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Theorie zu Magnetismus, Supraleitung und Elektronische Korrelation in Festkörpern

Sommersemester 2019

Blatt 7

(Abgabe: 04.06.2019)

Aufgabe 1 (**Integer Quantum Hall Effect**) (5=3+1+1 Punkte)

Consider an electron confined to the x-y-plane in a constant magnetic field along the z-axis. We solved this problem already by decoupling of the underlying differential equation. Now, we will use creation and annihilation operators to be able to evaluate the matrix elements of the current operator and calculate the conductivity.

a) Rewrite the Hamiltonian of the system $\hat{H}$, the momentum operators $\hat{p}_x, \hat{p}_y$ and position operators $\hat{x}, \hat{y}$ in terms of $\hat{a}_r, \hat{a}_l$, where $\hat{a}_r = \frac{1}{\sqrt{2}}(\hat{a}_x - i\hat{a}_y), \hat{a}_l = \frac{1}{\sqrt{2}}(\hat{a}_x + i\hat{a}_y), \hat{a}_x = \frac{1}{\sqrt{2}}(\beta\hat{x} + i\frac{\hat{p}_x}{\beta\hbar}), \hat{a}_y = \frac{1}{\sqrt{2}}(\beta\hat{y} + i\frac{\hat{p}_y}{\beta\hbar}), \beta = \sqrt{\frac{m\omega_0}{2\hbar}}$ and make use of the symmetric gauge $\vec{A} = -\frac{1}{2}\hat{\vec{r}} \times \vec{B}$.

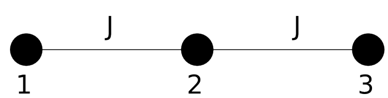
b) Calculate the matrix elements of the current operators $\hat{j}_x, \hat{j}_y$ in the eigenbasis of the Hamiltonian.

c) Use the Kubo formula

$$\sigma_{xy} = i\hbar \sum_{\{n_l, n_r\} \neq \{0,0\}} \frac{\langle 0,0|\hat{j}_y|n_l, n_r\rangle\langle n_l, n_r|\hat{j}_x|0,0\rangle - \langle 0,0|\hat{j}_x|n_l, n_r\rangle\langle n_l, n_r|\hat{j}_y|0,0\rangle}{(E_{l,r} - E_{0,0})^2}$$

to calculate the in-plane coductivity.

Aufgabe 2 (**3-Site Heisenberg Chain**) (5 Punkte)



a) Determine the energy spectrum for a spin-$\frac{1}{2}$ Heisenberg chain of three sites subject to the Hamiltonian

$$H^{chain} = J(\vec{S}_1 \cdot \vec{S}_2 + \vec{S}_2 \cdot \vec{S}_3), \quad (1)$$

with coupling constant J. (Hint: Rewrite the Hamiltonian in a proper way using the total-spin operator $\vec{S}_{123} = \sum_i \vec{S}_i$)

b) Discuss which are the ground and which are the excited states and their degeneracy in the ferromagnetic $J < 0$ and antiferromagnetic $J > 0$ case.

c) Write down explicitly the ferromagnetic ground state with total z-component of the spin $\vec{S}^z_{123} = \sum_i \vec{S}^z_i = \frac{3}{2}$ and the antiferromagnetic ground state with $\vec{S}^z_{123} = \frac{1}{2}$.

Hint: You may use the Clebsch-Gordan coefficients:

$$|J_a, J_b, J_c, M_c\rangle = \sum_{M_a, M_b} C(J_a, J_b, J_c, M_a, M_b, M_c)|J_a, M_a\rangle \otimes |J_b, M_b\rangle, \tag{2}$$

with constants $C(J_a, J_b, J_c, M_a, M_b, M_c)$, where J_i and M_i are the spin quantum numbers defined by the relations

$$J_i^2|J_i, M_i\rangle = J_i(J_i+1)|J_i, M_i\rangle \tag{3}$$

$$J_i^z|J_i, M_i\rangle = M_i|J_i, M_i\rangle, \tag{4}$$

In this particular case you have to consider the relations

$$|J_a = 1, J_b = \frac{1}{2}, J_c = \frac{3}{2}, M_c = \frac{3}{2}\rangle = |J_a, M_a = 1\rangle \otimes |J_b, M_b = \frac{1}{2}\rangle, \tag{5}$$

$$\begin{aligned} |J_a = 1, J_b = \frac{1}{2}, J_c = \frac{1}{2}, M_c = \frac{1}{2}\rangle = & \sqrt{\frac{2}{3}}|J_a, M_a = 1\rangle \otimes |J_b, M_b = -\frac{1}{2}\rangle \\ & - \sqrt{\frac{1}{3}}|J_a, M_a = 0\rangle \otimes |J_b, M_b = \frac{1}{2}\rangle. \end{aligned} \tag{6}$$