Inclusive and effective bulk viscosities of a multicomponent hadron gas

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Transport Meeting
ITP, Goethe University Frankfurt
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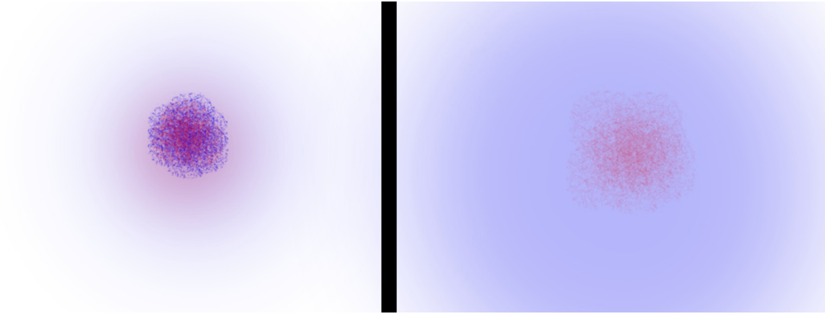
Outline

- **Motivation**
- **Bulk viscosity via Green-Kubo formula**
- **Box calculation in SMASH**
- **Performance and calibration**
- **Inclusive versus Effective bulk viscosity**
- **Results**
- **Conclusions**

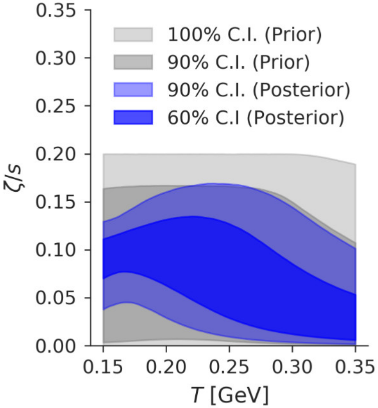
Efficient transport?

Consider a binary mixture of strongly and weakly interacting particles after a non-equilibrium disturbance e.g. a concentration gradient

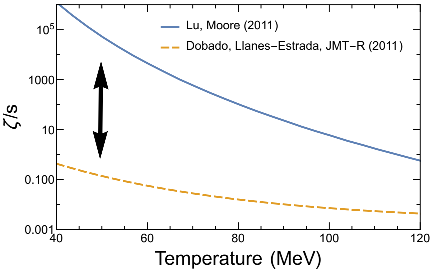


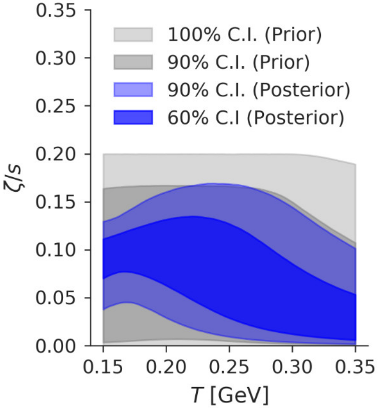
After some time, the blue component has relaxed. If density of red component is tiny, can we conclude that the system has (effectively) equilibrated?

Motivation for ζ



D. Everett *et al.* (JETSCAPE Coll.) arXiv:2011.01430 [Bayesian Analysis of RHICs]





Pion Gas

Lu, Moore (2011) [ChPT inelastic]

Dobado, Llanes-Estrada, JMT-R (2011) [UChPT elastic]

D. Everett *et al.* (JETSCAPE Coll.) arXiv:2011.01430 [Bayesian Analysis of RHICs]

Bulk viscosity

First order hydrodynamics:

$$T^{\mu\nu} = \epsilon u^\mu u^\nu + P h^{\mu\nu} + \pi^{\mu\nu} + \Pi h^{\mu\nu}$$

$$h^{\mu\nu} = u^\mu u^\nu - g^{\mu\nu} \quad , \quad \Delta^\mu = -h^{\mu\nu}\partial_\nu$$

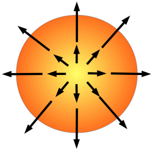
Shear and bulk viscosities

$$\pi^{\mu\nu} = \eta \left(\Delta^\mu u^\nu + \Delta^\nu u^\mu + \frac{2}{3} h^{\mu\nu} \partial_\lambda u^\lambda \right) \quad , \quad \Pi = -\zeta \partial_\lambda u^\lambda$$

Local rest frame

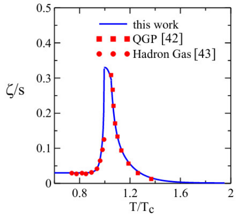
$$\frac{T^{ii}}{3} = P - \zeta \nabla \cdot \mathbf{V}$$

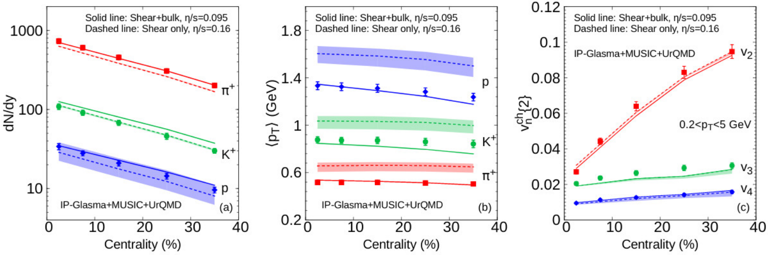
Total pressure ($T^{ii}/3$) is reduced wrt equilibrium pressure (P) under isotropic expansion



$\nabla \cdot \mathbf{V} > 0$

ζ/s is a parameter in hydrodynamic models simulating heavy-ion collisions





S. Ryu *et al.* Phys. Rev. Lett. 115, 132301 (2015)

Fluctuating hydrodynamics

Alternative approach to Green-Kubo: theory of hydrodynamic fluctuations

Pressure fluctuations
[Landau, Lifschitz (1987); Dobado, Llanes-Estrada, JMT-R (2011); Kapusta, Müller, Stephanov (2011)]

$$\Delta P(x) = -\zeta \nabla \cdot \mathbf{V}(x) + \xi(x)$$

where ΔP is local fluctuation wrt equilibrium pressure

$$\langle \xi(x) \rangle = 0 \quad , \quad \langle \xi(x_1)\xi(x_2) \rangle = 2T\zeta\delta^{(4)}(x_1 - x_2)$$

Without external compression/expansion

$$\zeta = \frac{1}{T} \int dt \int d^3x \, \langle \Delta P(t, \mathbf{x}) \Delta P(0, \mathbf{0}) \rangle$$

which coincides with the Green-Kubo formula.

Green-Kubo formula

Bulk viscosity
[Green (1954); Kubo (1957)...]

$$\zeta = \frac{V}{T} \int_0^\infty dt \, \langle \Delta\Pi(0) \Delta\Pi(t) \rangle \quad ; \quad \Delta\Pi(t) = \Pi(t) - \langle \Pi(t) \rangle$$

Bulk source
[Mori (1962); Luttinger (1964); Zwanzig (1965); Zubarev (1974)...]

$$\Delta\Pi(t) \equiv \Delta P(t) - \left(\frac{\partial P}{\partial \epsilon} \right)_n \Delta\epsilon(t) - \left(\frac{\partial P}{\partial n} \right)_\epsilon \Delta n(t)$$

$$P(t) = T^{ii}(t)/3 \quad , \quad \epsilon(t) = T^{00}(t) \quad , \quad n(t) = j^0(t)$$

Conservation of energy and particle number in V is **NOT** imposed

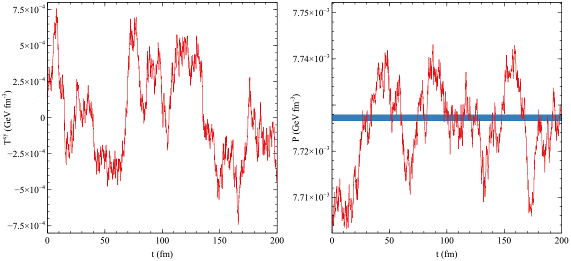
Averaged hydrodynamic fields

$$T^{\mu\nu}(t) = \frac{1}{V} \int d\mathbf{x} \; T^{\mu\nu}(t, \mathbf{x}) \, , \quad j^\mu(t) = \frac{1}{V} \int d\mathbf{x} \; j^\mu(t, \mathbf{x}) \, .$$

Energy-momentum tensor for a discrete system

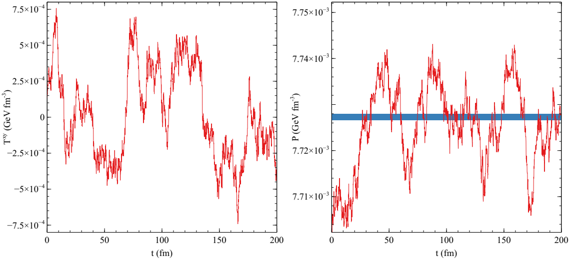
$$T^{\mu\nu}(t) = \frac{1}{V}\sum_{i=1}^{N} \frac{p_i^{\mu}(t)p_i^{\nu}(t)}{p_i^0(t)}$$

T^{xy} (GeV fm^{-3})



$T^{ii}/3$ (GeV fm^{-3})

t (fm)



- Strength of fluctuations (variance) $\sim \langle \Delta\Pi(0)^2 \rangle \equiv C_\zeta(0)$
- Tiny signal: $C_\zeta(0) \simeq C_\eta(0)/1000$

Bulk correlation function at $t = 0$

$$C_\zeta(0)V = \int \frac{d^3p}{(2\pi)^3} f^{eq}(p) \frac{1}{E_p^2} \left[\frac{p^2}{3} - E_p^2 \left(\frac{\partial P}{\partial \epsilon} \right)_n - E_p \left(\frac{\partial P}{\partial n} \right)_\epsilon \right]^2$$

$$f^{eq}(p) = g \exp[-(E_p - \mu)/T]$$

Correlation function

Bulk correlation function

$$C_\zeta(t) \equiv \langle \Delta\Pi(0)\Delta\Pi(t)\rangle$$

Exponential ansatz

$$C_\zeta(t) = C_\zeta(0) \exp\left(-\frac{t}{\tau_\zeta}\right)$$

with bulk relaxation time τ_ζ.

Bulk viscosity

$$\zeta = \frac{V}{T}\int_0^\infty dt\; C_\zeta(t) = \frac{C_\zeta(0) V \tau_\zeta}{T}$$

Entropy density with $\mu = 0$

$$s = (P + \epsilon)/T$$

SMASH transport code

J. Weil et al, PRC 94 (2016),
DOI: 10.5281/zenodo.3484711

- Simulating Many Accelerated Strongly-interacting Hadrons
- https://smash-transport.github.io
- Hadronic transport code: suitable for low-energy heavy-ion collisions (GSI-FAIR energies) and late, dilute stages of high-energy HICs
- Effectively solves Boltzmann equation: $p_\mu \partial^\mu_x f_i + F^i_\mu(x)\partial^\mu_p f_i = C[f_i, f_j]$
- Hadrons listed by PDG up to mass of $\sim$ 2 GeV

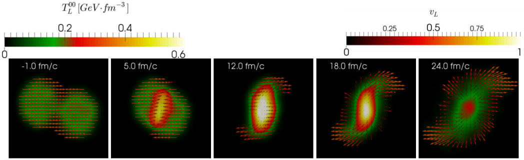


FIG. 24: Landau rest frame energy density T_L^{00} (background color) and velocity of Landau frame (arrows), both for baryons. Au+Au collision at $E_{\rm kin} = 0.8A$ GeV with impact parameter $b = 3$ fm, $N_{\rm test} = 20$. Color legends are given above.

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J. Weil et al, PRC 94 (2016),

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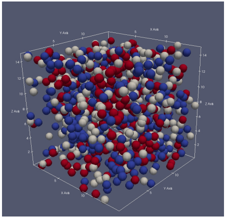


We use a static box setup with volume V. Requirements:

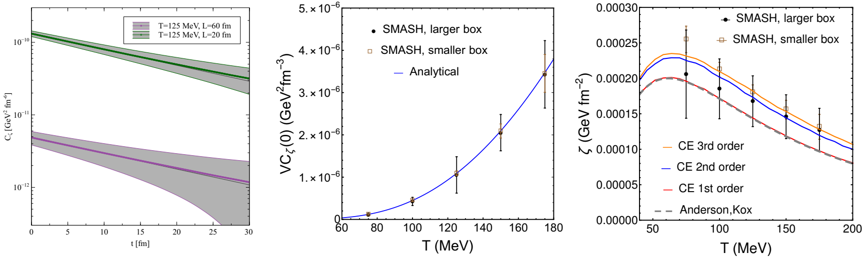
- Chemical equilibrium: fixed multiplicities
- Thermal equilibrium: well-defined temperature

We take V to be the entire box:

- Con: Smaller signal, as fluctuations $\propto 1/\sqrt{V}$
- Pro: Larger number of particles
- **Pro:** $\Delta\epsilon(t) = 0$, $\Delta n(t) = 0$ (for binary collisions)

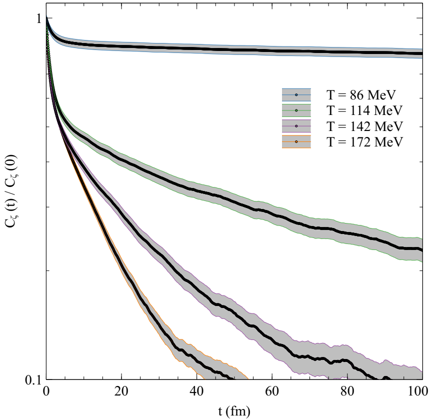
Relativistic hard-sphere gas

$T = 125$ MeV



- Hard-sphere gas: $m = 138$ MeV, $g = 3$, $\sigma = 20$ mb
- Comparison with Chapman-Enskog expansion of the Boltzmann equation [Anderson, Kox (1977); JMT-R (2011)]
- Lowest temperature achievable $T \sim 80$ MeV

We apply the method to the full hadron gas, neglecting fluctuations in particle number (approximation!).

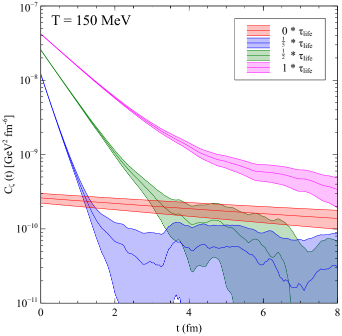


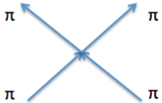
- Clear temperature dependence
- Error bars are relatively small
- **Correlation functions do not look like exponentials!**

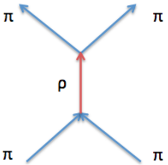
Let us analyze simpler systems...

Resonant system: $\pi + \rho$

Gas of pions interacting via ρ mesons, with lifetime tuned at will





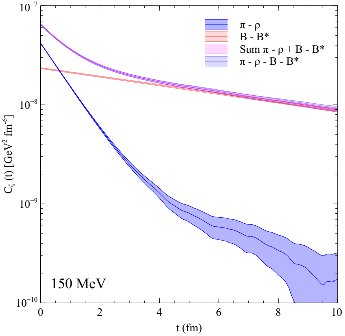


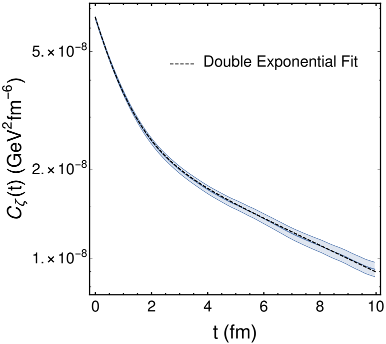
- Exponential decay with relaxation time proportional to lifetime
- Zero lifetime limit $\to$ elastic pion gas (difficult relaxation without number-changing processes).

Relaxation time reflects equilibration of microscopical process

Multiple resonances: no single exponential

Mixture of $\pi - \rho$ with physical lifetime, plus fictitious $B - B^*$ with same properties but $\times 7$ lifetime.





- Total correlation function is the direct sum of the 2 subprocesses
- **Two relaxation times** follow the same hierarchy as lifetimes
- $B - B^*$ system is a bottleneck for the system relaxation
- Similar situation for η in BAMPS (A. El *et al.* Eur.Phys.A,48,166 (2012))

General ansatz for correlation function

Assuming the **exponential relaxation** of individual processes:

$$C_\zeta(t) = \int_0^\infty d\tau \rho(\tau) \exp\left(-t/\tau\right)$$

- Single mode:

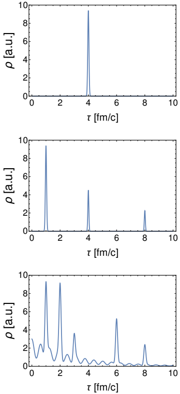
$$\rho(\tau) = 2C_\zeta(0)\delta(\tau - \tau_\zeta)$$

- Several modes:

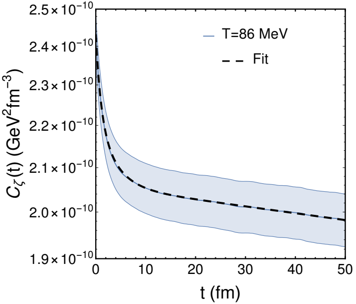
$$\rho(\tau) = 2\sum_i^N C_{\zeta,i}(0)\delta(\tau - \tau_{\zeta,i}) \; ; \sum_i^N C_{\zeta,i}(0) = C_\zeta(0)$$

- Continuum distribution:

$$\rho(\tau) \; ; \quad \int_0^\infty d\tau \rho(\tau) = C_\zeta(0)$$

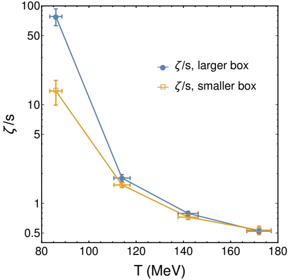


3-mode fitting function:



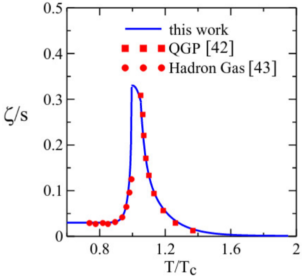
- $T = 86$ MeV, $V = (100\text{ fm})^3$
- Error band included in the fit
- Fit using *ROOT* and double checked with *Mathematica*

- 4 temperatures and 2 box sizes
- Lowest temperature point is difficult (we average over volumes)

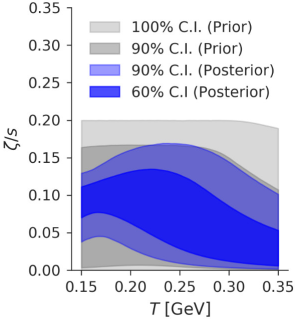


Size of ζ/s

Our $\zeta/s \sim \mathcal{O}(1)$ seems large in comparison to typical values used in HIC phenomenology...



S. Ryu *et al.* Phys. Rev. Lett. 115, 132301 (2015)



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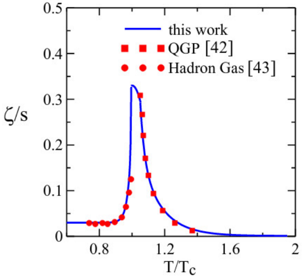

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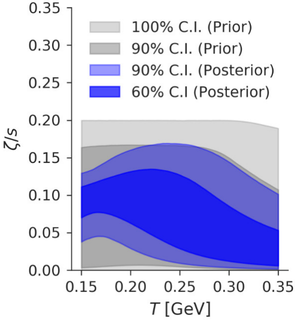

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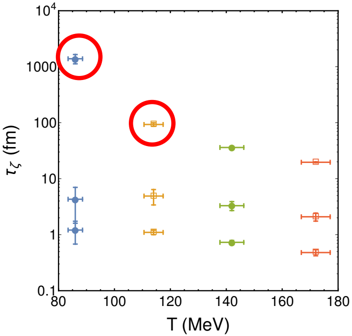
Slow processes

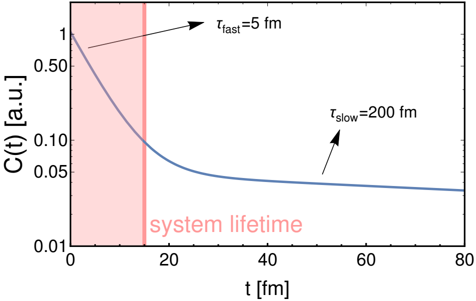
Why is ζ/s that large?



- **Inclusive viscosity** contains **all** physical processes
- Modes exceeding the lifetime of the hadronic phase
- Can this be reconciled with values from hydrodynamic codes?

Is long-time physics relevant?

- Inferred ζ/s upon comparison with **real** HICs e.g. Bayesian analyses [Bernhard, Moreland and Bass (2019), D. Everett *et al.* (2020)]
- Hadronic phase has limited lifetime $\sim 10 - 30$ fm
- A mode with large τ cannot be relevant in this scenario



Effective vs inclusive viscosity

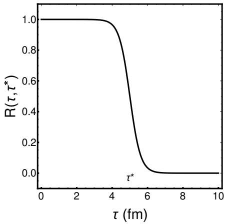
Regulator in the correlation function suppresses slow modes

$$C_{\zeta,\mathrm{eff}}(t,\tau^*) = \int_0^\infty d\tau \rho(\tau) \exp\left(-t/\tau\right) R(\tau,\tau^*)$$

e.g. a hard cutoff:

$$R(\tau,\tau^*) = \Theta(\tau^* - \tau)$$

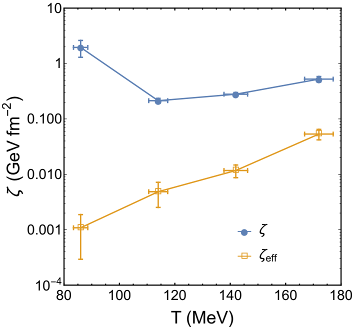
with $\tau^* \sim$ system lifetime.

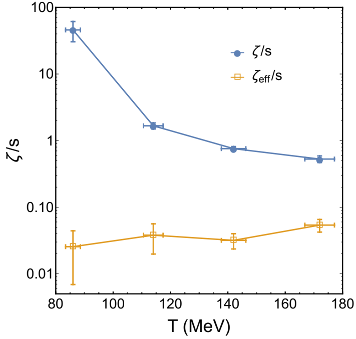


Effective viscosity

$$\zeta_{\mathrm{eff}} = \frac{V}{T}\int_0^\infty dt\ C_{\zeta,\mathrm{eff}}(t,\tau^*) \quad ; \quad C_{\zeta,\mathrm{eff}}(t,\tau^*) = \int_0^{\tau^*} d\tau \rho(\tau) \exp\left(-t/\tau\right)$$

Understood as a lower limit to the transport coefficient





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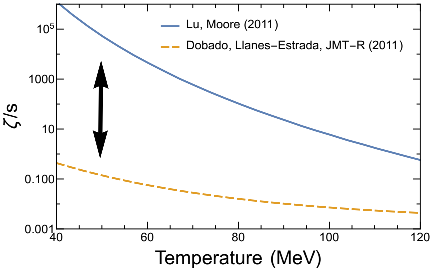
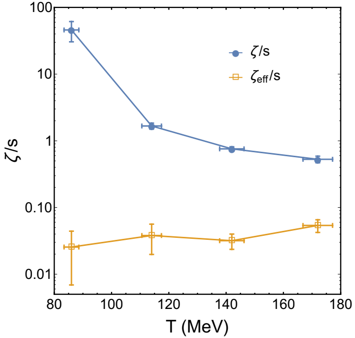
J. Weil et al, PRC 94 (2016),
DOI: 10.5281/zenodo.3484711

(volume averaged viscosity)

- Effective bulk viscosity is systematically smaller
- Slow modes dominate inclusive ζ. These modes not seen in other transport coefficients



Pion Gas



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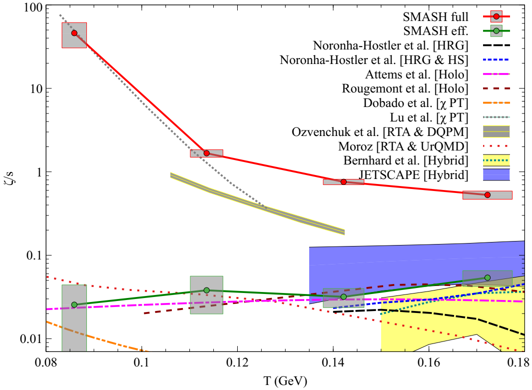
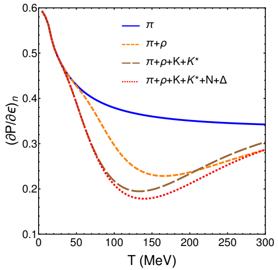
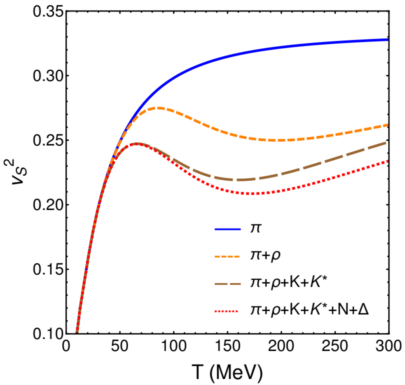
Comparison with other approaches

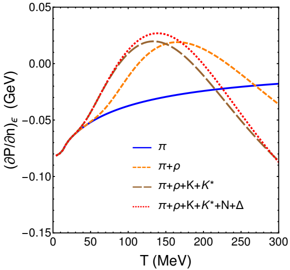


- **Inclusive** ζ/s similar within calculations allowing for slow processes
- **Effective** ζ/s close among those using relaxation time approximation, holographic approaches, and hybrid models for HICs

Adiabatic speed of sound

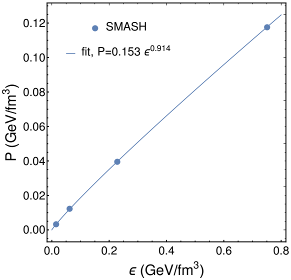


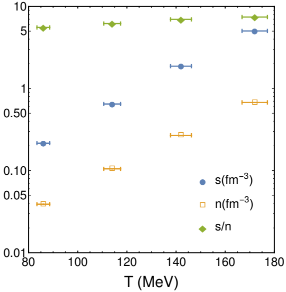




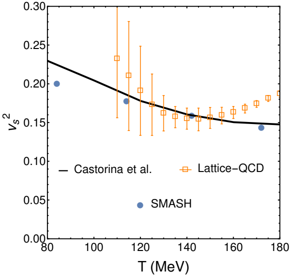
$$v_S^2 = \left(\frac{\partial P}{\partial \epsilon}\right)_{S=s/n} = \left(\frac{\partial P}{\partial \epsilon}\right)_n + \frac{n}{sT}\left(\frac{\partial P}{\partial n}\right)_\epsilon$$

Adiabatic speed of sound





$$v_S^2 = \left(\frac{\partial P}{\partial \epsilon}\right)_{S=s/n}$$



- Castorina *et al.* (2010): Hadron Resonance Gas, $m < 2.5$ GeV
- Lattice QCD, Borsanyi *et al.* (2014)

Conclusions

- Bulk viscosity is a subtle transport coefficient, very sensitive to microscopic details of interaction. This makes it very interesting to study.
- We addressed ζ of a hadron gas with SMASH using the Green-Kubo formalism. Single exponential ansatz is inadequate and several relaxation times are required.
- Slow processes dominate ζ and lead to a large **inclusive bulk viscosity**. It is a sensible coefficient in the thermodynamic limit (lattice-QCD calculations?)
- **Effective viscosity** discards very slow processes. It is compatible with the ζ/s extracted from HICs via hybrid models.

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DFG Deutsche Forschungsgemeinschaft

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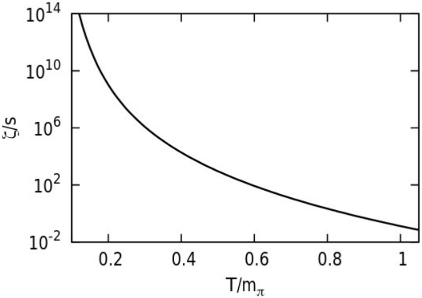
Can be ζ/s large?



Pion gas in chiral perturbation theory with $2\pi \leftrightarrow 4\pi$ processes [Lu, Moore (2011)]

References I

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