QCD matter with isospin-asymmetry

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Outline

- introduction: QCD with isospin asymmetry
- setup
  - pion condensation
- results
  - phase boundary for pion condensation
  - deconfinement
  - QCD phase diagram
  - further applications
- conclusions
Introduction
Isospin chemical potential

- isospin density \( n_I = n_u - n_d \)
- \( n_I < 0 \) → excess of neutrons over protons → excess of \( \pi^- \) over \( \pi^+ \)

- applications
  - neutron stars
  - heavy-ion collisions (RHIC isobaric runs)
Isospin chemical potential

- Isospin density \( n_I = n_u - n_d \)
- \( n_I < 0 \) → excess of neutrons over protons
  → excess of \( \pi^- \) over \( \pi^+ \)

- Chemical potentials (2-flavor)
  \[ \mu_B = \frac{3(\mu_u + \mu_d)}{2} \quad \mu_I = \frac{(\mu_u - \mu_d)}{2} \]

- Here: zero baryon number but nonzero isospin
  \[ \mu_u = \mu_I \quad \mu_d = -\mu_I \]
Methods

- QCD at low energies \( \approx \) pions
  chiral perturbation theory
- on the level of charged pions: \( \mu_\pi = 2\mu_I \)
  at zero temperature \( \mu_\pi < m_\pi \) vacuum state
  \( \mu_\pi = m_\pi \) Bose-Einstein condensation
- on the level of quarks: lattice simulations

\[
\mathcal{L}_{\text{QCD}} = \bar{\psi} M \psi + \frac{1}{4} \text{tr} F_{\mu \nu} F_{\mu \nu}
\]
Bose-Einstein condensate

- accumulation of bosonic particles in lowest energy state

[Anderson et al '95 JILA-NIST/University of Colorado]

- velocity distribution of Ru atoms at low temperature → peak at zero velocity (zero energy)
Bose-Einstein condensate

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- velocity distribution of Ru atoms at low temperature
  → peak at zero velocity (zero energy)

- phase transition, spontaneous symmetry breaking
Lattice setup

partition function

\[ Z = \int \mathcal{D}U \left( \det \mathcal{M}_\ell \det \mathcal{M}_s \ e^{-S_g} \right) \]

\[ \mathcal{M}_\ell = \begin{pmatrix} \mathcal{D}(\mu_I) + m_\ell & \lambda \gamma_5 \\ -\lambda \gamma_5 & \mathcal{D}(-\mu_I) + m_\ell \end{pmatrix} \quad \mathcal{M}_s = \mathcal{D}(0) + m_s \]
Lattice setup

- partition function

\[ Z = \int \mathcal{D}U \det \mathcal{M}_\ell \det \mathcal{M}_s e^{-S_g} \]

\[ \mathcal{M}_\ell = \begin{pmatrix} \bar{\Phi}(\mu I) + m_\ell & \lambda \gamma_5 \\ -\lambda \gamma_5 & \bar{\Phi}(-\mu I) + m_\ell \end{pmatrix} \quad \mathcal{M}_s = \bar{\Phi}(0) + m_s \]

- isospin chemical potential for the light quarks
Lattice setup

- partition function

\[
Z = \int \mathcal{D}U \text{ det } \mathcal{M}_\ell \text{ det } \mathcal{M}_s e^{-S_g}
\]

\[
\mathcal{M}_\ell = \begin{pmatrix}
\mathcal{D}(\mu_I) + m_\ell & \lambda \gamma_5 \\
-\lambda \gamma_5 & \mathcal{D}(-\mu_I) + m_\ell
\end{pmatrix}
\]

\[
\mathcal{M}_s = \mathcal{D}(0) + m_s
\]

- isospin chemical potential for the light quarks
- zero strangeness
Lattice setup

- partition function

\[ Z = \int \mathcal{D}U \det \mathcal{M}_\ell \det \mathcal{M}_s e^{-S_g} \]

with

\[ \mathcal{M}_\ell = \begin{pmatrix} \Phi(\mu_1) + m_\ell & \lambda \gamma_5 \\ -\lambda \gamma_5 & \Phi(-\mu_1) + m_\ell \end{pmatrix} \]

\[ \mathcal{M}_s = \Phi(0) + m_s \]

- isospin chemical potential for the light quarks
- zero strangeness
- degenerate light quark masses
Lattice setup

- partition function

\[ Z = \int \mathcal{D}U \det \mathcal{M}_\ell \det \mathcal{M}_s e^{-S_g} \]

\[ \mathcal{M}_\ell = \begin{pmatrix} \mathcal{D}(\mu_I) + m_\ell & \lambda \gamma_5 \\ -\lambda \gamma_5 & \mathcal{D}(-\mu_I) + m_\ell \end{pmatrix} \quad \mathcal{M}_s = \mathcal{D}(0) + m_s \]

- isospin chemical potential for the light quarks
- zero strangeness
- degenerate light quark masses
- pionic source: explicit symmetry breaking

necessary for spontaneous symmetry breaking in finite volume needs to be extrapolated as \( \lambda \rightarrow 0 \)
Lattice setup

- partition function

\[ Z = \int \mathcal{D}U \left[ \det M_{\ell} \det M_s e^{-S_g} \right] \]

\[ M_{\ell} = \begin{pmatrix} \bar{\Phi}(\mu_I) + m_\ell & \lambda \gamma_5 \\ -\lambda \gamma_5 & \bar{\Phi}(-\mu_I) + m_\ell \end{pmatrix} \quad M_s = \bar{\Phi}(0) + m_s \]

- isospin chemical potential for the light quarks
- zero strangeness
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- pionic source: explicit symmetry breaking
  necessary for spontaneous symmetry breaking in finite volume
  needs to be extrapolated \( \lambda \to 0 \)
Pion condensate from the lattice
Singular value representation

▶ pion condensate

\[
\langle \pi^{\pm} \rangle = \frac{\mathcal{T}}{\mathcal{V}} \frac{\partial \log \mathcal{Z}}{\partial \lambda} = \ldots = \frac{\mathcal{T}}{\mathcal{V}} \left\langle \text{Tr} \frac{2\lambda}{|\mathcal{D}(\mu_I) + m|^{2} + \lambda^{2}} \right\rangle
\]

▶ singular values

\[
|\mathcal{D}(\mu_I) + m|^{2} \psi_i = \xi_i^{2} \psi_i
\]

▶ spectral representation \cite{Brandt, Endrödi 1611.06758}

\[
\langle \pi^{\pm} \rangle = \frac{\mathcal{T}}{\mathcal{V}} \left\langle \sum_i \frac{2\lambda}{\xi_i^{2} + \lambda^{2}} \right\rangle \xrightarrow{\mathcal{V} \to \infty} \int d\xi \left\langle \rho(\xi) \right\rangle \frac{2\lambda}{\xi^{2} + \lambda^{2}} \xrightarrow{\lambda \to 0} \pi \left\langle \rho(0) \right\rangle
\]

first derived in \cite{Kanazawa, Wettig, Yamamoto '11}
Singular value representation

- pion condensate

\[ \langle \pi^\pm \rangle = \frac{T}{V} \frac{\partial \log Z}{\partial \lambda} = \ldots = \frac{T}{V} \left( \text{Tr} \frac{2\lambda}{|\mathcal{D}(\mu_I) + m_l|^2 + \lambda^2} \right) \]

- singular values

\[ |\mathcal{D}(\mu_I) + m|^2 \psi_i = \xi_i^2 \psi_i \]

- spectral representation [Brandt, Endrödi 1611.06758]

\[ \langle \pi^\pm \rangle = \frac{T}{V} \left( \sum_i \frac{2\lambda}{\xi_i^2 + \lambda^2} \right) \xrightarrow{V \to \infty} \int \! d\xi \left( \rho(\xi) \right) \frac{2\lambda}{\xi^2 + \lambda^2} \xrightarrow{\lambda \to 0} \pi \langle \rho(0) \rangle \]

first derived in [Kanazawa, Wettig, Yamamoto '11]

- compare to Banks-Casher-relation at \( \mu_I = 0 \)
Singular value density

- spectral densities at $\lambda/m = 0.17$

![Graph showing spectral densities at $\lambda/m = 0.17$]
Singular value density

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- compare density of states around zero ‘energy’ ($\xi \approx 0$) to velocity distribution around zero
Singular value density

- spectral densities at $\lambda/m = 0.17$

- compare density of states around zero ‘energy’ ($\xi \approx 0$) to velocity distribution around zero
- Bose-Einstein condensation!
Phase diagram
repeat this analysis for many different $T$ and $\mu_I$
Pion condensate

- repeat this analysis for many different $T$ and $\mu_I$
Phase boundary

- interpolate $\rho(0)$ as function of $\mu_I$ to find phase boundary

![Graph showing phase boundary with $T$ vs $\mu_I/m_\pi$ axes. The graph includes data points and a shaded region indicating the pion condensation phase, with a note about $36^3 \times 12$ preliminary results.](graph.png)
Phase boundary

- interpolate $\rho(0)$ as function of $\mu_I$ to find phase boundary

- compare to expectations from $\chi$PT [Son, Stephanov '00]

$\langle u\gamma_5 d \rangle = 0$

$\langle \pi^- \rangle \neq 0$

$\langle \bar{u}\gamma_5 d \rangle \neq 0$
Phase boundary

- interpolate $\rho(0)$ as function of $\mu_I$ to find phase boundary

- compare to expectations from $\chi$PT [Son, Stephanov '00]

- no pion condensate above $T \approx 160$ MeV [Brandt, Endrődi, Schmalzbauer 1709.10487]
Order of the transition - volume scaling

- Volume scaling of order parameter shows typical sharpening
- Collapse according to $O(2)$ critical exponents [Ejiri et al '09]
volume scaling of order parameter shows typical sharpening

collapse according to $O(2)$ critical exponents [Ejiri et al '09]

indications for a second order phase transition at $\mu_I = m_\pi/2$, in the $O(2)$ universality class
Deconfinement at high temperature

- deconfinement transition encoded in the Polyakov loop
  \[ P \sim \exp(-F_{q\bar{q}}/T) \]

- deconfined matter for high \(\mu_I\): BCS superconductor with \(u\bar{d}\) Cooper pairs \[\text{[Son, Stephanov '02]}\]
Phase diagram

- favored phase diagram schematically: hadronic, quark-gluon plasma, BEC, BCS phases
Further applications
Check Taylor-expansion

- isospin density via Taylor-expansion at $\mu_I = 0$
  \[ n_I(\mu_I) = \chi^I_2 \cdot \mu_I + \chi^I_4 \cdot \mu^3_I + \ldots \]
  using $\chi^I_{2,4}$ from [BMWc, 1112.4416]

- low $T$: breakdown of expansion at $\mu_I = m_\pi/2$
- high $T$: pin down validity range of LO and NLO expansion
Equation of state

- pressure

\[ p = \int_{m_{\pi}/2}^{\mu_f} d\mu'_I n_I(\mu'_I) \]

- energy density

\[ \epsilon = -p + \mu_I n_I \]

- application: mass-radius relation of compact stars
Summary

- Bose-Einstein condensation via singular value density
  \( \sim \) flat extrapolation in \( \lambda \)

- established second-order phase transition at \( \mu_I = \frac{m_\pi}{2} \)

- QCD phase diagram with isospin asymmetry