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Theorie zu Magnetismus, Supraleitung und Elektronische Korrelation in Festkörpern

Sommersemester 2019

Blatt 9

(Abgabe: 18.06.2019)

Aufgabe 1 (**Spin-wave theory for the 1D anti-ferromagnetic chain**) (10 Punkte)

Consider the anti-ferromagnetic Heisenberg model with N sites given by

$$H = J \sum_i \vec{S}_i \vec{S}_{i+1}, \tag{1}$$

where $J > 0$. We want to treat this Hamiltonian within the linear spin-wave theory approximation in the simple case of a 1D chain with N sites (see Fig. 1) and L is the lattice spacing.

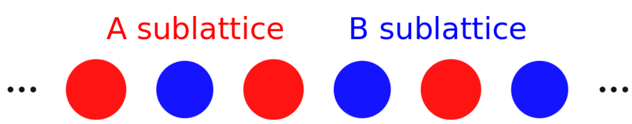

Figure 1: 1D Heisenberg chain divided into sublattices A and B.

a) For the antiferromagnetic Heisenberg model, the mean-field ground state has neighboring spins antiparallel, $|\uparrow\downarrow\uparrow \dots \downarrow\uparrow\rangle$. In contrast to the ferromagnetic model, we therefore define the Holstein-Primakoff transformation differently on the $\uparrow$ (A) and $\downarrow$ (B) sublattices:

$$S^+_{A,j} = \sqrt{2S - a^\dagger_j a_j}\, a_j, \qquad S^-_{A,j} = a^\dagger_j \sqrt{2S - a^\dagger_j a_j}, \qquad S^z_{A,j} = S - a^\dagger_j a_j, \tag{2}$$

$$S^+_{B,l} = b^\dagger_l \sqrt{2S - b^\dagger_l b_l}, \qquad S^-_{B,l} = \sqrt{2S - b^\dagger_l b_l}\, b_l, \qquad S^z_{B,l} = -S + b^\dagger_l b_l, \tag{3}$$

where a_j and b_l are bosonic operators on the respective sublattice.
Rewrite the Hamiltonian H in terms of those two bosonic operators and expand H up to the order of $a^\dagger_j a_j$ and $b^\dagger_l b_l$.

b) Use the Fouriertransform on each sublattice to rewrite the Hamiltonian in terms of

$$a_k = \frac{1}{\sqrt{N_A}} \sum_{j \in A} e^{-ikr_j} a_j, \qquad b_k = \frac{1}{\sqrt{N_B}} \sum_{j \in B} e^{-ikr_l} b_l. \tag{4}$$

c) A special case of the Bogoliubov transformation is given by

$$\alpha_k = \cosh(\theta_k) a_k - \sinh(\theta_k) b^\dagger_{-k} \tag{5}$$

$$\beta_k = \cosh(\theta_k) b_k - \sinh(\theta_k) a^\dagger_{-k}, \tag{6}$$

where θ is a even, real function.
Show that those operators are bosonic and commute with each other.

d) Find a condition on θ to be able to diagonalize the Hamiltonian H with the Bogoliubov transformation.
Hint: Find θ_k that removes all terms with two creation or annihilation operators, i.e. $\alpha\beta$ or $\alpha^\dagger\beta^\dagger$.

e) Show that the ground state energy of the quantum system is lower than the classical ground state energy.

f) Calculate the magnetization on the sublattice A and show that the correction to the classical result diverges in the limit $N \to \infty$.