Black Holes as Quantum Bound States

Tehseen Rug

Max-Planck-Institut für Physik, Ludwig-Maximilians Universität

November 12th 2014
Contents

1. Black Holes - The usual picture

2. Quantum Portrait of Black Holes

3. Quantum Bound State Description of Black Holes
   - Theoretical Framework
   - Results

4. Summary and Outlook
Classical Black Holes

General Relativity:

\[ S = M_p^2 \int d^4x \sqrt{-g} (R + g^{\mu\nu} T_{\mu\nu}) \]  \hspace{1cm} (1)

- Schwarzschild Black holes: Spherically symmetric solutions with source radius \( R = r_g = 2G_NM \)
- event horizon: "nothing can escape from a black hole"
- in general: Black Holes characterized by mass \( M \), charge \( Q \), angular momentum \( L \)
Semiclassical Black Holes

Basic Idea: Quantize matter field in classical background

- Consequence: Hawking radiation
  \[ T \sim \frac{1}{r_g}, \Gamma \sim \frac{1}{r_g} \]

- Remark:
  Quantum Corrections believed to be exponentially suppressed

- Mysteries:
  - negative heat capacity
  - no hair theorems
  - information paradox

**No resolution within semiclassical approach**
Graviton Bound States

- interpret GR as EFT of graviton on flat spacetime
- Black holes: Bound states of $N$ soft gravitons ($\lambda \sim r_g$)\(^1\) (analogy: Hadrons in QCD)

$$M = \sqrt{N} M_p, \quad r_g = \sqrt{N} L_p, \quad \alpha = 1/N$$  \(2\)

- e.g. for solar mass black hole: $N \sim 10^{71}$
- coupling weak, but large collective effect $\alpha N = 1$ (compare to baryons in large $N$ QCD)

\(^1\)Dvali, Gomez; 1112.3359 [hep-th]
known results recovered as $N \to \infty$

- e.g. Hawking radiation:

$$\Gamma \sim \frac{1}{r_g} + \mathcal{O}(1/N) \quad (3)$$

- new $1/N$ corrections large enough to resolve all the black hole mysteries!

- Question: Quantitative theoretical framework?
QCD Analogy

Use methods inspired from QCD to describe black hole bound states\(^2\)

- confining potential at low energies $\rightarrow$ large collective effects
- hadrons $\rightarrow$ large $N$ graviton bound states
- condensates of quarks and gluons $\rightarrow$ condensates of gravitons and curvature invariants
- hadronic currents, e.g. $\mathcal{J}(x) \sim \bar{q}q(x) \rightarrow$ black hole currents $\mathcal{J}(x) \sim h^N(x)$

---

\(^2\)Hofmann, Rug; 1403.3224 [hep-th]
Explicit Construction

Model black hole state $\mathcal{B}$ by auxiliary current $\mathcal{I}(x)$:

$$\langle \mathcal{B} | \mathcal{I}(x) | \Omega \rangle \neq 0$$

- $\mathcal{I}(x) | \Omega \rangle$: same quantum numbers as $| \mathcal{B} \rangle \Rightarrow \mathcal{I}(x) = \hbar^N(x)$ (take scalar fields for simplicity)

- consistency with isometries: $\mathcal{I}(x) = \mathcal{I}(r)$

Ward: implement symmetries at the end of computations

$$| \mathcal{B} \rangle = \Gamma_B^{-1} \int \frac{d^4 p}{(2\pi)^4} B(p) \int d^4 x e^{ipx} \mathcal{I}(x) | \Omega \rangle \quad (4)$$

- Remark:
  Generalization to other spacetimes (including perturbations) and topological defects possible!
Observables at Parton Level

- Light-cone constituent distribution:

\[ \mathcal{D}(r) = \int d^3k \ e^{-ik \cdot r} \langle B | n(k) | B \rangle \]  

(5)

- Energy density:

\[ \mathcal{E} = \langle B | T_{\mu \nu}(x) | B \rangle \]  

(6)

- Evaluation using (4) in \( N, M \to \infty, \ N/M \) fixed limit leads to

\[ M_B^2 = \frac{\langle \Phi^2(N-1) \rangle}{\langle \Phi^2(N-2) \rangle} N^2 \]  

(7)
Remarks

- scaling: \( M \sim N \) expected at parton level
  (compare to large \( N \) baryons in \( 1/N \) expansions)
- finite \( N \Rightarrow 1/N \) corrections
- higher-order corrections can be implemented in a
gauge-invariant way via Wilson lines:

\[
\mathcal{P} \exp \left( - \oint \! d\mathbf{z} \lambda \Gamma_{\mu\lambda}^{\mu} (z) \right)
\]  
(8)

- \( x^\mu x^\nu \Gamma_{\mu\nu}^\lambda \) gauge: all condensates automatically gauge-invariant
  (analogue: External field methods and Fock-Schwinger gauge
  in QCD sum rule calculations)
Scattering

\[ \langle B' \Phi' | B \Phi \rangle \text{ in tree approximation} \]

\[ r_g^{-2} \ll q^2 \ll M_p^2 \]: EFT description valid, but resolution of bound state possible

ACD and OPE lead to

\[ k' \sigma  \frac{d^3 k'}{d^3 k'} \sim \mathcal{D}(r) \] 

---

\[3\] Gruending, Mueller, Hofmann, Rug; 1407.1051 [hep-th]
Summary:

- treat black holes as large $N$ bound state of gravitons
- employ QCD inspired methods
- $1/N$ corrections as solution to black hole mysteries
- scaling $M \sim N$, embedding of observables in scattering experiments
Summary:

- treat black holes as large $N$ bound state of gravitons
- employ QCD inspired methods
- $1/N$ corrections as solution to black hole mysteries
- scaling $M \sim N$, embedding of observables in scattering experiments

Outlook:

- large $N$ baryons
- black hole formation
- application to different spacetimes
Thank You for Your Attention