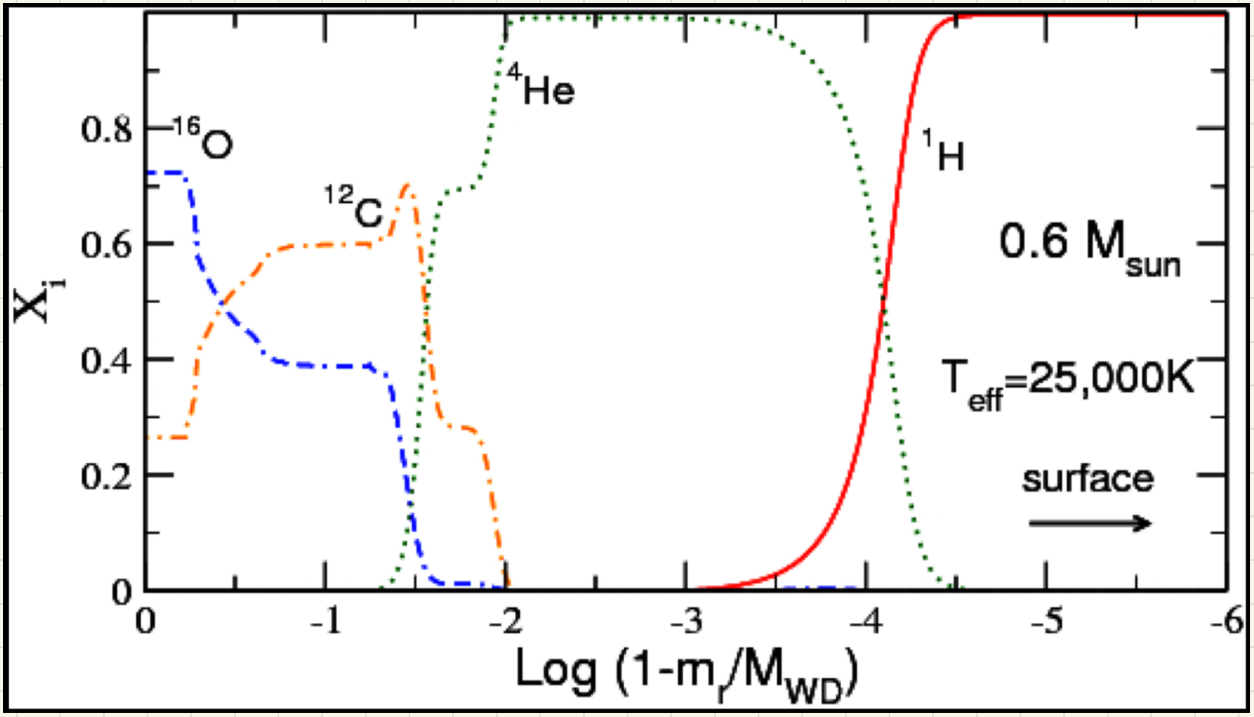
18. Weiße Zwerge

Struktur:



Quelle: www.lsw.uni-heidelberg.de/users/mcamenzi/HD_WD_Struct.pdf , 25.6.2020

Definiere $\sigma \equiv \frac{A}{Z} \simeq 2$ für leichte Atomkerne (He, C, O, ...)

Massendichte: $\rho = n_N m_N = \sigma n_e m_N$ (18.1)

Fermi-Impuls: $p_F = \hbar (3\pi^2 n_e)^{1/3} = \hbar \left(\frac{3\pi^2 \rho}{\sigma m_N} \right)^{1/3}$ (18.2)

Fermi-Energie: (i) nicht-relativistisch $E_F = \frac{p_F^2}{2m_e} = \frac{\hbar^2}{2m_e} \left(\frac{3\pi^2 \rho}{\sigma m_N} \right)^{2/3}$ (18.3)

(ii) ultra-relativistisch $E_F = p_F c = \hbar c \left(\frac{3\pi^2 \rho}{\sigma m_N} \right)^{1/3}$ (18.4)

Druck: (i) nicht-rel. (Gl. (14.17)) $P_{nr} = \frac{2}{5} n_e E_F = \frac{2}{5} \frac{\rho}{\sigma m_N} \frac{\hbar^2}{2m_e} \left(\frac{3\pi^2 \rho}{\sigma m_N} \right)^{2/3}$

$$= \underbrace{\frac{\hbar^2}{15\pi^2 m_e} \left(\frac{3\pi^2}{\sigma m_N} \right)^{5/3}} \rho^{5/3}$$

$$\equiv K_1 \rho^{5/3} \quad (18.5)$$

(ii) ultra-rel. (Gl. (14.18)) $P_{ur} = \frac{1}{4} n_e E_F = \frac{1}{4} \frac{\rho}{\sigma m_N} \hbar c \left(\frac{3\pi^2 \rho}{\sigma m_N} \right)^{1/3}$

$$= \underbrace{\frac{\hbar c}{12\pi^2} \left(\frac{3\pi^2}{\sigma m_N} \right)^{4/3}} \rho^{4/3}$$

$$\equiv K_2 \rho^{4/3} \quad (18.6)$$

Übergang nicht-rel. → ultra-rel.: $p_{F,c} \overset{!}{=} m_e c$

$\Rightarrow$ charakteristische Dichte: $\rho_c \equiv \sigma m_N n_{e,c} = \frac{\sigma m_N}{3\pi^2\hbar^3} p_{F,c}^3 = \frac{\sigma m_N}{3\pi^2\hbar^3} (m_e c)^3$ (18.7)

(i) $\gamma = \frac{5}{3}$: $K_1 \rho_c^{-1/3} = \frac{\hbar^2}{15\pi^2 m_e} \left(\frac{3\pi^2}{\sigma m_N}\right)^{5/3} \rho_c^{-1/3} \overset{(18.7)}{=} \frac{\hbar^2}{15\pi^2 m_e} \frac{(m_e c)^5}{\hbar^5 \rho_c^{5/3}} \rho_c^{-1/3}$

$\rho_0 \ll \rho_c$

$= \frac{m_e^4 c^5}{15\pi^2\hbar^3} \rho_c^{-2} = \frac{1}{5} m_e c^2 \frac{\rho_c^{-1}}{\sigma m_N}$

$= \frac{1}{5} m_e c^2 \frac{3\pi^2\hbar^3}{(m_e c)^3} \left(\frac{1}{\sigma m_N}\right)^2 = \frac{3\pi^2}{5} \frac{(\hbar c)^3}{(m_e c^2)^2 m_N^2} \frac{1}{\sigma^2}$ (18.8)

$\Rightarrow$ $R \overset{(17.14)}{=} x_1 \sqrt{\frac{K_1 \rho_c^{-1/3}}{4\pi G} \frac{5}{2}} \left(\frac{\rho_c}{\rho_0}\right)^{1/6}$ mit $x_1 = 3.654$

$\overset{(18.8)}{=} 3.654 \left(\frac{3\pi}{8} \frac{\hbar c}{G} \frac{1}{m_N^2}\right)^{1/2} \frac{\hbar c}{m_e c^2} \frac{1}{\sigma} \left(\frac{\rho_c}{\rho_0}\right)^{1/6}$

$= 3.966 \underbrace{\frac{M_P}{m_N}}_{1.302 \cdot 10^{19}} \underbrace{\frac{\hbar c}{m_e c^2}}_{3.862 \cdot 10^{-13}\,m} \frac{1}{\sigma} \left(\frac{\rho_c}{\rho_0}\right)^{1/6} \simeq \frac{1.994}{\sigma} \left(\frac{\rho_c}{\rho_0}\right)^{1/6} \cdot 10^4\,\text{km}$ (18.9)

$$K_1 \rho_c^{1/3} = \frac{\hbar^2}{15\pi^2 m_e}\left(\frac{3\pi^2}{\sigma m_N}\right)^{5/3}\rho_c^{1/3} \overset{(18.7)}{=} \frac{\hbar^2}{15\pi^2 m_e}\frac{(m_e c)^5}{\hbar^5 \rho_c^{5/3}}\rho_c^{1/3}$$

$$= \frac{m_e^4 c^5}{15\pi^2\hbar^3}\rho_c^{-4/3} = \frac{1}{5} m_e c^2 \frac{\rho_c^{-1/3}}{\sigma m_N} = \frac{1}{5} m_e c^2 (3\pi^2)^{1/3}\frac{\hbar c}{m_e c^2}\left(\frac{1}{\sigma m_N}\right)^{4/3}$$

$$= \frac{(3\pi^2)^{1/3}}{5}\hbar c\left(\frac{1}{\sigma m_N}\right)^{4/3} \qquad (18.10)$$

$$\Rightarrow \boxed{M} \overset{(17.16)}{=} 4\pi \cdot \underbrace{2.714}_{x_1^2\left|\frac{d\theta}{dx}(x_1)\right|} \left(\frac{K_1 \rho_c^{1/3}}{4\pi G}\frac{5}{2}\right)^{3/2}\left(\frac{\rho_0}{\rho_c}\right)^{1/2}$$

$$\overset{(18.10)}{=} 34.105\left(\frac{(3\pi^2)^{1/3}}{8\pi}\frac{\hbar c}{G}\frac{1}{m_N^2}\right)^{3/2} m_N \frac{1}{\sigma^2}\left(\frac{\rho_0}{\rho_c}\right)^{1/2}$$

$$\simeq 1.473\left(\frac{M_P}{m_N}\right)^3 \frac{1}{\sigma^2}\left(\frac{\rho_0}{\rho_c}\right)^{1/2} m_N = \boxed{\frac{2.735}{\sigma^2}\left(\frac{\rho_0}{\rho_c}\right)^{1/2} M_\odot} \qquad (18.11)$$

(ii) $\gamma = \frac{4}{3}$: $K_2 \rho_c^{-2/3} = \frac{\hbar c}{12\pi^2}\left(\frac{3\pi^2}{\sigma m_N}\right)^{4/3}\rho_c^{-2/3} = \frac{\hbar c}{12\pi^2}\frac{(m_e c)^4}{\hbar^4 \rho_c^{4/3}}\rho_c^{-2/3}$

$\rho_0 \gg \rho_c$

$$= \frac{m_e^4 c^5}{12\pi^2\hbar^3}\rho_c^{-2} = \frac{1}{4} m_e c^2 \frac{\rho_c^{-1}}{\sigma m_N}$$

$$= \frac{3\pi^2}{4}\frac{(\hbar c)^3}{(m_e c^2)^2 m_N^2}\frac{1}{\sigma^2} \qquad (18.12)$$

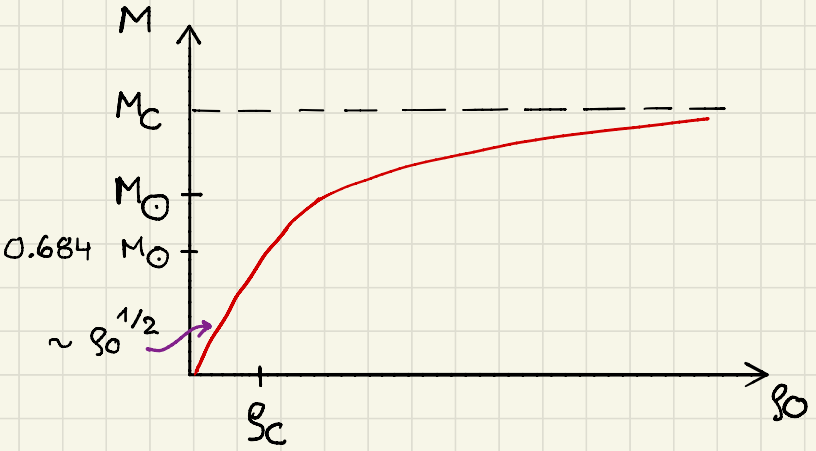
$\Rightarrow$ $R \overset{(17.14)}{=} \overset{x_1}{6.897}\left(\frac{K_2 \rho_c^{-2/3}}{4\pi G}4\right)^{1/2}\left(\frac{\rho_c}{\rho_0}\right)^{1/3}$

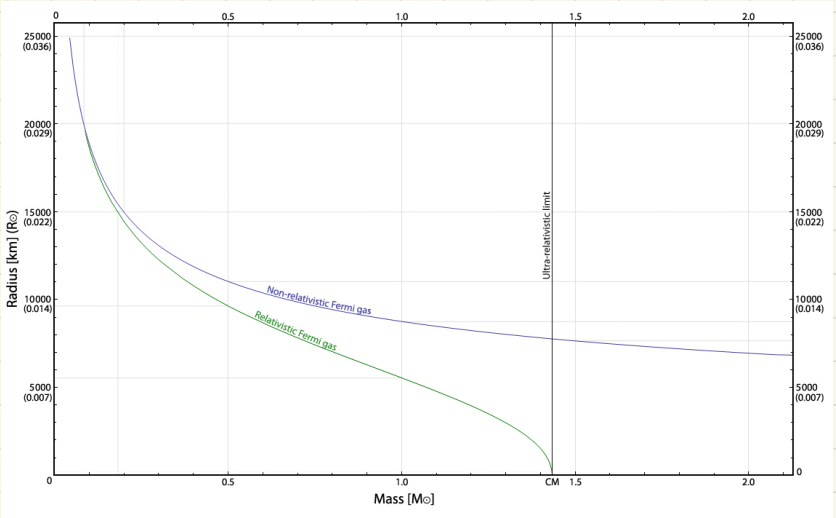
$$\overset{(18.12)}{=} 6.897\left(\frac{3\pi^2}{4\pi}\frac{\hbar c}{G}\frac{1}{m_N^2}\right)^{1/2}\frac{\hbar c}{m_e c^2}\frac{1}{\sigma}\left(\frac{\rho_c}{\rho_0}\right)^{1/3}$$

$$\simeq 10.587\,\frac{M_P}{m_N}\frac{\hbar c}{m_e c^2}\frac{1}{\sigma}\left(\frac{\rho_c}{\rho_0}\right)^{1/3} \simeq \boxed{\frac{5.323}{\sigma}\left(\frac{\rho_c}{\rho_0}\right)^{1/3}\cdot 10^4 \text{ km}} \qquad (18.13)$$

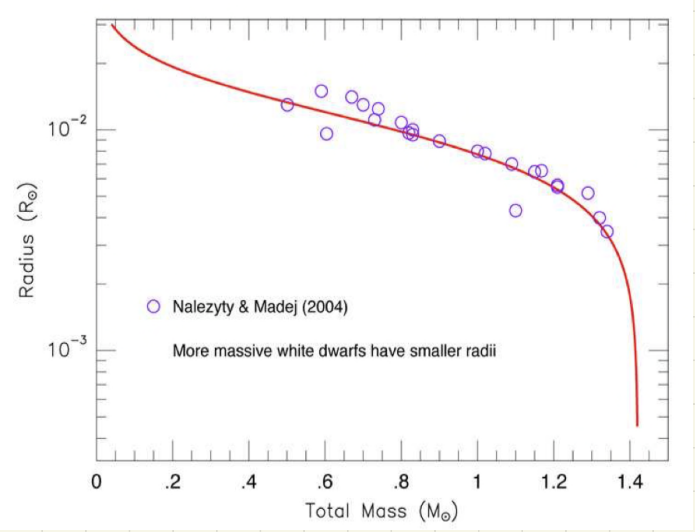
$M \overset{(17.16)}{=} 4\pi \cdot \overset{x_1^2\left|\frac{d\Theta}{dx}(x_1)\right|}{2.018}\left(\frac{K_2}{\pi G}\right)^{3/2} \simeq 25.359\left(\frac{(3\pi^2)^{4/3}}{12\pi^3}\frac{\hbar c}{G}\frac{1}{m_N^2}\right)^{3/2}\frac{m_N}{\sigma^2}$

$$\simeq 3.098\left(\frac{M_P}{m_N}\right)^3\frac{m_N}{\sigma^2} = \boxed{\frac{5.751}{\sigma^2}M_\odot \simeq 1.438\,M_\odot \equiv M_c} \qquad (18.14)$$





Quelle: https://deacademic.com/pictures/dewik/87/WhiteDwarf_mass-radius.jpg , 25.6.2020



Quelle: www.physik.uni-regensburg.de/forschung/gebhardt/gebhardt_files/skripten/WeisseZwerge.pdf , 25.6.2020