Tests of Chiral Perturbation Theory with K_{e4} decays at NA48/2

Excited QCD 09 / Zakopane

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On behalf of NA48 Collaboration:
Cambridge, CERN, Chicago, Dubna, Edinburgh, Ferrara, Firenze, Mainz, Northwestern, Perugia, Pisa, Saclay, Siegen, Torino, Vienna

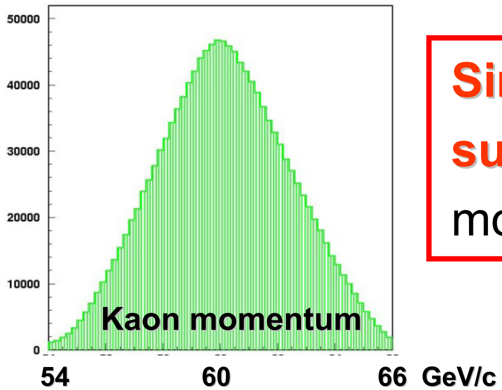
$\pi\pi$ scattering - Introduction

- ***Motivation:***
  - In χPT the **quark condensate $\langle\bar{q}q\rangle_0$** determines the relative size of mass and momentum terms in the perturbative expansion.
  - $\langle\bar{q}q\rangle_0$ is a free parameter and must be determined **experimentally**.
  - **a_0 and a_2** are S-wave $\pi\pi$ scattering lengths in isospin states I=0 and I=2, correspondingly. They enter into all $\pi\pi$ scattering amplitudes.
  - The relation between $\langle\bar{q}q\rangle_0$ and the scattering lengths a_0 and a_2 is known from this theory [*] with a high precision (2%), so **the experimental measurement of a_0 and a_2 provides an important constraint for χPT Lagrangian parameters**

- ***How to measure a_0 and a_2?***
  - **In $K^\pm \to \pi\pi e\nu$ (K_{e4}):** the hadronic effects are described in terms of formfactors; the relative phase between them depends on a_0 and a_2 (no theoretical uncertainty on the form factors).
  - **In $K^\pm \to \pi^\pm\pi^0\pi^0$:** "cusp" at $M_{00}^2 = 4m_{\pi+}^2$ due to rescattering.

[*] see G. Colangelo AIP Conf. Proc. 756, 60 (2005) and Nucl. Phys. B 457, 513 (1995)

NA48/2 Beam Line



Simultaneous K⁺ and K⁻ beams, **superimposed** in space, with momentum spectra **(60±3) GeV/c**.

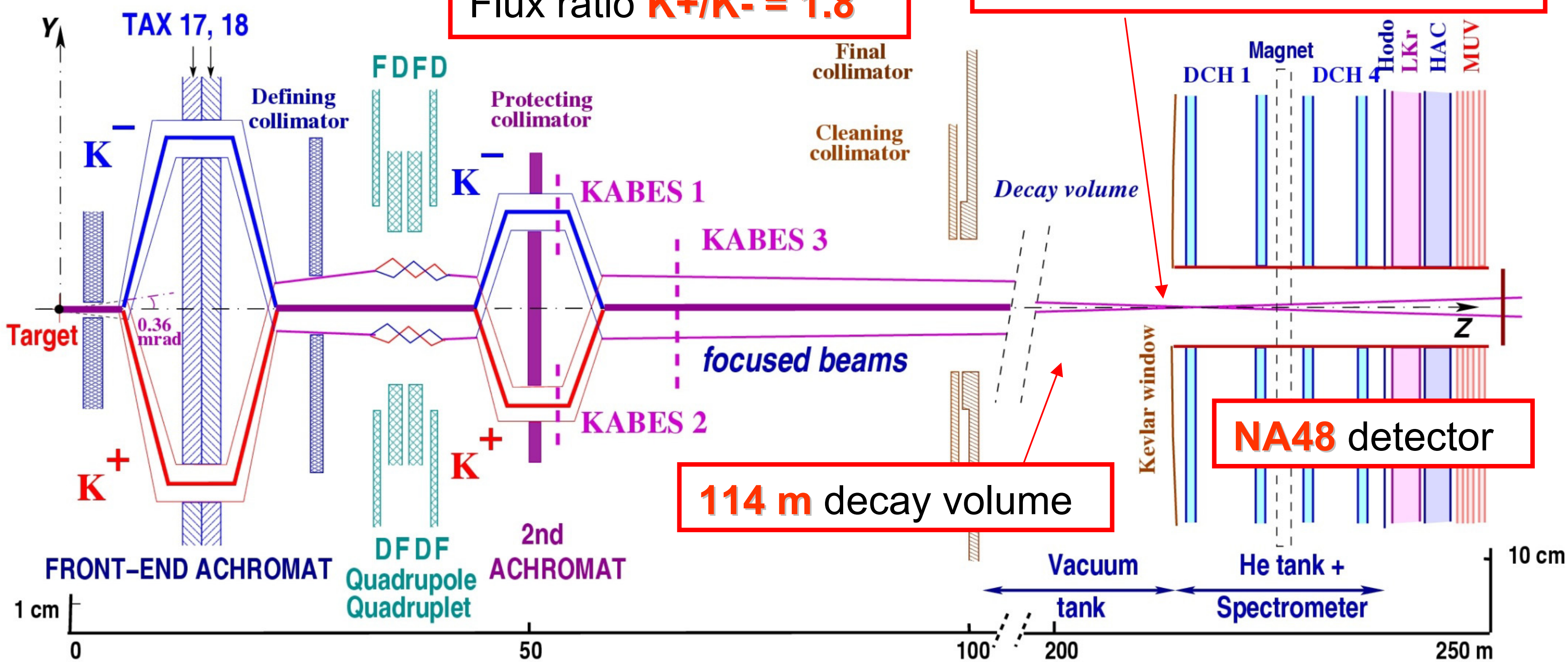
Data-taking periods:
2003 – 50 days
2004 – 60 days

Flux ratio **K+/K- = 1.8**

Beams coincide within 1 mm

114 m decay volume

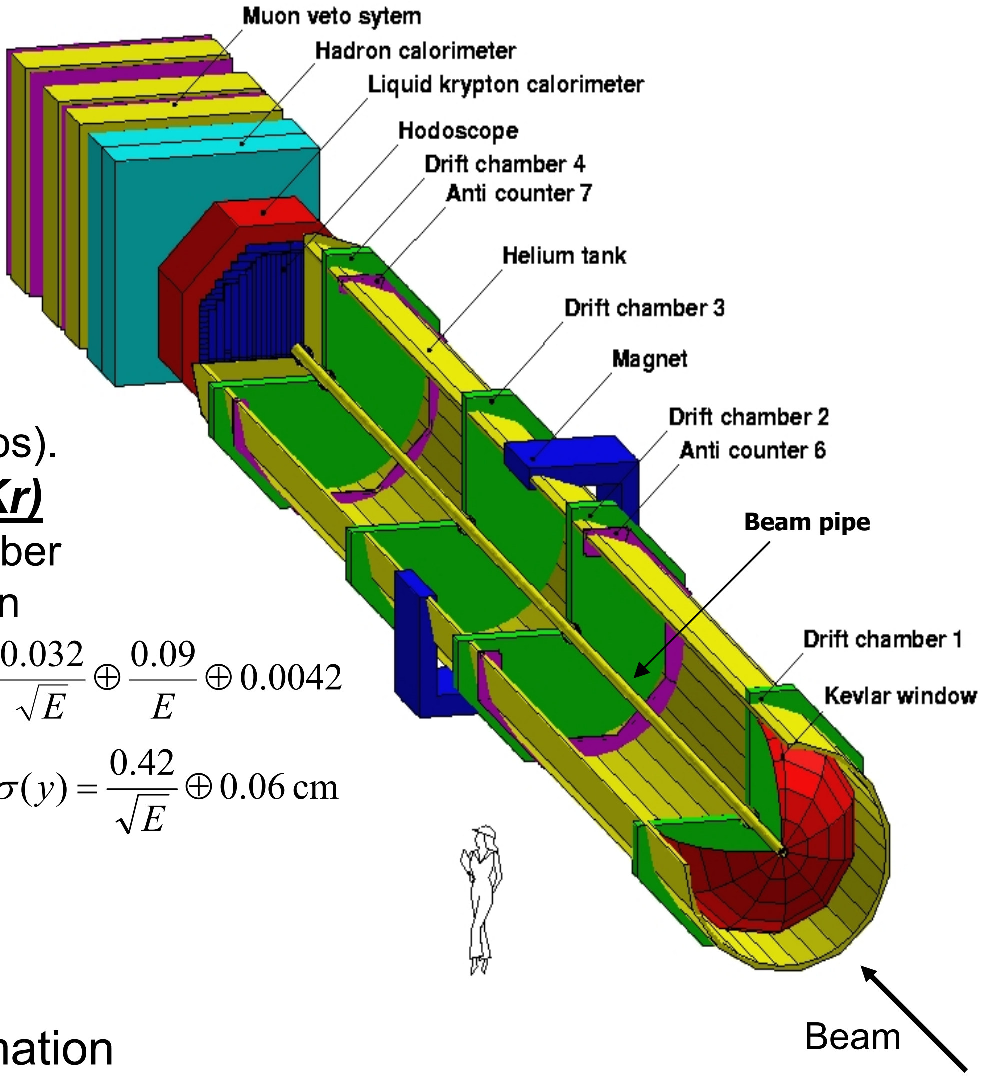
NA48 detector



NA48 detector

Main subdetectors:

- ***Magnetic spectrometer (4 DCHs):***
  - high efficiency;
  - $\Delta p/p = 1.0\% \oplus 0.044\% * p$ [GeV/c]
  - Very good resolution for charged masses $M(\pi^{\pm}\pi^{+}\pi^{-}) = 1.7$ MeV/c^2
- ***Hodoscope***
  - fast trigger;
  - precise time measurement ($\sigma_t = 150$ ps).
- ***Liquid Krypton EM calorimeter (LKr)***
  - Quasi-homogeneous ionization chamber
  - Active volume of ~10 m^3 liquid krypton
  - Energy resolution (E in GeV): $$\frac{\sigma(E)}{E} = \frac{0.032}{\sqrt{E}} \oplus \frac{0.09}{E} \oplus 0.0042$$
    ($\sigma(E) \approx 142$ MeV for $E = 10$ GeV)
  - Space resolution (E in GeV): $$\sigma(x) = \sigma(y) = \frac{0.42}{\sqrt{E}} \oplus 0.06 \text{ cm}$$
    ($\sigma(x) = \sigma(y) \approx 1.5$ mm for $E = 10$ GeV)



Trigger:

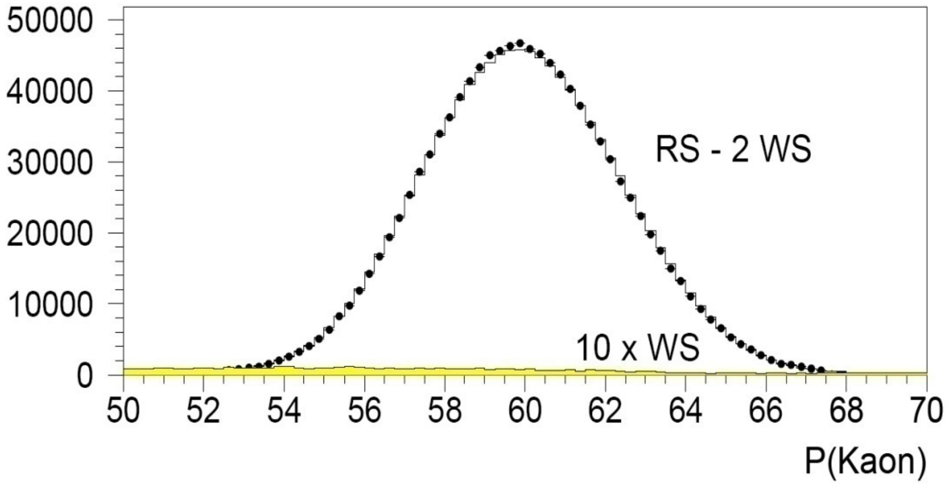
- ***L1*** Hodoscope, DCH multiplicity
- ***L2*** ON-line processing of DCH information

Ke4 decays selection

- ***Ke4 topology:***
  - 3 tracks and 1 vertex
  - 1 electron (E_{LKr}/p_{DCH}~1)
  - two π with opposite signs
  - reconstruct PK → pick the solution closer to 60 GeV/c
  - (or alternatively one can reconstruct the missing mass under PK=60 GeV/c assumption)

- ***Background estimations:***
  - "wrong sign" selection (with electron opposite to the charge of the vertex)
  - Suppressed by 10^{-10} by the $\Delta S=\Delta Q$ rule
  - Need to be rescaled according to the relative misID probability:
    - RS/WS=2 if coming from $K3\pi$
    - RS/WS=1 if coming from $K2\pi(\pi^0)$

- ***Sources of backgrounds:***
  - $K^\pm \rightarrow \pi^\pm\pi^+\pi^-$ (dominant) with $\pi \rightarrow e\nu$ decay or misID
  - $K^\pm \rightarrow \pi^\pm\pi^0\pi^0$ with $\pi^0 \rightarrow e^+e^-\gamma$ with e misID as π



Total background is ~0.5% from the signal

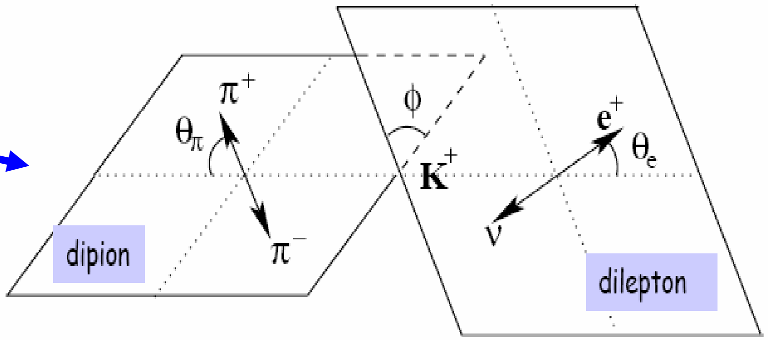
Estimated from WS events and crosschecked with MC simulation

Total statistics 1.15 Million K_{e4} decays

Analysis procedure

- 5 kinematic variables **[Phys.Rev. 137, B438 (1965)]**:

$S_\pi(M^2_{\pi\pi})$ | $S_e(M^2_{e\nu})$ | $\cos\theta_\pi$ | $\cos\theta_e$ | ϕ



- 3 form factors (F,G,H) – expanding in partial waves **[Pais-Treiman1968]**:

axial:

$$F = F_s e^{i\delta_s} + F_p e^{i\delta_p} \cos\theta_\pi$$

$$G = G_p e^{i\delta_g}$$

vector:

$$H = H_p e^{i\delta_h}$$

+ *d*-wave …

- Expanding in terms of $q^2=(S_\pi/4m_\pi-1)$ and S_e **[Amoros-Bijnens1999]**:

$$F_s = f_s + f_s' q^2 + f_s'' q^4 + f_e\left(S_e/4m_\pi^2\right) + \ldots$$

$$F_p = f_p + f_p' q^2 + \ldots$$

$$G_p = g_p + g_p' q^2 + \ldots$$

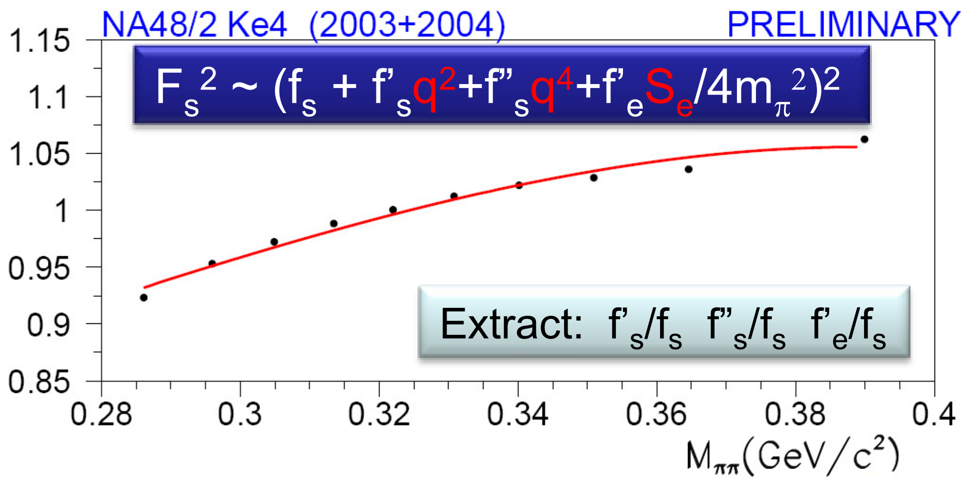
$$H_p = h_p + h_p' q^2 + \ldots$$

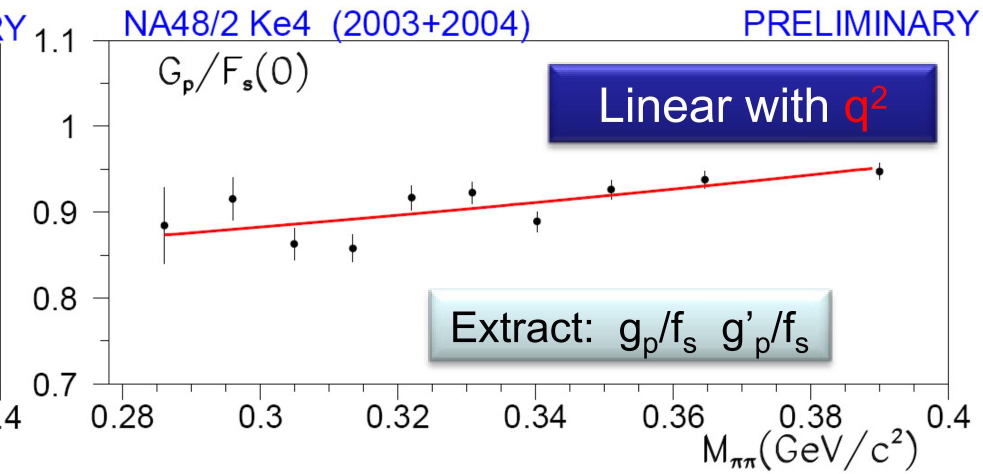
- Find F_s, F_p, G_p , H_p and $\delta = \delta_s - \delta_p$ in q^2 bins
- Extract the slopes from these dependencies
- **Extract a_0 from $\delta(q^2)$ dependence**

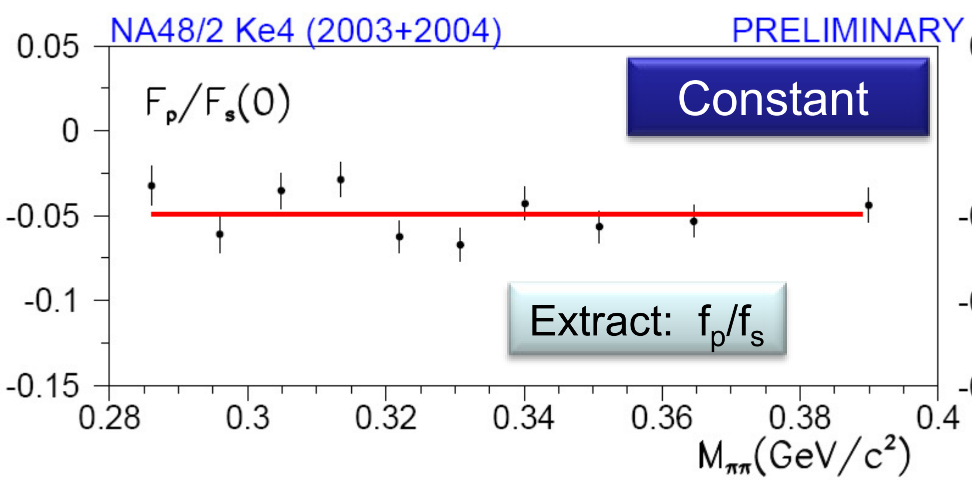
Fitting of form factors:

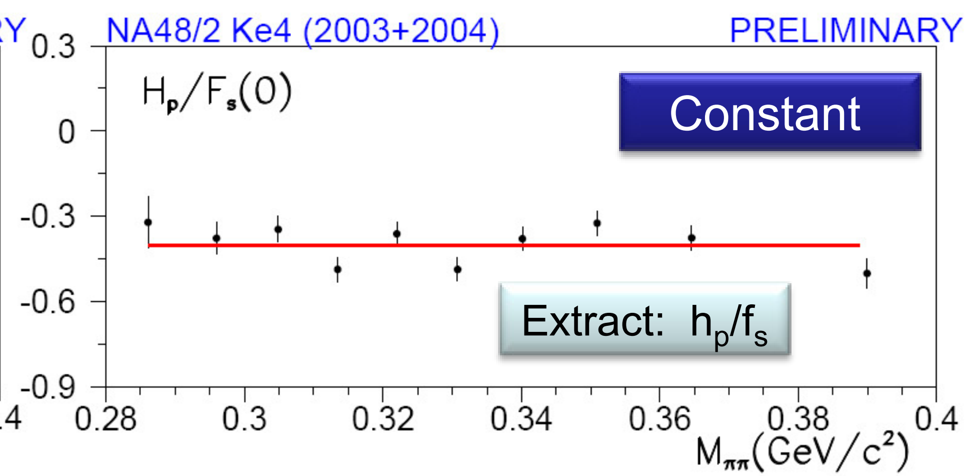
- Define 10x5x5x5x12 iso-populated bins in ($M_{\pi\pi}$, $M_{e\nu}$, $\cos\theta_\pi$, $\cos\theta_e$, ϕ)
  49 K+/box and **27 K-/box** (**DATA**)
  1180 K+/box and **650 K-/box** (**MC**)
- 10 independent fits (one fit per $M_{\pi\pi}$ bin) for F_s, F_p, G_p, H_p, δ in 4D space
- The 4 sets of points fitted with polynomial in **powers of q^2** (Taylor expansion valid in isospin symmetry limit)
- **Model independent analysis** → no assumption is made on variation of δ and FF from one bin to the next
- Without the overall normalization from branching fraction, only **relative form factors** (F_p, G_p, H_p)/F_s are measured.

FF extraction

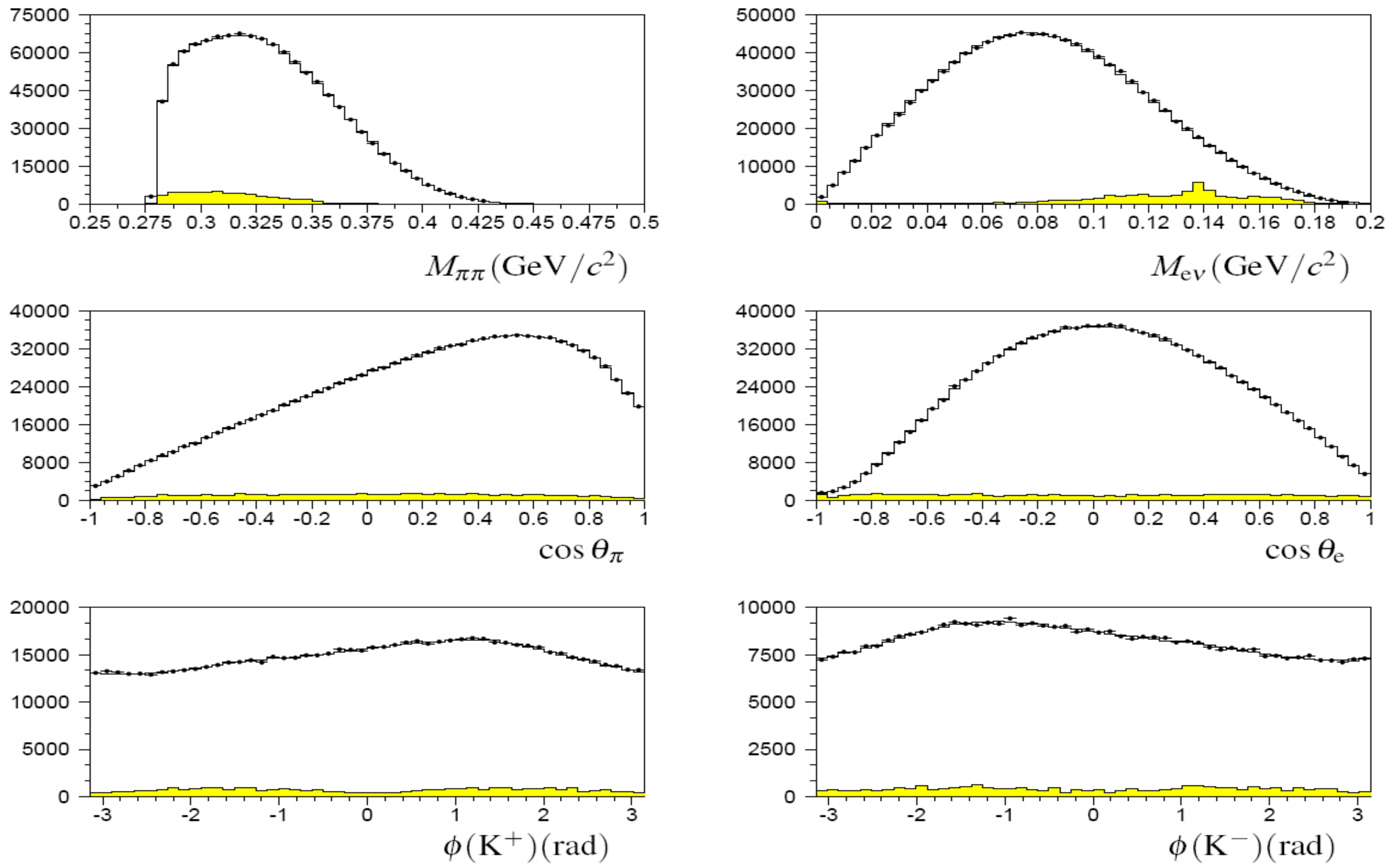








DATA/MC comparison after fit



• data histogram – MC after fit WSx10

FF – results

Preliminary 2003+2004

f'_s/f_s	$0.158 \pm 0.007_{stat} \pm 0.006_{syst}$
f''_s/f_s	$-0.078 \pm 0.007_{stat} \pm 0.007_{syst}$
f'_e/f_s	$0.067 \pm 0.006_{stat} \pm 0.009_{syst}$
f_p/f_s	$-0.049 \pm 0.003_{stat} \pm 0.004_{syst}$
g_p/f_s	$0.869 \pm 0.010_{stat} \pm 0.012_{syst}$
g'_p/f_s	$0.087 \pm 0.017_{stat} \pm 0.015_{syst}$
h_p/f_s	$-0.402 \pm 0.014_{stat} \pm 0.008_{syst}$

- Results in agreement with the published 2003 data → **EPJ C54 (2008)**
- Systematics conservatively taken as for 2003 data → mostly statistically limited → now being reevaluated for the whole sample
- Among the systematic effects studied:

  beam geometry, acceptance, particle identification, backgrounds, radiative corrections…

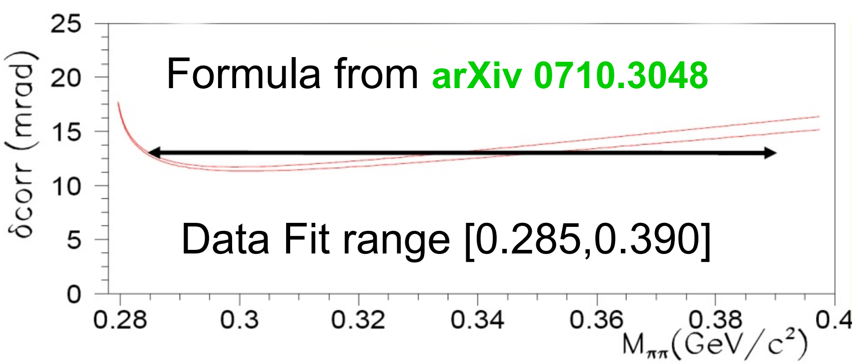
Scattering lengths extraction

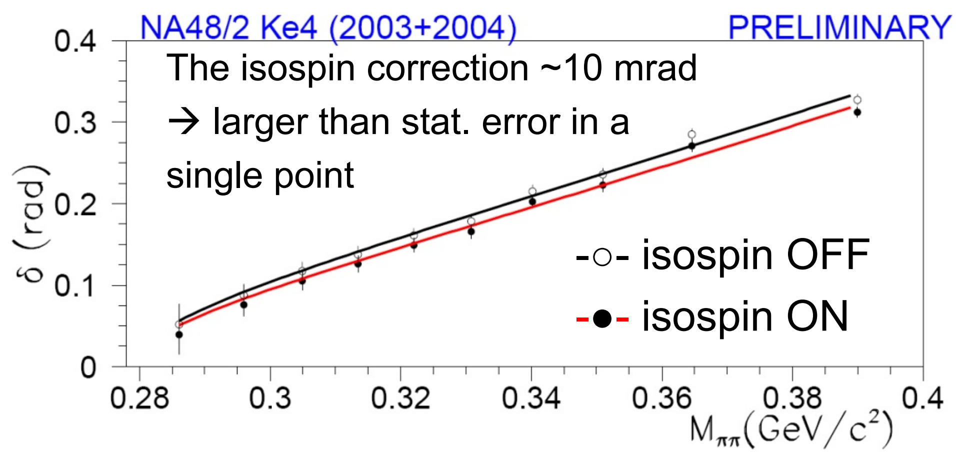
- External inputs for extracting of the S-wave scattering lengths from $\delta=(\delta_0^0-\delta_1^1)$ variation:
  - **Theoretical:** numerical solutions of **Roy equations** **[ACGL Phys. Rep.353 (2001), DFGS EPJ C24 (2002)]** relate δ and (a_0,a_2)
  - **Universal band** → allowed solutions from the experimental inputs at high energy
- χPT precise prediction **[CGL NPB603(2001)]**:

  $a_0 = 0.220 \pm 0.005$ $\qquad$ $a_2 = -0.0444 \pm 0.0008$
- **2p fit** → find a_0 and a_2 as free parameters
- **1p fit** → find a_0 by using χPT constraint
- Radiative effects implemented in MC simulation
- **Isospin corrections considered!**

2p fit	Isospin corr OFF	Isospin corr ON
a_0	0.244 ± 0.013	0.218 ± 0.013
a_2	-0.0385 ± 0.0084	-0.0457 ± 0.0084

Large 2σ effect!





Scattering lengths result

Preliminary 2003+2004:

2p fit:

$a_0 = 0.218 \pm 0.013_{stat} \pm 0.007_{syst} \pm 0.017_{theor}$

$a_2 = -0.0457 \pm 0.0084_{stat} \pm 0.0041_{syst} \pm 0.0030_{theor}$

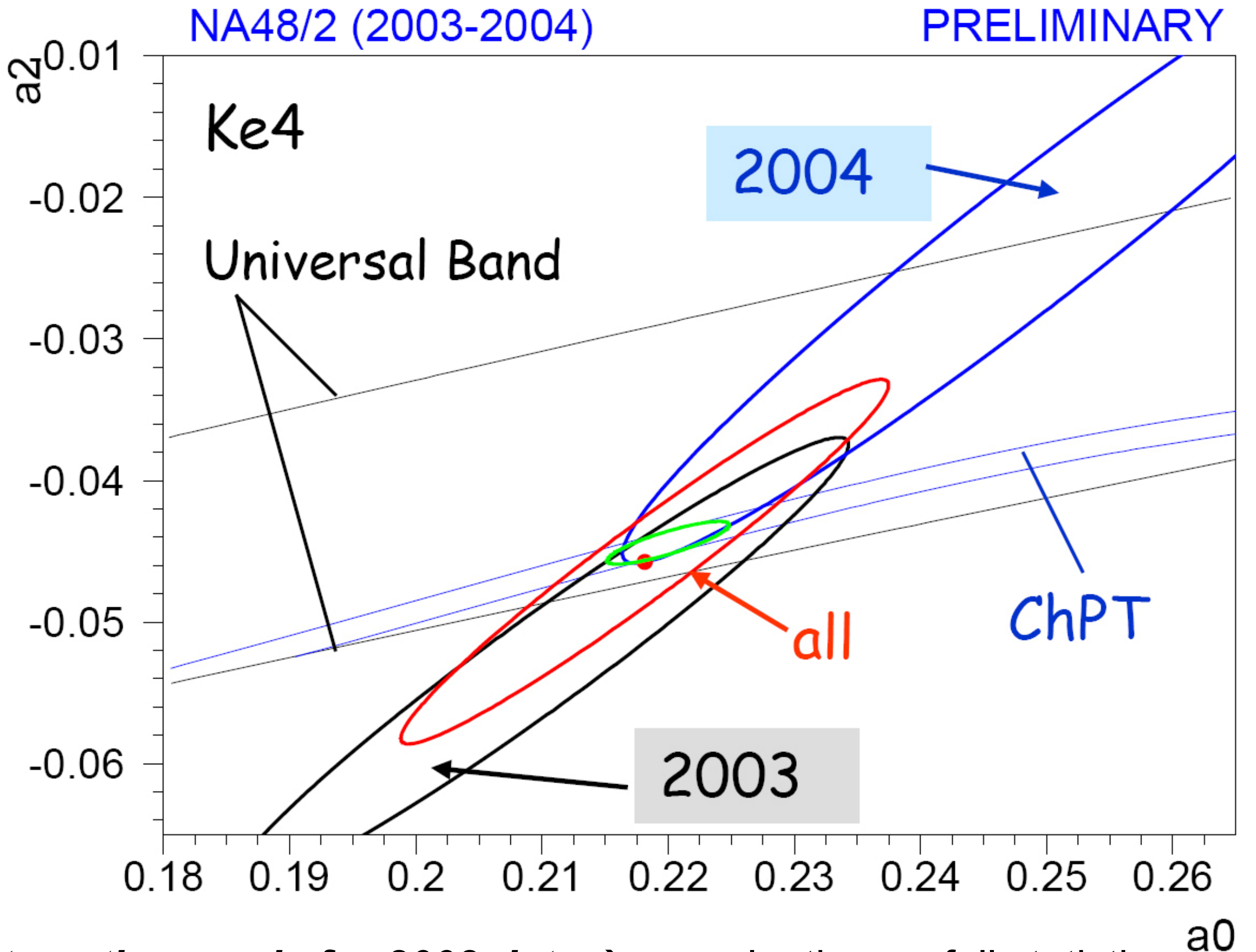
1p ChPT fit:

$a_0 = 0.220 \pm 0.005_{stat} \pm 0.002_{syst} \pm 0.006_{theor}$

χPT prediction:

$a_0 = 0.220 \pm 0.005$

$a_2 = -0.0444 \pm 0.0008$



Systematic error is for 2003 data → reevaluation on full statistics.

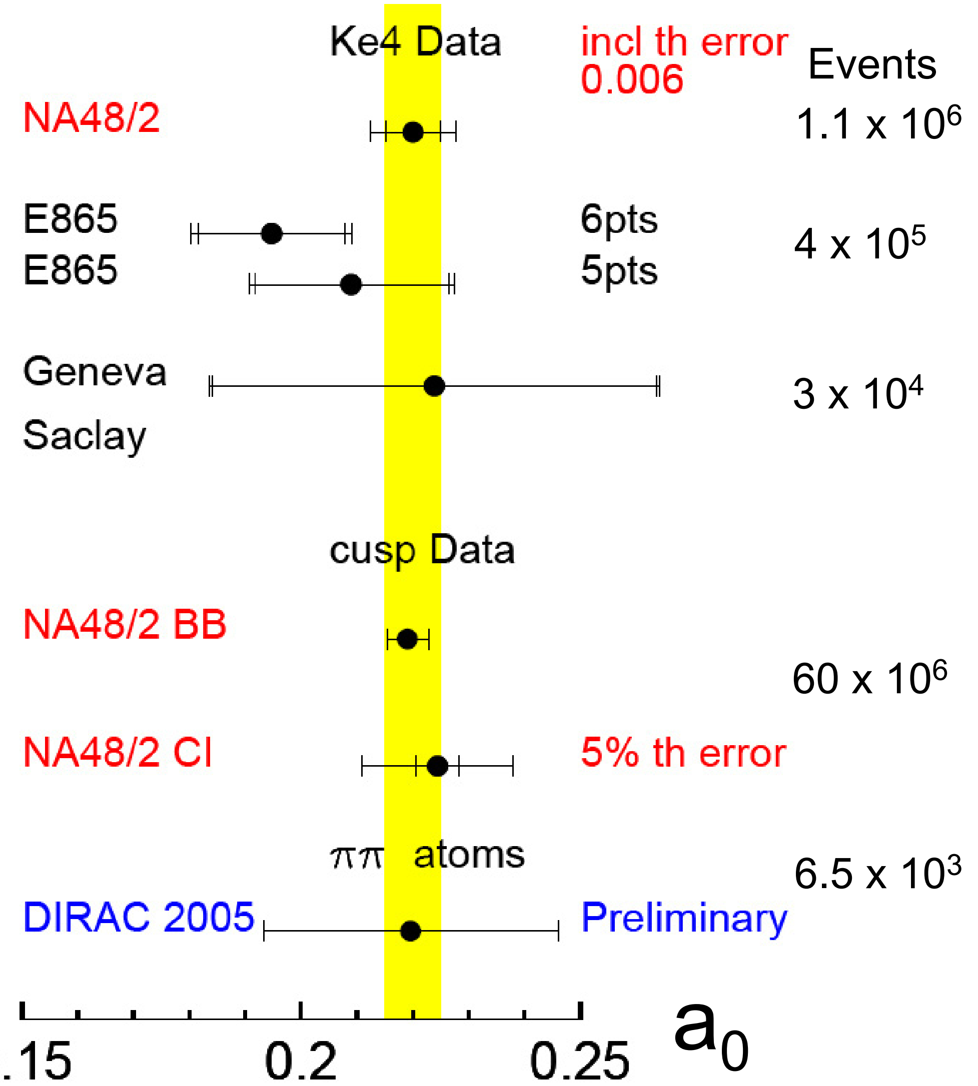
Theoretical error → from control of the isospin corr. and inputs to Roy equations.

Conclusions

- ***Comparisons with world data:***
  - **Ke4:** apply isospin corrections to published phase points of all experiments and perform a_0 χPT fit
  - **Cusp:** (a_0-a_2) χPT fit with two models
  - **$\pi\pi$ atoms Dirac:** preliminary
  - NA48/2 results: preliminary for 2003+2004
- χPT prediction (yellow band):

$$a_0 = 0.220 \pm 0.005$$

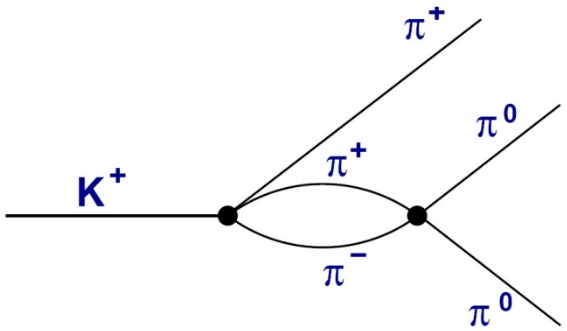
- NA48/2 experimental precision now at the same level
- Final publications coming soon.

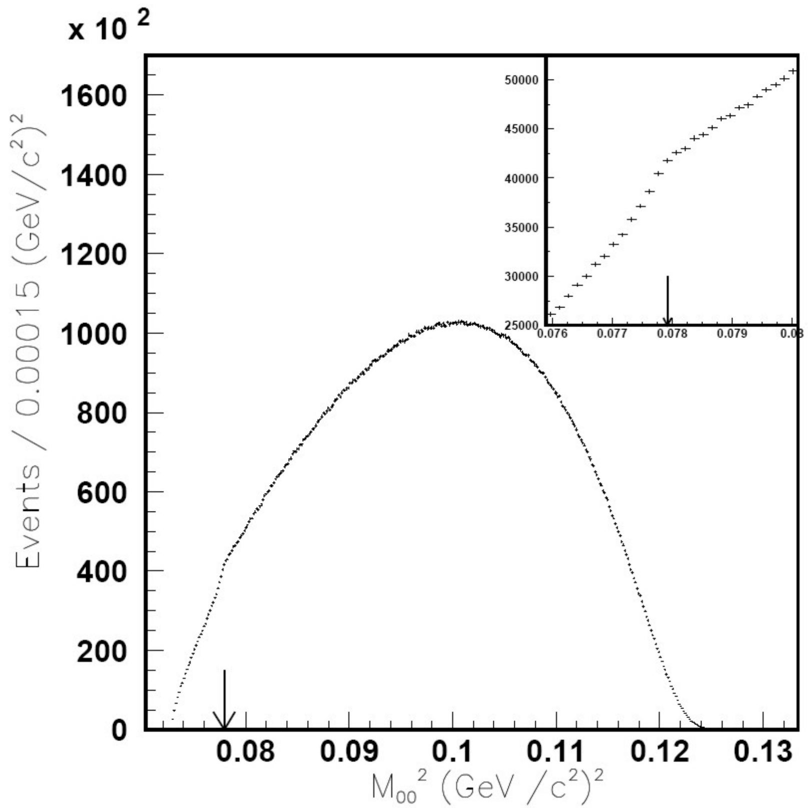


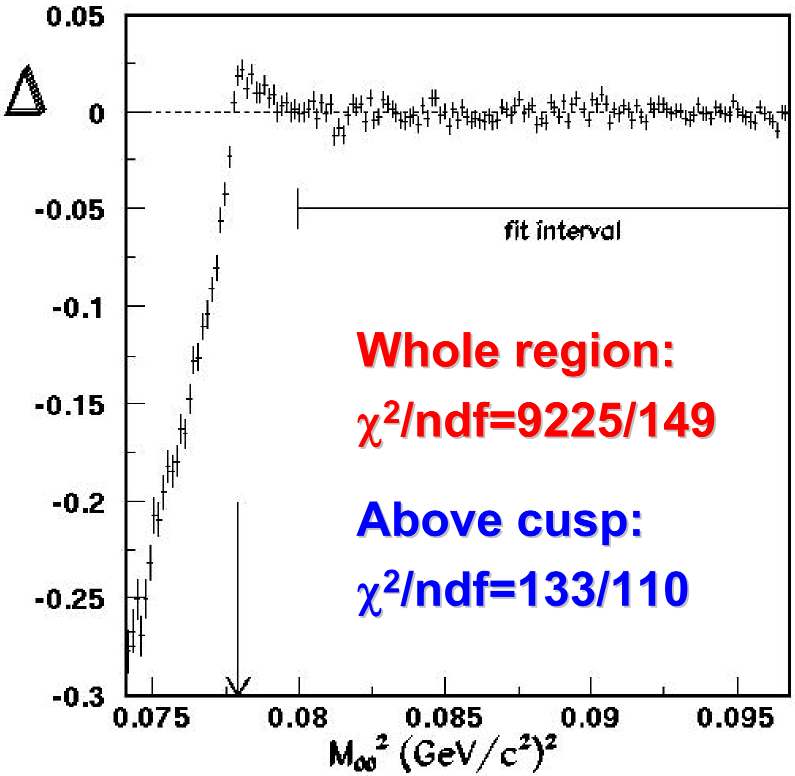
SPARE SLIDES

$K^{\pm} \rightarrow \pi^{\pm}\pi^{0}\pi^{0}$: "cusp" in M_{00}^2 distribution

- Selected events (2003+2004): **~60 x 10^6**
- M_{00}^2 computed imposing the mean vertex of the π_0s (improved resolution close to the threshold)
- Evidence for a **cusp** at $M_{00}^2 = 4m_{\pi+}^2$ due to $\pi\pi$ rescattering

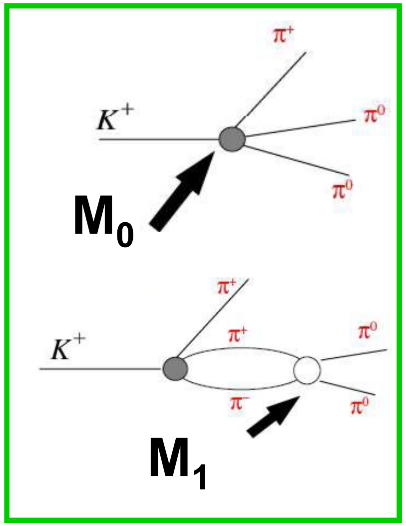






$K^{\pm}\rightarrow\pi^{\pm}\pi^0\pi^0$: "cusp" in M_{00}^2 distribution

- ***N. Cabibbo – Rescattering model at one loop***
  - Direct emission, given by $M_0=1+gu/2+h'u^2/2$
  - Charge exchange ($\pi^+\pi^-\rightarrow\pi^0\pi^0$) in final state of $K\rightarrow\pi\pi^+\pi^-$, given by M_1
  - **M_0 and M_1 interfere** destructively below the threshold



- ***Cabibbo-Isidori (CI) model:*** More complete formulation of the model includes all rescattering processes at one loop and two loop level **[JHEP 0503 (2005) 21]** has been used to extract NA48/2 result. **Theoretical uncertainty** → **5%**

- ***Bern-Bonn model*** (Colangelo, Gasser, Kubis, Rusetsky): effective field theory approach based on non-relativistic Lagrangian; electromagnetic effects included in the amplitudes; two loop formulation **[arXiv:0807.0515]**

Using χPT constraint:

CI: $a_0-a_2 = 0.268 \pm 0.003_{stat} \pm 0.002_{syst} \pm 0.001_{ext} \pm 0.013_{th}$

BB: $a_0-a_2 = 0.266 \pm 0.003_{stat} \pm 0.002_{syst} \pm 0.001_{ext}$

a_2 free:

CI: $a_0-a_2 = 0.266\pm0.005_{stat}\pm0.002_{syst} \pm0.001_{ext} \pm0.013_{th}$

$a_2 = -0.039\pm0.009_{stat}\pm0.006_{syst} \pm0.002_{ext}$

BB: $a_0-a_2 = 0.273\pm0.005_{stat}\pm0.002_{syst} \pm0.001_{ext}$

$a_2 = -0.065\pm0.015_{stat}\pm0.010_{syst} \pm0.002_{ext}$