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Theorie zu Magnetismus, Supraleitung und Elektronische Korrelation in Festkörpern

Sommersemester 2019

Blatt 10

(Abgabe: 25.06.2019)

Aufgabe 1 (**London theory of superconductivity**) (4=3+1 Punkte)

We want to derive a (classical) phenomenological theory to explain the Meissner effect in superconductors. We assume that the superconducting charge carriers underlie the Newtonian equation of motion

$$m_e \frac{d^2}{dt^2}\vec{r} = e\vec{E},$$

where $\vec{E}$ is the applied electric field. Together with the superconducting carrier denisty n_s and the definition of the current density $\vec{j} = n_s e \frac{d}{dt}\vec{r}$ we derive

$$\frac{d}{dt}\vec{j} = \frac{n_s e}{m_e}\vec{E}.$$

i) We assume the magnetic field $\vec{B}$ to be zero at $t = -\infty$. Use Maxwells equation and neglect the second time derivative of the magnetic field $\partial_t^2 \vec{B}$ to derive the equation

$$\Delta\vec{B} = \frac{1}{\lambda_L}\vec{B},$$

where we definded the London penetration depth as $\lambda_L^{-2} = \frac{4\pi n_s e^2}{m_e c^2}$.

ii) Consider a superconducting slab in the $x-y-$plane, where the slab is located only on the positive x-axis. We apply a constant magnetic field along the y-direction, i.e. the magnetic field in the slab can only depend on x. Calculate the magnetic field within the slab.

Aufgabe 2 (**Complex Integration and the Residue Theorem**) (6=2+2+2 Punkte)

Calculate the following integrals by choosing a proper integration contour and using the residue theorem. Draw the contour path that you chose and show why some parts of the contour integral vanish.

$$\text{i)}\ \int_{-\infty}^{\infty} \frac{e^{-ias}}{s^2+2}\,ds, \quad a > 0$$

$$\text{ii)}\ \int_{0}^{\infty} \frac{1}{(s^2+a^2)^2}\,ds, \quad a > 0$$

$$\text{iii)}\ \int_{0}^{\infty} \frac{s^{1/2}}{s^2+3s+2}\,ds$$

Hints:

i) *The residue of a pole c of order n of a function f is given by*

$$Res(f,c) = \frac{1}{(n-1)!}\lim_{z\to c}\frac{d^{n-1}}{dz^{n-1}}\left[(z-c)^n f(z)\right]$$

ii) *Put the branch cut of the function $f(z) = z^{1/2} = \exp(1/2\log(z))$ onto the positive real axis.*