Glueballs and statistical mechanics of the gluon plasma

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Thermal QCD: Basic facts

Introducing the QGP

- « Standard » QCD: $T = 0$
- Important physics at $T > 0$
  - Critical temperature, T_c

- Interior of some neutron stars
- Early stage of the Universe
- Challenging problem in field theory

J. C. Collins and M. J. Perry, Phys. Rev. Lett. **34**, 1353 (1975);
E. V. Shuryak, Phys. Rep. **61**, 71 (1980).

Quark-gluon plasma

T_c

- QGP: after deconfinement
  - Strongly coupled stage
    - Color interactions still strong
    - Surviving bound states
    - Experimental evidences at RHIC

      Nucl. Phys. A **757**, 1 (2005)
    - Effective models and lattice QCD

This work

  - « True » deconfined medium $T \gg T_c$
    - Perturbative QCD

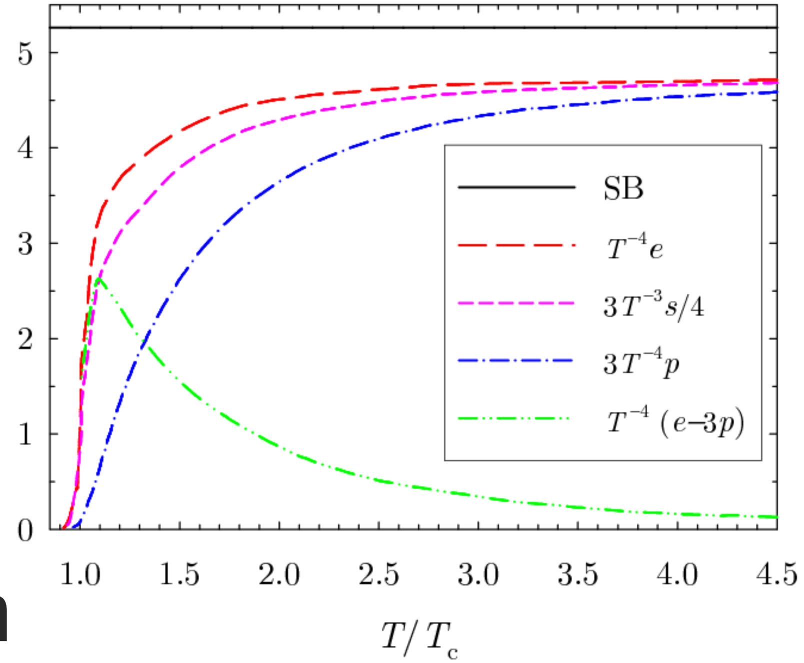
Lattice QCD (I)

- Finite T QCD can be implemented on the lattice
- Various results
  - Equation of state
    - Quenched and unquenched
    - Zero chemical potential or not
  - Static potentials
- In particular: Gluon plasma
  - Unquenched pure gauge simulations at zero chemical potential

Lattice QCD (II)

- Gluon plasma equation of state

G. Boyd *et al.*,
PRL **75**, 4169 (1995)



- Observation
  - Saturation below Stefan-Boltzmann
  - Strong increase near T_c

Quasiparticle models (I)

- How to understand the lattice results?
  - First step towards the full QGP
- Idea: Gluon plasma is an ideal gas of free transverse bosons (16 d.o.f.)
  - Well-known in statistical physics
  - $m = 0$ excluded, only Stefan-Boltzmann
  - $m = m_0 \neq 0$ in qualitative disagreement

Need for a temperature-dependent gluon mass $m(T)$ and corresponding statistical mechanics

Quasiparticle models (II)

- What is expected for $m(T)$?
  - $T \gg T_c$, perturbative results
    K. Kajantie *et al*., Phys. Rev. Lett. **79,** 3130 (1997);
    P. Lévai and U. Heinz, Phys. Rev. C **57**, 1879 (1998)

$$m(T) \propto m_D(T) \propto \sqrt{\alpha_s(T)}\,T \sim T$$

  - $T \simeq T_c,\ m(T)$ phenomenological
    - Color interactions

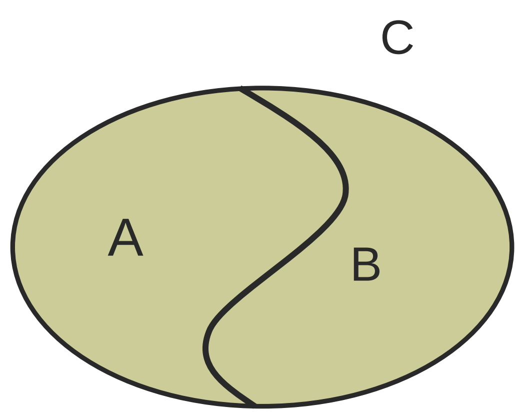
$$\sqrt{p^2 + m(T)^2} \quad \text{instead of} \quad \sqrt{p^2 + \bar{m}(T)^2} + V(r, T)$$

⟹ $m(T)$ stands for $\langle V \rangle$

Extended statistical mechanics

First principles (I)

- Classical systems at equilibrium
  - Probability density



$$\rho_C(H_A + H_B) = \rho_A(H_A)\,\rho_B(H_B)$$

$$\rho(H) = \frac{e^{-\beta H}}{\int e^{-\beta H}\,d\lambda} = \frac{e^{-\beta H}}{Z(\beta)}$$

  - Energy and entropy

$$E = \overline{H} = \int H\rho\,d\lambda \qquad S = \overline{-\ln\rho} = -\int \rho\ln\rho\,d\lambda$$

  - H may depend on T

First principles (II)

- Thermodynamical consistency
  - First and second laws of thermodynamics
  - Constraints

$$\partial_\beta S = \frac{1}{T}\partial_\beta E \qquad \text{and} \qquad p = \frac{TS-E}{V}$$

- A priori, in these relations,

$$T = \frac{1}{f(\beta)}$$

How to satisfy the constraints?

General solution

- $\beta(T)$ is found through $f(\beta) = \frac{1}{T}$ where

$$f(\beta) = \beta\left[1 + \frac{\overline{\partial_\beta H(T{=}1/f(\beta))}}{\partial_\beta E(\beta, T{=}1/f(\beta))}\right]$$

  - Unambiguous if $H(T)$ is known
  - Physical quantities formally unchanged
    - β has no physical meaning a priori
      Standard case: $\partial_\beta H = 0 \;\Rightarrow\; \beta = \frac{1}{T}$
- Difficult for numerical computations

Alternative solutions

- Modified formulas with $\beta = 1/T$ and
  - Energy preserved
    V. M. Bannur, Phys. Lett. B **647**, 271 (2007)

$$\tilde{S} = S + B^{(1)},\ \tilde{p} = p + \frac{B^{(1)}}{\beta},\ B^{(1)} = \int_{\beta_\star^{(1)}}^{\beta} \nu\, \overline{\partial_\beta H}|_{\beta=\nu} d\nu$$

  - Entropy preserved
    P. Lévai and U. Heinz, Phys. Rev. C **57**, 1879 (1998)

$$\tilde{E} = E - B^{(2)},\ \tilde{p} = p + B^{(2)},\ B^{(2)} = \int_{\beta_\star^{(2)}}^{\beta} \overline{\partial_\beta H}|_{\beta=\nu} d\nu$$

  - Pressure preserved
    V. Goloviznin and H. Satz, Z. Phys. C**57**, 671 (1993).
- Inequivalent solutions but convenient for phenomenology if $H(T)$ can be fitted

Ideal gas of bosons

- Same formalism with:
  - Probability density $\rho(H) = [e^{\beta\, H} - 1]^{-1}$
  - Hamiltonian $H = \epsilon(k,T) = \sqrt{k^2 + m(T)^2}$
  - Basic formulas

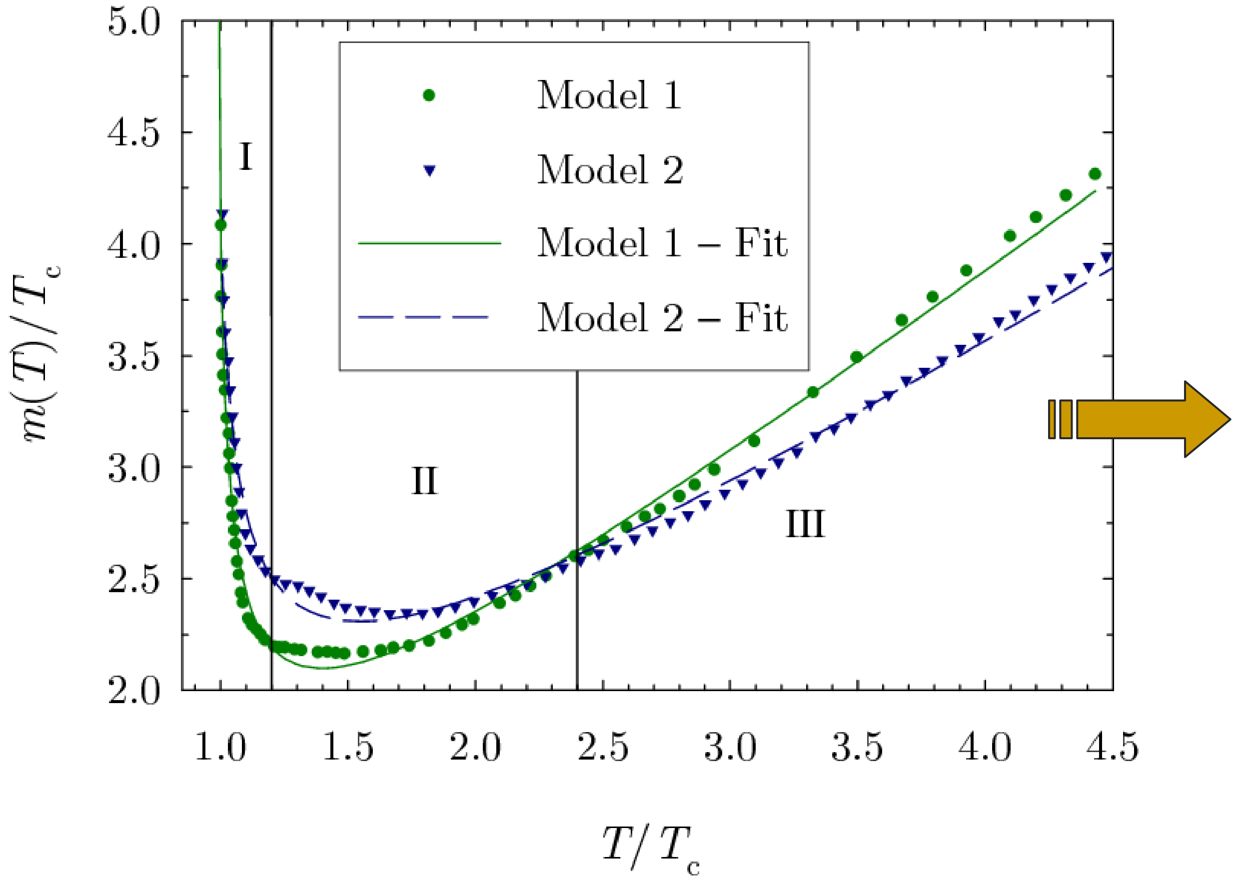
$$\begin{cases} e = \frac{8}{\pi^2} \int_0^\infty dk\, k^2 q(\epsilon/T)\, \epsilon \\ s = \frac{8}{3\pi^2 T} \int_0^\infty dk\, k^2 q(\epsilon/T)[k\partial_k \epsilon + 3\epsilon] \\ p = \frac{8}{3\pi^2} \int_0^\infty dk\, k^3 q(\epsilon/T)\, \partial_k \epsilon \end{cases}$$

- $m(T)$ unknown
  - Use of alternative solutions

The gluon plasma

Thermal gluon mass (I)

- Model 1: energy preserved, fit on lattice energy
- Model 2: entropy preserved, fit on lattice entropy



Compatible behaviors

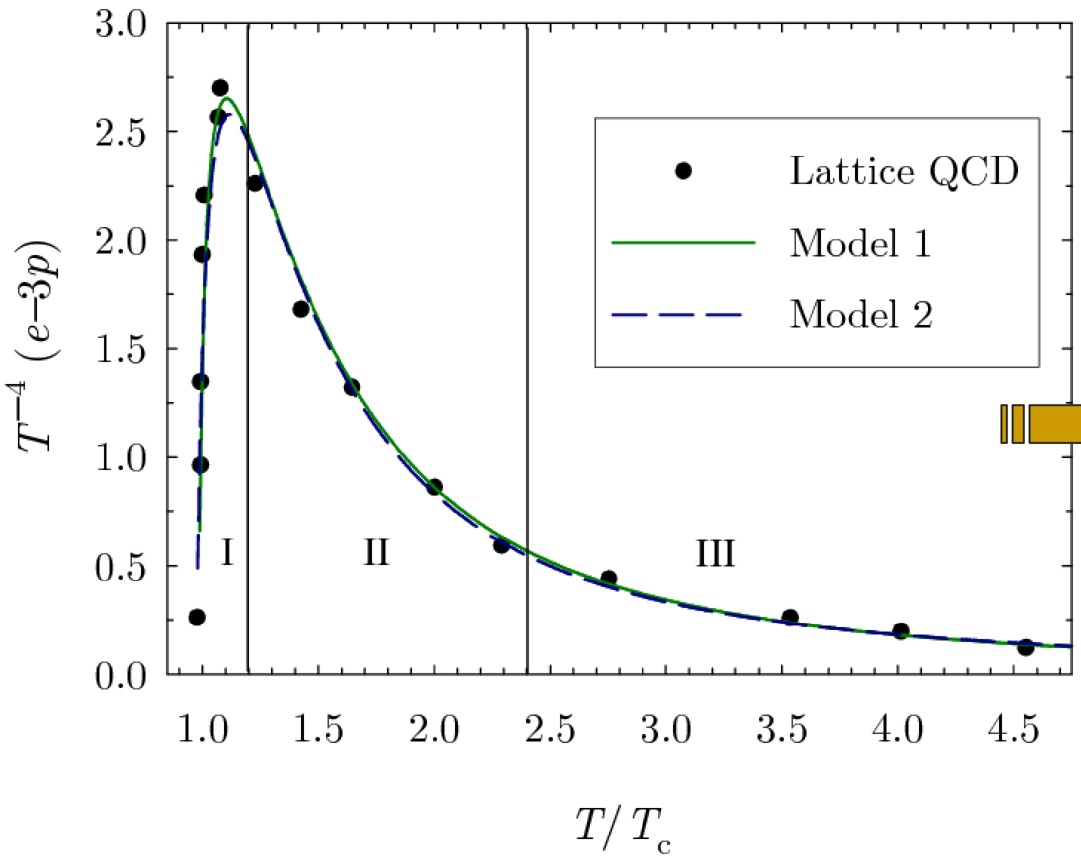
Thermal gluon mass (II)

Qualitatively $m(T) \simeq \frac{m_1}{(T/T_c - 1)^{m_3}} + m_0 T$

- Singular near T_c
  - Region I
  - Needed to reproduce the strong increase of e, p, s
  - Strong color interactions
- Linear at large T
  - Region III
  - Needed to reproduce the saturation below Stefan-Boltzmann
  - Perturbative QCD

Comparison with lattice (I)

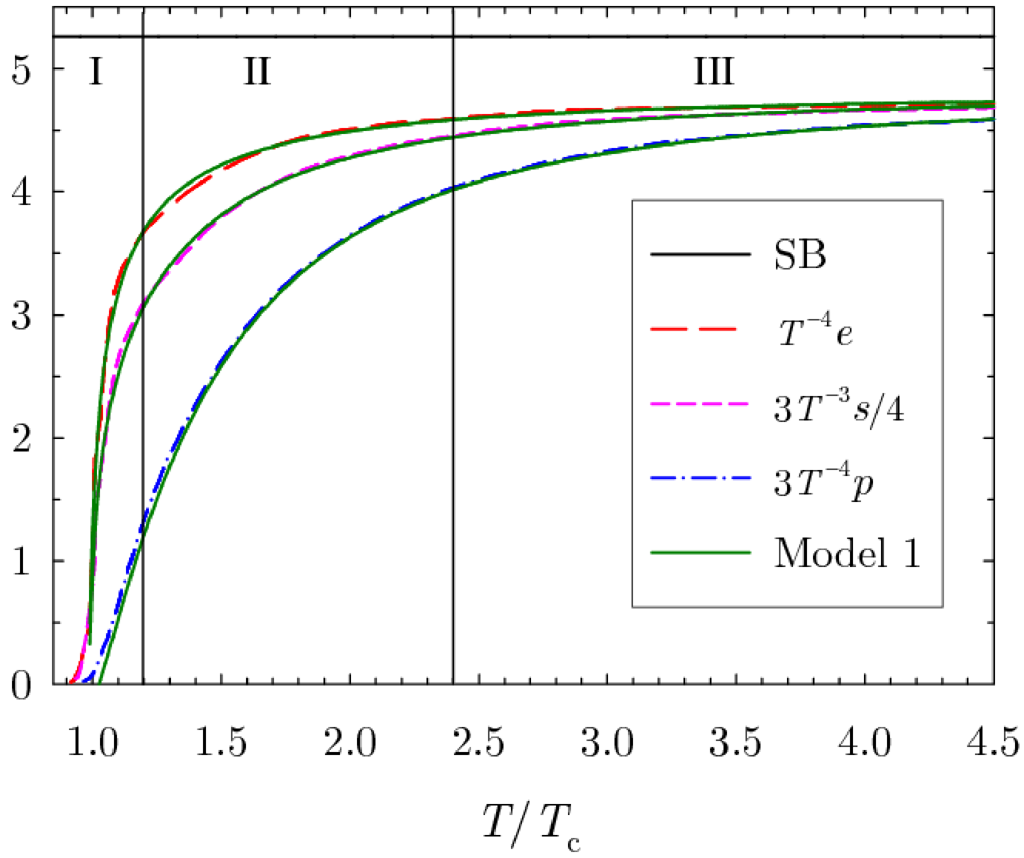
- Interaction measure



Identical results with similar fitted parameters

Comparison with lattice (II)

- Excellent agreement



Glueballs in the gluon plasma

Constituent model (I)

- Lightest glueballs
  - Two transverse gluons
  - Color singlet: most attractive

$$8 \otimes 8 = 1 \oplus 8 \oplus 8 \oplus 10 \oplus \overline{10} \oplus 27$$

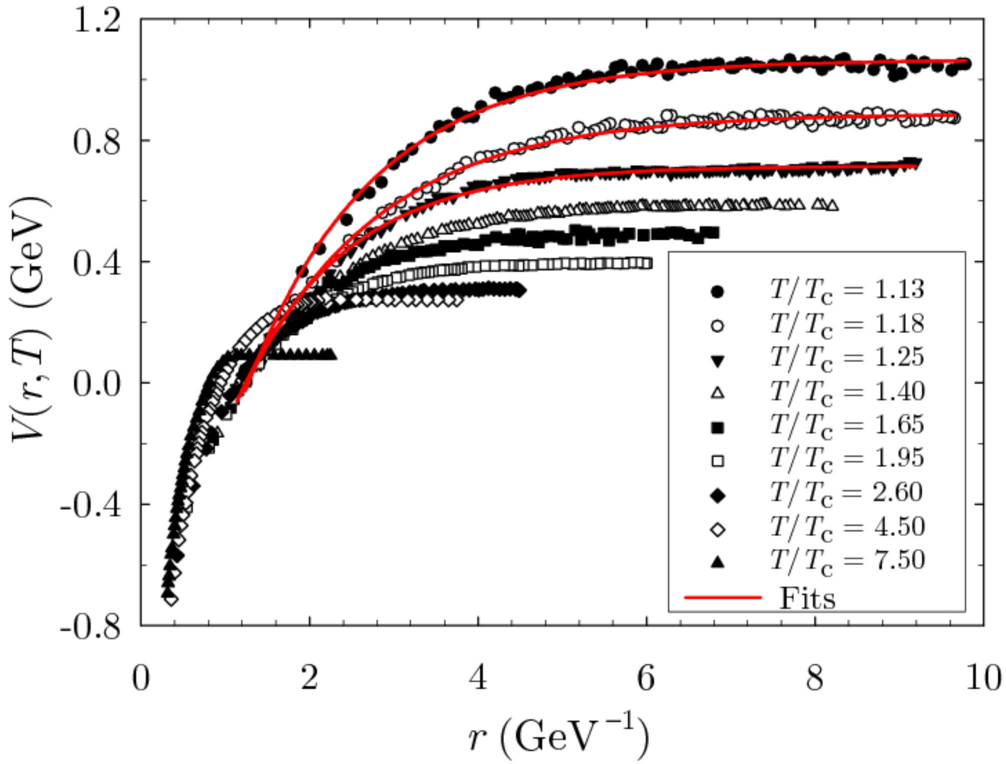
  - $0^{\pm +}$ channels: formally $\langle L^2 \rangle = 2$

V. Mathieu, F. Buisseret and C. Semay, Phys. Rev. D **77**, 114022 (2008)

- Hamiltonian $H = 2\sqrt{p^2 + \bar{m}(T)^2} + V_{gg}(r,T)$
  - Free gluon mass $\bar{m}(T) \simeq T$
  - Input: potential term $V_{gg} = (9/4)\, V_{q\bar{q}}$

Constituent model (II)

- Potential: internal energy of the gluons
  - Lattice data for quark-antiquark

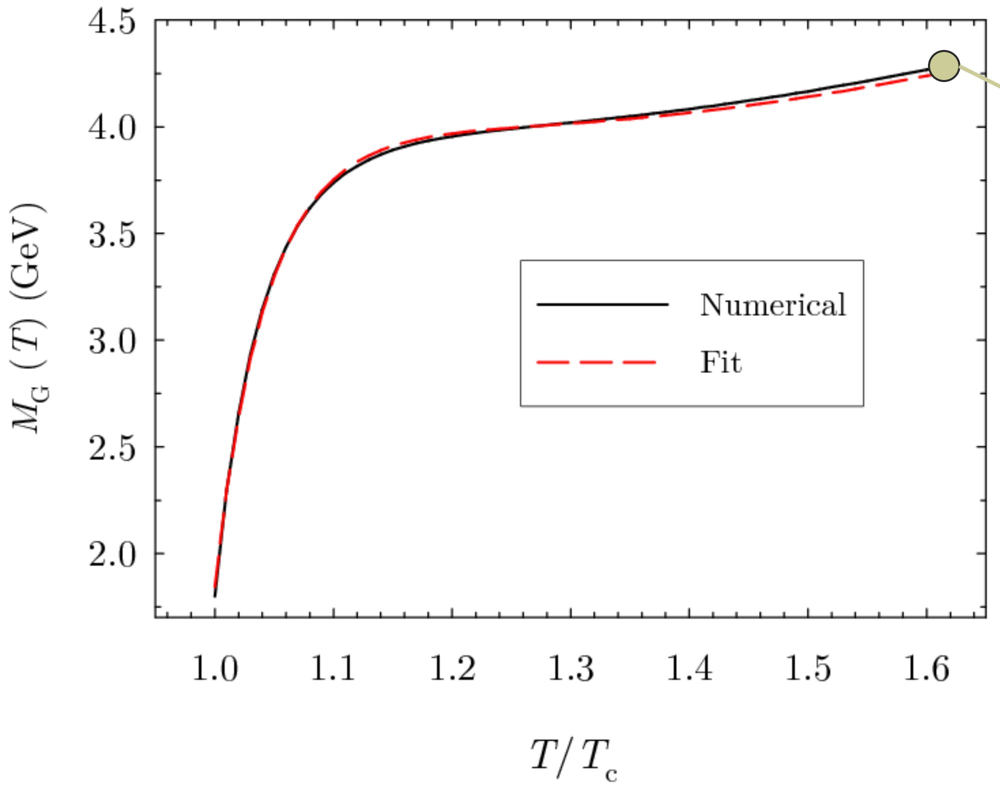


$$V_{q\bar{q}}(r,T) = -a(T)\ \mathrm{e}^{-b(T)\,r} + c(T)$$

O. Kaczmarek *et al*., Nucl. Phys. Proc. Suppl. **129**, 560 (2004) [hep-lat/ 0309121]

Mass spectrum

- Lightest glueballs
  - Dissociation temperature



$$T = 1.6\,T_c$$

Gluon plasma as a mixture (I)

- Idea: Gluon plasma is an ideal mixture of free gluons and glueballs
  - Energy preserved

$$e = [1 - n(T)]\, e(16, \bar{m}T) + n(T)\, e(2, M_G(T))$$

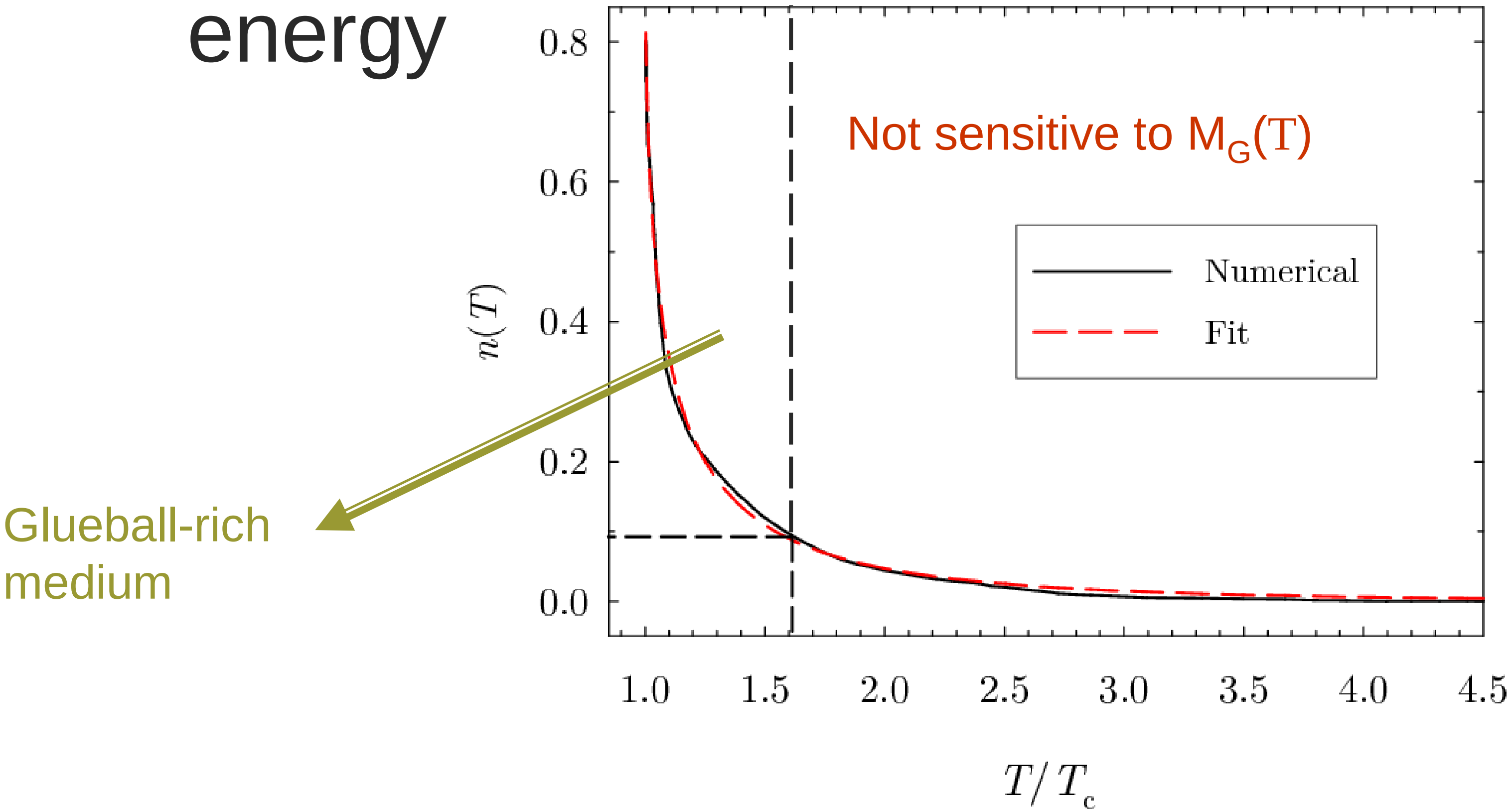
Glueball abundance

Free transverse gluons

Scalar and pseudoscalar glueballs

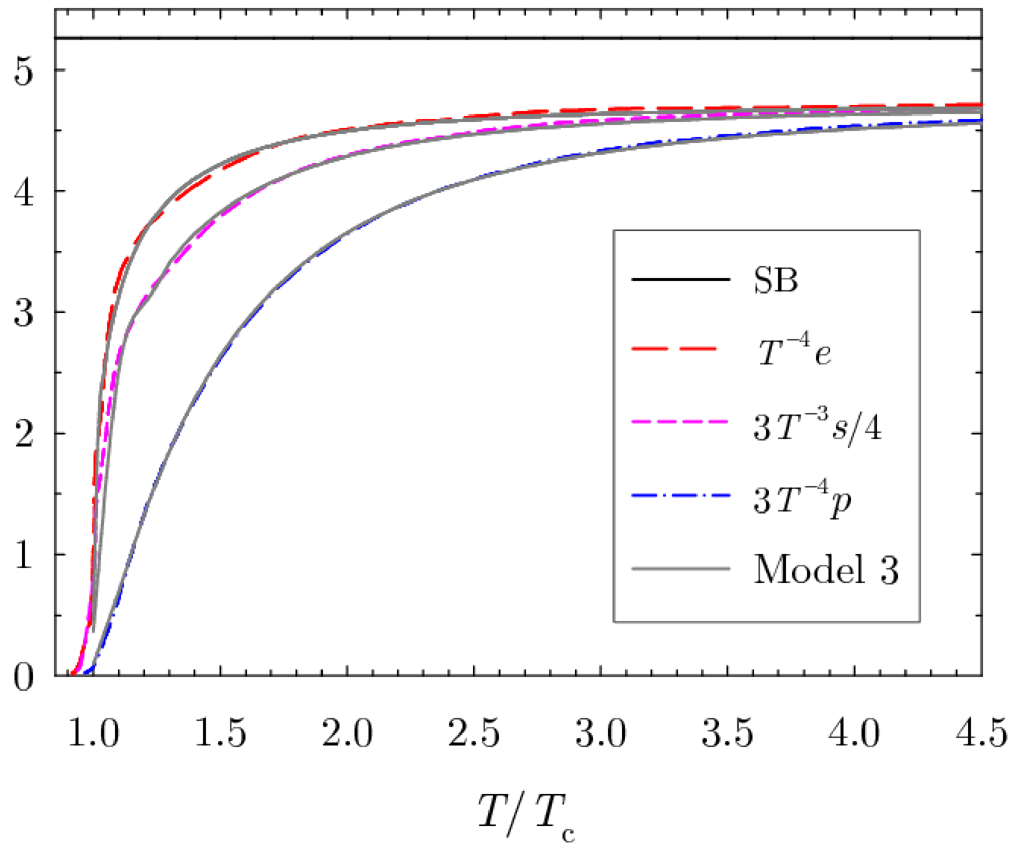
Gluon plasma as a mixture (II)

- Glueball abundance fitted on lattice energy



Gluon plasma as a mixture (III)

- Agreement with lattice data
  - Different physical picture



Conclusions

Summary

- Quasiparticle models of gluon plasma
  - Thermal gluon mass
  - Reconsider statistical mechanics
- Strong color interactions near T_c
  - Glueballs up to 1.6 T_c
  - Mixture of gluons and glueballs
  - Experimental observation of glueballs?
- Soon on the arXiv

Outlook

- Inclusion of quarks in the model
  - Comparison with lattice QCD
  - Many bound states
    - Experimental predictions
- Nonzero chemical potential
- Something I hope
  - Lattice computation of the static energy between two color octet sources at finite T