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Theorie zu Magnetismus, Supraleitung und Elektronische Korrelation in Festkörpern

Sommersemester 2019

Blatt 5

(Abgabe: 21.05.2019)

Aufgabe 1 (**Alternative derivation of Curie's law**) (5 Punkte)

If one writes the free energy F_S in the form

(1) $$e^{-\beta F_S} = \sum_n e^{-\beta E_n} = \sum_n \langle n|e^{-\beta\mathcal{H}}|n\rangle = \mathrm{Tr}\, e^{-\beta\mathcal{H}},$$

then it is easy to deduce Curie's law directly at high temperature without going through the algebra of Brillouin functions. Indeed, when $\mathcal{H} \ll k_B T$, one can expand $e^{-\beta\mathcal{H}} = 1 - \beta\mathcal{H} + (\beta\mathcal{H})^2/2 - \ldots$ and evaluate the free energy up to second order in the magnetic field, using the fact that ($\vec{J}$ is an operator here)

(2) $$\mathrm{Tr}(J_\mu J_\nu) = \frac{1}{3}\delta_{\mu\nu}\mathrm{Tr}J^2.$$

Then, the high-temperature susceptibility can be extracted using the relation

(3) $$\chi = -\left.\frac{\partial^2 F_S}{\partial B^2}\right|_{B=0}.$$

Calculate the high-temperature susceptibility χ within this approximation for the effective Hamiltonian $\mathcal{H} = \sum_{i=1}^N g\mu_B B J_z^{(i)}$ with $\vec{B} = (0,0,B)$.

Aufgabe 2 (**Temperature-dependent correction to the Pauli susceptibility**) (5 Punkte)

Show that if T is small compared with the Fermi temperature, the temperature-dependent correction to the Pauli susceptibility $\chi_{\mathrm{Pauli}}(0) = 2\mu_B^2\rho_0(\varepsilon_F)$ is given by

(4) $$\chi_{\mathrm{Pauli}}(T) = \chi_{\mathrm{Pauli}}(0)\left(1 - \frac{\pi^2}{6}(k_B T)^2\left[\left(\frac{\rho_0'}{\rho_0}\right)^2 - \frac{\rho_0''}{\rho_0}\right]\right),$$

where $\rho_0 \equiv \rho_0(\varepsilon = \varepsilon_F)$, $\rho_0' \equiv \left.\frac{d\rho_0}{d\varepsilon}\right|_{\varepsilon=\varepsilon_F}$ and $\rho_0'' \equiv \left.\frac{d^2\rho_0}{d\varepsilon^2}\right|_{\varepsilon=\varepsilon_F}$. Show that for free electrons this reduces to

(5) $$\chi_{\mathrm{Pauli}}(T) = \chi_{\mathrm{Pauli}}(0)\left(1 - \frac{\pi^2}{12}\left(\frac{k_B T}{\varepsilon_F}\right)^2\right).$$

(*Hints:* The derivation starts from the general formula for the Pauli susceptibility

$$\chi_{Pauli} = -2\mu_B^2\int_{-\infty}^{\infty} d\epsilon\ \rho_0(\epsilon)\frac{df(\epsilon)}{d\epsilon},\ f(\epsilon) = \frac{1}{e^{\frac{\epsilon-\mu}{kT}}+1}$$

and then using (i) integration by parts, (ii) the Sommerfeld expansion and (iii) some other formulas from Sec. 5.8 of the 'Solid State Physics I' script.)