Short Note

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Why No PlaneWaves of Macroscopic Bodies? A Micro-Macro Threshold?

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Why No Plane Waves of Macroscopic Bodies? A Micro-Macro Threshold?

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Abstract: According to Quantum Mechanics a narrow wave-package of the center of mass of macroscopic objects, due to their heavy mass, remains stable over astronomical times. However Quantum Mechanics allows and energetically even prefers largely smeared out c.m. states that never been seen. Why is this the real state of the world we know? Does it suggest a micro-macro threshold with different theoretical descriptions?

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1. Introduction

One of the strange features of Quantum Mechanics is that for its formulation, one needs the classical physics that actually should emerge as its macroscopic limit. All experiences with quantum objects have to be analyzed through classical "glasses".

Naturally, then the question arises: where is the threshold (if any) between the microscopic and the macroscopic worlds?

In the first half of the twentieth century the big successes of quantum theory were obtained mainly in the field of ("elementary") microscopic particles: electrons, atoms, nuclei. Thereafter, the most spectacular evolution occurred in understanding the properties of condensed matter on the basis of many-body quantum mechanics. This step revolutionized our technology. Now we witness another exceptional development in producing ever smaller pieces of solid matter. We are approaching the microscopic world from above. Therefore, the answer to above stated question might be very close.

Quantum Mechanics is an exceptionally successful theory, nevertheless the relation between macroscopic classical and microscopic quantum mechanical description is still subject of worry. I want to underline some simple but often ignored aspects in this context. The theory of condensed matter is conceived as the quantum mechanical description of bound states of an enormous number of Coulomb interacting electrons and ions. In the absence of an external potential the center of mass (c.m.) motion is fully separable from the relative (internal) one [1] and its wave function satisfies the free Schrödinger equation of a particle with the total mass m of the system. All we are treating is however, only the relative motion, while the center of mass motion is considered irrelevant and left to the classical description. In what follows we shall however discuss only the center of mass motion.

In the real world the center of mass states of the macroscopic objects are sharply defined wave packages. According to Quantum Mechanics (Schrödinger equation) these should spread out in time. A somewhat appeasing argument is given in the frame of Ehrenfest's theorem in almost all handbooks about their stability for macroscopic masses.

In Section 2 we recapitulate the standard quantum mechanical description of an ideal wave package (here the free motion of the c.m), while in Section 3 we comment on the not understood absence of broad wave packages of macroscopic objects in the real word.

2. Evolution of a Wave-packet of the c.m.

Let us consider a solution of the free 1D Schrödinger equation for an object of mass m in the form of a Gaussian wave packet (having the ideal uncertainty $\Delta x \Delta p = \hbar$) travelling with a constant average velocity. It may represent the free motion of the c.m. of a macroscopic object of total mass m. Using the

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Galilean invariance of the free Schrödinger equation [2] one may always go to its own frame (in our case the c.m. frame of reference) defined by the vanishing of the average velocity and at some initial time $t = 0$ one has the wave function

$$\psi(x,0) = \frac{1}{(2\pi)^{\frac{1}{4}}\sqrt{d}} e^{-\frac{x^2}{4d^2}} \quad .$$

Then after a lapse of time t it decays

$$\psi(x,t) = \frac{1}{(2\pi)^{\frac{1}{4}}\sqrt{d + \frac{\imath\hbar t}{2dm}}} e^{-\frac{x^2}{4(d^2+\frac{\imath\hbar t}{2m})}} .$$

This means that the average quadratic width grows as

$$\langle x^2 \rangle_t = d^2 + (\frac{\hbar t}{2md})^2$$

One may see, that for $m \to \infty$ the width does not change at all. However, the decisive parameter is not the mass m alone but the product md. For any finite mass m at $d \to 0$ the wave package decays instantly.

Leaving aside these ideal limits, let us consider a numerical example for sake of illustration. A body of $m = 1g$ and an initial linear imprecision of its center of mass of 1 Angström after one million years will be smeared out as wide as to 100 Angström. This is worth to think about. Of course, the discussion may be extended to the motion in the presence of a slowly varying external potential on the scale of the macroscopic object as in Ehrenfest's Theorem.

All these aspects are well-known. My own short comments follow in the next sections.

3. What about the "initial" state?

The provoking question is: Why all macroscopic objects we know have very precise center of mass position? Ultimately, why are there no plane waves of macroscopic objects, although energetically more favorable. In our example of the Gaussian package the average kinetic energy is

$$\frac{\hbar^2}{2m}\langle k^2 \rangle = \frac{\hbar^2}{8md^2} \quad .$$

For non-ideal wave-packets ($\Delta x \Delta p > \hbar$) the average kinetic energy is even greater. Therefore the more smeared is the wave packet, the smaller its conserved average energy is.

Although energetically preferred, no smeared out (on macroscopic scale) macroscopic objects have ever been seen. It looks like a "super-selection" rule for macroscopic objects. "God allowed them only in this state of precise c.m.?" On the other hand, a world with smeared out center of masses is hard to imagine. It won't function. The macroscopic world, as we know it, is the only possible world!?

On our current way toward micro-miniaturization we might get an answer to this question. Where is the threshold between micro and macro? One cannot exclude the possibility, that the macroscopic physics is not just the simple limit of the quantum theory. (In a mathematical sense it is surely not the $\hbar \to 0$ limit.) This should solve also the "bootstrap" aspect of Quantum Mechanics that needs for its formulation the existence of macroscopic laws.

4. Comments

I am aware that just touching this point is a heresy. In our human-centered world-image motivated by the exceptional scientific progresses of the last centuries we cheer to construct a unified picture of the world without loopholes and crevices. One looks even for "God's equations" forgetting that mathematics has to be connected to experiment by some interpretation and this is the most difficult one.

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However, all theories of physics of the past were just fragments of knowledge with loose connections between them. Of course, thermodynamics has to do with statistical mechanics, but this is a rather subtle one. Whether one may derive every-day quantum theory of condensed matter from the modern quantum field theory of elementary particles remains a question of belief. The unification of gravitation and quantum theory looks merely a dream. I am afraid, that we are misled by our need for harmony and exaggerated confidence in our brains.

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