Bound states in gauge theory I

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Standard Model @ LHC & Paul Hoyer Fest

literature

D.D.Dietrich, P.Hoyer & M.Jarvinen, “Towards a Born term for hadrons,” Phys. Rev. D 87 (2013) 065021 [arXiv:1212.4747 [hep-ph]]

D.D.Dietrich, P.Hoyer & M.Jarvinen, “Boosting equal time bound states,” Phys. Rev. D 85 (2012) 105016 [arXiv:1202.0826 [hep-ph]]

P.Hoyer, “Introduction to QCD - A Bound State Perspective,” arXiv:1106.1420 [hep-ph]

P.Hoyer, “Hadron Structure,” Acta Phys. Polon. B 41 (2010) 2701 [arXiv:1010.5431 [hep-ph]]

S.J.Brodsky and P.Hoyer, “The hbar Expansion in Quantum Field Theory,” Phys. Rev. D 83 (2011) 045026 [arXiv:1009.2313 [hep-ph]]

P.Hoyer, “The quark model via an hbar expansion of QCD,” PoS EPS-HEP2009 (2009) 073

P.Hoyer, “Bound states at lowest order in hbar,” arXiv:0909.3045 [hep-ph]

D.D.Dietrich, P.Hoyer, M.Jarvinen & S.Peigne, “Perturbative gauge theory in a background,” JHEP 0703 (2007) 105 [hep-ph/0608075]

P.Hoyer, “Novel Lorentz Covariance for Bound States,” Phys. Lett. B172 (1986) 101

outline

- motivation & introduction
- bound state equation
    - Poincaré invariance
    - solutions
        - spectrum
- conclusions

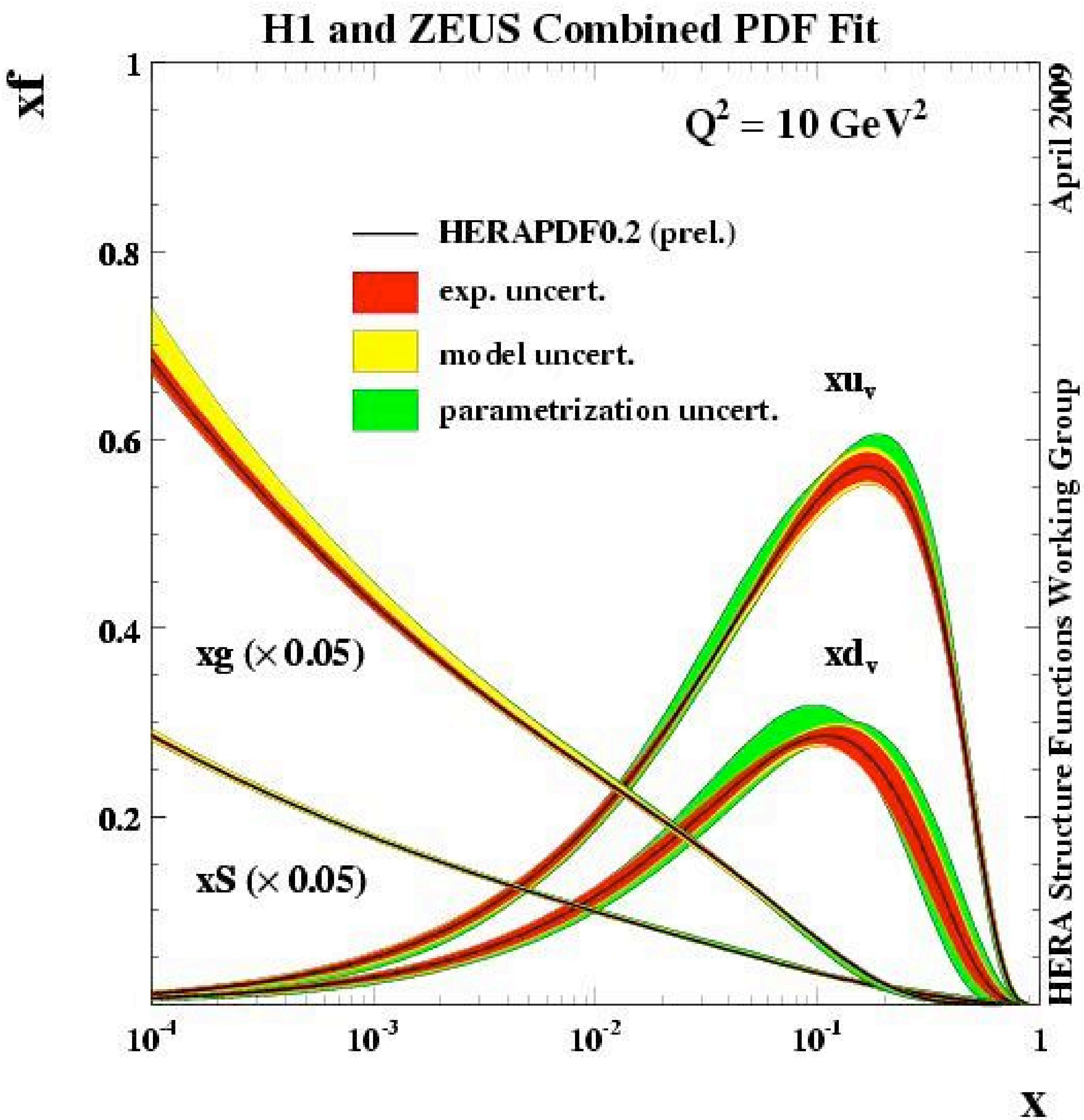
motivation

- understanding of
  - bound state dynamics
  - bound state spectra
  - qcd & field theory
- wanted:
  hadronic basis for scattering amplitudes

observations

- QCD bound states
  ∞ # of constituents vs. few valence quarks

HERA



https://www.desy.de/h1zeus/combined_results/proton_structure/combined_plots_comp/fig1a.pdf

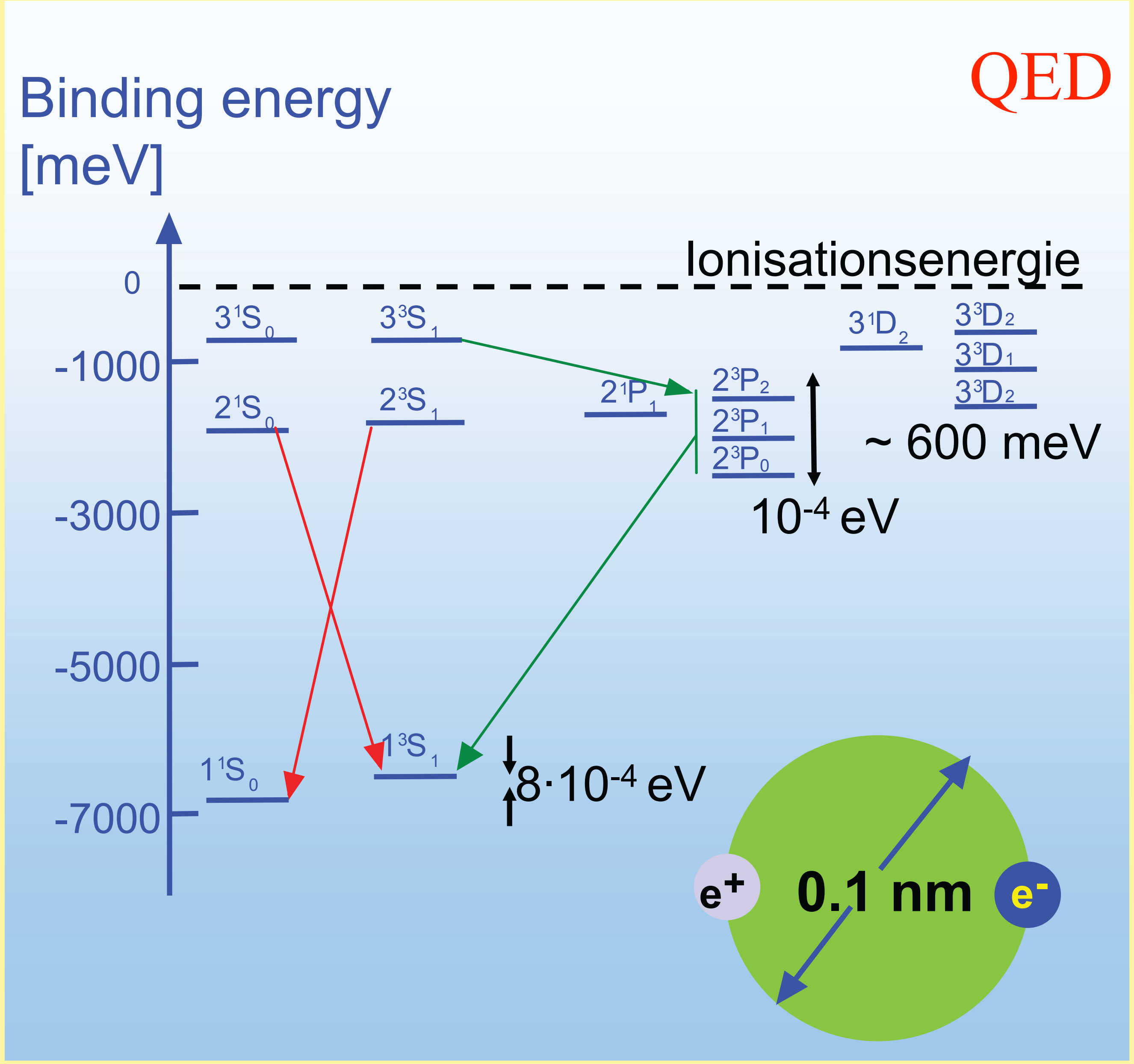
DDD

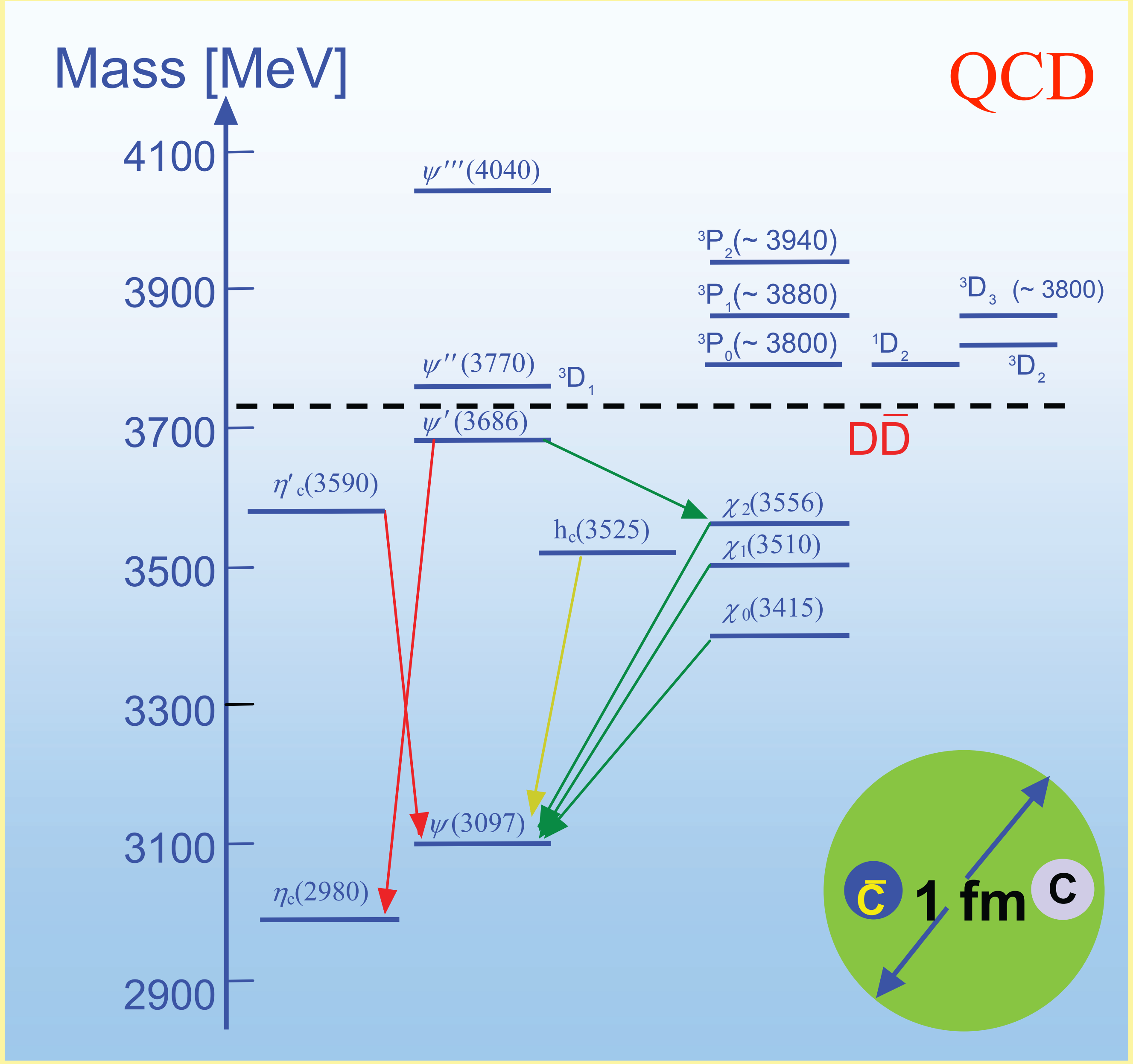
observations

- QCD bound states
  ∞ # of constituents vs. few valence quarks
- bound state spectra
  e.g. charmonium and positronium qualitatively similar

spectra

- Positronium
- Charmonium





from one of Paul's talks

observations

- QCD bound states
  ∞ # of constituents vs. few valence quarks
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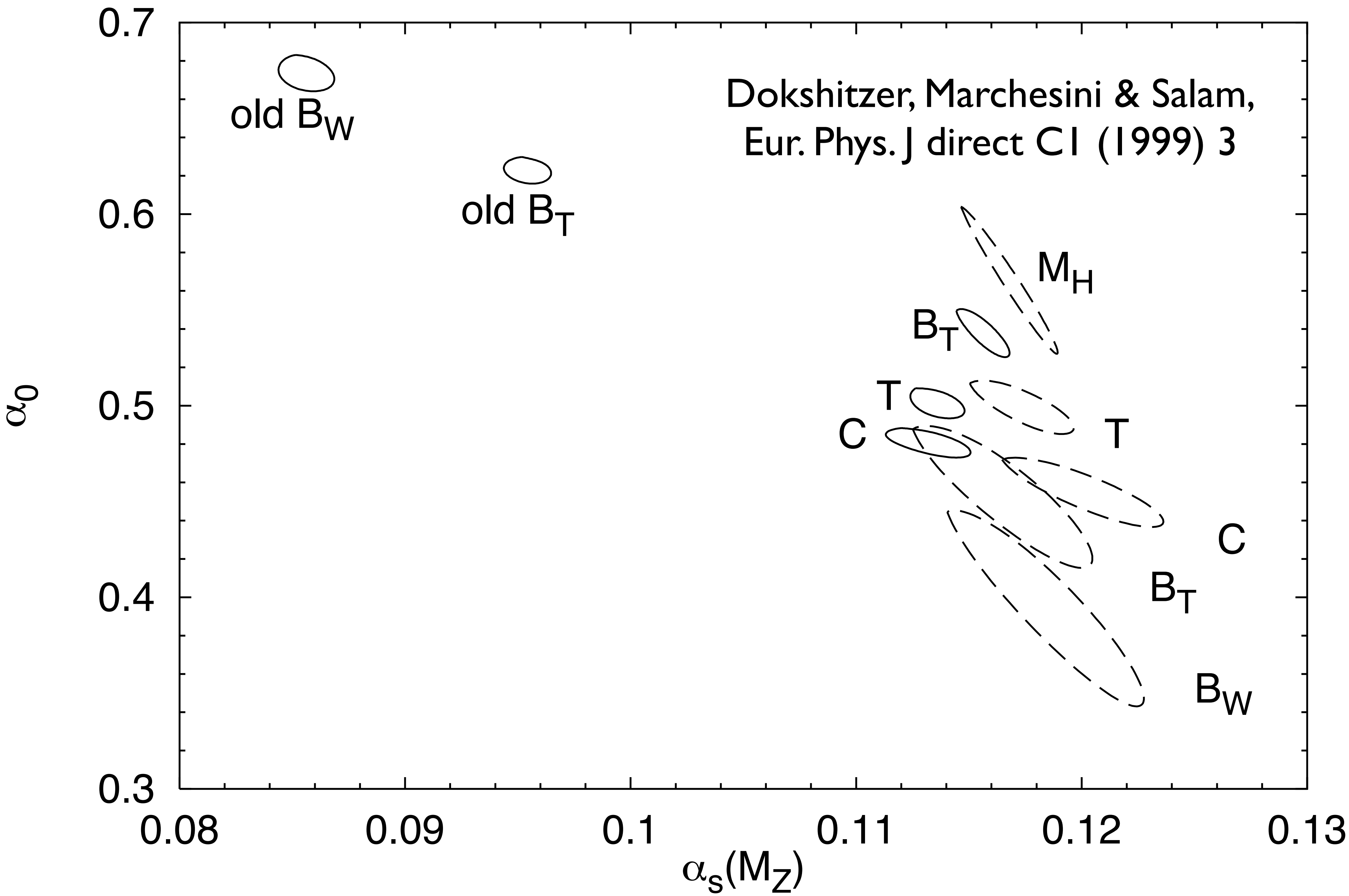
  e.g. charmonium and positronium qualitatively similar
- α_s might freeze in already for moderate virtualities

DDD

coupling



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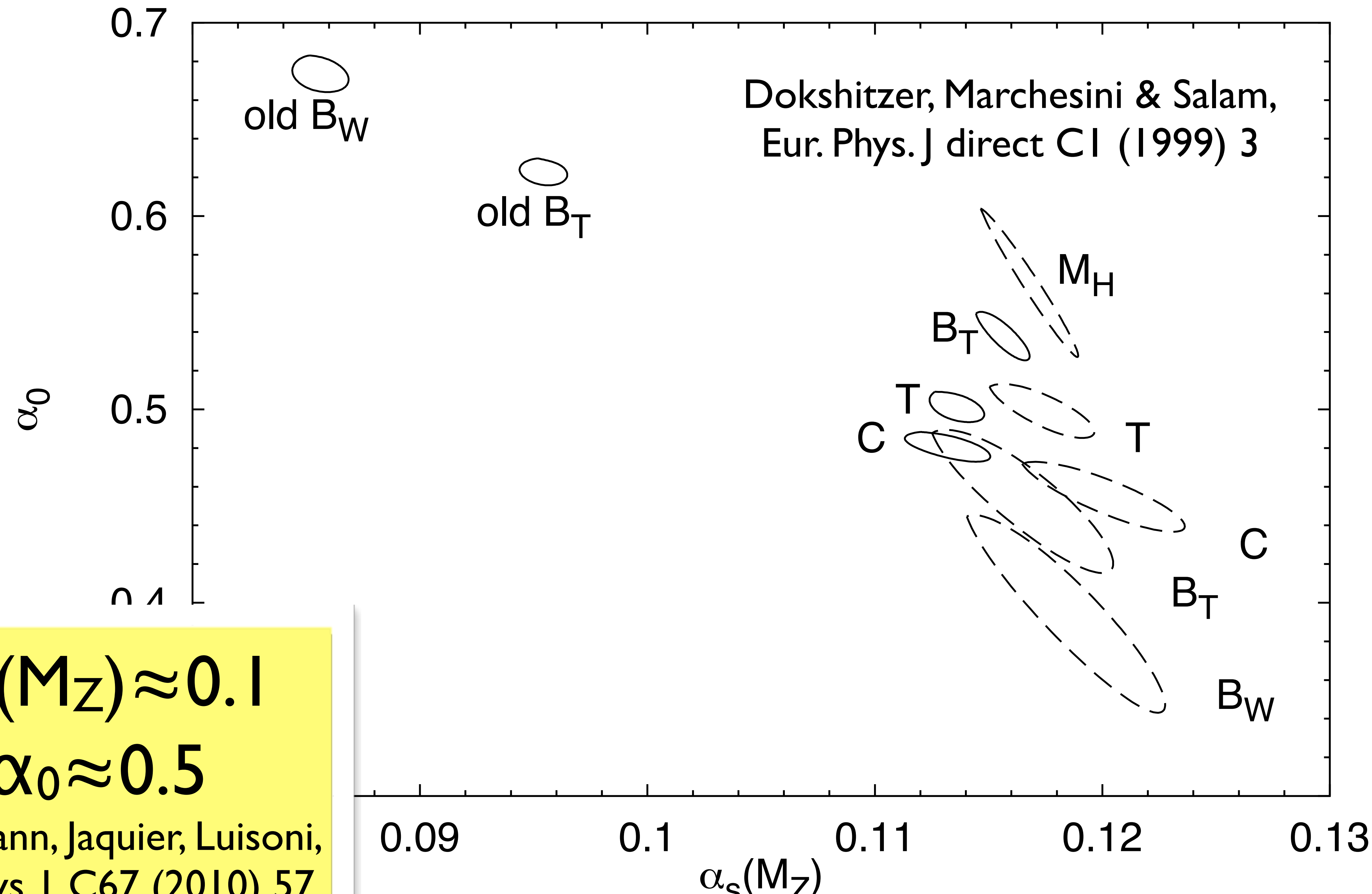
coupling



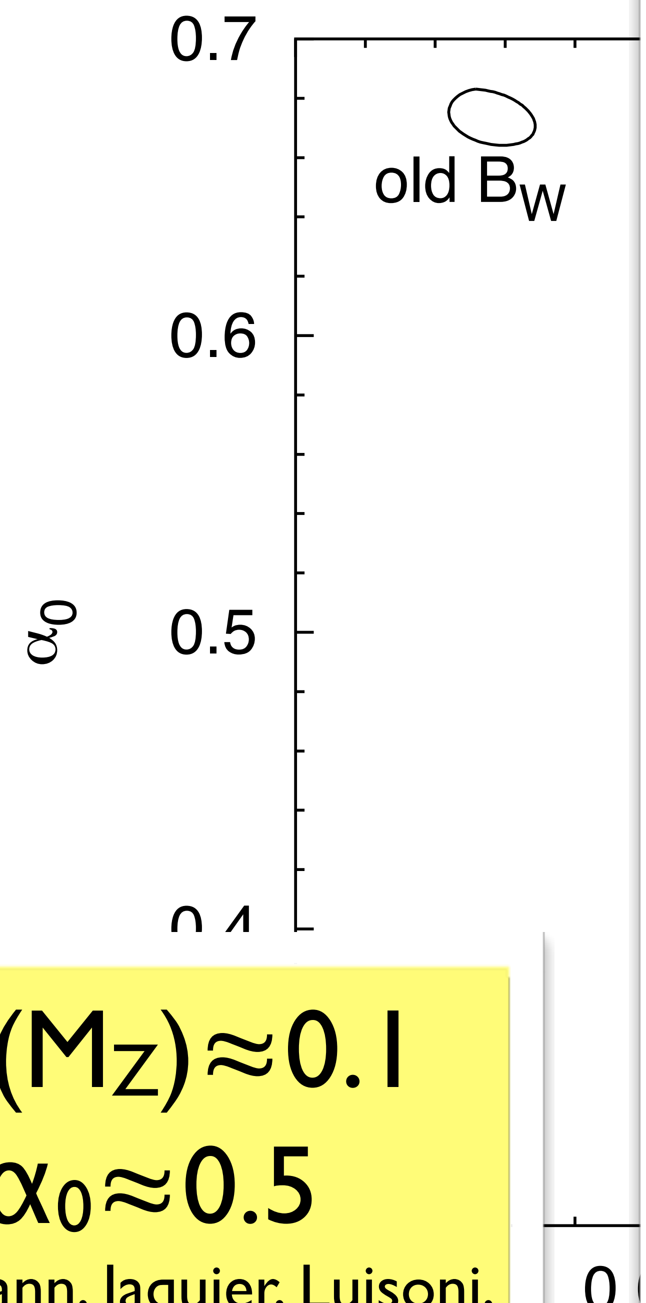
$\alpha_S(M_Z)\approx 0.1$

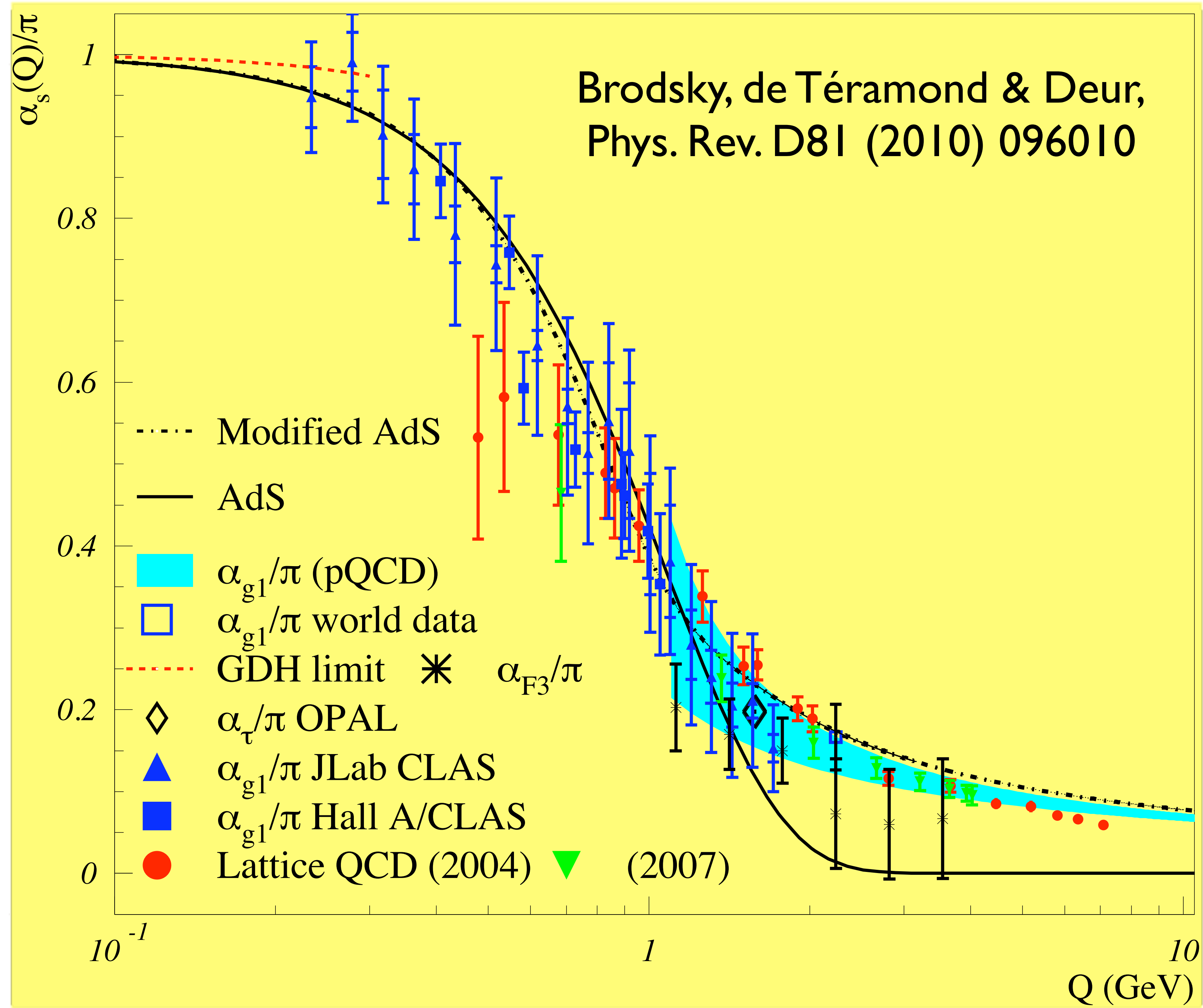
$\alpha_0\approx 0.5$

Gehrmann, Jaquier, Luisoni, Eur. Phys. J. C67 (2010) 57

DDD

coupling





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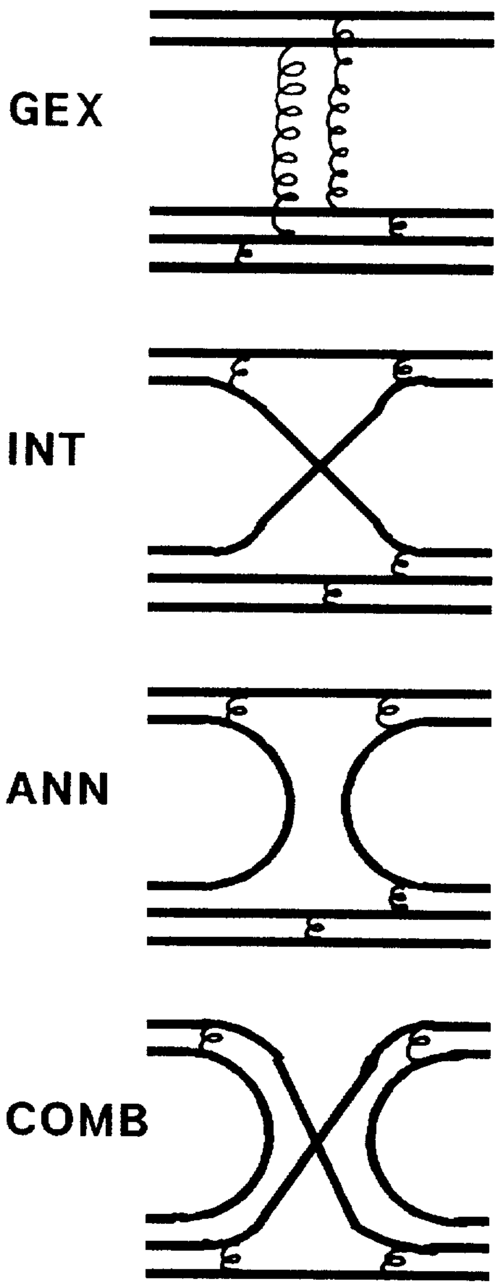
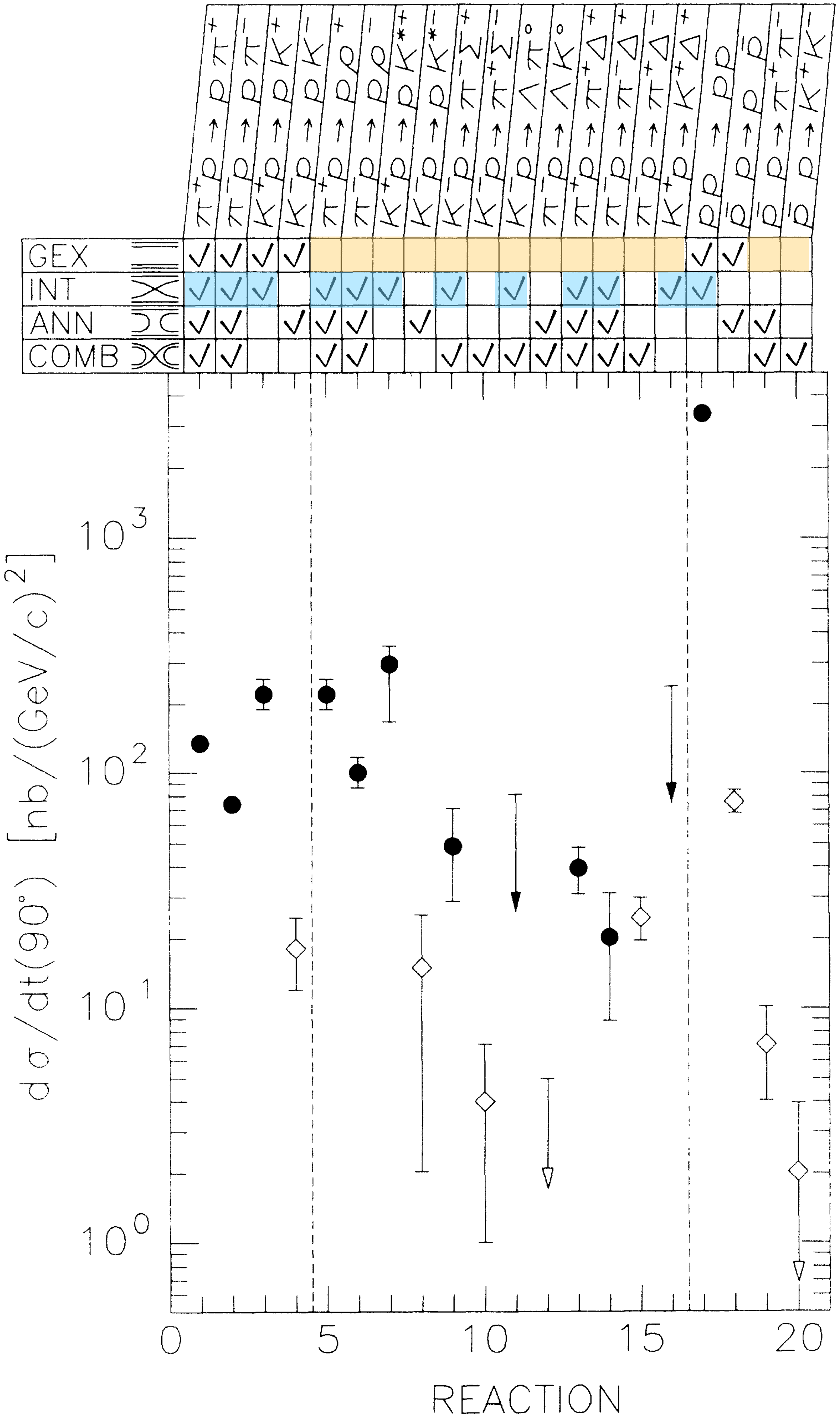
- α_s might freeze in already for moderate virtualities
- virtual gluons more costly than anticipated
  $\Leftarrow$ large-angle scattering, Okubo-Zweig-Iizuka rule, ...

large-ar hadron





White et al.
PRD 49 (1994) 58

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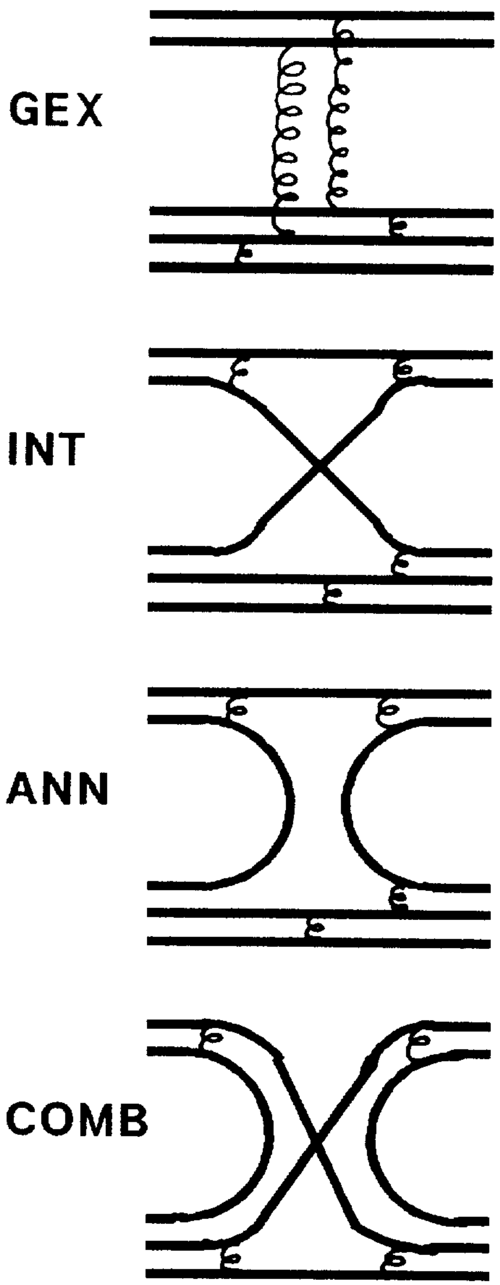
large-ar adron



White et al.
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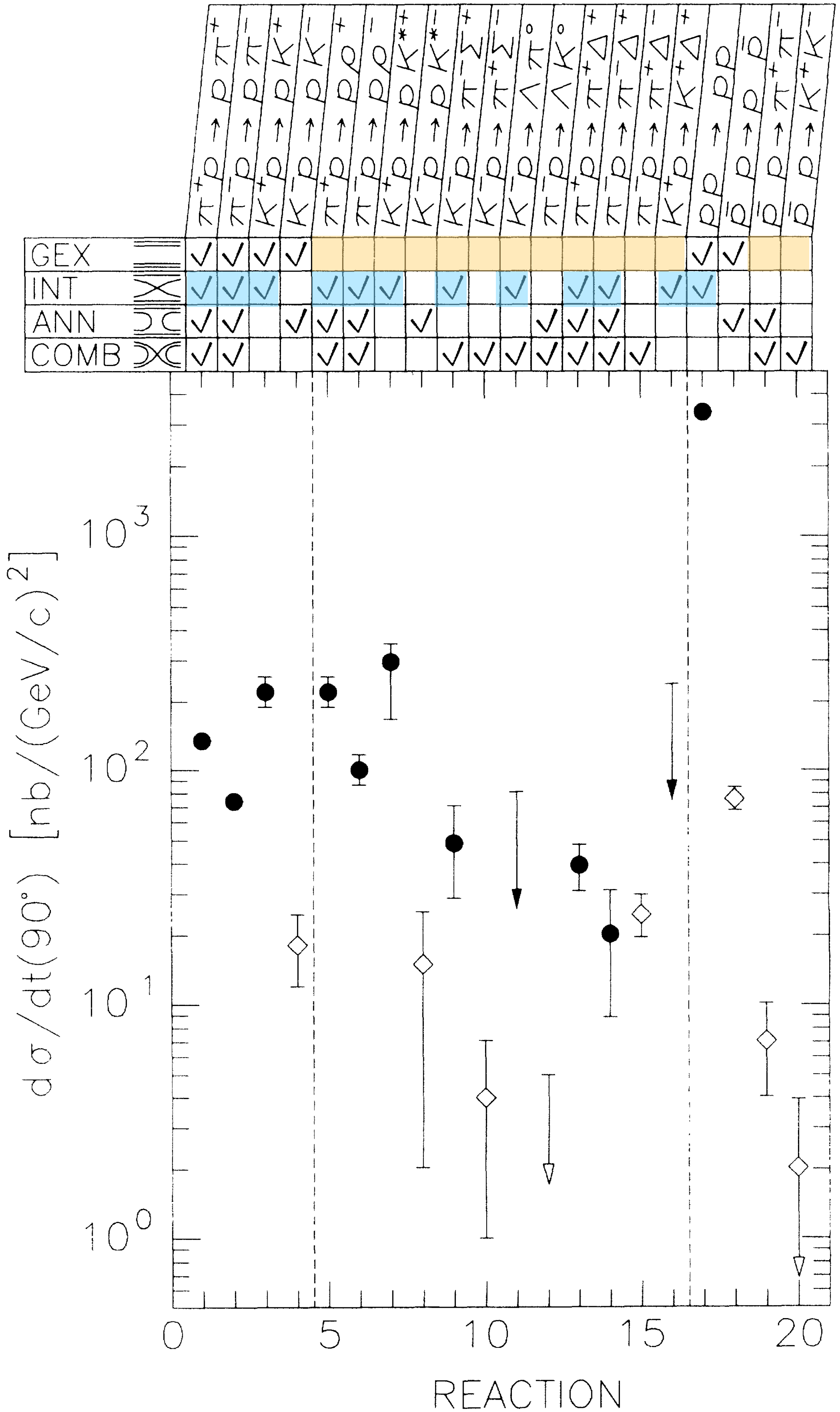

also
Baller et al.
PRL 60 (1988) 1118

scaling (pp):

Gunion, Brodsky & Blankenbecler
PRD 6 (1972) 2652

$$\frac{d\sigma}{dt} \propto \frac{1}{s^2 u^4 t^4}$$

$$\cancel{\frac{d\sigma}{dt} \propto \frac{1}{t^8}}$$

Landshoff
PRD 10 (1974) 1024

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observations

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  e.g. charmonium and positronium qualitatively similar
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DDD

observations

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  ⇐ large-angle scattering, Okubo-Zweig-Iizuka rule, ...

⇒ try valence fermions
+ low-order instantaneous interactions

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not less !

$$|e^-(\boldsymbol{x}_1)e^+(\boldsymbol{x}_2)\rangle$$

$$\boldsymbol{\nabla}\cdot\boldsymbol{A}=0$$

$$-\boldsymbol{\nabla}^2 A^0(\boldsymbol{x})=e\left[\delta(\boldsymbol{x}-\boldsymbol{x}_1)-\delta(\boldsymbol{x}-\boldsymbol{x}_2)\right]$$

$$A^0(\boldsymbol{x})=\frac{e}{4\pi}\left[\frac{1}{|\boldsymbol{x}-\boldsymbol{x}_1|}-\frac{1}{|\boldsymbol{x}-\boldsymbol{x}_2|}\right]$$

vanishing field is no solution in presence of charges !

$$\tfrac{1}{2}[eA^0(\boldsymbol{x}_1)-eA^0(\boldsymbol{x}_2)]=-\alpha/|\boldsymbol{x}_1-\boldsymbol{x}_2|$$

$\Rightarrow$ in non-relativistic limit: hydrogen Schrödinger problem

DDD

actually, a bit more

$$-\boldsymbol{\nabla}^2 A^0(\boldsymbol{x}) = e\,[\delta(\boldsymbol{x}-\boldsymbol{x}_1) - \delta(\boldsymbol{x}-\boldsymbol{x}_2)]$$

$$A^0(\boldsymbol{x}) = \frac{e}{4\pi}\left[\frac{1}{|\boldsymbol{x}-\boldsymbol{x}_1|} - \frac{1}{|\boldsymbol{x}-\boldsymbol{x}_2|}\right]$$

$$A^0_\Lambda(\boldsymbol{x}) = \Lambda^2\,\hat{\boldsymbol{\ell}}\cdot\boldsymbol{x}$$

homogeneous solution

$$\hat{\boldsymbol{\ell}} \parallel \boldsymbol{x}_1 - \boldsymbol{x}_2$$

$$V(\boldsymbol{x}_1-\boldsymbol{x}_2) = \tfrac{1}{2}[eA^0_\Lambda(\boldsymbol{x}_1) - eA^0_\Lambda(\boldsymbol{x}_2)] = \tfrac{1}{2}e\Lambda^2|\boldsymbol{x}_1-\boldsymbol{x}_2|$$

DDD

1+1d qed

- no transverse gauge bosons
- linear potential
- more tractable

1+1d qed

$$A^1 = 0$$

Leibbrandt,
Introduction to noncovariant gauges
Rev. Mod. Phys. 59 (1987) 1067

$$S = \int d^2x \Big[-\frac{1}{2}\left(\partial_1 A^0\right)\left(\partial^1 A^0\right) + \sum_f \psi_f^\dagger(x)\gamma^0\left(i\partial\!\!\!/ - m_f - e\gamma^0 A^0\right)\psi_f(x) \Big]$$

$$-\partial_1^2 A^0(x) = e\sum_f \psi_f^\dagger \psi_f(x)$$

$$A^0(x) = -\frac{e}{2}\sum_f \int dy^1 |x^1 - y^1| \psi_f^\dagger \psi_f(x^0, y^1)$$

DDD

eliminate A

$$S = \int d^2x \Big[-\frac{1}{2}(\partial_1 A^0)(\partial^1 A^0) + \sum_f \psi_f^\dagger(x) \gamma^0 (i\partial\!\!\!/ - m_f - e\gamma^0 A^0) \psi_f(x) \Big]$$

$$A^0(x) = -\frac{e}{2} \sum_f \int dy^1 |x^1 - y^1| \psi_f^\dagger \psi_f(x^0, y^1)$$

$$S \equiv S_F + S_V = \sum_f \int d^2x \, \psi_f^\dagger(x) \gamma^0 (i\partial\!\!\!/ - m_f) \psi_f(x) + \\ + \frac{e^2}{4} \sum_{f,f'} \int d^2x \, d^2y \, \delta(x^0 - y^0) \psi_f^\dagger \psi_f(x) |x^1 - y^1| \psi_{f'}^\dagger \psi_{f'}(y)$$

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Poincaré invariance

$$[P^0, P^1] = 0$$

$$[P^0, M^{01}] = iP^1$$

$$[P^1, M^{01}] = iP^0$$

DDD

spatial translation

$$\psi_f(x^0, x^1) \to \psi_f\left(x^0, x^1 - \epsilon(x^0)d\ell\right)$$

$$\delta S = d\ell \int dx^0 \; \epsilon'(x^0) P^1$$

$$P^1 = -i \sum_f \int dx^1 \; \psi_f^\dagger(x) \partial_1 \psi_f(x)$$

momentum

spatial translation

$$\psi_f(x^0, x^1) \to \psi_f\left(x^0, x^1 - \epsilon(x^0)d\ell\right)$$

total momentum conserved
in potential interaction

$$P^1 = -i\sum_f \int dx^1\, \psi_f^\dagger(x)\partial_1\psi_f(x)$$

momentum

DDD

temporal translation

$$\psi(x^0, x^1) \to \psi\left(x^0 - \epsilon(x^0)dt, x^1\right)$$

$$P_F^0 \;=\; \sum_f \int dx^1 \; \psi_f^\dagger(x)(-i\gamma^0\gamma^1\partial_1 + m_f\gamma^0)\psi_f(x),$$

$$P_V^0 \;=\; -\frac{e^2}{4}\sum_{f,f'}\int dx^1 dy^1 \; \psi_f^\dagger\psi_f(x^0, x^1)|x^1 - y^1|\psi_{f'}^\dagger\psi_{f'}(x^0, y^1)$$

Hamiltonian

DDD

temporal translation

$$\psi(x^0, x^1) \to \psi(x^0 - \epsilon(x^0)dt, x^1)$$

balance between potential and kinetic energy

$$P_F^0 = \sum_f \int dx^1 \, \cdots \, \psi_f(x),$$

$$P_V^0 = -\frac{e^2}{4} \sum_{f,f'} \int dx^1 dy^1 \, \psi_f^\dagger \psi_f(x^0, x^1) |x^1 - y^1| \psi_{f'}^\dagger \psi_{f'}(x^0, y^1)$$

Hamiltonian

DDD

boost

$$x^0 \to x^0 + d\xi x^1 \qquad\qquad x^1 \to x^1 + d\xi x^0$$

$$(A^0, A^1 = 0) \to (A^0, d\xi A^0)$$ **but:** $A^1 = 0$

$$\psi(x) \to \exp(-id\xi\,\theta)\psi(x)$$

$$\partial_1 \theta(x) = eA^0(x) = -\frac{e^2}{2}\int d^2y\,\delta(x^0 - y^0)|x^1 - y^1|\psi^\dagger\psi(y)$$

$$\psi(x^0, x^1) \to \left[1 + \tfrac{1}{2}\epsilon(x^0)\gamma^0\gamma^1 d\xi - i\epsilon(x^0)\theta(x^0, x^1)d\xi\right]$$
$$\psi\big(x^0 - \epsilon(x^0)x^1 d\xi, x^1 - \epsilon(x^0)x^0 d\xi\big)$$

DDD

boost

$$x^0 \to x^0 + d\xi x^1 \qquad\qquad x^1 \to x^1 + d\xi x^0$$

but: $A^1 = 0$

non-covariant gauge

$\Rightarrow$

must restore gauge condition

$$\partial_1 \theta(x) = eA^0(x) = -\frac{\;}{2}\int d^2y\, \delta(x^0 - y^0)|x^1 - y^1| \psi^\dagger\psi(y)$$

$$\psi(x^0, x^1) \to \left[1 + \tfrac{1}{2}\epsilon(x^0)\gamma^0\gamma^1 d\xi - i\epsilon(x^0)\theta(x^0, x^1)d\xi\right]$$
$$\psi\left(x^0 - \epsilon(x^0)x^1 d\xi, x^1 - \epsilon(x^0)x^0 d\xi\right)$$

DDD

Poincaré invariance

$$M_F^{01} = x^0 P^1 + \sum_f \int dx^1 \, \psi_f^\dagger(x) \Big[x^1 (i\gamma^0\gamma^1\partial_1 - \gamma^0 m_f) + \frac{i}{2}\gamma^0\gamma^1 \Big] \psi_f(x)$$

$$M_V^{01} = \frac{e^2}{8} \sum_{f,f'} \int dx^1 dy^1 \, \psi_f^\dagger \psi_f(x^0, x^1)(x^1 + y^1)|x^1 - y^1| \psi_{f'}^\dagger \psi_{f'}(x^0, y^1)$$

as densities

$$\mathcal{P}^0 = \bar{\psi}\big(-\tfrac{1}{2} i\gamma^1 \overleftrightarrow{\partial}_1 + m\big)\psi - \frac{e^2}{4} \int dy^1 \, \psi^\dagger \psi(x^0, x^1)|x^1 - y^1| \psi^\dagger \psi(x^0, y^1)$$

$$\mathcal{P}^1 = \bar{\psi}\big(-\tfrac{1}{2} i\gamma^0 \overleftrightarrow{\partial}_1\big)\psi,$$

$$\mathcal{M}^{01} = x^0 \mathcal{P}^1 - x^1 \mathcal{P}^0$$

cross check ✔

DDD

Poincaré invariance

$$M^{01}_F = x^0 P^1 + \sum_f \int dx^1\, \psi_f^\dagger(x)\Big[x^1(i\gamma^0\gamma^1\partial_1 - \gamma^0 m_f) + \frac{i}{2}\gamma^0\gamma^1\Big]\psi_f(x)$$

$$M^{01}_V = \frac{e^2}{8}\sum_{f,f'}\int dx^1 dy^1\, \psi_f^\dagger\psi_f(x^0,x^1)(x^1+y^1)|x^1-y^1|\psi_{f'}^\dagger\psi_{f'}(x^0,y^1)$$

as densities

$$\mathcal{P}^0 = \bar\psi\big(-\tfrac{1}{2}i\gamma^1\overset{\leftrightarrow}{\partial}_1 + m\big)\psi - \frac{e^2}{4}\int dy^1\, \psi^\dagger\psi(x^0,x^1)|x^1-y^1|\psi^\dagger\psi(x^0,y^1)$$

$$\mathcal{P}^1 = \bar\psi\big(-\tfrac{1}{2}i\gamma^0\overset{\leftrightarrow}{\partial}_1\big)\psi,$$

$$\boxed{\mathcal{M}^{01} = x^0\mathcal{P}^1 - x^1\mathcal{P}^0}$$

cross check ✔

DDD

Poincaré invariance

$$[P^0, P^1] = 0$$

$$[P^0, M^{01}] = iP^1$$

$$[P^1, M^{01}] = iP^0$$

check using

$$\left\{\psi_{f\alpha}(x^0, x^1), \psi^\dagger_{f'\beta}(x^0, y^1)\right\} = \delta(x^1 - y^1)\delta_{ff'}\delta_{\alpha\beta}$$

DDD

Poincaré invariance

$$[P^0, P^1] = 0$$

for QCD it closes only in the singlet sector

$$[P^1, M^{01}] = iP^0$$

check using

$$\left\{\psi_{f\alpha}(x^0, x^1), \psi^\dagger_{f'\beta}(x^0, y^1)\right\} = \delta(x^1 - y^1)\delta_{ff'}\delta_{\alpha\beta}$$

DDD

bound state equation

valence quarks only!

$$|0\rangle_R = N^{-1} \prod_{p^1} d^\dagger(p^1)\,|0\rangle$$

retarded vacuum

$$\psi(x)\,|0\rangle_R = 0$$

low orders only ! ⇒ no loops

$$|E,k\rangle \equiv \int dx_1 dx_2\, \exp\left[\tfrac{1}{2} ik(x_1+x_2)\right] \boxed{\bar{\psi}_1(0,x_1)} e^{i\varphi}\Phi(x_1-x_2) \boxed{\psi_2(0,x_2)}\,|0\rangle_R$$

$$\psi_1(x)\,|0\rangle_R = \psi_2^\dagger(x)\,|0\rangle_R = 0$$

DDD

bound state equation

$$P^0 |E,k\rangle = E |E,k\rangle$$

$$P_F^0 = \sum_f \int dx^1 \, \psi_f^\dagger(x)(-i\gamma^0\gamma^1\partial_1 + m_f\gamma^0)\psi_f(x),$$

$$P_V^0 = -\frac{e^2}{4}\sum_{f,f'}\int dx^1 dy^1 \, \psi_f^\dagger\psi_f(x^0,x^1)|x^1-y^1|\psi_{f'}^\dagger\psi_{f'}(x^0,y^1)$$

$$|E,k\rangle \equiv \int dx_1 dx_2 \, \exp\left[\tfrac{1}{2}ik(x_1+x_2)\right]\bar{\psi}_1(0,x_1)e^{i\varphi}\Phi(x_1-x_2)\psi_2(0,x_2)\,|0\rangle_R$$

DDD

$$\gamma^0 = \sigma_3, \qquad \gamma^1 = i\sigma_2, \qquad \gamma^0\gamma^1 = \sigma_1$$

bound state equation

$$P^0 \left|E, k\right\rangle = E \left|E, k\right\rangle$$

$$P_F^0 = \sum_f \int dx^1 \, \boxed{\psi_f^\dagger(x)} (-i\gamma^0\gamma^1\partial_1 + m_f\gamma^0) \boxed{\psi_f(x)},$$

$$P_V^0 = -\frac{e^2}{4} \sum_{f,f'} \int dx^1 dy^1 \, \boxed{\psi_f^\dagger \psi_f(x^0, x^1)} |x^1 - y^1| \boxed{\psi_{f'}^\dagger \psi_{f'}(x^0, y^1)}$$

$$\left|E, k\right\rangle \equiv \int dx_1 dx_2 \, \exp\left[\tfrac{1}{2} ik(x_1 + x_2)\right] \boxed{\bar{\psi}_1(0, x_1)} e^{i\varphi} \Phi(x_1 - x_2) \boxed{\psi_2(0, x_2)} \left|0\right\rangle_R$$

DDD

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bound state equation

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$$\left|E, k\right\rangle \equiv \int dx_1 dx_2 \, \exp\left[\tfrac{1}{2}ik(x_1 + x_2)\right]\bar{\psi}_1(0, x_1)e^{i\varphi}\Phi(x_1 - x_2)\psi_2(0, x_2)\left|0\right\rangle_R$$

$$i\partial_x\left\{\sigma_1, \Phi(x)\right\} - (\partial_x\varphi)\left\{\sigma_1, \Phi(x)\right\} - \tfrac{1}{2}k\left[\sigma_1, \Phi(x)\right] + m_1\sigma_3\Phi(x) - m_2\Phi(x)\sigma_3$$
$$= \left[E - V(x)\right]\Phi(x)$$

DDD

$$\gamma^0 = \sigma_3, \qquad \gamma^1 = i\sigma_2, \qquad \gamma^0\gamma^1 = \sigma_1$$

bound state equation

$$P^0 |E,k\rangle = E\,|E,k\rangle$$

$$P_F^0 = \sum_f \int dx^1\, \psi_f^\dagger(x)(-i\gamma^0\gamma^1\partial_1 + m_f\gamma^0)\psi_f(x),$$

$$P_V^0 = -\frac{e^2}{4}\sum_{f,f'}\int dx^1 dy^1\, \psi_f^\dagger\psi_f(x^0,x^1)|x^1-y^1|\psi_{f'}^\dagger\psi_{f'}(x^0,y^1)$$

$$|E,k\rangle \equiv \int dx_1 dx_2 \exp\left[\tfrac{1}{2}ik(x_1+x_2)\right]\bar{\psi}_1(0,x_1)e^{i\varphi}\Phi(x_1-x_2)\psi_2(0,x_2)\,|0\rangle_R$$

$$i\partial_x\{\sigma_1,\Phi(x)\} - (\partial_x\varphi)\{\sigma_1,\Phi(x)\} - \tfrac{1}{2}k\,[\sigma_1,\Phi(x)] + m_1\sigma_3\Phi(x) - m_2\Phi(x)\sigma_3$$
$$= [E - V(x)]\,\Phi(x)$$

DDD

$$\gamma^0 = \sigma_3, \qquad \gamma^1 = i\sigma_2, \qquad \gamma^0\gamma^1 = \sigma_1$$

bound state equation

$$P^0 |E,k\rangle = E |E,k\rangle$$

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$$i\partial_x\{\sigma_1,\Phi(x)\} - (\partial_x\varphi)\{\sigma_1,\Phi(x)\} - \tfrac{1}{2}k\left[\sigma_1,\Phi(x)\right] + m_1\sigma_3\Phi(x) - m_2\Phi(x)\sigma_3$$
$$= \left[E - V(x)\right]\Phi(x)$$

DDD

$$\gamma^0 = \sigma_3, \qquad \gamma^1 = i\sigma_2, \qquad \gamma^0\gamma^1 = \sigma_1$$

bound state equation

$$P^0 |E,k\rangle = E |E,k\rangle$$

$$P^0_F = \sum_f \int dx^1 \, \psi_f^\dagger(x)(-i\gamma^0\gamma^1\partial_1 + m_f\gamma^0)\psi_f(x),$$

$$P^0_V = -\frac{e^2}{4}\sum_{f,f'}\int dx^1 dy^1 \, \psi_f^\dagger\psi_f(x^0,x^1)|x^1 - y^1|\psi_{f'}^\dagger\psi_{f'}(x^0,y^1)$$

$$|E,k\rangle \equiv \int dx_1 dx_2 \, \exp\left[\tfrac{1}{2}ik(x_1+x_2)\right]\bar{\psi}_1(0,x_1)e^{i\varphi}\Phi(x_1-x_2)\psi_2(0,x_2)\,|0\rangle_R$$

$$i\partial_x\{\sigma_1,\Phi(x)\} - (\partial_x\varphi)\{\sigma_1,\Phi(x)\} - \tfrac{1}{2}k\left[\sigma_1,\Phi(x)\right] + m_1\sigma_3\Phi(x) - m_2\Phi(x)\sigma_3$$
$$= \left[E - V(x)\right]\Phi(x)$$

DDD

decomposition

$$i\partial_x \{\sigma_1, \Phi(x)\} - (\partial_x \varphi)\{\sigma_1, \Phi(x)\} - \tfrac{1}{2}k\,[\sigma_1, \Phi(x)] + m_1\sigma_3\Phi(x) - m_2\Phi(x)\sigma_3$$

$$\Phi(x) \;\equiv\; \Phi_0(x) + \sum_{j=1}^{3} \Phi_j(x)\sigma_j$$

no derivatives on #2 & #3

change of variable

$$s(x) \;\equiv\; \frac{1}{2}\int_0^x du\big[E - V(u)\big] = \frac{\varepsilon(x)}{2e^2}\left[2EV(x) - V(x)^2\right]$$

$$= \; \frac{\varepsilon(x)}{2e^2}(M^2 - p^2)$$

$$M \equiv \sqrt{E^2 - k^2}$$

$$p^2 = (E - V)^2 - k^2$$

k independent @
fixed s & M !

$$i\partial_s \Phi_1(s) = \left[1 - \frac{(m_1 - m_2)^2}{p^2}\right]\Phi_0(s)$$

$$i\partial_s \Phi_0(s) = \left[1 - \frac{(m_1 + m_2)^2}{p^2}\right]\Phi_1(s)$$

DDD

change of variable

boosts:

equations form invariant in terms of s

relation between s & x frame dependent

$$dx/ds = 2/(E - V(x))$$

$$V \ll E \qquad\qquad 1/E$$

cross check: direct boosting ✔

$$i\partial_s \Phi_0(s) = \left[1 - \frac{(m_1+m_2)^2}{p^2}\right] \Phi_1(s)$$

change of variable

$s(x) \equiv \; \ldots \; V(x)^2]$

boosts:

equations form invariant in terms of s

relation between s & x frame dependent

$$dx/ds = 2/(E - V(x))$$

$V \ll E$ $\qquad$ $1/E$

cross check: direct boosting ✔

$$i\partial_s \Phi_1(s) = \left[1 \; \ldots \; \frac{\ldots}{p^2}\right] \Phi_0(s)$$

works only for the linear potential

$$= \left[1 - \frac{(m_1 + m_2)^2}{p^2}\right] \Phi_1(s)$$

solutions

- possible analytically ☺
- selection of spectrum ?

AUGUST 1, 1932 PHYSICAL REVIEW VOLUME 41

The Dirac Electron in Simple Fields*

By MILTON S. PLESSET
Sloane Physics Laboratory, Yale University

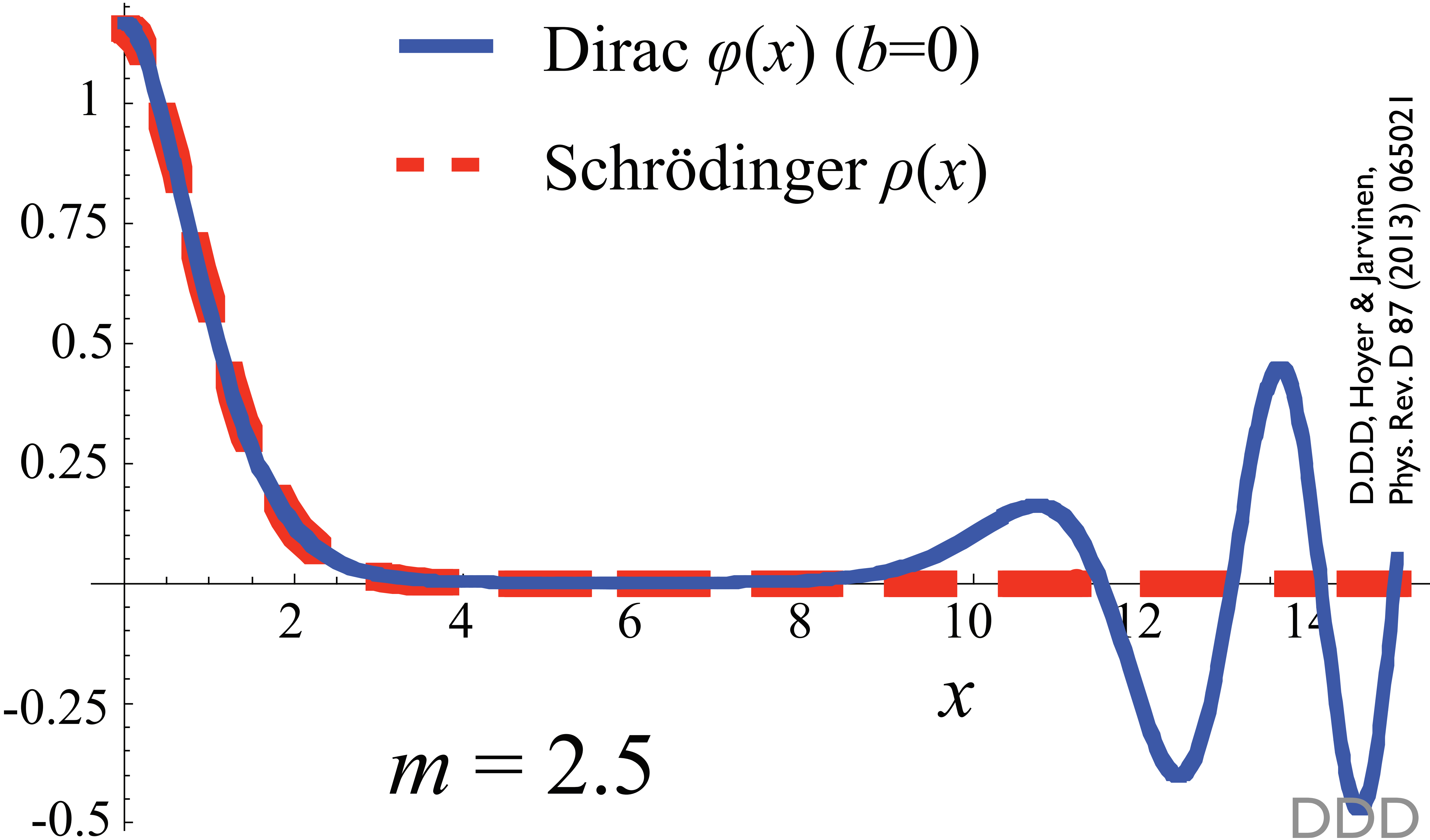
(Received June 6, 1932)

The relativity wave equations for the Dirac electron are transformed in a simple manner into a symmetric canonical form. This canonical form makes readily possible the investigation of the characteristics of the solutions of these relativity equations for simple potential fields. If the potential is a polynomial of any degree in x, a continuous energy spectrum characterizes the solutions. If the potential is a polynomial of any degree in $1/x$, the solutions possess a continuous energy spectrum when the energy is numerically greater than the rest-energy of the electron; values of the energy numerically less than the rest-energy are barred. When the potential is a polynomial of any degree in r, all values of the energy are allowed. For potentials which are polynomials in $1/r$ of degree higher than the first, the energy spectrum is again continuous. The quantization arising for the Coulomb potential is an exceptional case.

See also: E. C. Titchmarsh, Proc. London Math. Soc. (3) 11 (1961) 159 and 169; Quart. J. Math. Oxford (2), 12 (1961), 227.

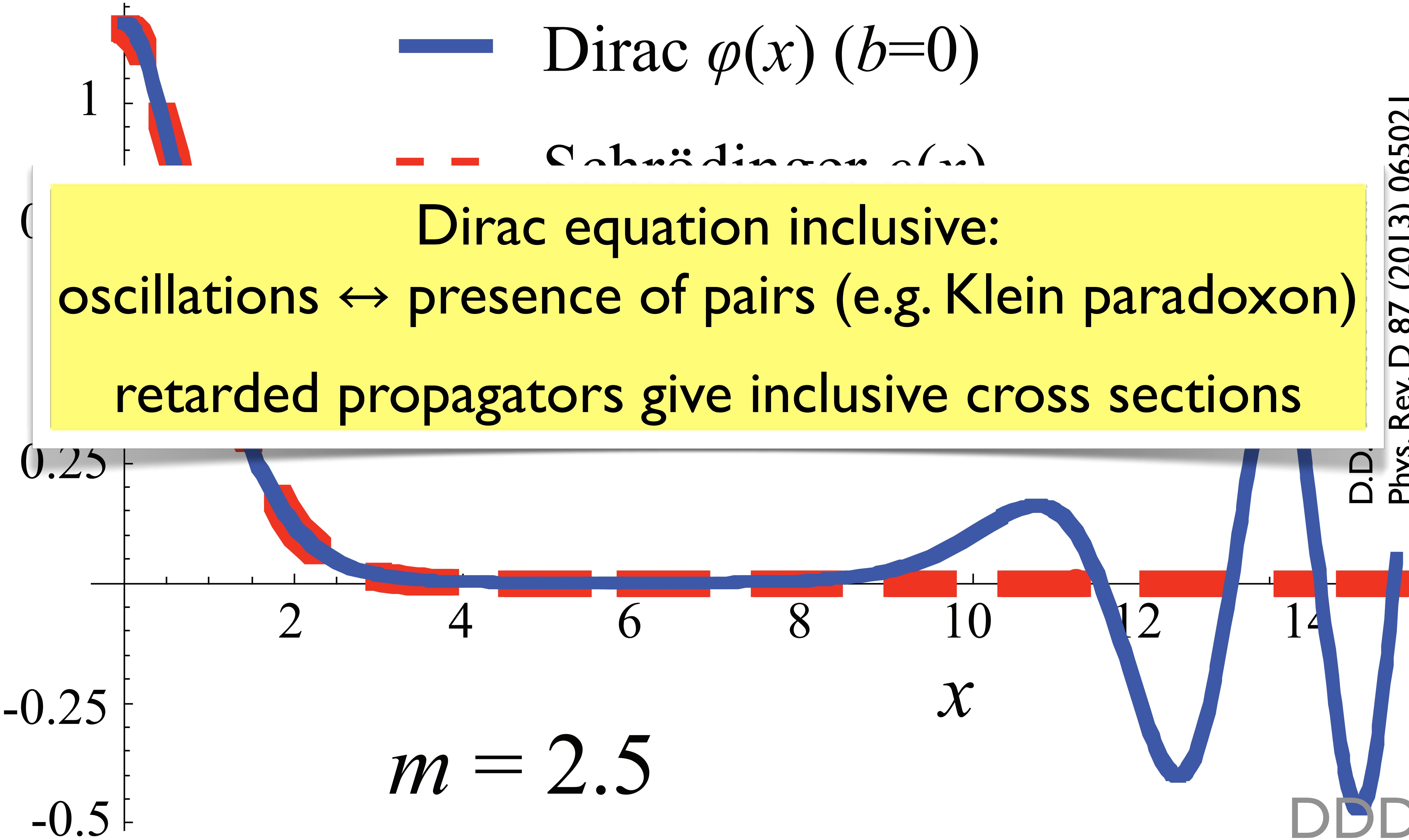
$$\varphi(x \to \infty) \sim \exp(\pm i x^2/4)$$

oscillations



$$\varphi(x \to \infty) \sim \exp(\pm i x^2/4)$$

oscillations



Dirac equation inclusive:

oscillations ↔ presence of pairs (e.g. Klein paradoxon)

retarded propagators give inclusive cross sections

D.D. Phys. Rev. D 87 (2013) 065021

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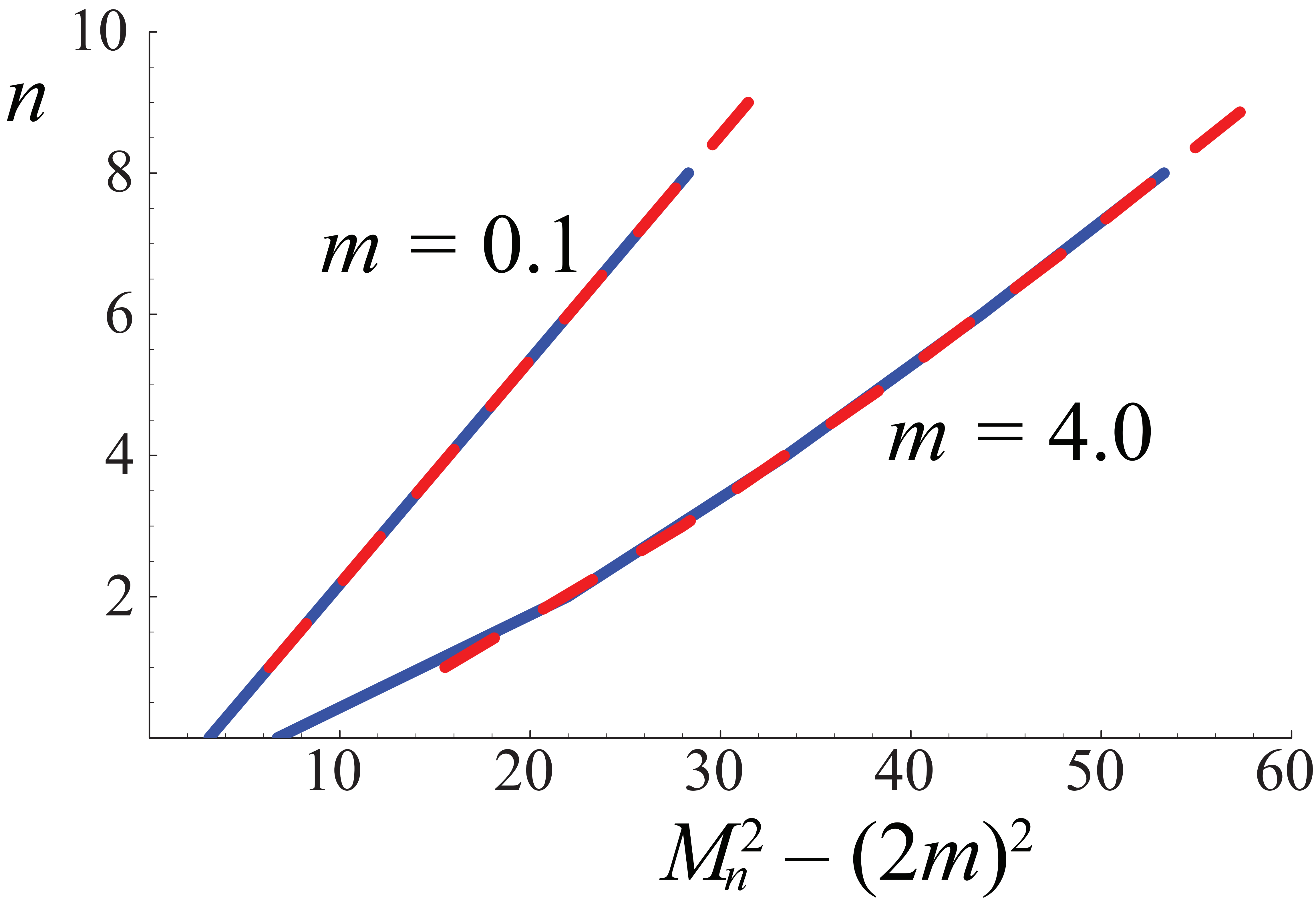
solutions ($m_1=m_2$)

- oscillations as in Dirac
- only 1d subspace of solutions regular @ s=0 opposed to Dirac
- selection of parity even or odd $\Rightarrow$ spectrum

DDD

Regge trajectories



D.D.D, Hoyer & Jarvinen,
Phys. Rev. D 87 (2013) 065021

solutions ($m_1 \neq m_2$)

- no solutions regular @ s=0
- orthogonality relations ✔
- duality normalisation for highly excited states

boosting

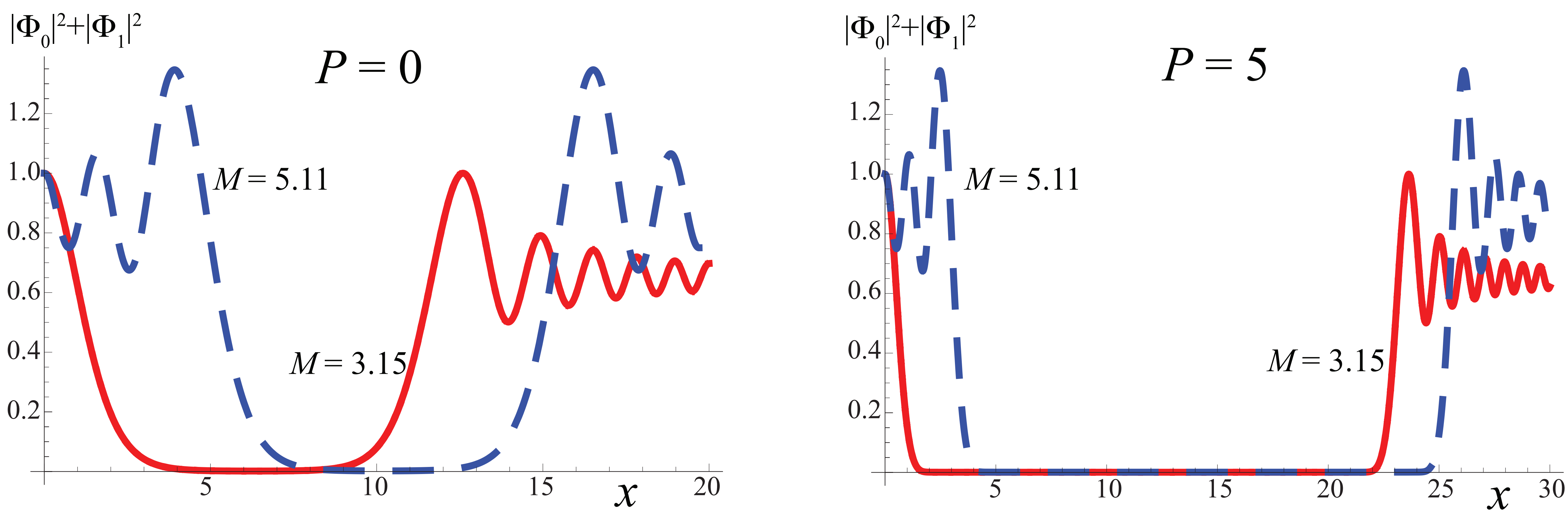
Ground state

Excited state

$m_1/e = 1.0$

$m_2/e = 1.5$

D.D.D, Hoyer & Jarvinen,
Phys. Rev. D 87 (2013) 065021



DDD

summary

- objective: towards a Born term for hadrons
  - valence quarks only
  - no transverse gluons
  - perturbation theory
  - Poincaré invariance
- 1+1 qed as model

DDD

Thank yo much

for attention!

Thanks, Paul !

DDD