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Theorie zu Magnetismus, Supraleitung und Elektronische Korrelation in Festkörpern

Sommersemester 2019

Blatt 13

(Abgabe: 16.07.2019)

Note: All points on this sheet are for bonus points. You may earn up to 15 bonus points.

Aufgabe 1 (**Non-interacting Pauli Susceptibility**) (5=1+1+2+1 Punkte)

Consider a non-interacting Fermi gas at zero temperature ($T = 0$) in a uniform external magnetic field B. The Zeeman interaction can be written as:

(1)
$$\mathcal{H}_{\text{Zee}}(B) = \frac{g\mu_B B}{2} \sum_{\vec{k}} \left(c^\dagger_{\vec{k},\uparrow} c_{\vec{k},\uparrow} - c^\dagger_{\vec{k},\downarrow} c_{\vec{k},\downarrow} \right)$$

a) Assuming a chemical potential μ, describe the exact ground state of the non-interacting $S = 1/2$ fermions in the uniform magnetic field.

b) Write down exact expressions for the spin-dependent Green's functions $G^B_{\sigma,\sigma'}(\vec{k},\omega)$, where σ and σ' indicate the spin ($+1 =\uparrow$ or $-1 =\downarrow$).

c) The magnetization can be expressed as $M = B^{-1}\langle \mathcal{H}_{\text{Zee}} \rangle$. Find the magnetization of the ground state using the expressions obtained for $G^B_{\sigma,\sigma'}(\vec{k},\omega)$.

d) Calculate the Pauli susceptibility $\chi^0_{\text{Pauli}} = \lim_{B\to 0}(\partial M/\partial B)$ for the non-interacting gas at $T = 0$.

Aufgabe 2 (**Diagrams**) (5 Punkte)

Obtain the Green's functions in a magnetic field $G^B_{\sigma,\sigma'}$ for the previous question, but starting from the Green's functions without an external field ($B = 0$), denoted $G^0_{\sigma,\sigma'}$. Consider $\mathcal{H}_{\text{Zee}}$ as an interaction part, and construct the perturbation theory in magnetic field. For example, show that:

(2)
$$G^B_{\sigma,\sigma} = G^0_{\sigma,\sigma} + \left(\frac{g\mu_B B\sigma}{2}\right) \left(G^0_{\sigma,\sigma}\right)^2 + \left(\frac{g\mu_B B\sigma}{2}\right)^2 \left(G^0_{\sigma,\sigma}\right)^3 + ...$$

where $\sigma = \pm 1$. Create a diagrammatic notation for representing the terms in the perturbation series. Evaluate the perturbation sum exactly using your diagrammatic method, and show that the results are the same as the exact expressions from part b).
Hint: Consider a Geometric Series.

Aufgabe 3 (**Interacting Susceptibility**) (5 Punkte)

Consider an interacting Fermi gas at $T = 0$ in a uniform magnetic field B. In addition to the Zeeman interaction (1), there is also a time- and momentum-independent interaction:

(3)
$$V = \frac{U}{2} \sum_{\vec{k},\vec{k}',\vec{q}} c^\dagger_{\vec{k}+\vec{q},\uparrow}\, c^\dagger_{\vec{k}'-\vec{q},\downarrow}\, c_{\vec{k}',\downarrow}\, c_{\vec{k},\uparrow}$$

a) Calculate the Green's functions $G^B_{\sigma,\sigma'}(\vec{k},\omega)$ in the Hartree-Fock approximation. For this purpose, calculate the proper self-energy up to first order in U.

Hint: Remember your results from Problem Set 10.

b) Use your expressions for $G^B_{\sigma,\sigma'}(\vec{k},\omega)$ to write down coupled equations for the densities $n_\sigma = \sum_{\vec{k}}\langle c^\dagger_{\vec{k},\sigma} c_{\vec{k},\sigma}\rangle$ of spin-up and spin-down particles in the ground state.

c) Using $M = B^{-1}\langle \mathcal{H}_{\text{Zee}}\rangle$, show for small U:

$$\chi = \lim_{B\to 0}\left(\frac{\partial M}{\partial B}\right) \approx \frac{\chi^0_{\text{Pauli}}}{1 - \frac{2U}{(g\mu_B)^2}\chi^0_{\text{Pauli}}} \tag{4}$$

where χ^0_{Pauli} is the non-interacting susceptibility for the same particle density.

Hint: Consider $n_\sigma = n/2 + \Delta n_\sigma$, where the total particle number is $n_\uparrow + n_\downarrow = n$. In the limit of small U and B, you may assume $\mu \gg \Delta n_\sigma U$.