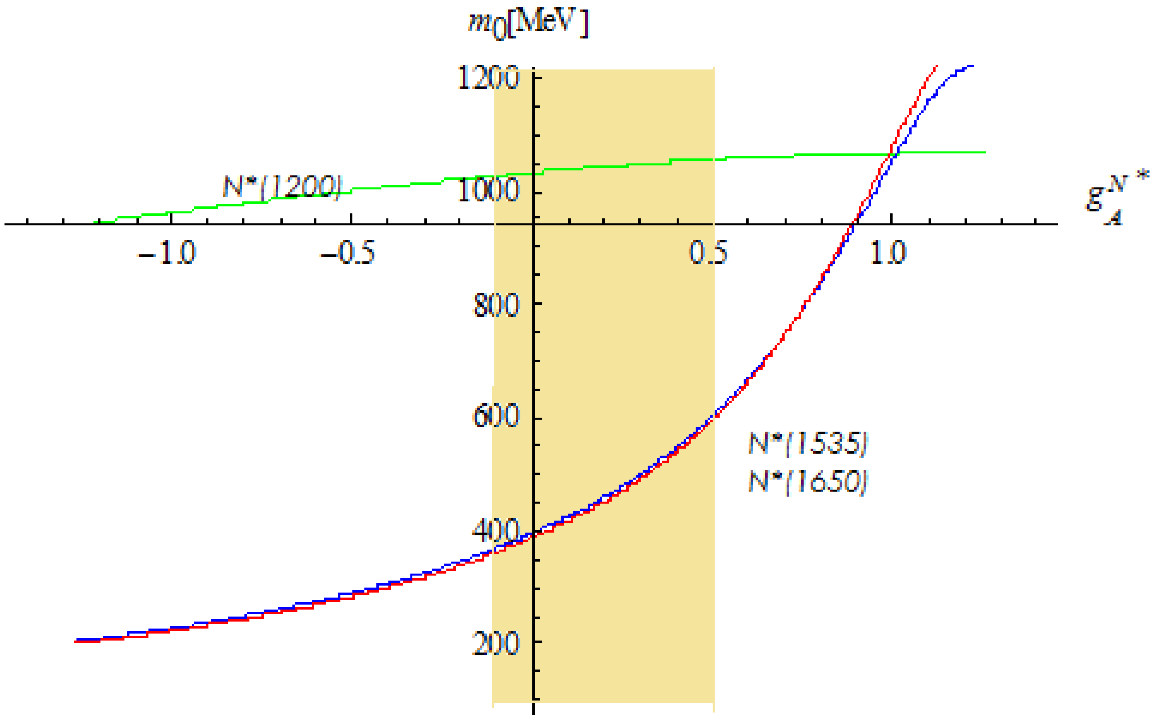
m_0 vs. the axial coupling constant of N*(1535)



350 MeV < m_0 < 600 MeV

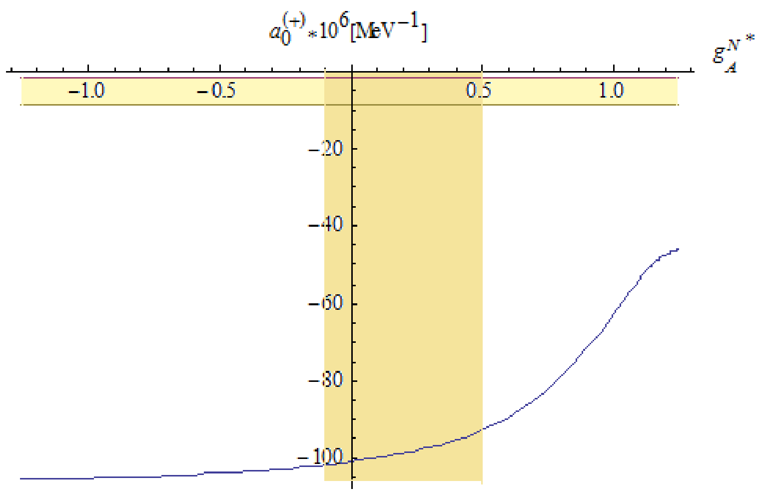
S.G., F. Giacosa, D.Rischke,e-Print: hep-ph/**0901.4043**

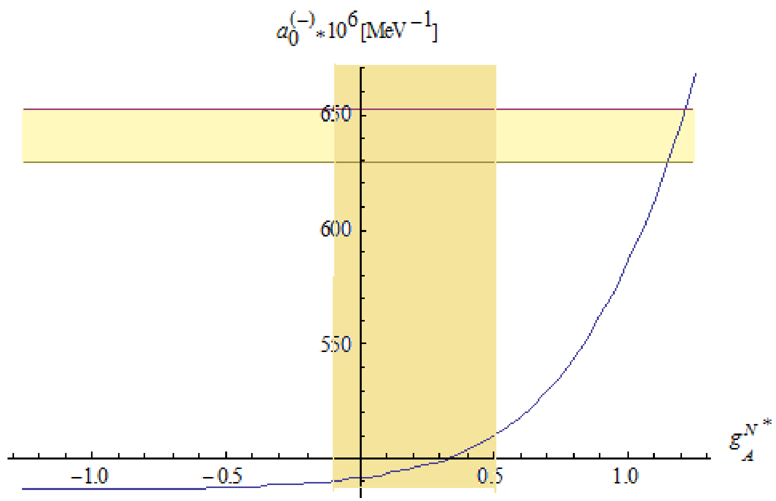
Results: globally symmetric model



Results: globally symmetric model

s-wave scattering lengths

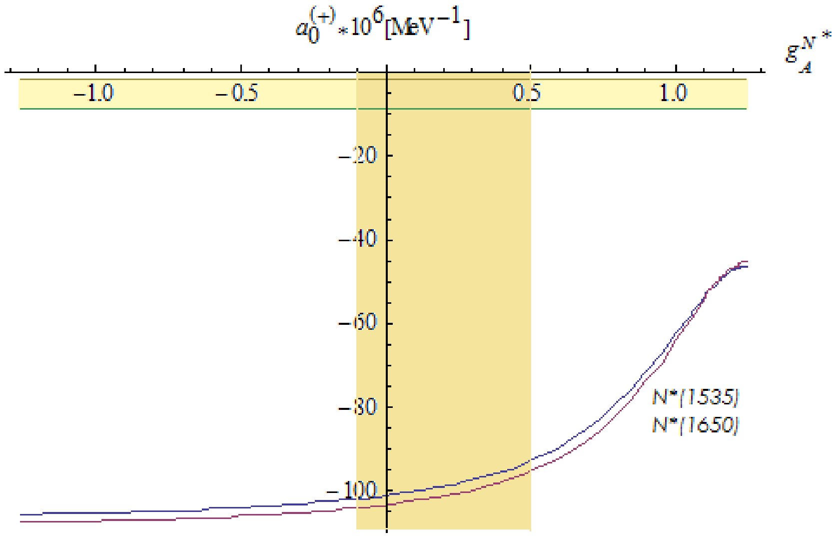


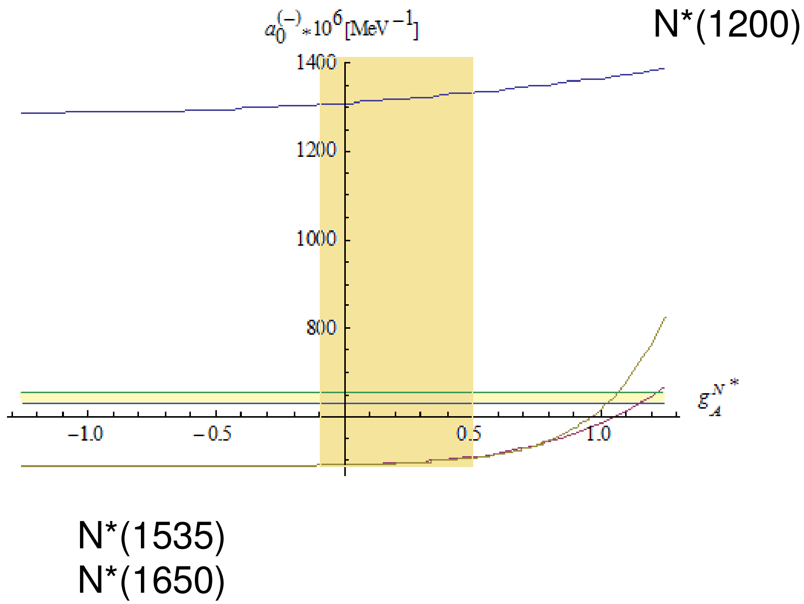


m_{N^*} = 1535 MeV

Results: globally symmetric model

s-wave scattering lengths





N*(1200) is off by over a factor 1000!

Introducing a scalar tetraquark

(preliminary)

Introducing the tetraquark

(almost) as before:

$$L = L_{mes} + h_2 Tr[\Phi^+ L_\mu L^\mu \Phi + \Phi R_\mu R^\mu \Phi^+]$$
$$+ 2h_3 Tr[\Phi R_\mu \Phi^+ L^\mu] +$$
$$\overline{\Psi}_{1L} i\gamma_\mu \partial^\mu \Psi_{1L} + c_1 \overline{\Psi}_{1L} \gamma_\mu L^\mu \Psi_{1L} + \overline{\Psi}_{1R} i\gamma_\mu \partial^\mu \Psi_{1R} + c_1 \overline{\Psi}_{1R} \gamma_\mu R^\mu \Psi_{1R}$$
$$+ \overline{\Psi}_{2L} i\gamma_\mu \partial^\mu \Psi_{2L} + c_2 \overline{\Psi}_{2L} \gamma_\mu R^\mu \Psi_{2L} + \overline{\Psi}_{2R} i\gamma_\mu \partial^\mu \Psi_{2R} + c_2 \overline{\Psi}_{2R} \gamma_\mu L^\mu \Psi_{2R}$$
$$- \hat{g}_1 (\overline{\Psi}_{1L} \Phi \Psi_{1R} + \overline{\Psi}_{1R} \Phi^+ \Psi_{1L}) - \hat{g}_2 (\overline{\Psi}_{2L} \Phi^+ \Psi_{2R} + \overline{\Psi}_{2R} \Phi \Psi_{2L})$$
$$- m_0 \frac{\chi}{\chi_0} (\overline{\Psi}_{1L} \Psi_{2R} - \overline{\Psi}_{1R} \Psi_{2L} - \overline{\Psi}_{2R} \Psi_{1L} + \overline{\Psi}_{2L} \Psi_{1R})$$

and add:

$$L_\chi = \frac{1}{2} (\partial_\mu \chi)^2 - \frac{1}{2} m_\chi^2 \chi^2 + g\chi(\sigma^2 + \pi^2) + b\chi(R_\mu^2 + L_\mu^2)$$

Introducing the tetraquark

$$\chi = \frac{1}{2}[u,d][\bar{u},\bar{d}] \qquad \sigma = \frac{1}{2}(u\bar{u} + \bar{d}d)$$

$$\begin{pmatrix} H \\ S \end{pmatrix} = \begin{pmatrix} \cos\vartheta & \sin\vartheta \\ -\sin\vartheta & \cos\vartheta \end{pmatrix} \begin{pmatrix} \chi \\ \sigma \end{pmatrix}$$

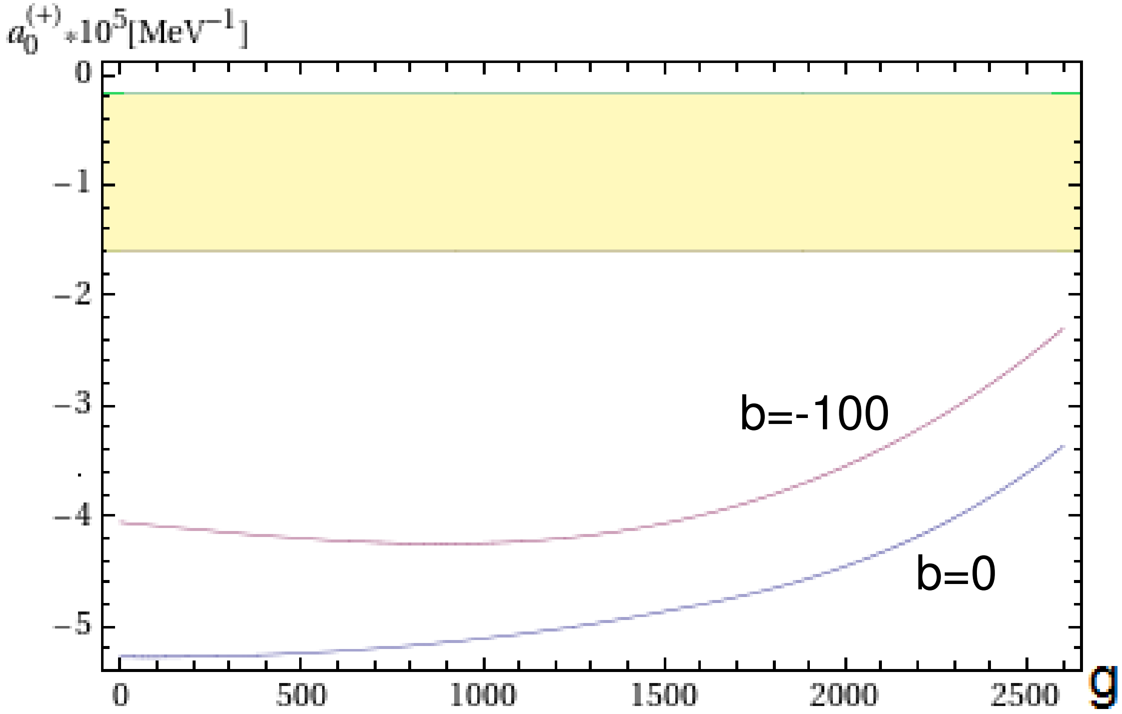
H predominantly $f_0(600)$

S predominantly $f_0(1370)$

$$\vartheta = \frac{1}{2}\arctan\frac{4g\varphi}{m_\sigma^2 - m_\chi^2} \qquad L_{\chi\sigma} = g\chi(\sigma^2 + \pi^2)$$

Results: the tetraquark

The isospin even scattering length as a function of g



$$+ g\chi(\sigma^2 + \pi^2) + b\chi(R_\mu^2 + L_\mu^2)$$

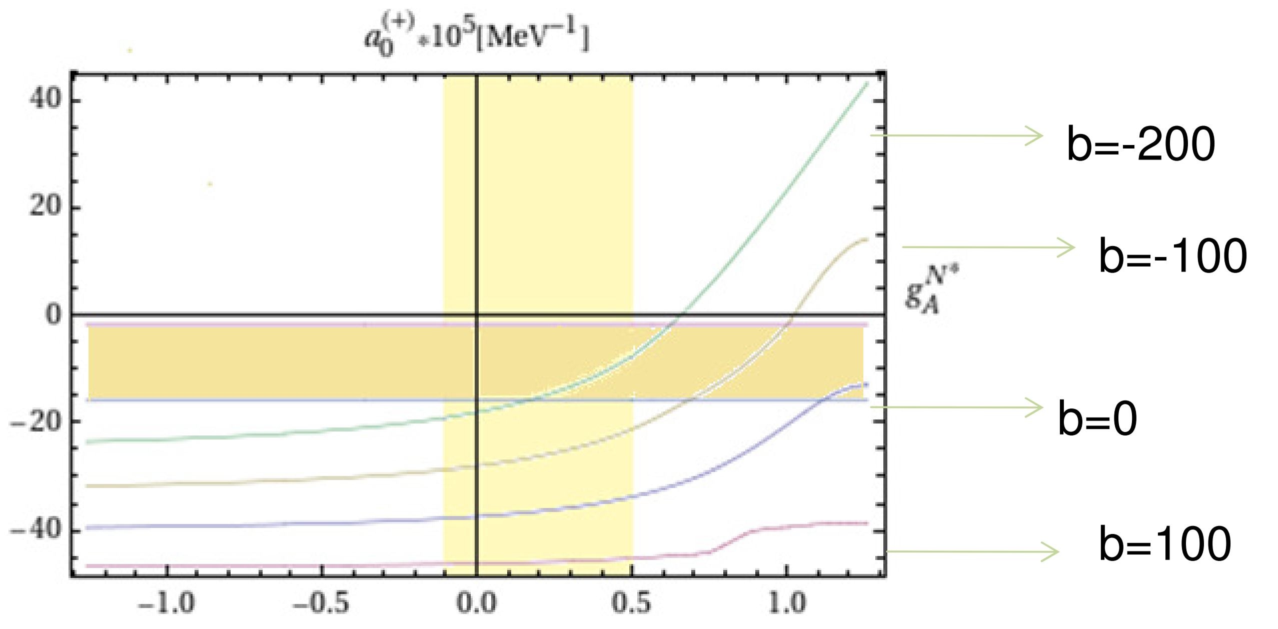
Results: the tetraquark

The isospin even scattering length as a function of the axial charge of the chiral partner of the nucleon for g=2500 MeV and different values of b.

$g \approx 2.5 GeV$ by A. Heinz et. al in finite temperature studies (hep-ph 08051134)

$$g = 2500\,\mathrm{MeV},\, h_1 = h_2 = 0$$

$$\frac{h_1}{2} Tr[\Phi^+\Phi] Tr[(L^\mu)^2 + (R^\mu)^2\ ] + h_2 Tr[\Phi^+ L_\mu L^\mu \Phi + \Phi R_\mu R^\mu \Phi^+]$$



Conclusions and outlook

- $N^*(1535)$ is a good candidate to be the chiral partner of the nucleon in the mirror assignment, although the theoretical decay into nucleon - eta and one scattering length are too small.

- The result points to $m_0 \approx 500$ MeV : half of the nucleon mass survives in the chiral limit (relevant also at finite temperature and densities studies)

- The hypothesis of speculative, very broad partner with mass 1200 MeV is not favored by our study

- The scattering length $a_0^{(+)}$ depends on the scalars $\Rightarrow$ a scalar tetraquark is included.

  The scalar tetraquark field shifts the isospin even scattering length in the "right direction". It has no influence on isospin odd scattering length.

  A large value of the coupling constant g is favoured. i.e. a strong mixing between the quarkonium and tetraquark fields.

- The delta resonance should be included also in the future.

Thank you!