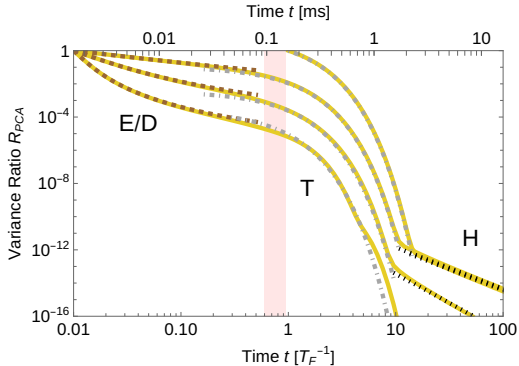
Lessons on the general form of hydrodynamic attractors from cold atoms and classical Yang-Mills theory

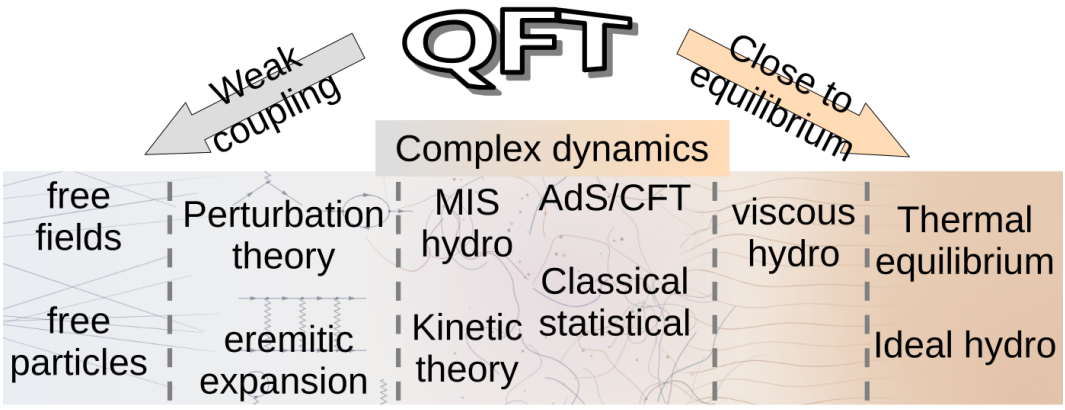
Clemens Werthmann
Ghent University

based on 2507.02838
and 2608.28106
see also: 2609.09114



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erc
European Research Council
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How to effectively describe QFT



Away from simple limits, even effective theories for QFT are still complicated.

Can one identify further organizing principles that simplify dynamics?

Attractors: convergence to universal behaviour $\Rightarrow$ Can this be exploited?

Overview

Sections of this talk:

1. Review of relevant aspects of the Bjorken flow attractor
2. Hydrodynamic attractor in ultracold atomic gases
3. Expansion attractor in classical Yang-Mills theory

Relation to planned Emmy Noether project

Effective descriptions of the early far-from-equilibrium stage in heavy ion collisions

high energies

Hydrodynamic attractors

- explore further generalizations
- construct simplified descriptions

low energies

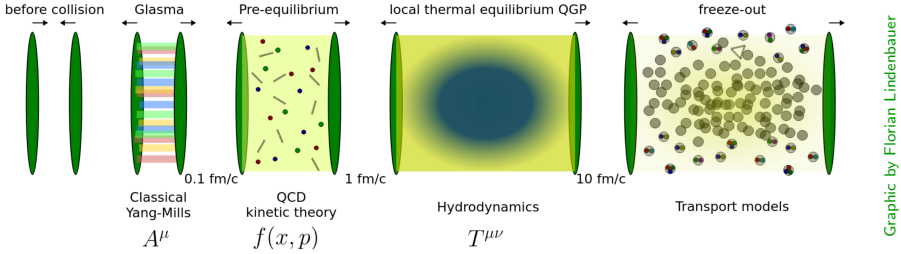
Multi-fluid dynamics

- derive equations from kinetic theory
  see e.g. CW, Iurii Karpenko, Pasi Huovinen, PRC 113 034908
- verify modeling assumptions

Hydrodynamic Attractor in Bjorken flow

High energy heavy ion collisions

High speed and contracted nuclei $\Rightarrow$ timescale separation of dynamical regimes:
each non-equilibrium effective theory applied in some regime (AdS/CFT: alternative)



Bjorken flow: simplified model of longitudinally boost-invariant and transversely homogeneous and isotropic system. Displays attractor behaviour!

What is a hydrodynamic attractor?

Convergence to universal time evolution long before equilibrium

prototypical example:

conformal Bjorken flow, in scaled time $\tilde{w} = \tau/\tau_{\rm int}$

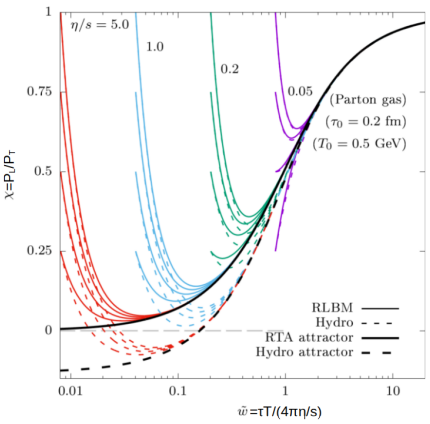
- pressure anisotropy P_L/P_T
- energy density $\epsilon\tau^{4/3}/\lim_{\tau\to\infty}\epsilon\tau^{4/3}$
- moment ratios to equilibrium $\mathcal{M}_{mn}/\mathcal{M}_{mn}^{(\rm eq)}$

mechanism:

information loss through

- equilibration at late times
- expansion at early times

... and lost information *stays* lost

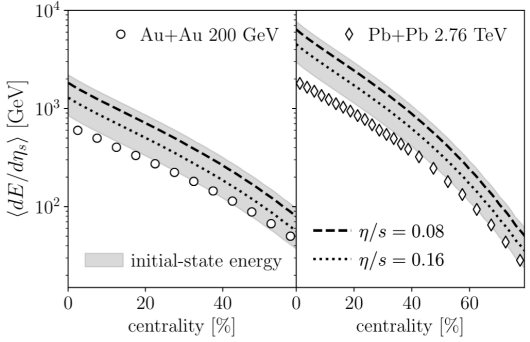


Victor Ambrus, Lorenzo Bazzanini, Alessandro Gabbana, Daniele Simeoni, Raffaele Tripiccione, Sauro Succi, Nature Computat.Sci. 2 (2022) 641-654

Practical applications in phenomenology

- even with transverse profile, at early times "$\tau \ll \langle \nabla_\perp \rangle^{-1}$", transverse dynamics can be neglected $\rightarrow$ *local* Bjorken flow attractor
- final state Multiplicity $\sim$ Entropy $\rightarrow$ attractor links multiplicity to initial energy!
  Giuliano Giacalone, Aleksas Mazeliauskas, Sören Schlichting, PRL 123 (2019) 262301

$$\frac{\mathrm{d}N_{\mathrm{ch}}}{\mathrm{d}\eta} = \frac{4}{3}\frac{N_{\mathrm{ch}}}{S} C_\infty^{\frac{3}{4}} \left(4\pi\frac{\eta}{s}\right)^{\frac{1}{3}} \left(\frac{\pi^2}{30}\nu_{\mathrm{eff}}\right)^{\frac{1}{3}} \int \mathrm{d}^2\mathbf{x}\, [e(\mathbf{x})\tau]_0^{\frac{2}{3}}$$

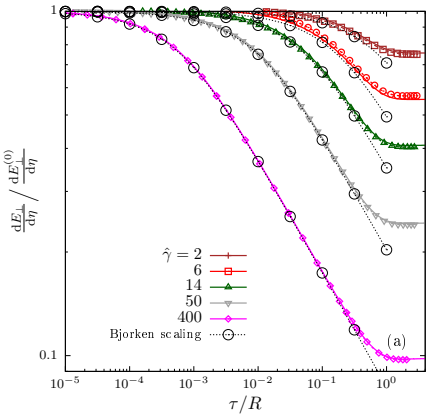


Practical applications in phenomenology

Can also predict early time evolution of

- transverse energy $\mathrm{d}E_\perp/\mathrm{d}\eta$
- flow velocity u^μ
- eccentricity ϵ_n

Victor Ambruș, Sören Schlichting, CW, PRD 105 (2022) 014031 and PRD 107 (2023) 094013



beyond that:

- linear perturbations to attractor: "KøMPøST"
  Kurkela, Mazeliauskas, Paquet, Schlichting, Teaney, PRL 122 (2019) 122302
- extra ideal d.o.f. in hydro: "Attractodynamics"
  Du Plessis, Scheihing-Hitschfeld, Steinhorst, 2607.16393

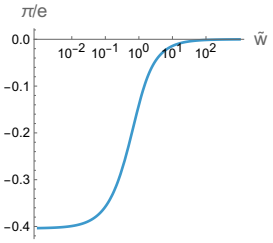
Early and late time attractors

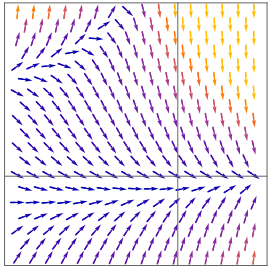
Attractor curve transitions between early and late time "fixed points", which reflect two different mechanisms
Jean-Paul Blaizot, Li Yan, Annals Phys. 412 (2020) 167993 and PRC 104 (2021) 5, 055201

in the hydrodynamic evolution of shear $\pi = (P_L - P)$, different terms dominate at different times:

Late time interaction domination $\partial_\tau(\pi/e) = -\tau_{\rm int}^{-1}(\pi/e)$
$\Rightarrow$ non-equilibrium quantity π/e decays away, no special curve

Early time expansion domination
$\partial_\tau(\pi/e) = \tau^{-1}[(\pi/e)^2 + \lambda(\pi/e) - 64/15]$
$\Rightarrow$ two fixed points $(\pi/e)_\pm = \frac{\lambda}{2} \pm \sqrt{\frac{\lambda^2}{4} + \frac{64}{15}}$
$(\pi/e)_+$ is repulsive, $(\pi/e)_-$ is attractive
$\Rightarrow$ **uniquely defines starting point of the attractor curve!**





Complication in the non-conformal case

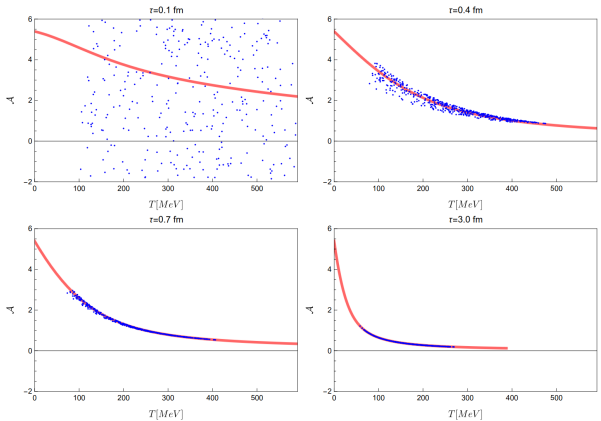
MIS hydro with non-conformal eos

Michał Spaliński, arXiv: 2508.21039

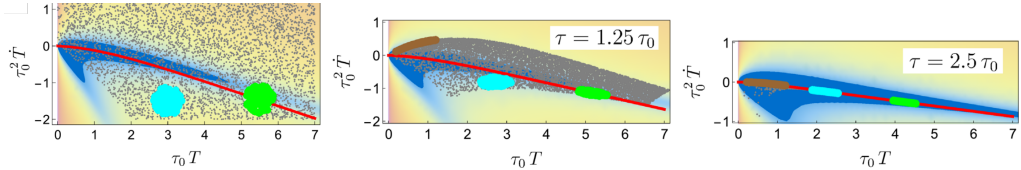
Evolution equations no longer decouple in terms of $\tilde{w} = \tau/\tau_{\rm int}$, collapse not evident in single observables, but in $(\pi(\tau), T(\tau))$ $\Rightarrow$ 2D attractor surface

special case of conformality

all interaction timescales are $\propto T^{-1}$ $\Rightarrow$ in state space (including τ), any direction orthogonal to $T\tau$ is trivial!



Dimensional reduction

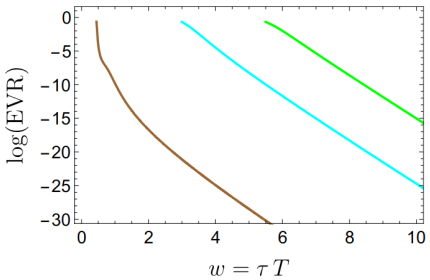


Michał P. Heller, Ro Jefferson, Michał Spaliński, Viktor Svensson, PRL 125 (2020) 13, 132301

Diagnose attractor behaviour in state space:

- track directional contraction of local ensembles in state space (π, T)
- at each time, perform "principal component analysis": construct covariance matrix Σ of the ensemble spread in state space and diagonalize

$\Rightarrow$ directional variance components: decay indicates loss of sensitivity to that state space direction!



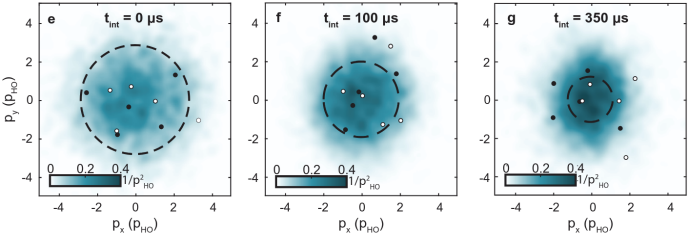
Hydrodynamic attractor in ultracold atomic gases

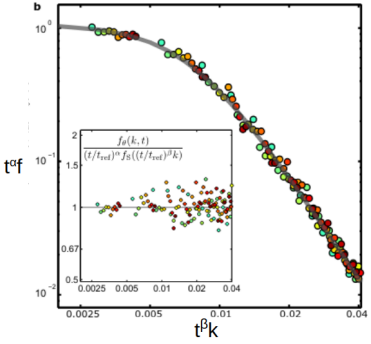
2507.02838

Cold atoms as “Quantum Many-Body Simulators”

Ultracold atomic gas:

- trapped cloud of 10–10^7 atoms at $T \sim 10\,\mathrm{nK} \Rightarrow$ large quantum effects
- high degree of control over interactions, can simulate phenomena in other systems, e.g. in heavy ion collisions:
  - non-thermal fixed point (bottom up thermalization)
    $f(k,t) = t^{\alpha} f_s(t^{\beta} k)$
    Maximilian Prüfer et al., Nature 563 (2018) 7730, 217
  - elliptic flow, $N \sim 10$
    Sandra Brandstetter et al., Nature Phys. 21 (2025) 1, 52



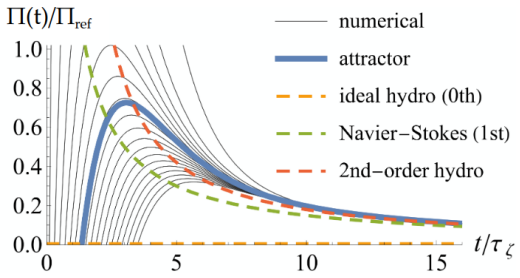


Cold atom attractor: prior works

recent discovery: hydrodynamic attractor also replicated in ultracold quantum gases

Keisuke Fujii, Tilman Enss, PRL 133 (2024) 173402

- drive via external magnetic field modulating spin contact interaction
- competition of excitation and relaxation of bulk pressure Π



Cold atom attractor: prior works

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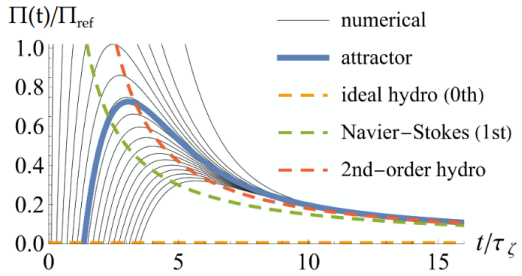
Keisuke Fujii, Tilman Enss, PRL 133 (2024) 173402

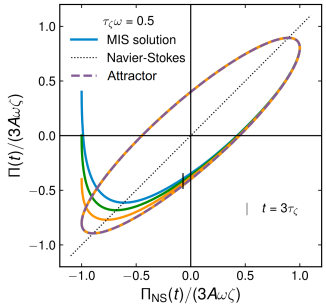
- drive via external magnetic field modulating spin contact interaction
- competition of excitation and relaxation of bulk pressure Π

cold atom attractor in periodic drive

Aleksas Mazeliauskas, Tilman Enss, PRL 136 (2026) 10

- relaxation makes system lag behind Navier-Stokes description, which instantaneously follows drive
- precision study of relaxation dynamics at given ω





Distinctions of attractors

Bjorken flow	cold atoms
ultrarelativistic	nonrelativistic
longitudinal expansion	mimicked isotropic contraction
evolution of shear	evolution of bulk pressure
experimentally inaccessible	**directly accessible in experiment**
expansion driven part	**?**
early time fixed point	**?**

Our questions

Is there a distinct expansion dominated memory loss mechanism**?**

Does it drive the system to a specific constant state**?**

Setup of the attractor in cold atoms

- pointlike interaction between "↑" and "↓", rate quantified by "contact density"
  Shina Tan, Ann.Phys. 323 (2008) 2952 and 2971

$$C = \lim_{k\to\infty} k^4 n_\sigma(k)$$

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  small $|a^{-1}|$: change $\partial_t a^{-1}$ acts like isotropic expansion/contraction
  Keisuke Fujii, Yusuke Nishida, PRA 98 (2018) 063634
  $$\Pi = -3\zeta\, a\partial_t a^{-1} \qquad C = C_{\rm eq} + 4\pi m a \Pi$$
  bulk viscosity $\zeta \overset{a\to\infty}{\sim} a^{-2}$ (simple time dependence: power law drive $a^{-1}(t) \sim t^{-\alpha}$)

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  bulk viscosity $\zeta \overset{a\to\infty}{\sim} a^{-2}$ (simple time dependence: power law drive $a^{-1}(t) \sim t^{-\alpha}$)
- numerical observation: low frequency bulk viscosity is of Drude form
  Tilmann Enss, PRL 123 (2019) 205301
  $$\zeta(\omega) = \frac{i\chi}{\omega + i\tau_\Pi^{-1}} \quad (\zeta = \chi\tau_\Pi)$$
  $\Rightarrow$ promote Π-equation to MIS type
  Keisuke Fujii, Tilman Enss, PRL 133 (2024) 173402
  $$\tau_\Pi \dot{\Pi} + \Pi = -3\zeta\, a\partial_t a^{-1}$$

Early time behaviour of bulk pressure

Are there early and late time attractors in $\tau_\Pi \dot{\Pi} = -\Pi - 3\zeta\ a^{-1}\partial_t a$, $\zeta = \zeta^{(2)} a^{-2}$?

Distinction via separation of scales:

"Expansion domination" $\hat{=}$ drive source term dominates over relaxation term

"Interaction domination" $\hat{=}$ vice versa

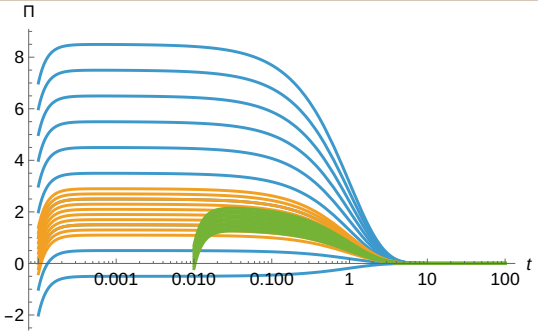
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"Interaction domination" $\hat{=}$ vice versa



$\Rightarrow$ starting power law drive of $a(t)$ at $t_0 \ll \tau_\Pi$:

1. $\dot{\Pi} = -3\tau_\Pi^{-1}\zeta\, a^{-1}\partial_t a$ only builds up a fixed amount of bulk pressure Π!
2. $\dot{\Pi} = -\tau_\Pi^{-1}\Pi$ causes decay to zero $\Rightarrow$ trivial attractor

Nontrivial late time attractor: competition of the two effects; here only for very small Π

How to get early time attractor?

idea: like in Bjorken case, consider ratio $\Pi/\mathcal{E}$: effect of expansion on energy will introduce additional phenomenology

evolution of energy density contains a state-dependent drive term

Shina Tan, AoP 323 (2008) 2971

$$\partial_t \mathcal{E} = \frac{C}{4\pi m}\partial_t a^{-1} = -3\Pi\ a\partial_t a^{-1} + \frac{C_{\rm eq}}{4\pi m}\partial_t a^{-1}$$

$\partial_t(\Pi/\mathcal{E})$ inherits this term: $(\chi = \tau_\Pi^{-1}\zeta)$

$$\partial_t(\Pi/\mathcal{E}) = \Big[3(\Pi/\mathcal{E})^2 - 3(\chi/\mathcal{E})\Big]\ a\partial_t a^{-1} - \tau_\Pi^{-1}(\Pi/\mathcal{E})$$

$\Rightarrow$ if $\chi/\mathcal{E} \sim$ const., the two drive terms have an analogous form to the Bjorken case!

This equation has early time attractor and repulsor at

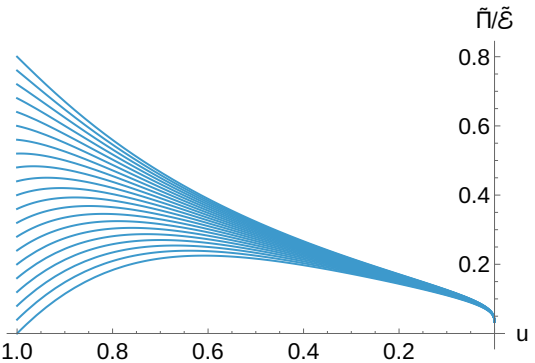
$$\Pi/\mathcal{E} = \pm\sqrt{\chi/\mathcal{E}}$$

Simplified evolution in expansion dominated case

Early attractor behaviour independent of form of $a^{-1}(t)$:
dropping relaxation term $\propto \tau_\Pi^{-1}$, can substitute $u = a^{-1}(t)$, $X(t) = \tilde{X}[u(t)]$

$$\partial_u(\tilde{\Pi}/\tilde{\mathcal{E}}) = 3(\tilde{\Pi}/\tilde{\mathcal{E}})^2 u^{-1} - (\chi a^2/\tilde{\mathcal{E}})\ u$$

$\Rightarrow$ evolution depends only on range covered in a^{-1}



1st caveat: no early time fixed point

calculations show (close to unitarity): $\chi \sim a^{-2}$

K.Dusling, T.Schäfer, PRL 111 (2013) 120603 T.Enss, PRL 123 (2019) 205301

$$\partial_t(\Pi/\mathcal{E}) = \left[3(\Pi/\mathcal{E})^2 a - (\chi a^2/\mathcal{E})\ a^{-1}\right] \partial_t a^{-1} - \tau_\Pi^{-1}(\Pi/\mathcal{E})$$

$\Rightarrow$ terms compete if $a \sim \mathcal{O}(1-10)$, but **no "fixed point"**: $\partial_t(\Pi/\mathcal{E}) \neq 0$,

i.e. no special attractor curve

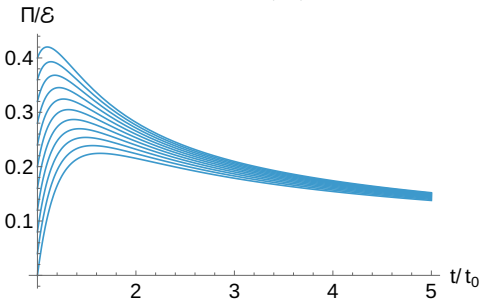
2nd caveat: no attractor in single observable

temperature dependence of χ introduces nontrivial coupling of two equations

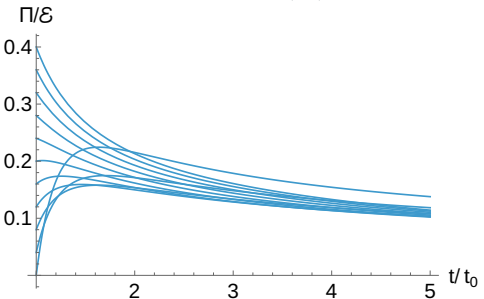
$$\partial_t(\Pi/\mathcal{E}) = \left[3(\Pi/\mathcal{E})^2 a - \chi(a,\mathcal{E})a/\mathcal{E}\right]\partial_t a^{-1} - \tau_\Pi^{-1}(\Pi/e)$$

$$\partial_t \mathcal{E} = -3\Pi a \partial_t a^{-1} + C_{\text{eq}}\text{-term}$$

same $\mathcal{E}(t_0)$



different $\mathcal{E}(t_0)$

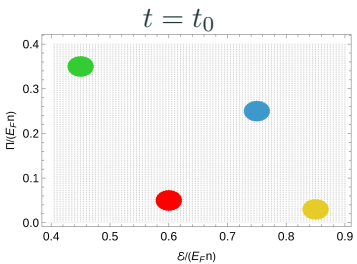


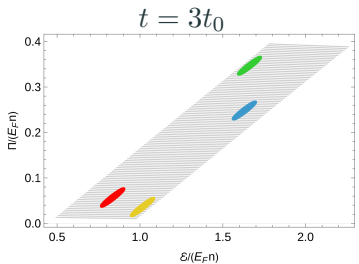
even for same $\mathcal{E}(t_0)$, different $\partial_t\mathcal{E}$ blocks collapse $\Rightarrow$ attraction only in $(\Pi, \mathcal{E}, t)$-space!

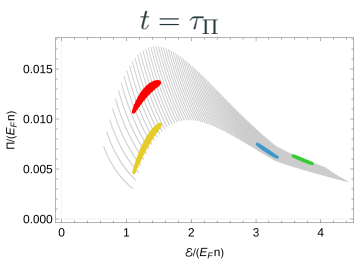
Attractor in state space

Ensembles in 2D state space dynamically contract towards 1D subset!

<u>But:</u> orientation of contracting direction depends on position and time







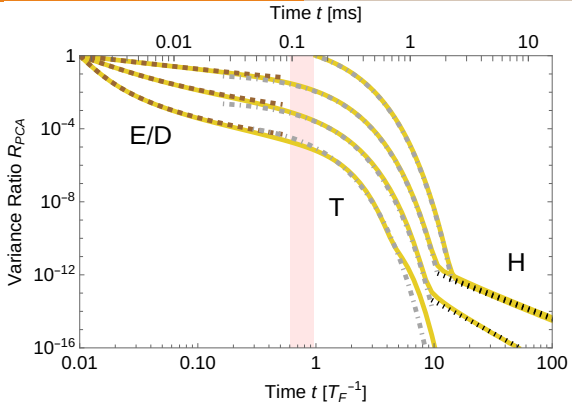
Principal component variance ratio

recover all expected behaviour (squared because of variance):

E/D) expansion driven convergence, same as in absence of interaction

T) thermalization driven convergence $\sim [\exp(-t/\tau_{\Pi})]^2$

H) hydrodynamic Navier-Stokes tail $\sim [a^{-1}(t)\partial_t a^{-1}(t)]^2$



By measuring $(\mathcal{E}, \Pi)$ of a local ensemble for several evolution times, this diagnostic can be employed in experiment!

Experimental techniques for fast measurement have been developed recently

Kevin G.S. Xie et al., PRL 136 (2026) 083402

Expansion attractor in classical Yang-Mills theory

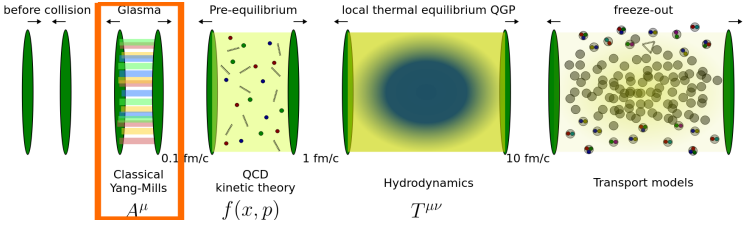
2608.28106

Generalize attractor to non-equilibrating interaction

Central questions of this project

What does attractor behaviour look like in a theory without a late time equilibrium fixed point? Can information loss be reversed?

Fitting dynamical description, related to heavy ion collisions: classical Yang-Mills!



Graphic by Florian Lindenbauer

Field strength tensor $F^{\mu\nu} = \nabla^\mu A^\nu - \nabla^\mu A^\nu - ig[A^\mu, A^\nu]$

evolves according to $D_\mu F^{\mu\nu} = \nabla_\mu F^{\mu\nu} - ig[A_\mu, F^{\mu\nu}] = 0$

Simplifying assumptions

Working in $SU(2) \Rightarrow$ YM fields have 3 colors

No ensemble, but *single* classical trajectory: strong potentials instead of particles

Bjorken-like symmetries: $A^\mu_a(\tau, x, y, \eta) = A^\mu_a(\tau)$

- longitudinal boost invariance
- transverse homogeneity
- **no** transverse isotropy
  (renders dynamics trivial)

What to expect?

- no particle-like excitations $\Rightarrow$ no nonthermal fixed point
  J. Berges, K. Boguslavski, S. Schlichting, R. Venugopalan, PRD 89, 074011
- no transverse dependence $\Rightarrow$ no Nielsen-Olesen instabilities
  N. K. Nielsen, P. Olesen, Nucl. Phys. B 144, 376
- information loss from strong expansion

Evolution and Constraints

A priori 24 d.o.f. A_a^μ, $\partial_\tau A_a^\nu$

Gauge fixing:

- temporal gauge $A^\tau = 0$ (removes 6)
- fix global gauge (removes 3)

Evolution and Constraints

A priori 24 d.o.f. A^μ_a, $\partial_\tau A^\nu_a$

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- fix global gauge (removes 3)

Evolution equations:

$$0 = f^{bc}_a(A^i_b \partial_\tau A^i_c + \tau^2 A^\eta_b \partial_\tau A^\eta_c) = C_a$$

$$0 = \partial^2_\tau A^1_a + \tau^{-1}\partial_\tau A^1_a + g^2 f^{bc}_a f^{de}_c A_{b,\mu} A^\mu_d A^1_e$$

$$0 = \partial^2_\tau A^2_a + \tau^{-1}\partial_\tau A^2_a + g^2 f^{bc}_a f^{de}_c A_{b,\mu} A^\mu_d A^2_e$$

$$0 = \partial^2_\tau A^\eta_a + 3\tau^{-1}\partial_\tau A^\eta_a + g^2 f^{bc}_a f^{de}_c A_{b,\mu} A^\mu_d A^\eta_e$$

Too many dynamical equations? No: if $C_a(t_0) = 0$, then it stays 0!

$\Rightarrow$ first equation constrains state space (removes 3)

The system has 12 physical d.o.f.

Evolution and Constraints

A priori 24 d.o.f. A_a^μ, $\partial_\tau A_a^\nu$

Gauge fixing:

- temporal gauge $A^\tau = 0$ (removes 6)
- fix global gauge (removes 3)

Field strength tensor:

$$F_a^{\tau i} = \partial_\tau A_a^i + 2\delta_\eta^i \tau^{-1} A_a^\eta \qquad \text{(abelian)}$$

$$F_a^{jk} = g f_a^{bc} A_b^j A_c^k \qquad \text{(non-abelian)}$$

electric: $E_a^i = F_a^{\tau i}, \quad E_{L,a} = \tau F_a^{\tau\eta}$,

magnetic: $B_a^i = \tau F_a^{\bar{i}\eta}, \quad B_{L,a} = F^{xy}$

Evolution equations:

$$0 = f_a^{bc}(A_b^i \partial_\tau A_c^i + \tau^2 A_b^\eta \partial_\tau A_c^\eta) = C_a$$

$$0 = \partial_\tau^2 A_a^1 + \tau^{-1}\partial_\tau A_a^1 + g^2 f_a^{bc} f_c^{de} A_{b,\mu} A_d^\mu A_e^1$$

$$0 = \partial_\tau^2 A_a^2 + \tau^{-1}\partial_\tau A_a^2 + g^2 f_a^{bc} f_c^{de} A_{b,\mu} A_d^\mu A_e^2$$

$$0 = \partial_\tau^2 A_a^\eta + 3\tau^{-1}\partial_\tau A_a^\eta + g^2 f_a^{bc} f_c^{de} A_{b,\mu} A_d^\mu A_e^\eta$$

Too many dynamical equations? No: if $C_a(t_0) = 0$, then it stays 0!

$\Rightarrow$ first equation constrains state space (removes 3)

The system has 12 physical d.o.f.

Expansion dominated regime at early times

Consider the case where τ^{-1}-terms dominate over g^2-terms: each field reacts to expansion on its own!

$$0 = \partial_\tau^2 A_a^i + \tau^{-1}\partial_\tau A_a^i$$
$$0 = \partial_\tau^2 A_a^\eta + 3\tau^{-1}\partial_\tau A_a^\eta$$

$$E_a^i = \partial_\tau A_a^i(\tau_0)\frac{\tau_0}{\tau} \qquad E_{L,a} = 2A_a^\eta(\tau_0) + \tau_0\partial_\tau A_a^\eta(\tau_0)$$

$$B_a^i = g\left[A_a^{\bar{i}}(\tau_0) - \tau_0\partial_\tau A_a^{\bar{i}}(\tau_0)\ln\left(\frac{\tau}{\tau_0}\right)\right]\left\{\tau A_a^\eta(\tau_0) - \frac{1}{2}\partial_\tau A_a^\eta(\tau_0)\left[\frac{\tau_0^3}{\tau} - \tau_0\tau\right]\right\}$$

$$B_{L,a} = g\left[A_a^x(\tau_0) - \tau_0\partial_\tau A_a^x(\tau_0)\ln\left(\frac{\tau}{\tau_0}\right)\right]\left[A_a^y(\tau_0) - \tau_0\partial_\tau A_a^y(\tau_0)\ln\left(\frac{\tau}{\tau_0}\right)\right]$$

- longitudinal fields (mostly) constant
- transverse fields $\propto \tau_0/\tau$
- transv. magn. fields transition to $\propto \tau$

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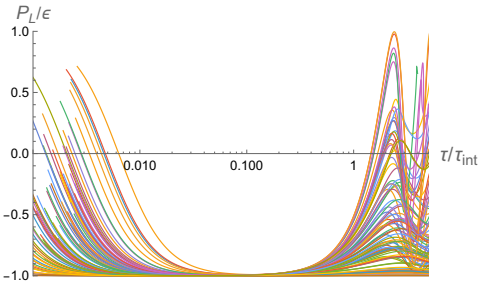
- longitudinal fields (mostly) constant
- transverse fields $\propto \tau_0/\tau$
- transv. magn. fields transition to $\propto \tau$

$$e = (E^2 + B^2)/2,$$
$$P_L = (E_T^2 + B_T^2 - E_L^2 - B_L^2)/2$$

- $E_T^2 + B_T^2 \gg E_L^2 + B_L^2$: $P_L/\epsilon = 1$
- $E_T^2 + B_T^2 \ll E_L^2 + B_L^2$: $P_L/\epsilon = -1$

Pressure anisotropy

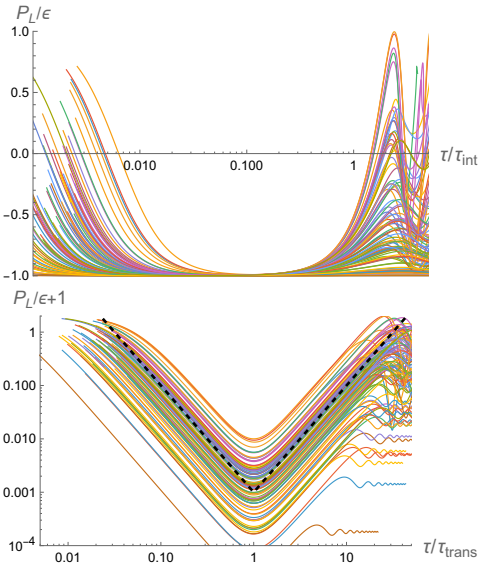
- $P_L/\epsilon \to -1$ at early times
- interaction terms are of the form $M^e_a A^i_e \Rightarrow$ interaction timescale $\tau_{\rm int}$ is inverse square root of largest eigenvalue of $M^e_a = g^2 f^{bc}_a f^{de}_c A_{b,\mu} A^\mu_d$
- $\tau \gtrsim \tau_{\rm int}$: oscillations



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- $P_L/\epsilon + 1$ goes from $\propto \tau^{-2}$ to $\propto \tau^2$
- transition timescale $\tau_{\rm trans} = \left(\frac{\tau_0^2 \sum_{i=x,y} \partial_\tau A^i_a \partial_\tau A^{i,a}(\tau_0)}{g^2 \sum_{i=x,y} f^{bc}_a A^i_b A^\eta_c f^{ade} A^i_d A^\eta_e(\tau_0)} \right)^{1/4}$
  $\Rightarrow$ **three** dynamical regimes!
  but also P_L/ϵ isn't everything!

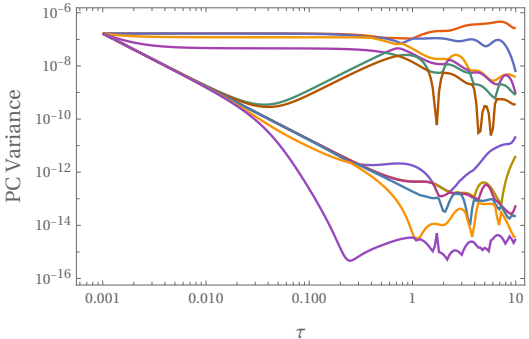


Dimensional Reduction in classical Yang-Mills theory

Consider spherical ensemble around simple initial condition for easier interpretation:

$$B_{L,3} = B^x_{T,1} = B^x_{T,2} = B^y_{T,2} = 1\,, \quad \text{others} = 0$$

- constant variances, $\propto \tau^{-2}$ variances splitting into $\propto \tau^2$ and continued $\propto \tau^{-2}$
- late time oscillations scramble information in state space, no change in hierarchy



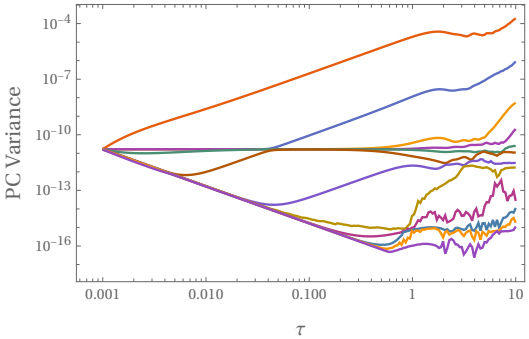
Dimensional Reduction in classical Yang-Mills theory

Now: spherical ensemble around a state with particularly small final P_L/P_T:

mostly same behaviour **but:** quadratic growth in one field already at initial time, another follows soon

$\Rightarrow$ macroscopic state defined fully by a few fields

$\Rightarrow$ initial state dependence in dimensional reduction



Lessons for general attractor behaviour

Knowledge of attractor behaviour leads to simplified descriptions of otherwise complex non-equilibrium dynamics (e.g. early time evolution in heavy ion collisions)

But general attractor lacks some simplifications of conformal Bjorken flow:

1. no collapse in single observables
2. no fixed points
3. partially reversible memory loss

problem that already exists in Bjorken flow
(especially in AdS/CFT W. van der Schee et al., PRL 124 (2020) 102301):

4. extent of memory loss is state dependent

A potential general effective description of attractor dynamics needs to account for these complications!