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Theorie zu Magnetismus, Supraleitung und Elektronische Korrelation in Festkörpern

Sommersemester 2019

Blatt 4

(Abgabe: 14.05.2019)

Aufgabe 1 (**Pauli Spin matrices**) (4 Punkte)

The Pauli matrices ($\vec{\sigma} = (\sigma_1\ \sigma_2\ \sigma_3)^t$) fulfil the simple identity

(1) $$(\vec{a} \cdot \vec{\sigma})(\vec{b} \cdot \vec{\sigma}) = \vec{a} \cdot \vec{b} + i(\vec{a} \times \vec{b}) \cdot \vec{\sigma}$$

if all components of $\vec{a}, \vec{b}$ commute with all components of $\vec{\sigma}$ for arbitrary operators $\vec{a}, \vec{b}$. In case that $[a_i, a_j] = 0, \forall i, j \in \{1, 2, 3\}$ we have $\vec{a} \times \vec{a} = \vec{0}$.
Since the components of the momentum operator $\vec{p}$ fulfils such a commutation relation we can write the Hamiltonian H of a spin-1/2 particle without external magnetic field as

$$H = \frac{\vec{p}^2}{2m} = \frac{\vec{p}^2}{2m} + i \cdot \vec{0} \cdot \vec{\sigma} \overset{(1)}{=} \frac{(\vec{p} \cdot \vec{\sigma})^2}{2m}.$$

In the presence of a magnetic field $\vec{H}$ the components of $\vec{p} + e\vec{A}/c$ do not commute with each other. Show that one can use (1) to derive the following expression for the Hamiltonian

$$H = \frac{1}{2m}\left(\vec{p} + \frac{e\vec{A}}{c}\right)^2 + \frac{e\hbar}{2mc}\sigma \cdot \vec{H} = \frac{1}{2m}\left[\left(\vec{p} + \frac{e\vec{A}}{c}\right) \cdot \vec{\sigma}\right]^2.$$

In this compact form not only contains the spin contribution but also the orbit contribution to the magnetic part of the Hamiltonian (assumption: $g_0 = 2$).

Aufgabe 2 (**Langevin theory of paramagnetism**) (6=2+2+2 Punkte)

Consider a system of N atoms with intrinsic magnetic moment μ. The Hamiltonian for this system with external magnetic field $\vec{B}$ is given as

$$H = H_0 - \mu B \sum_{i=1}^{N} \cos \alpha_i,$$

with H_0 being the Hamiltonian without external field and α_i is the angle between the external field $\vec{B}$ and the magnetic moment of the i-th atom.

Show that:

a) The induced magnetic moment is given by

(2) $$M = N\mu \left(\coth \theta - \frac{1}{\theta}\right),$$

with $\theta = \mu B / k_B T$.
Hint:

$$\coth x = 1/\tanh x$$

b) The magnetic susceptibility per atom is given by

$$\chi = \frac{\mu^2}{k_B T} \left(\frac{1}{\theta^2} - \frac{1}{\sinh^2 \theta} \right). \tag{3}$$

c) For high temperature T the susceptibility χ obeys the Curie law, i.e. $\chi \propto T^{-1}$. Find the corresponding Curie constant C which fulfills $\chi = C/T$ for high temperatures.
Hint:

$$\frac{1}{\sinh^2 x} \approx \frac{1}{x^2} - \frac{1}{3} \quad \text{for } |x| \ll 1$$