Pion FF in QCD Sum Rules with NLCs

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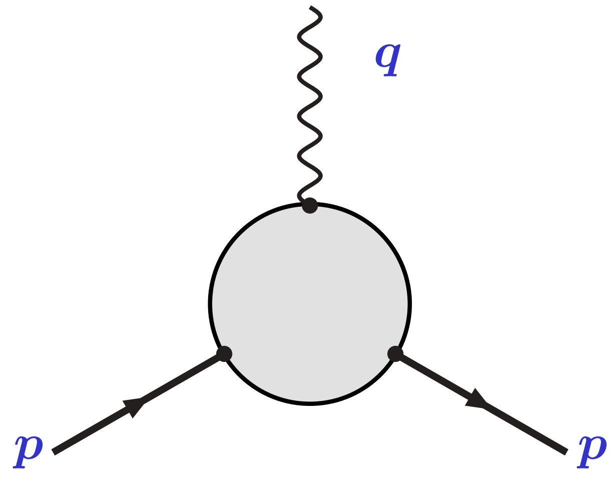
- QCD SR vs experimental data.
- Local Duality approach and NLC SRs.
- Conclusions.

Definition of pion FF

Pion FF F_π is defined by the matrix element

$$\langle \pi^+(p') | J_\mu(0) | \pi^+(p) \rangle = \left(p + p'\right)_\mu F_\pi(Q^2)\,,$$



where J_μ is the electromagnetic current, $(p' - p)^2 = q^2 \equiv -Q^2$ is the photon virtuality, and pion FF is normalized to $F_\pi(0) = 1$.

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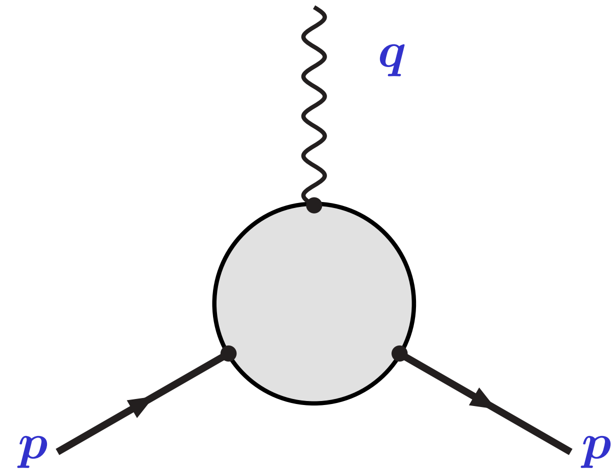

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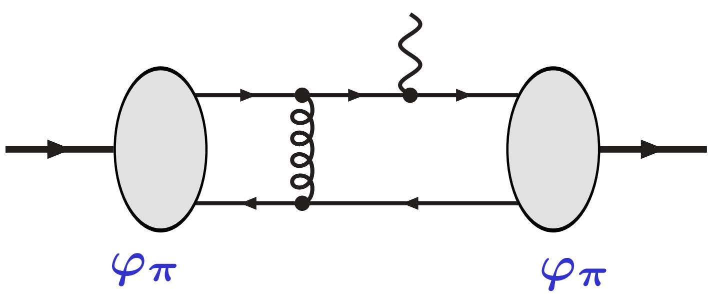
At asymptotically large Q^2 pQCD factorization gives pion FF



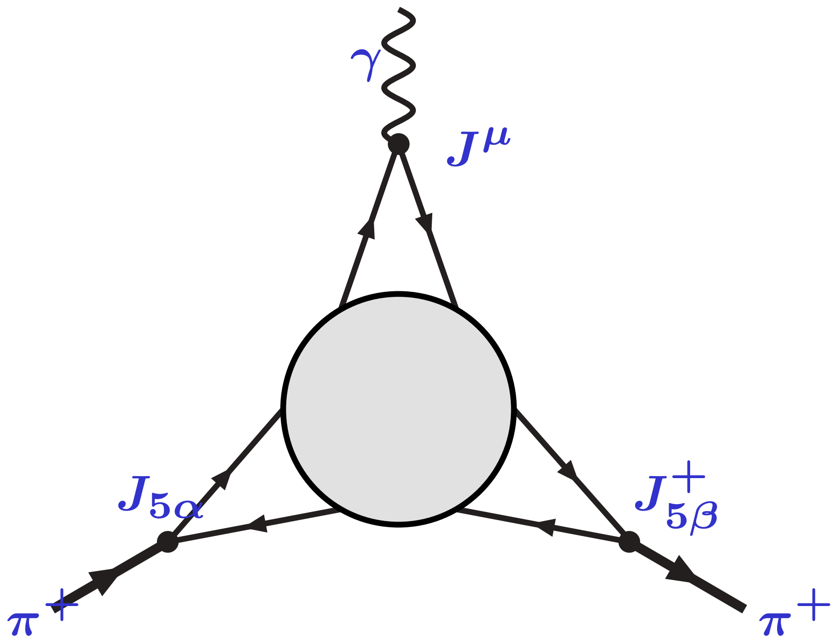
$$F_\pi(Q^2) = \frac{8\pi\alpha_s(Q^2) f_\pi^2}{9Q^2}\left|\int_0^1 \frac{\varphi_\pi(x,Q^2)}{x}\,dx\right|^2$$

in terms of twist-2 pion DA $\varphi_\pi(x,Q^2)$.

AAV correlator

Axial-Axial-Vector correlator can be used for studying pion FF by QCD SR technique:

$$\int\!\!\int d^4x\, d^4y\, e^{i(qx-p_2y)} \langle 0|T\left[J^{+}_{5\beta}(y) J^{\mu}(x) J_{5\alpha}(0)\right]|0\rangle$$



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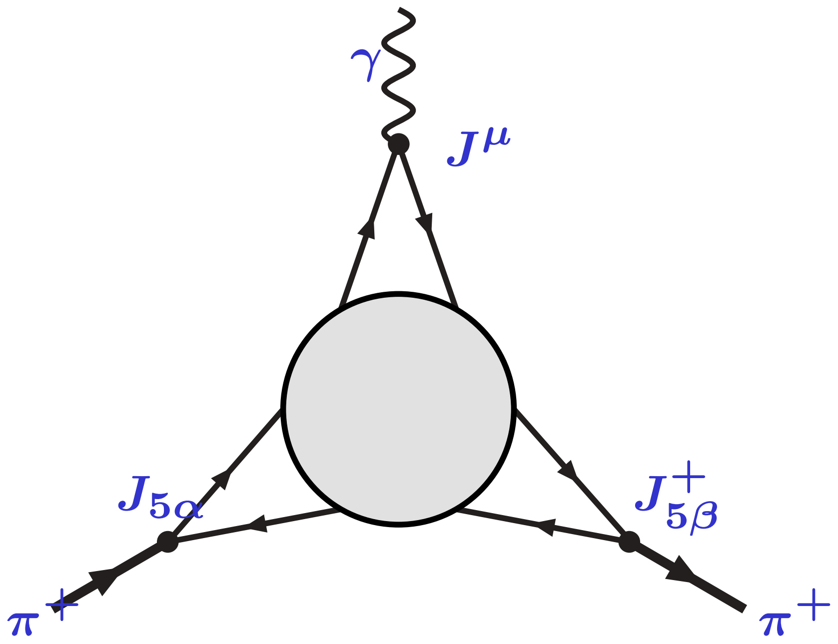

where EM current $J^{\mu}(x) = e_u\, \overline{u}(x)\gamma^{\mu}u(x) + e_d\, \overline{d}(x)\gamma^{\mu}d(x)$

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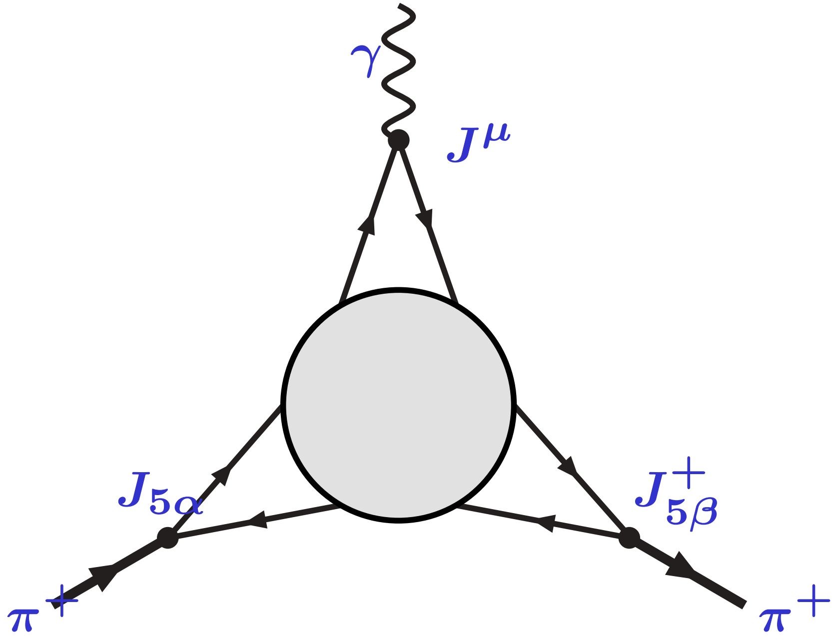
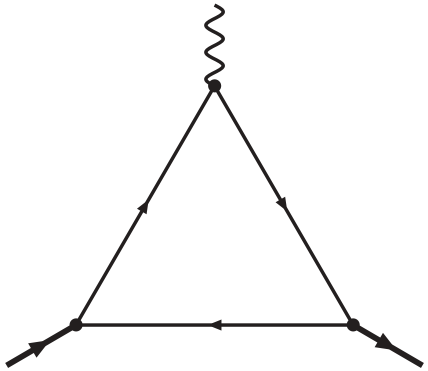


where EM current $J^\mu(x) = e_u\, \overline{u}(x)\gamma^\mu u(x) + e_d\, \overline{d}(x)\gamma^\mu d(x)$ and axial-vector current: $J_{5\alpha}(x) = \overline{d}(x)\gamma_5\gamma_\alpha u(x)$.

Diagramms for AAV-correlator



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Nesterenko&Radyushkin
$\oplus$**Ioffe&Smilga [1982]**

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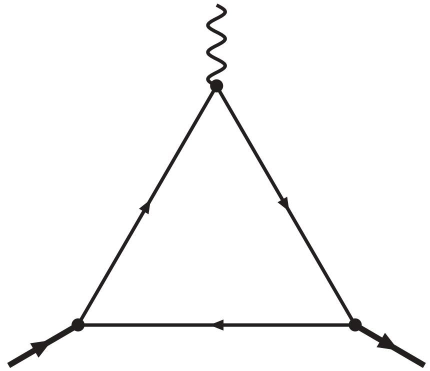

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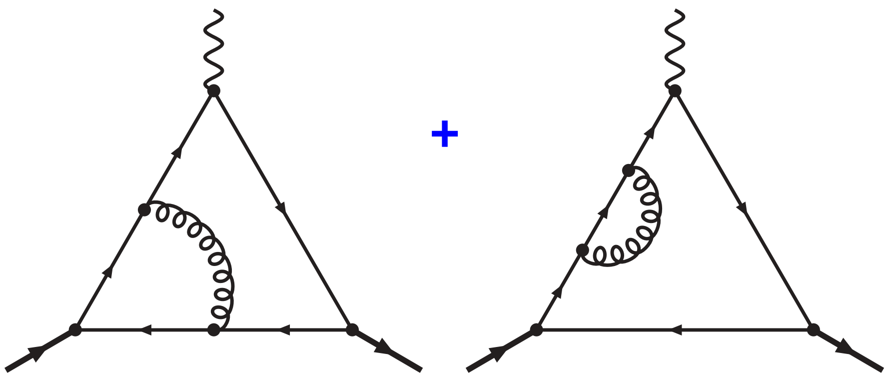

Perturbative NLO terms

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Braguta&Onishchenko [2004]

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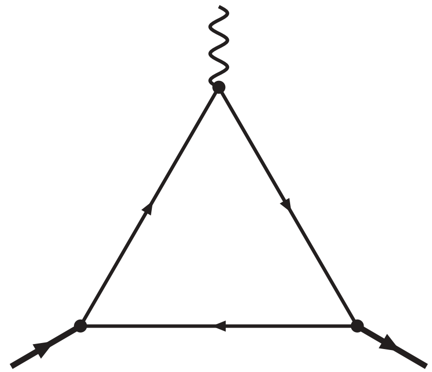

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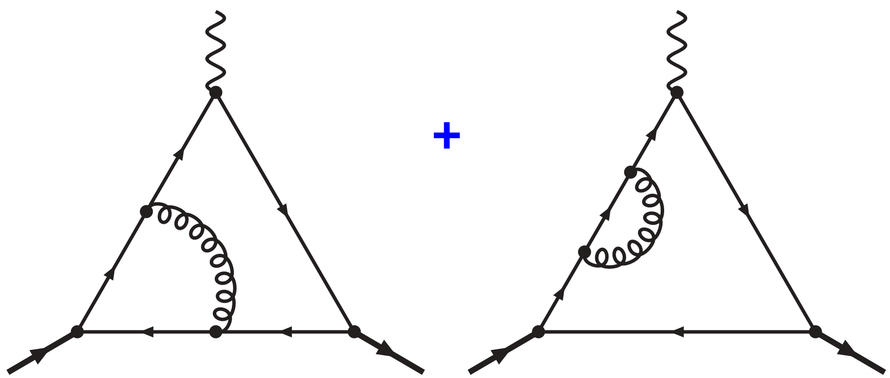

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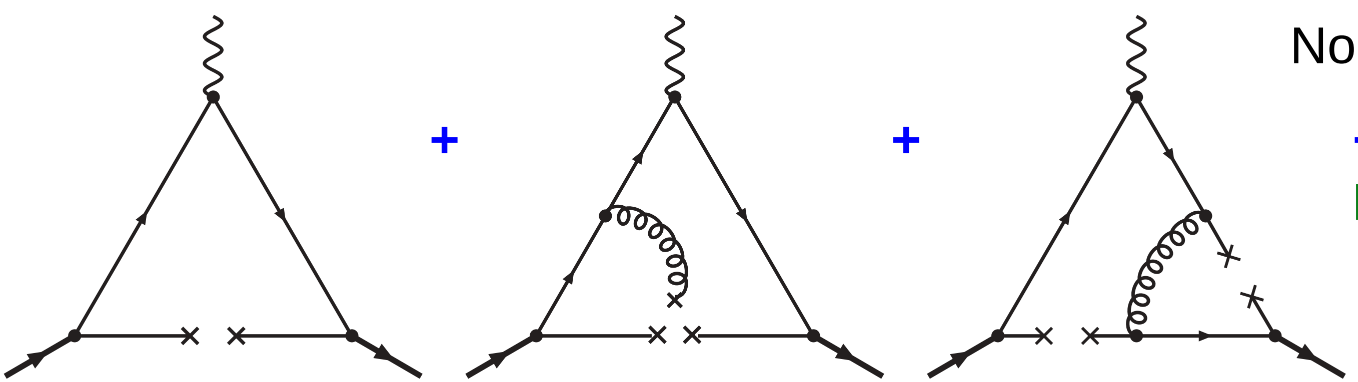

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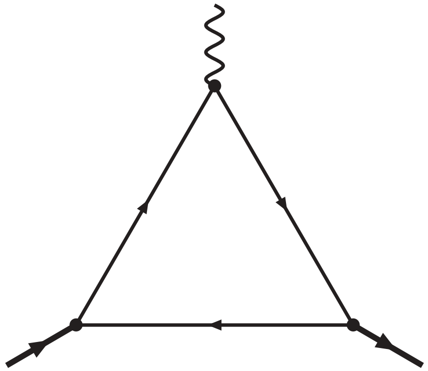

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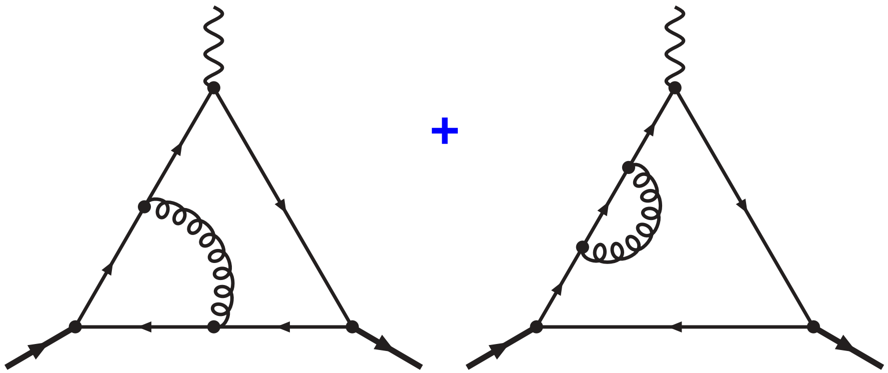

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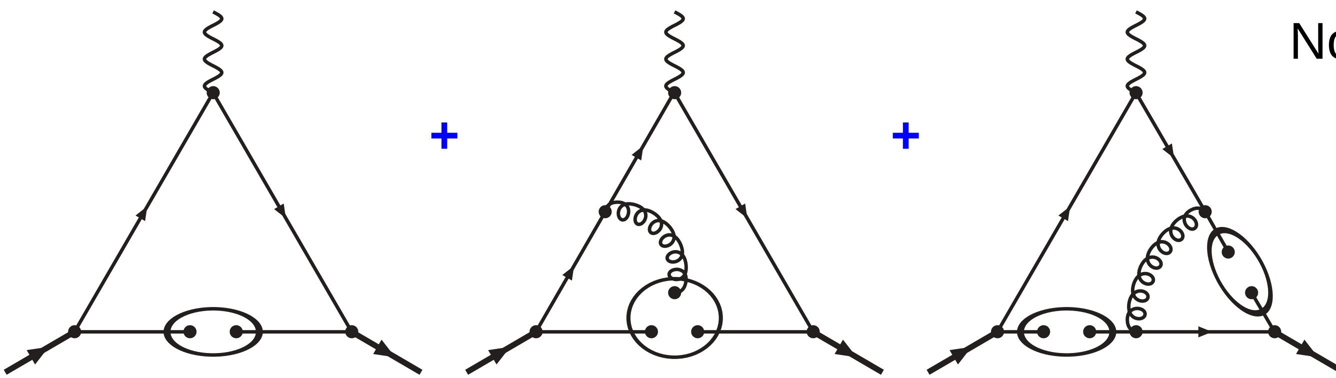

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nonlocal condensates

Introducing NLC in QCD calculations

$$T\left(\overline{\psi}\psi\right) = \overline{\psi}\psi + :\overline{\psi}\psi: \text{ (Wick theorem)}$$

$$\langle 0|T\left(\overline{\psi}\psi\right)|0\rangle = i^{-1}\hat{S}_0(x) + \boxed{?}$$

QCD PT

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$$\langle:\overline{\psi}(0)\psi(0):\rangle = \langle\overline{q}q\rangle$$

CONST $\neq 0$

[SVZ'79]

Condensate

Decay constants,

masses of hadrons

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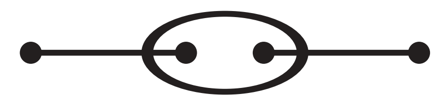
NLC QCD SR

$$\langle:\overline{\psi}(0)\psi(z):\rangle$$

$$F_S(z^2) + \hat{z}F_V(z^2)$$



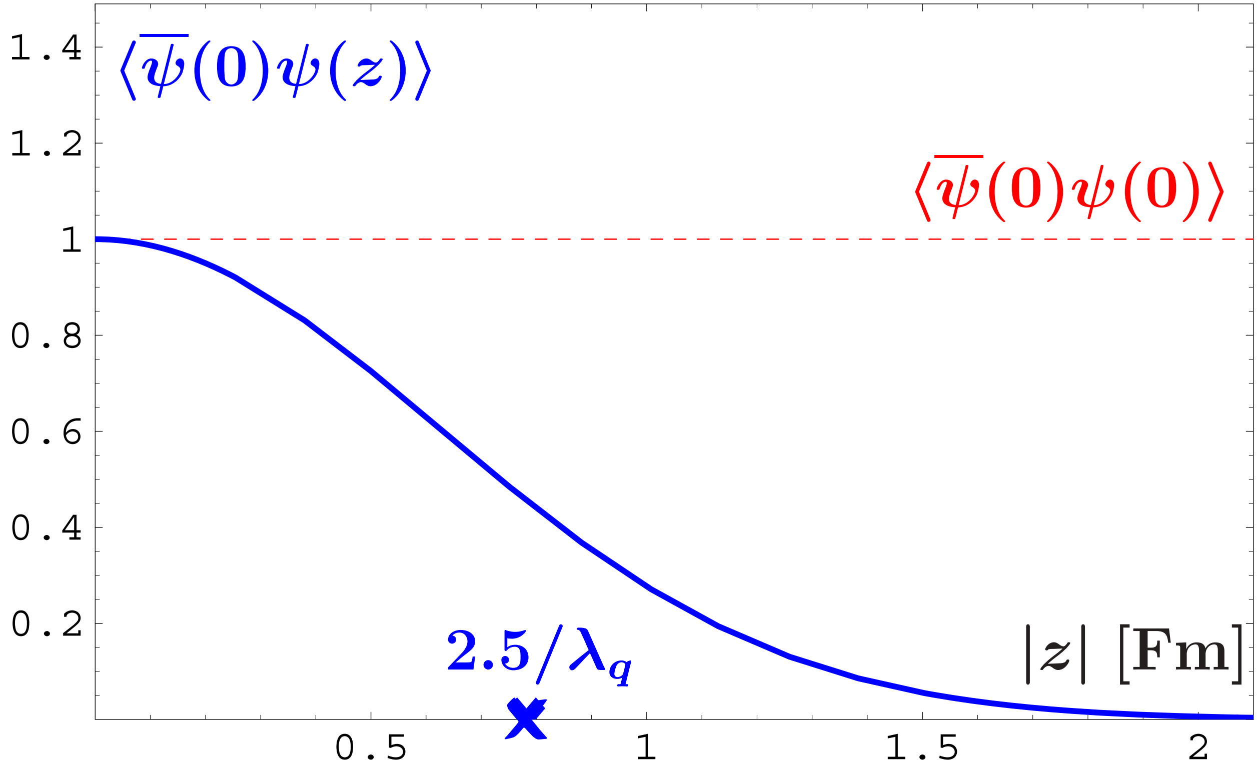
M&R '86

Nonlocal condensate

Distribution Amplitudes,

Form Factors

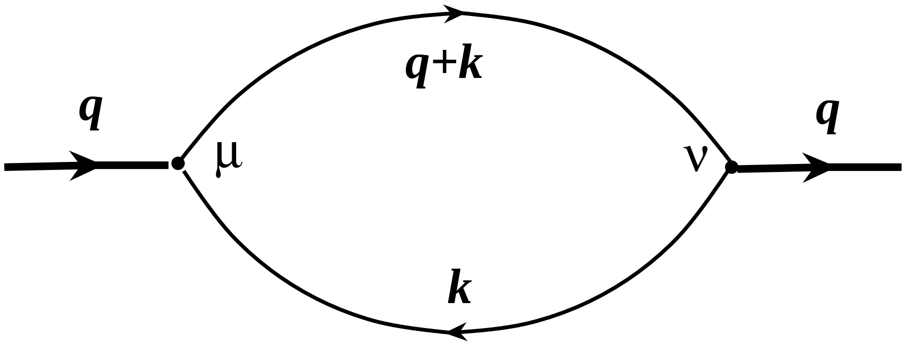
Lattice data of Pisa group



Nonlocality of quark condensates from lattice data of Pisa group in comparison with **local limit**.

Even at $|z| \simeq 0.5$ **Fm** nonlocality is quite important!

Diagrams for $\langle T\left(J_1(z)J_2(0)\right)\rangle$



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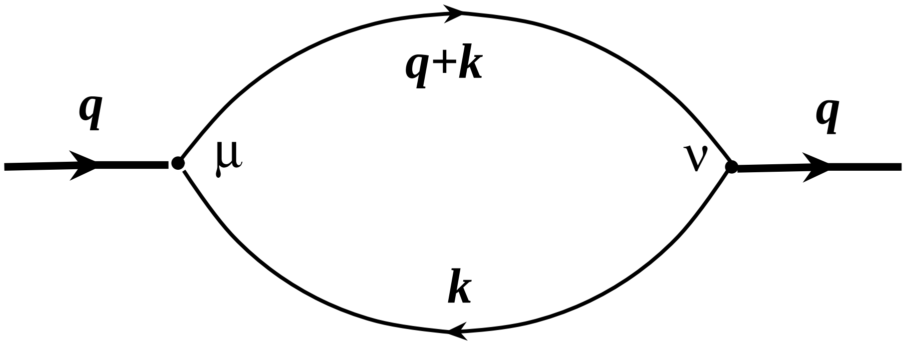

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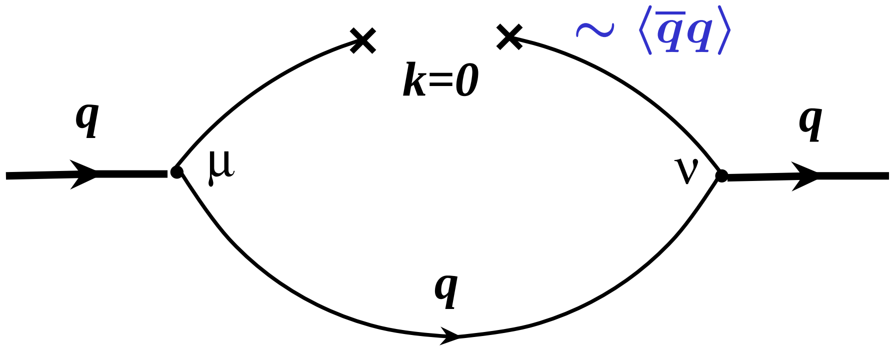

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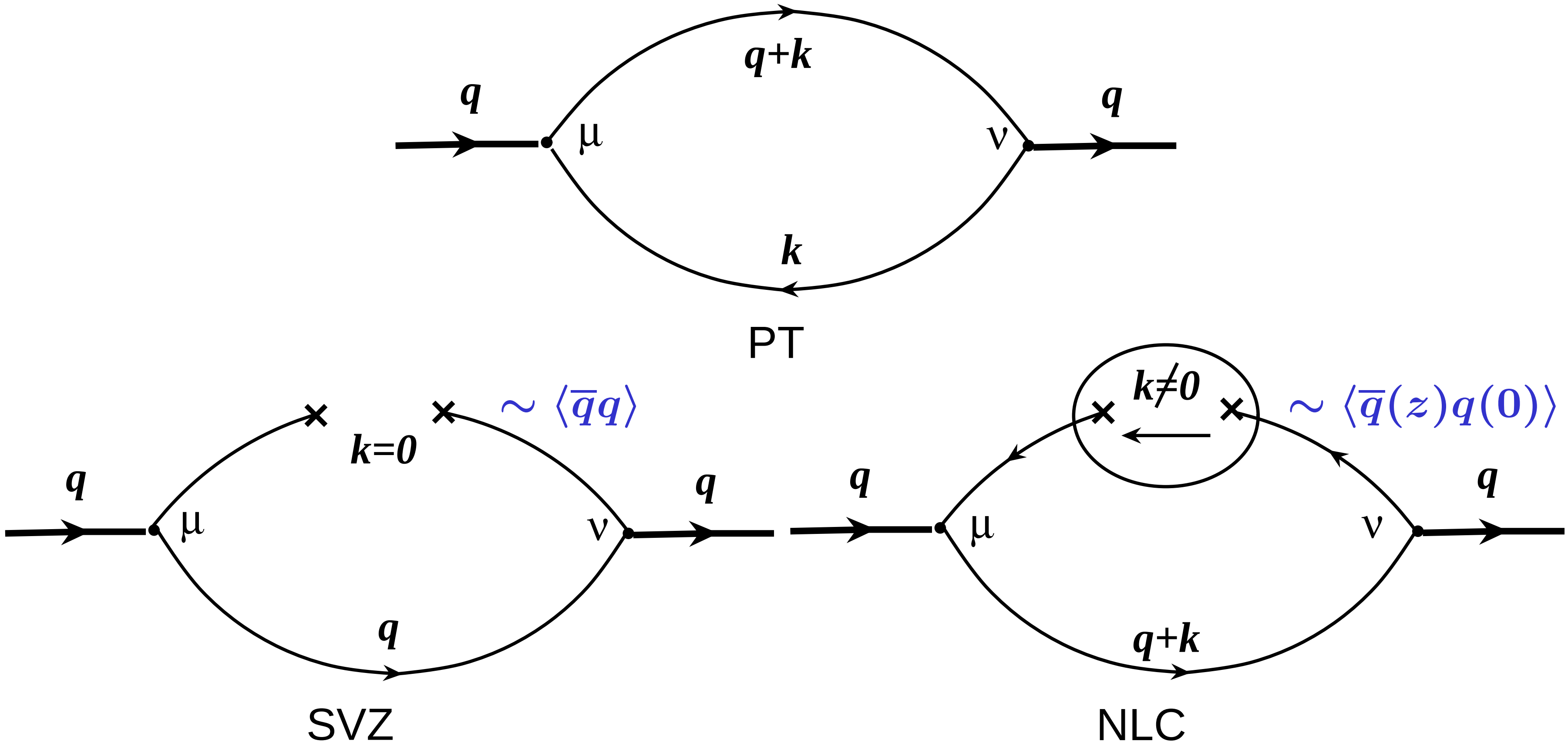

Quarks run through vacuum with nonzero momentum $k \neq 0$:

$$\langle k^2\rangle = \frac{\langle\overline{\psi}D^2\psi\rangle}{\langle\overline{\psi}\psi\rangle} = \lambda_q^2 = 0.4 - 0.5\,\text{GeV}^2$$

Non-Local Condensates in QCD SR

- Illustration of NLC-model: $\langle\bar{q}(0)q(z)\rangle = \langle\bar{q}(0)q(0)\rangle e^{-|z^2|\lambda_q^2/8}$

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- A **single scale** parameter $\lambda_q^2 = \langle k^2\rangle$ characterizing the average momentum of quarks in QCD vacuum:

$$\lambda_q^2 = \begin{cases} 0.4 \pm 0.1\ \text{GeV}^2 & [\text{ QCD SRs, 1987 }] \\ 0.5 \pm 0.05\ \text{GeV}^2 & [\text{ QCD SRs, 1991 }] \\ 0.4 - 0.5\ \text{GeV}^2 & [\text{ Lattice, 1998-2002 }] \end{cases}$$

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- Correlation length $\lambda_q^{-1} \sim \rho$-meson size
- Possible to include second ($\Lambda \simeq 450$ MeV) scale with $\langle\bar{q}(0)q(z)\rangle\Big|_{|z|\gg 1\text{ Fm}} \sim \langle\bar{q}q\rangle e^{-|z|\Lambda}$ (not included here)

Non-Local Condensates in QCD SR

Parameterization for scalar and vector condensates:

$$\langle \bar{\psi}(0)\psi(x)\rangle = \langle \bar{\psi}\psi\rangle \int_0^\infty \boxed{f_S(\alpha)}\, e^{\alpha x^2/4}\, d\alpha\,;$$

$$\langle \bar{\psi}(0)\gamma_\mu\psi(x)\rangle = -ix_\mu A_0 \int_0^\infty \boxed{f_V(\alpha)}\, e^{\alpha x^2/4}\, d\alpha\,,$$

where $A_0 = 2\alpha_s\pi\langle\bar{\psi}\psi\rangle^2/81$.

Non-Local Condensates in QCD SR

Convenient to parameterize the 3-local condensate in fixed-point gauge by introduction of three scalar functions:

$$\begin{aligned}\langle\bar{\psi}(0)\gamma_\mu(-g\widehat{A}_\nu(x))\psi(y)\rangle &= (x_\mu y_\nu - g_{\mu\nu}(xy))\overline{M}_1 \\ &+ (x_\mu x_\nu - g_{\mu\nu}x^2)\overline{M}_2\,; \\ \langle\bar{\psi}(0)\gamma_5\gamma_\mu(-g\widehat{A}_\nu(x))\psi(y)\rangle &= i\varepsilon_{\mu\nu xy}\overline{M}_3\,,\end{aligned}$$

with

$$\overline{M}_i(y^2, x^2, (x-y)^2) =$$

$$A_i \iiint_0^\infty d\alpha_1\, d\alpha_2\, d\alpha_3 \boxed{f_i(\alpha_1,\alpha_2,\alpha_3)}\, e^{(\alpha_1 y^2 + \alpha_2 x^2 + \alpha_3 (x-y)^2)/4}\,.$$

where $A_i = \{-\frac{3}{2}, 2, \frac{3}{2}\}A_0$ **[Mikhailov&Radyushkin'89]**.

Non-Local Condensates in QCD SR

The minimal Gaussian ansatz:

$$f_S(\alpha) = \delta\,(\alpha - \Lambda)\ ; \quad f_V(\alpha) = \delta\,'(\alpha - \Lambda)\ ; \quad \Lambda \equiv \lambda_q^2/2\,;$$

$$f_i\,(\alpha_1, \alpha_2, \alpha_3) = \delta\,(\alpha_1 - \Lambda)\ \delta\,(\alpha_2 - \Lambda)\ \delta\,(\alpha_3 - \Lambda)\ .$$

Only one parameter $\lambda_q^2 = 0.35 - 0.55$ **GeV^2**.

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Problems:

- **QCD equations of motion are violated**
- **Vector current correlator is not transverse**
  $\Rightarrow$ **gauge invariance is broken**

Improved Gaussian model

We modify functions f_i: $f_i^{\text{imp}}(\alpha_1, \alpha_2, \alpha_3) =$

$$(1 + X_i \partial_x + Y_i \partial_y + Z_i \partial_z)\, \delta(\alpha_1 - x\Lambda)\, \delta(\alpha_2 - y\Lambda)\, \delta(\alpha_3 - z\Lambda)$$

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What does it give us?

- **If** $12\,(X_2 + Y_2) - 9\,(X_1 + Y_1) = 1\,,\; x + y = 1\,,$

  than QCD equations of motion are satisfied;

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  than QCD equations of motion are satisfied;

- **We minimize nontransversity of polarization operator by special choice of model parameters:**

$$\begin{aligned} X_1 &= -0.082\,;\ Y_1 = Z_1 = -2.243\,;\ x = 0.788\,; \\ X_2 &= -1.298\,;\ Y_2 = Z_2 = -0.239\,;\ y = 0.212\,; \\ X_3 &= +1.775\,;\ Y_3 = Z_3 = -3.166\,;\ z = 0.212\,. \end{aligned}$$

NLC QCD SR

The Borel SR for the pion FF based on three-point AAV correlator:

$$f_\pi^2\, F_\pi(Q^2) = \iint\limits_0^{s_0} ds_1\, ds_2\; \rho_3(s_1, s_2, Q^2)\, e^{-(s_1+s_2)/M^2} + \Phi_{\text{OPE}}(Q^2, M^2)\,.$$

Approach	Acc	Condensates	Q^2-behavior of Φ_{OPE}
N&R, I&S 82	LO	local	**const** $+\,Q^2$

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B&O 04 - LD	NLO	NO $M^2 \to \infty$	$\Phi_{\text{OPE}} \to 0$, $s_0 =$? (f_π LD SR)

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- Nonlocality improves Q^2 behavior of OPE $\Rightarrow$ widens region of applicability up to $Q^2 \simeq 10$ **GeV**^{2}.

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B&O 04 - LD	NLO	NO $M^2 \to \infty$	$\Phi_{\text{OPE}} \to 0$, $s_0 =$? (f_π LD SR)
Here	NLO	nonlocal	$(\text{const} + Q^2)\, e^{-Q^2\lambda_q^2}$

- Nonlocality improves Q^2 behavior of OPE $\Rightarrow$ widens region of applicability up to $Q^2 \simeq 10$ **GeV**^{2}.
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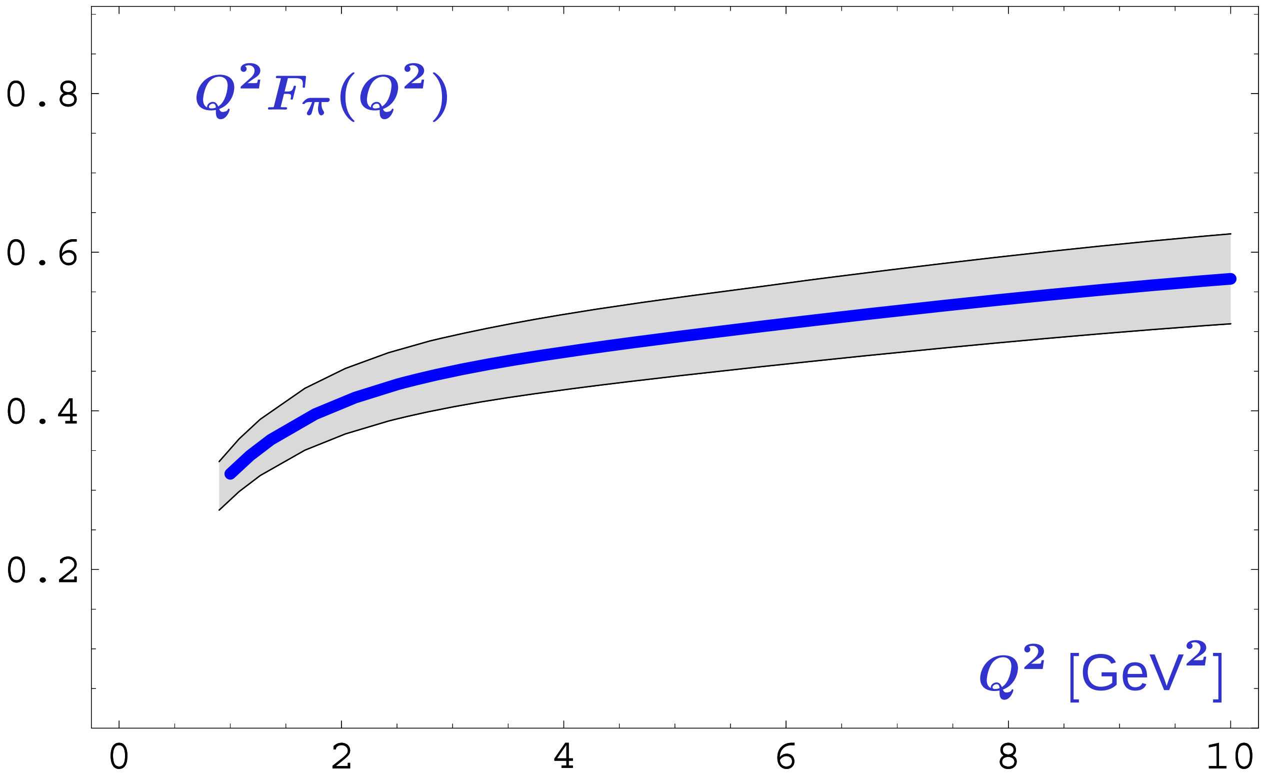
NLC QCD SR Result for $Q^2 F_\pi(Q^2)$



curve	approach
blue solid line	NLC QCD SR

Pion FF from: SRs with NLC **(blue solid line)**,

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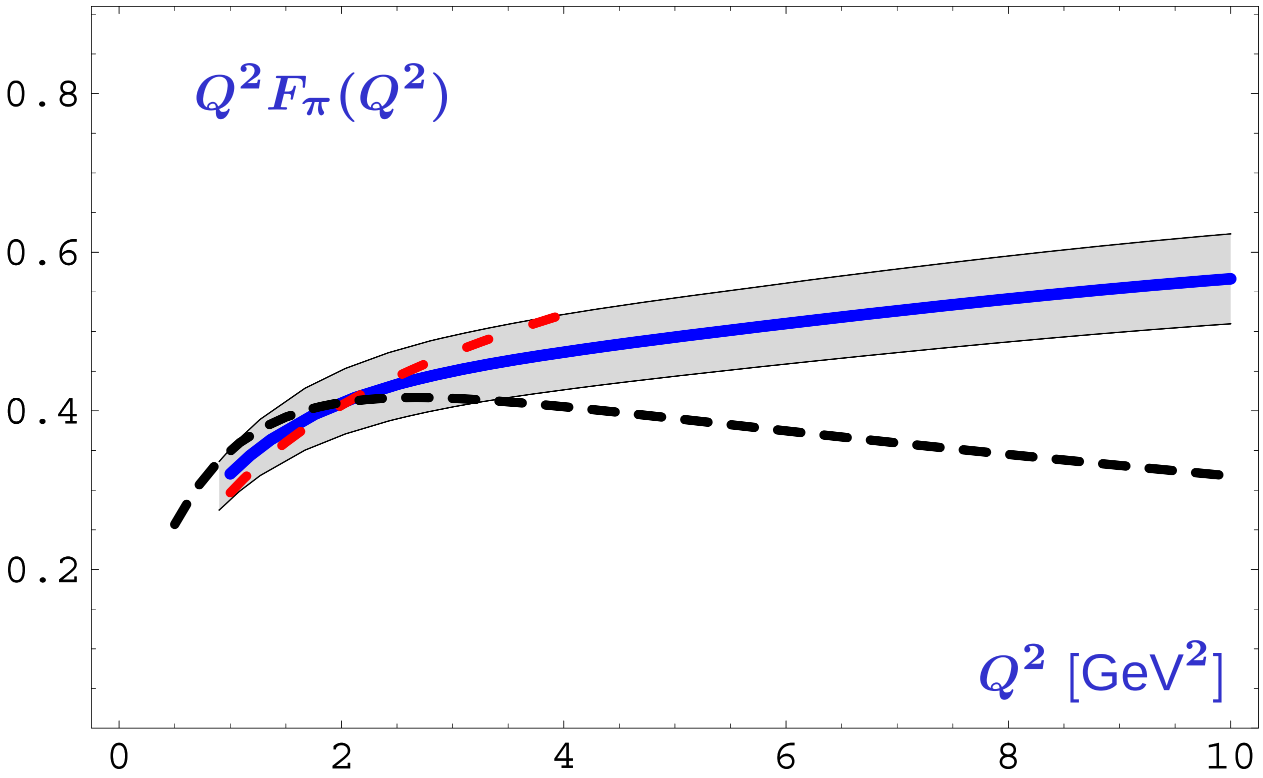
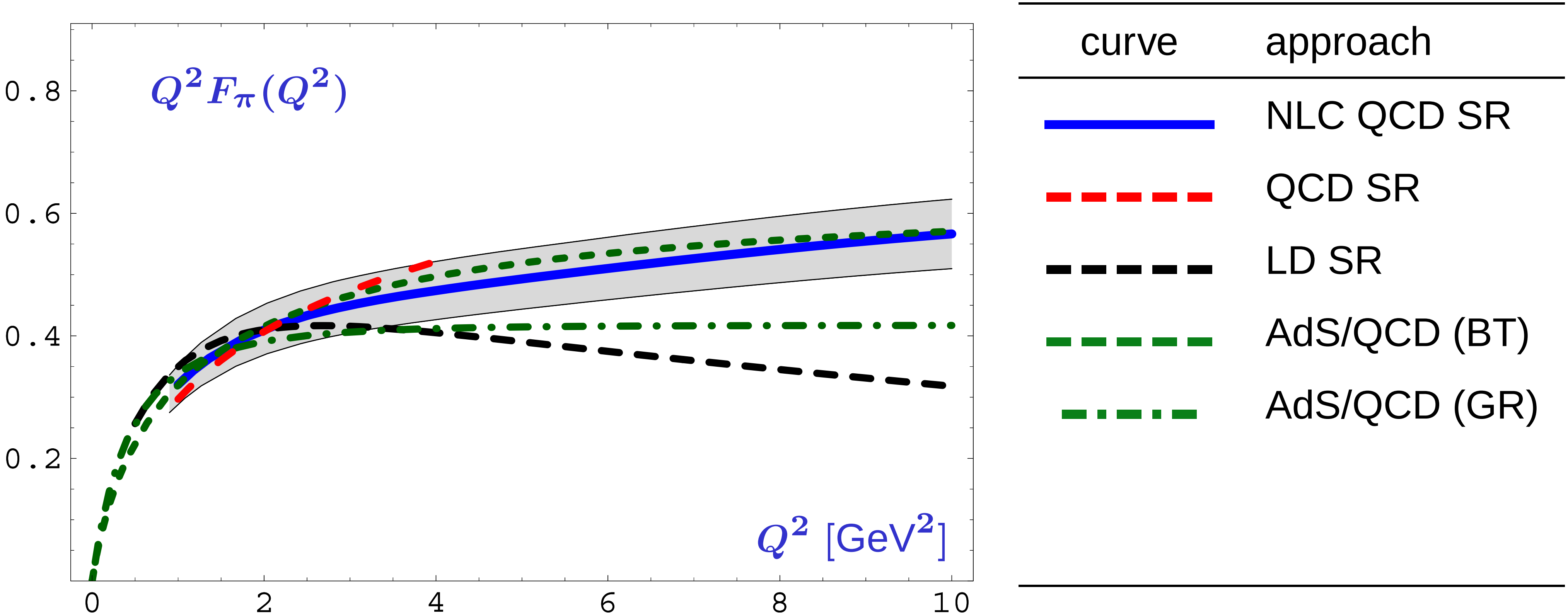


curve	approach
	NLC QCD SR
	QCD SR
	LD SR

Pion FF from: SRs with NLC **(blue solid line)**,
standard QCD SRs **(red dashed line)** **[N&R⊕I&S 82]**,
$O(\alpha_s)$ Local Duality **(black dashed line)** **[B&O 04]**,

NLC QCD SR Result for $Q^2\, F_\pi(Q^2)$



Pion FF from: SRs with NLC **(blue solid line)**, standard QCD SRs **(red dashed line)** **[N&R⊕I&S 82]**, $O(\alpha_s)$ Local Duality **(black dashed line)** **[B&O 04]**, AdS/QCD **(green dashed line)** **[B&T 07]** and **(green dot-dashed line)** **[G&R 07]**

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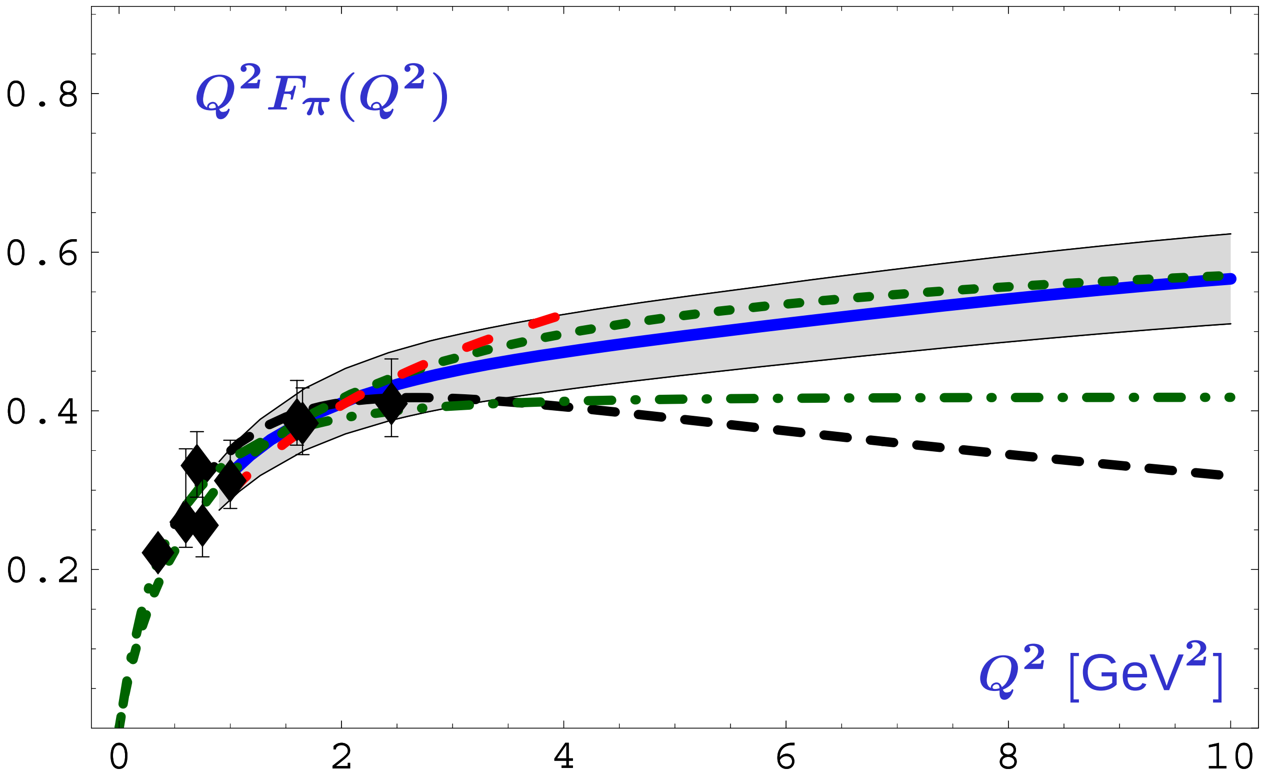

curve	approach
blue solid line	NLC QCD SR
red dashed line	QCD SR
black dashed line	LD SR
green dashed line	AdS/QCD (BT)
green dot-dashed line	AdS/QCD (GR)
♦	**[JLab 08]**

Pion FF from: SRs with NLC **(blue solid line)**, standard QCD SRs **(red dashed line)** **[N&R⊕I&S 82]**, $O(\alpha_s)$ Local Duality **(black dashed line)** **[B&O 04]**, AdS/QCD **(green dashed line)** **[B&T 07]** and **(green dot-dashed line)** **[G&R 07]** in comparison with **[JLab 08]** (♦) experimental data.

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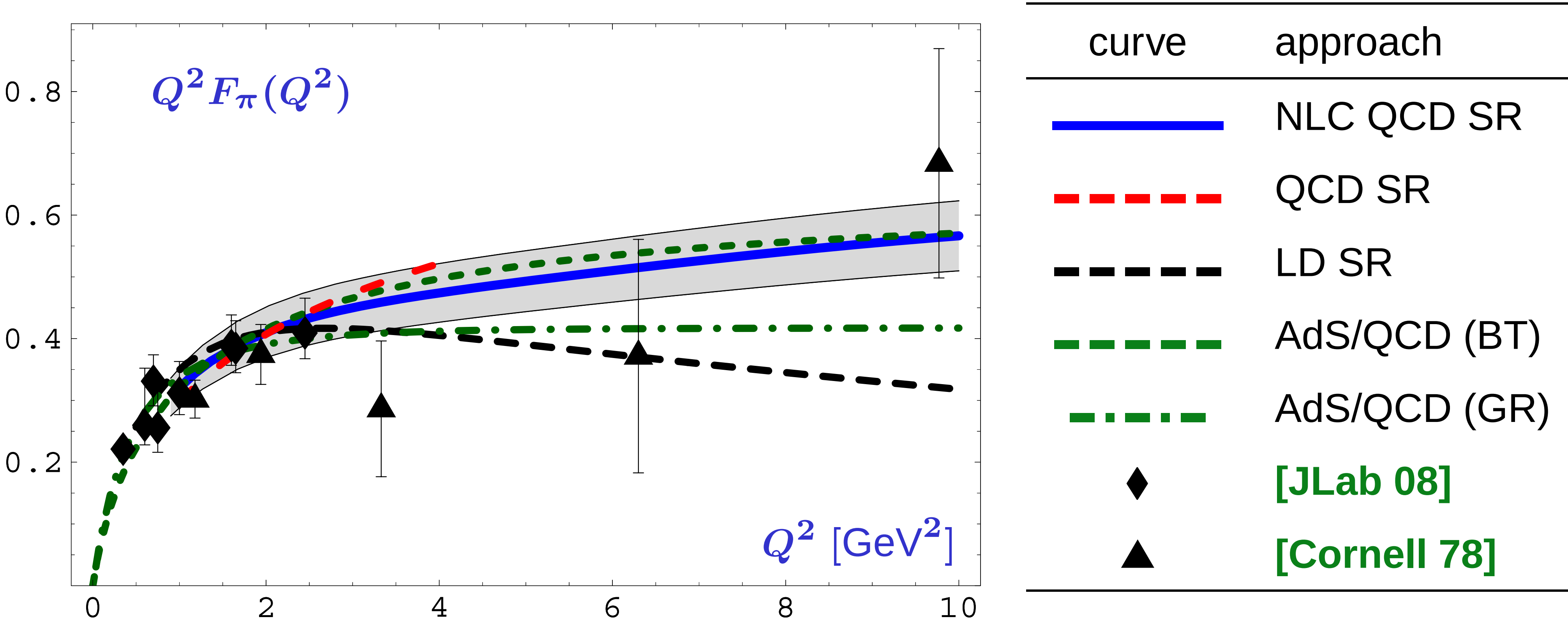

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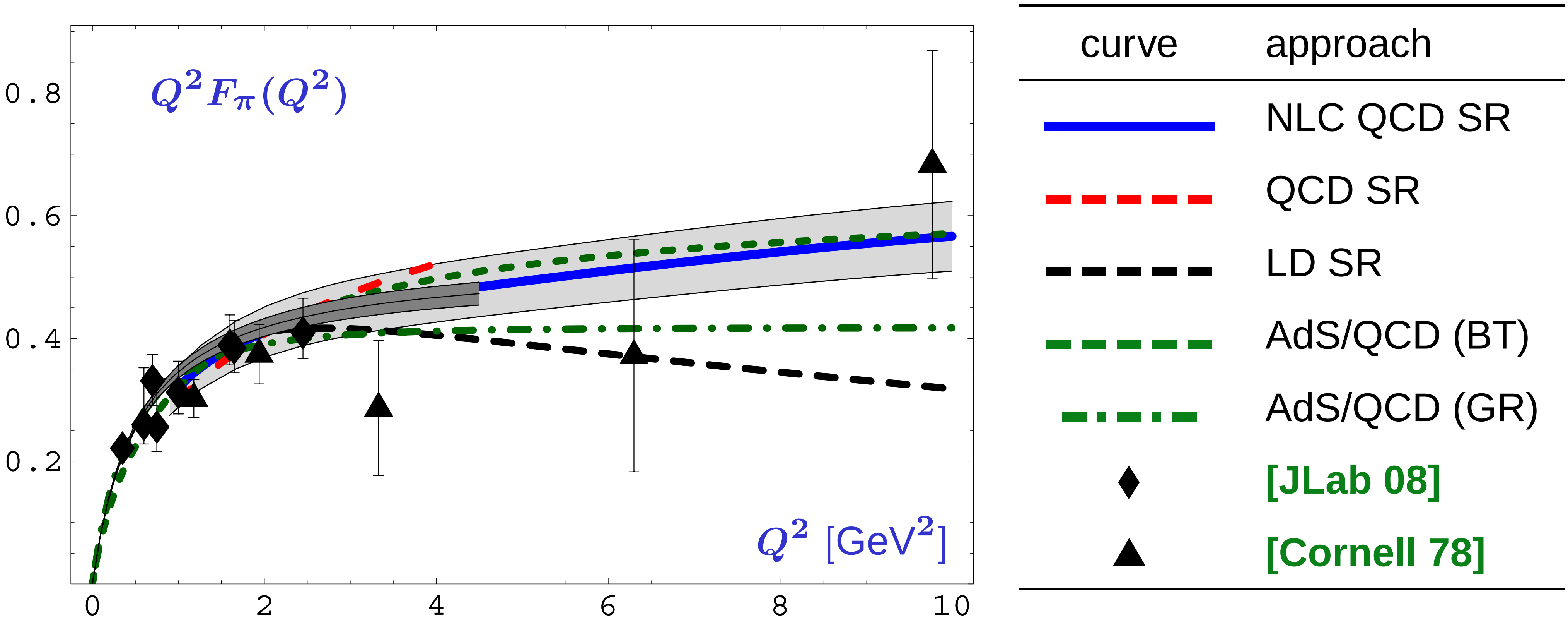
NLC QCD SR vs. Lattice QCD results



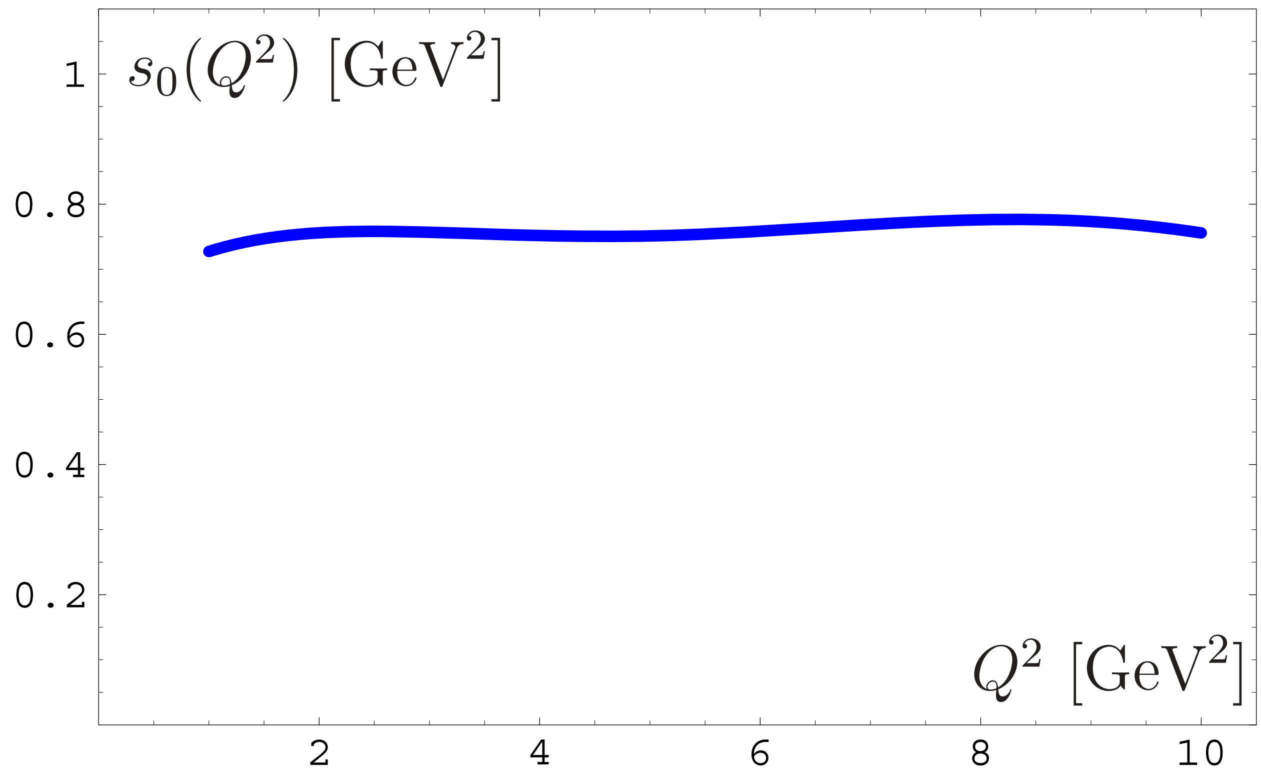
Pion FF from: SRs with NLC **(blue solid line)**,

in comparison with recent lattice results by **D. Brommel et al. [Eur. Phys. J., C51 (2007) 335]**.

Local Duality vs Sum Rules for pion FF

Borel SR:

$$f_\pi^2 F_\pi(Q^2) = \iint_0^{s_0} ds_1\, ds_2\, \rho_3(s_1, s_2, Q^2)\, e^{-(s_1+s_2)/M^2} + \Phi_{\text{OPE}}(Q^2, M^2)\,.$$



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Approximation of LD result

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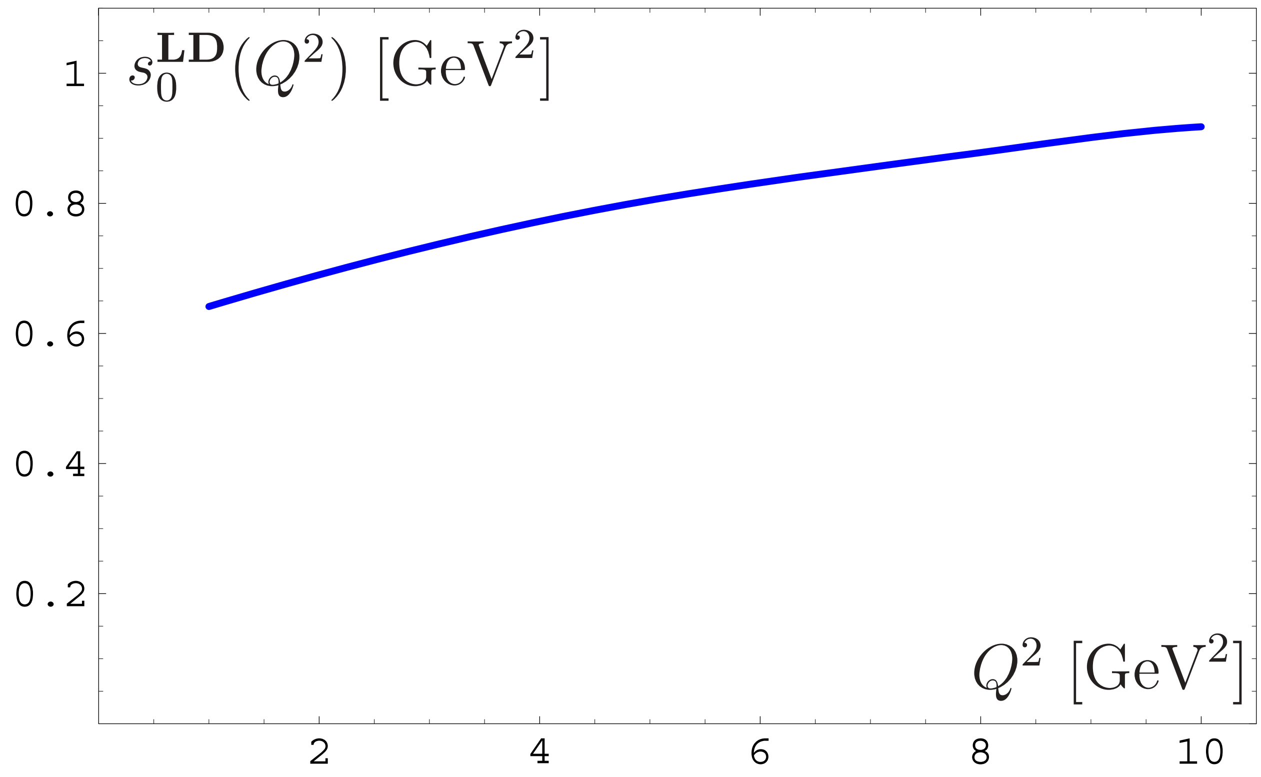
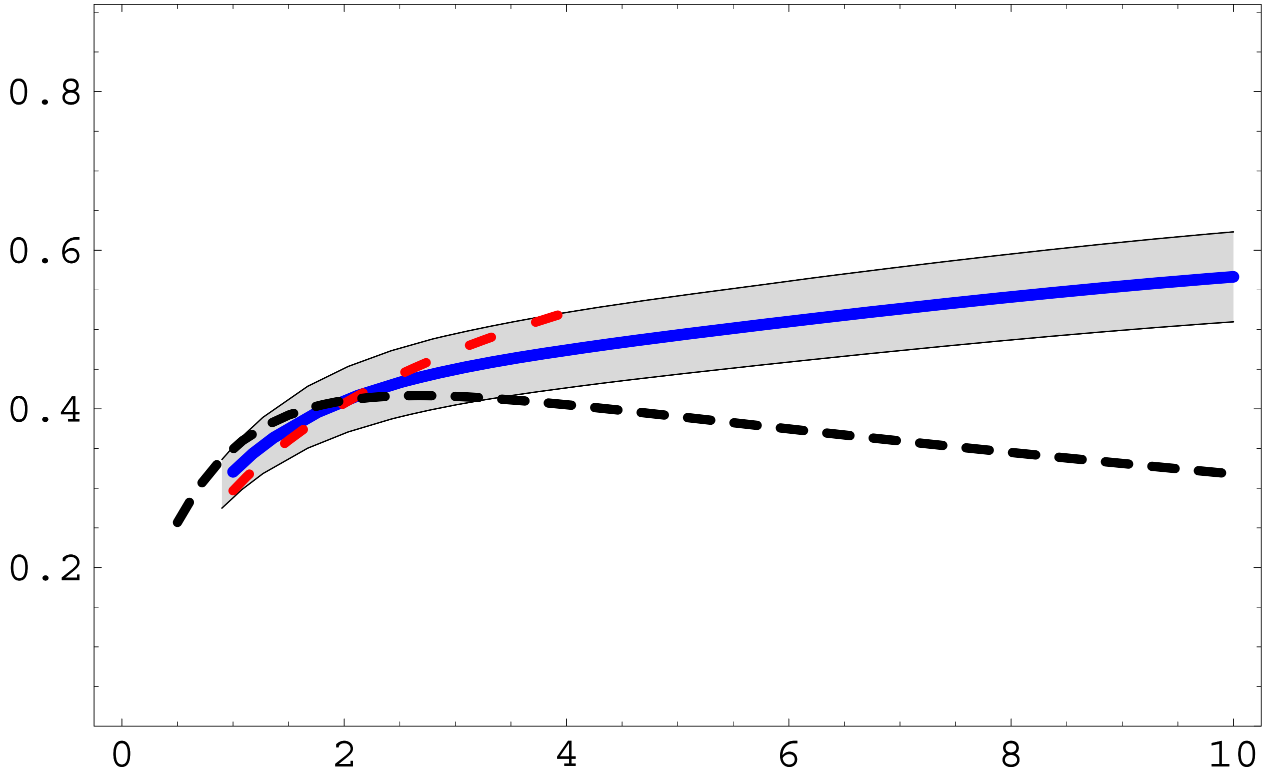


Approximation of LD result

This is the reason for pion FF underestimation in **Braguta–Lucha–Melikhov (2008)** approach:



Conclusion

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- QCD SR method with NLCs for the pion FF gives us a strip of predictions. This strip appears to be in a good agreement with existing experimental data of **JLab** and **Cornell**, as well as with lattice data.