



Untangling the evolution of heavy ion collisions using direct photon interferometry

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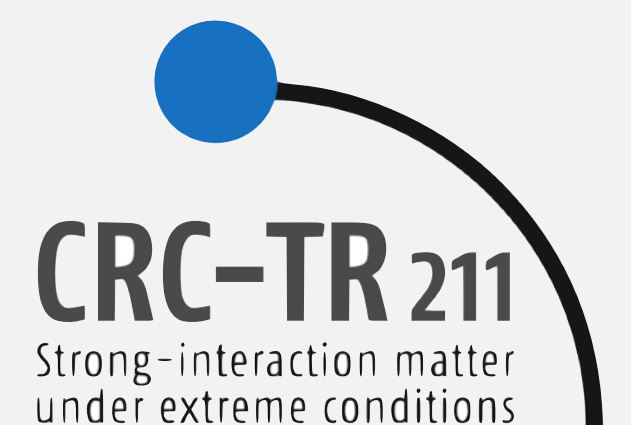
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In collaboration with J. Berges, Nicole Löhner,
A. Mazeliauskas and K. Reygers

Based on: Garcia-Montero *et al* , [arXiv:1909.12246](https://arxiv.org/abs/1909.12246)

Garcia-Montero, [arXiv:1909.12294](https://arxiv.org/abs/1909.12294)

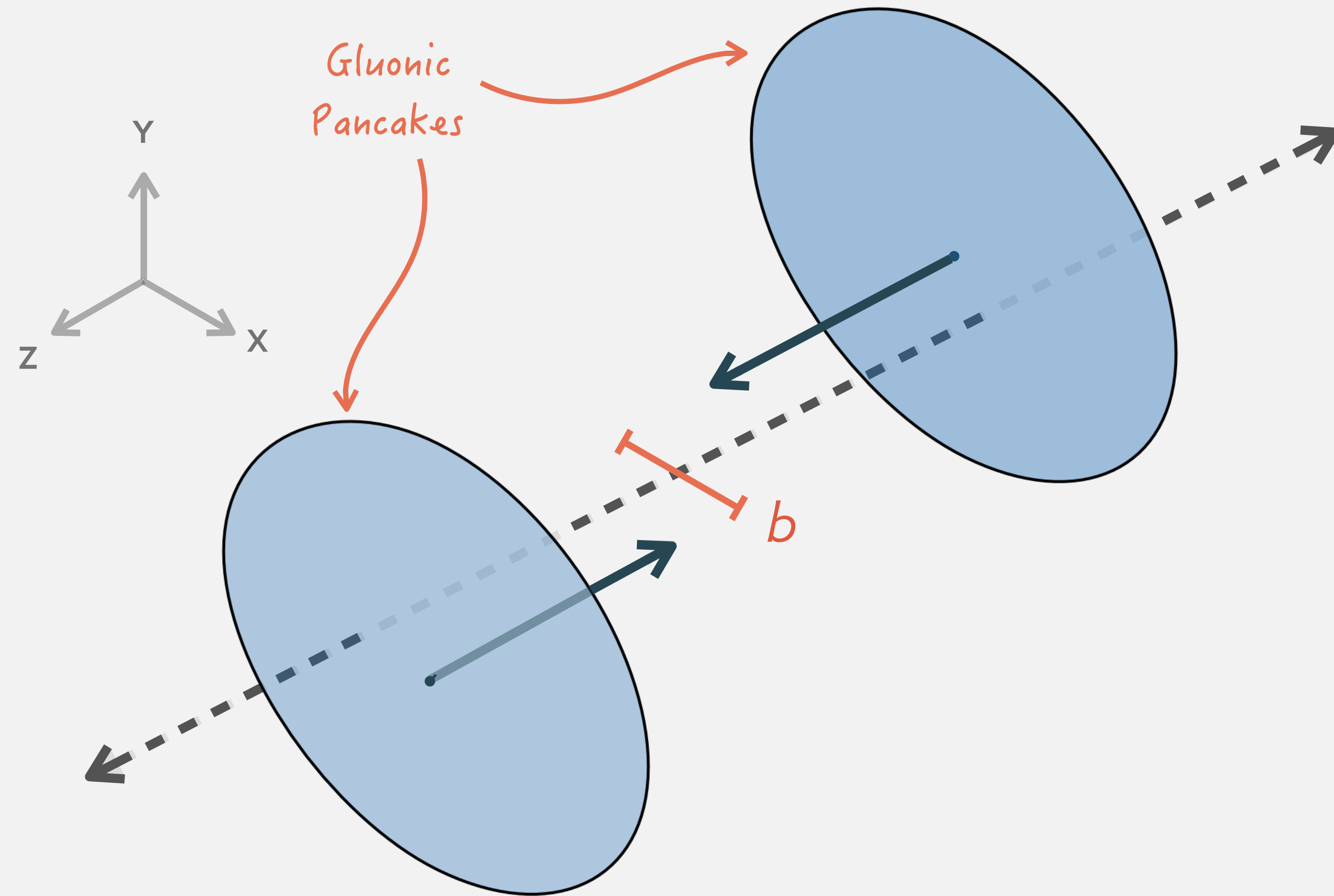


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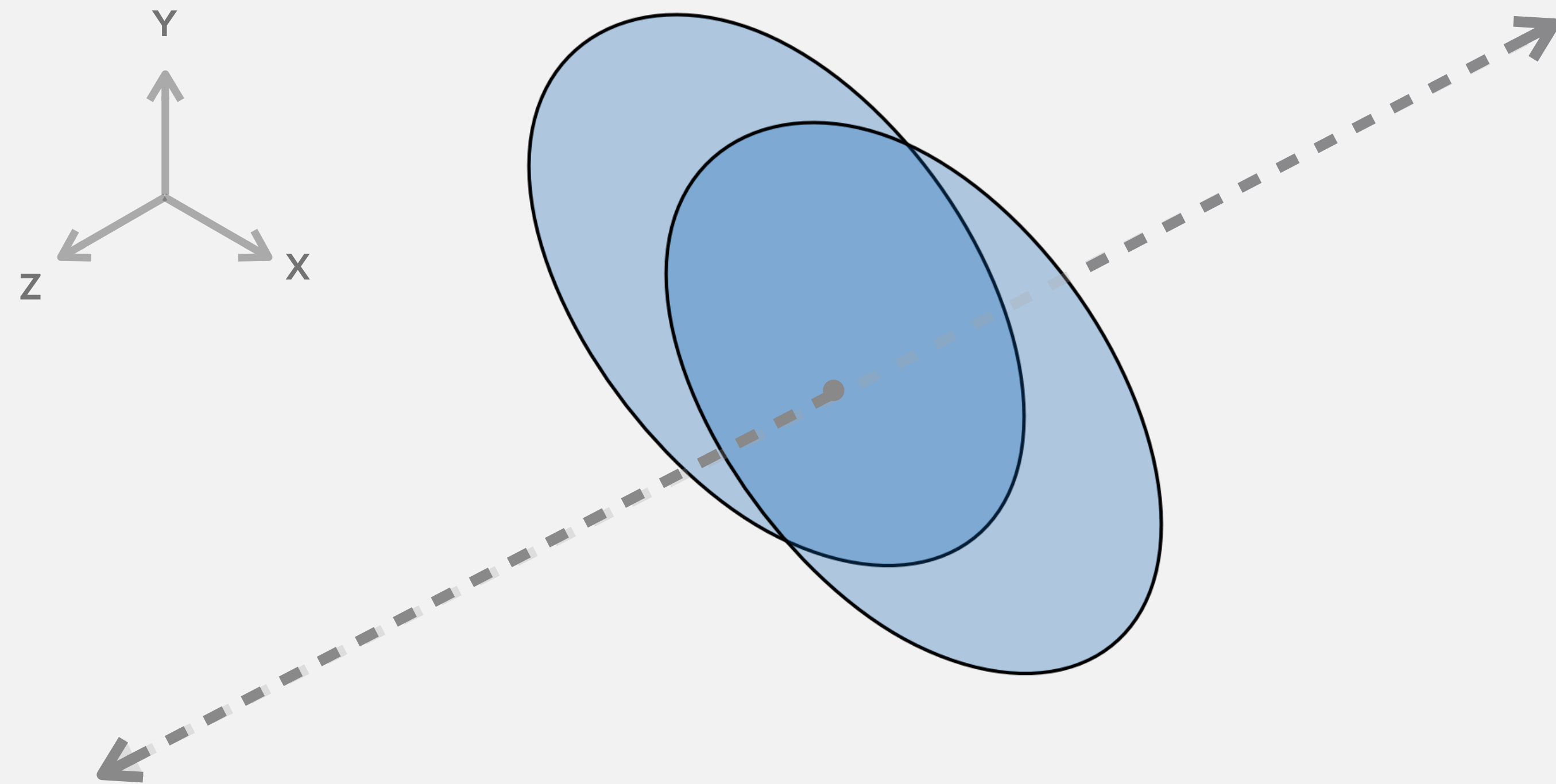
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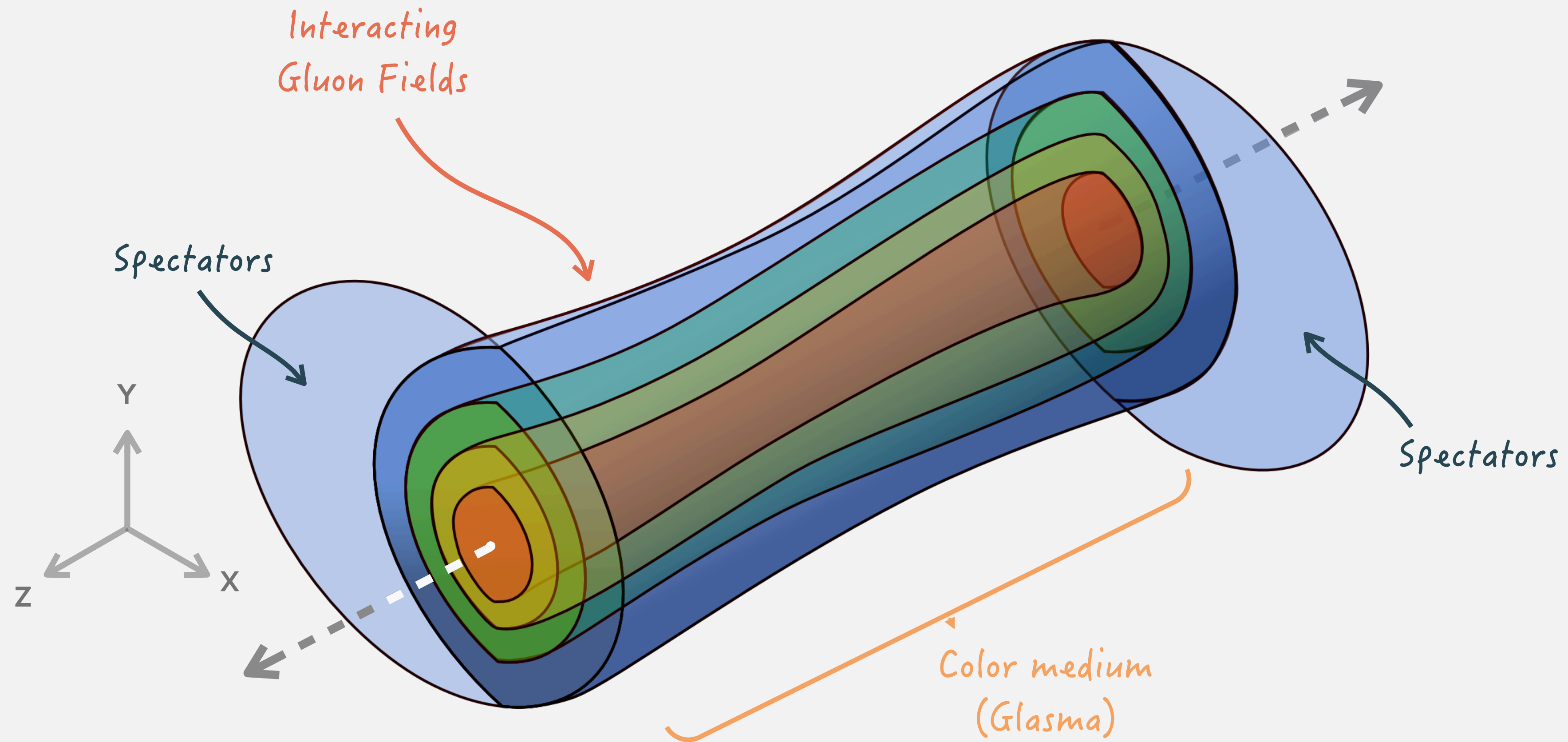
Ultra-relativistic Nucleus-Nucleus (A+A) Collisions



Ultrarelativistic Nucleus-Nucleus (A+A) Collisions



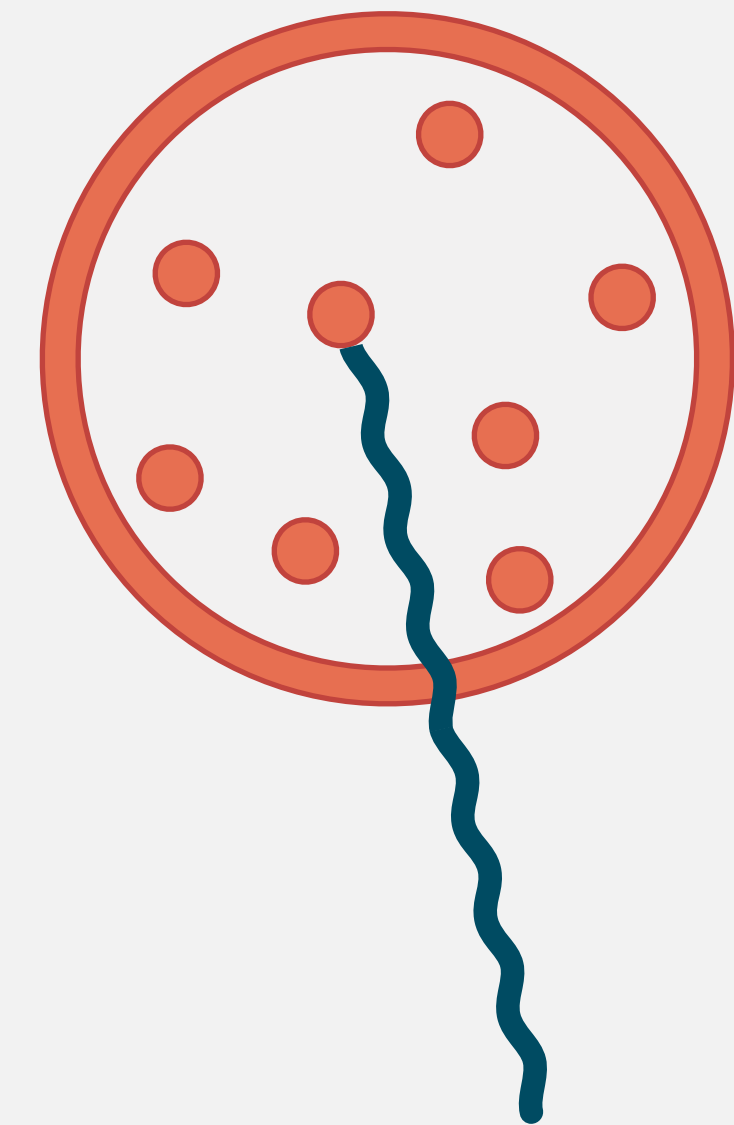
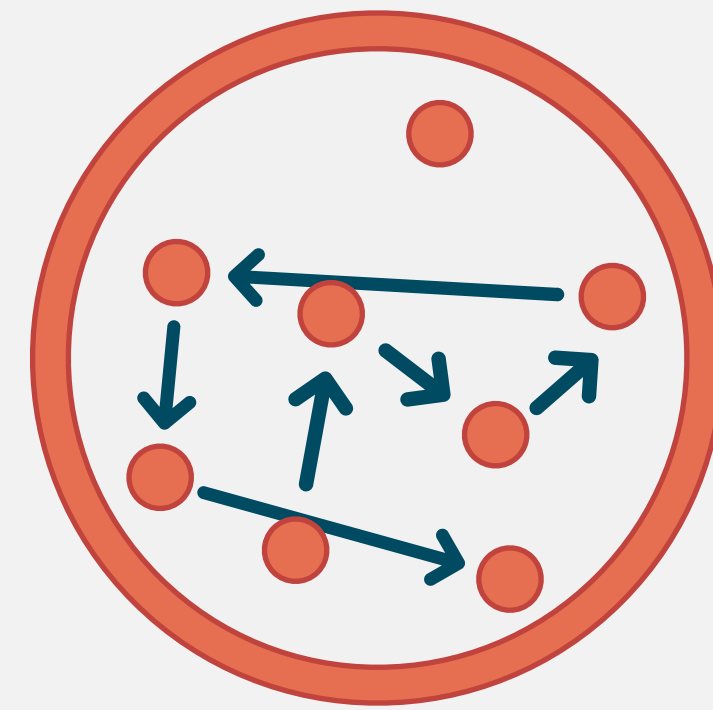
Ultrarelativistic Nucleus-Nucleus (A+A) Collisions



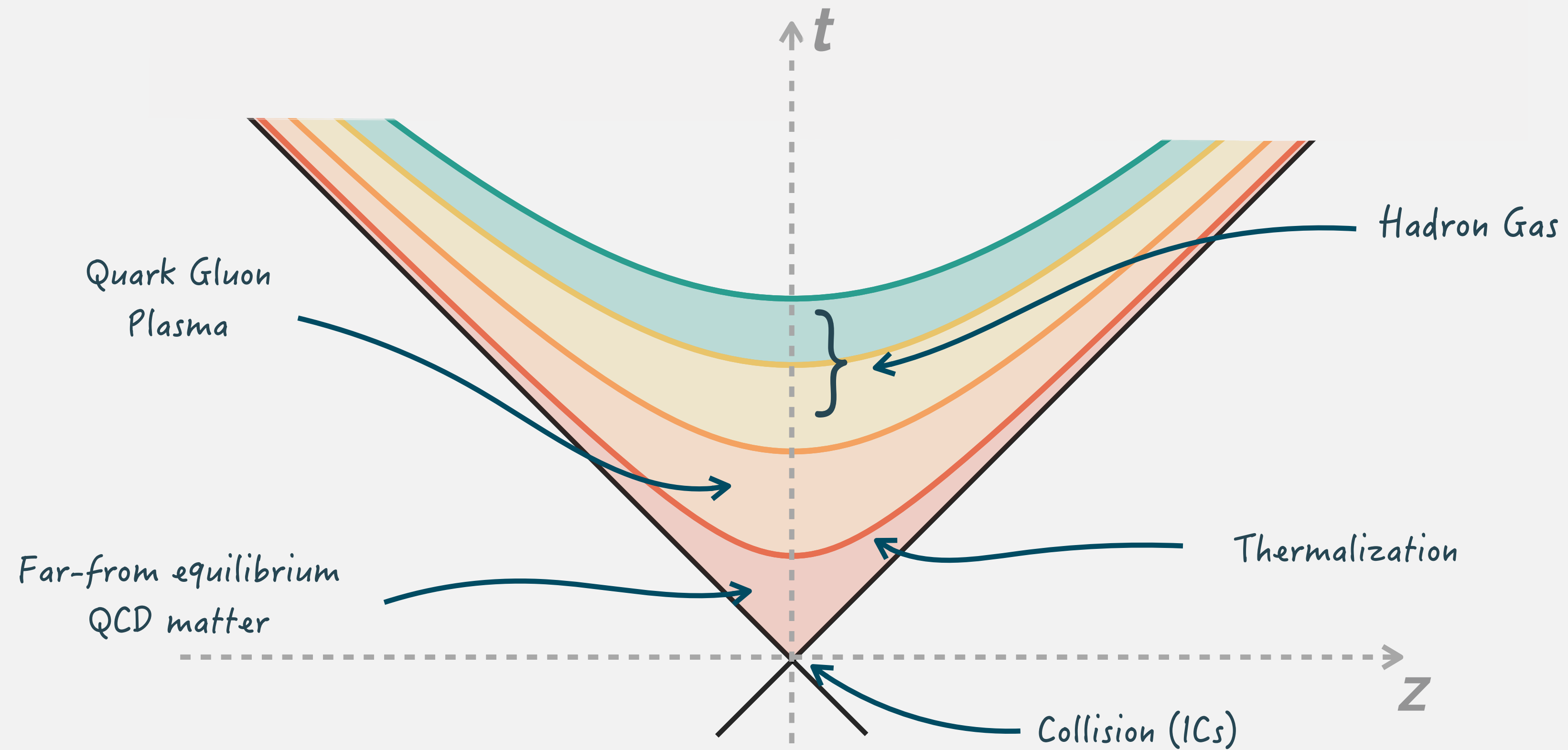
Why photons?

- Produced at every stage
- No strong interactions
- Mean free path in medium $>$ medium size

 Photons escape, virtually unscathed

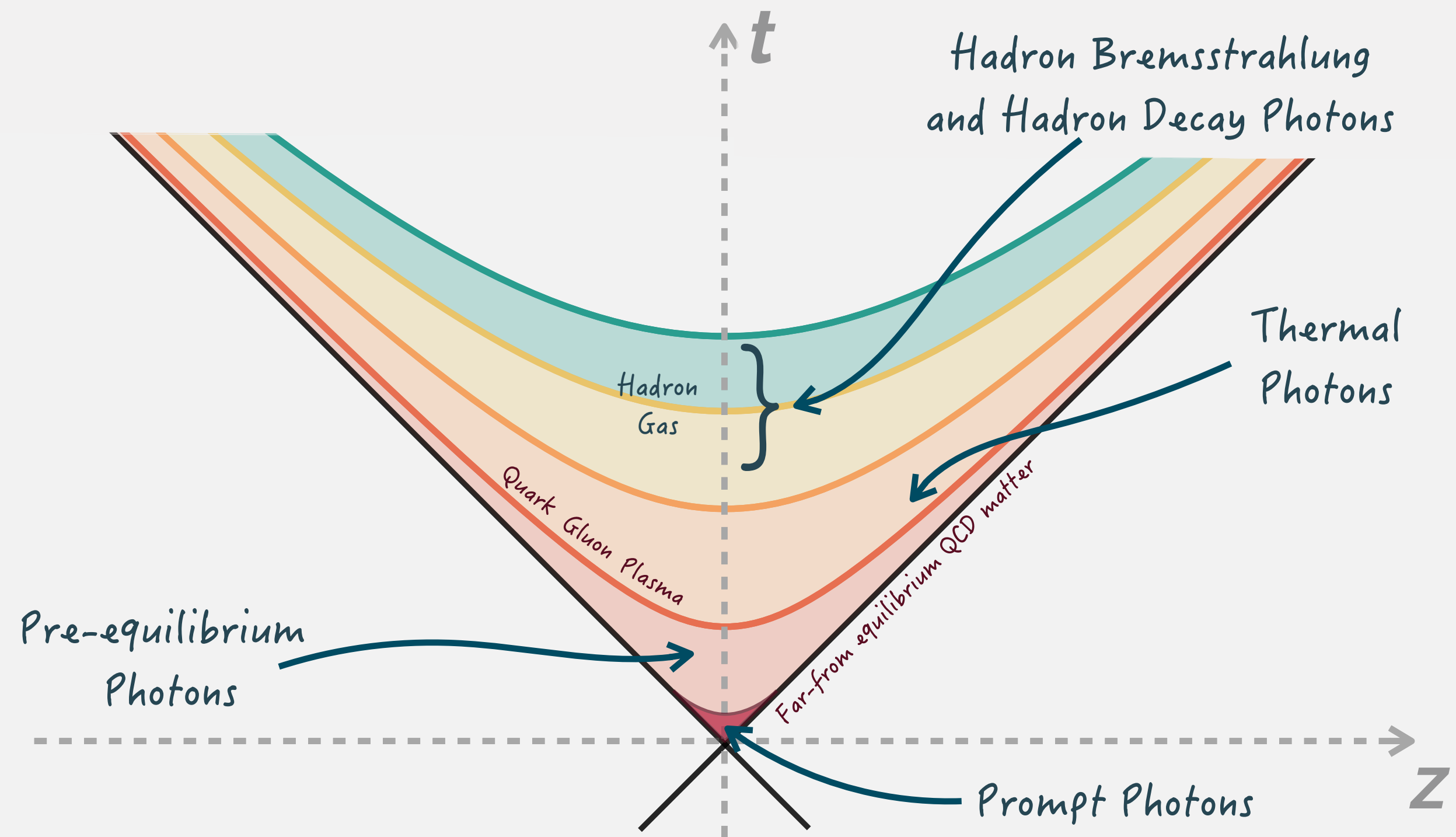


The Standard model of Heavy Ion Collisions



Photon Production

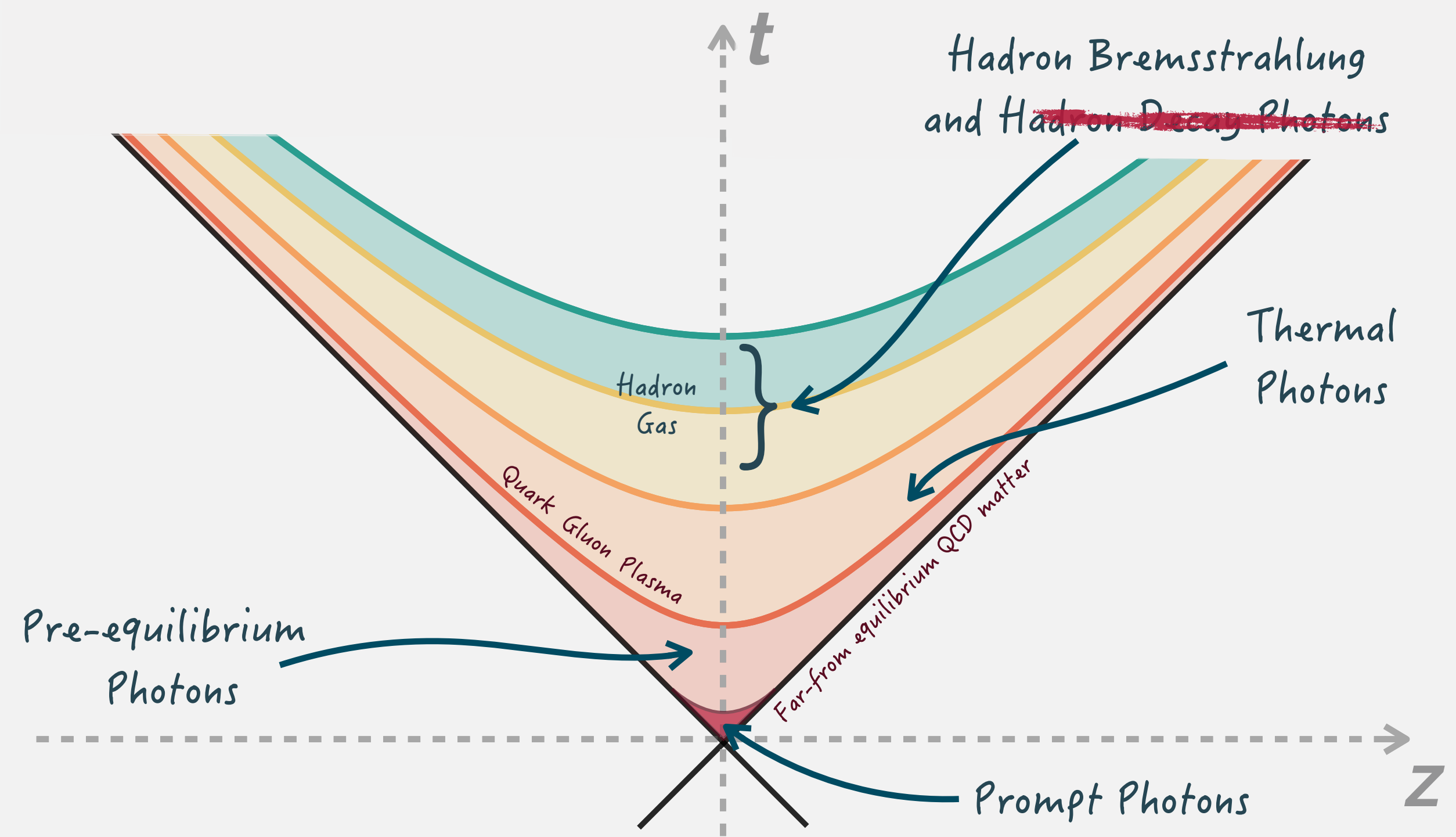
Photons are produced throughout the evolution by different processes, and fly away to the detectors



Photon Production

Photons are produced throughout the evolution by different processes, and fly away to the detectors

Direct Photons: Produced NOT by decays (In medium)



Direct Photon Puzzle

Photons @ RHIC

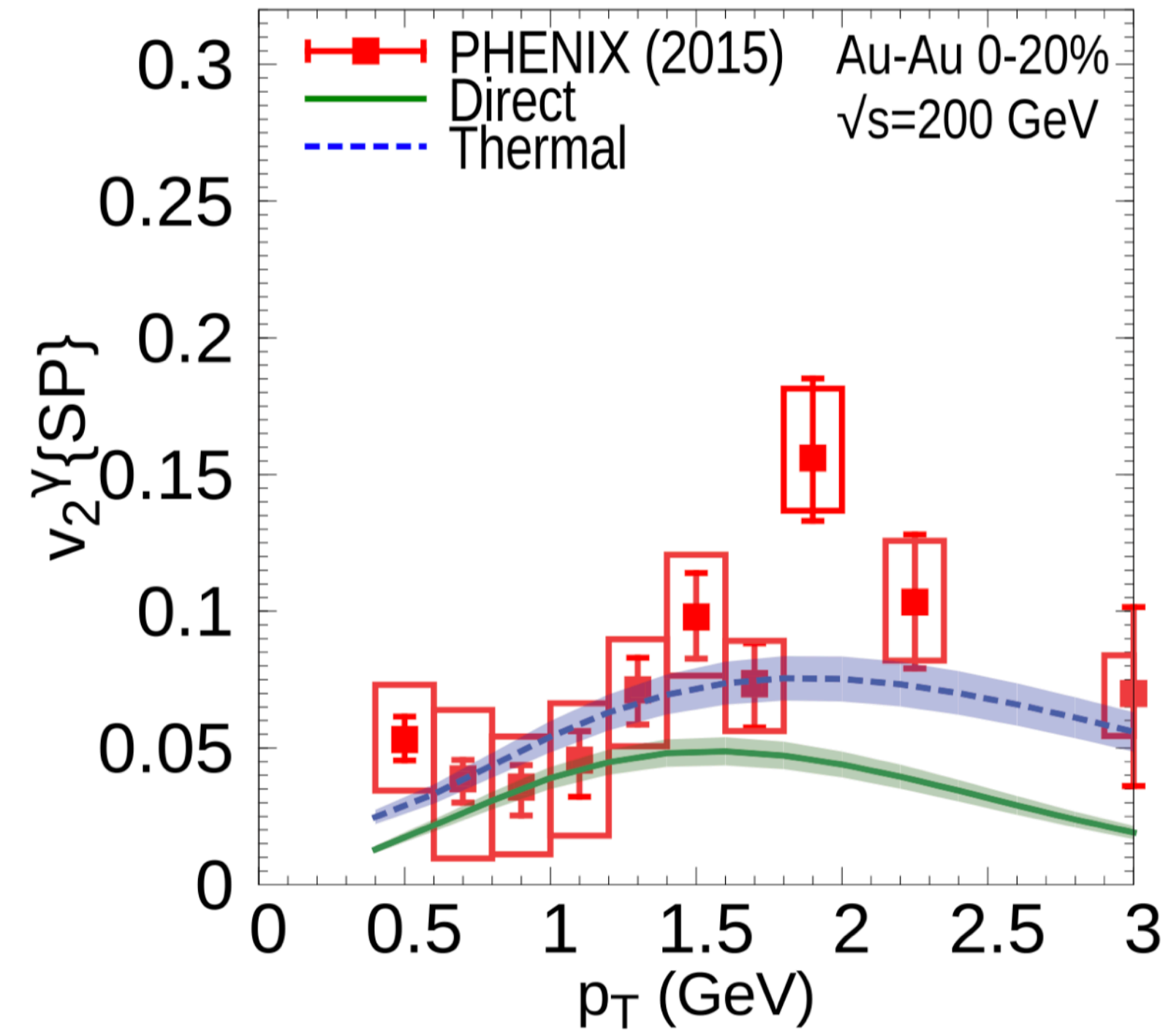
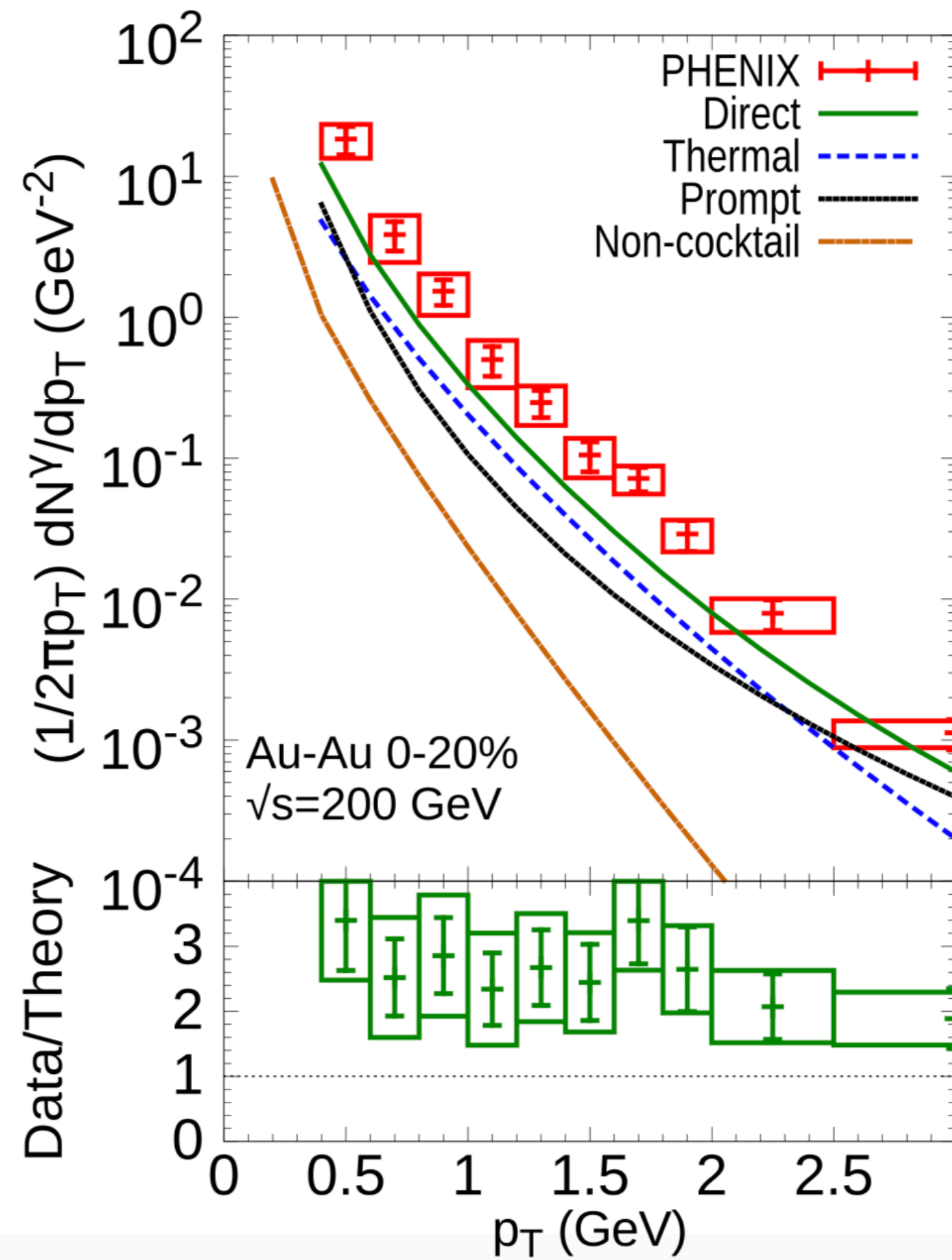


Figure from Paquet *et al*, Phys.Rev. C93 (2016) no.4, 044906

Photons @ LHC

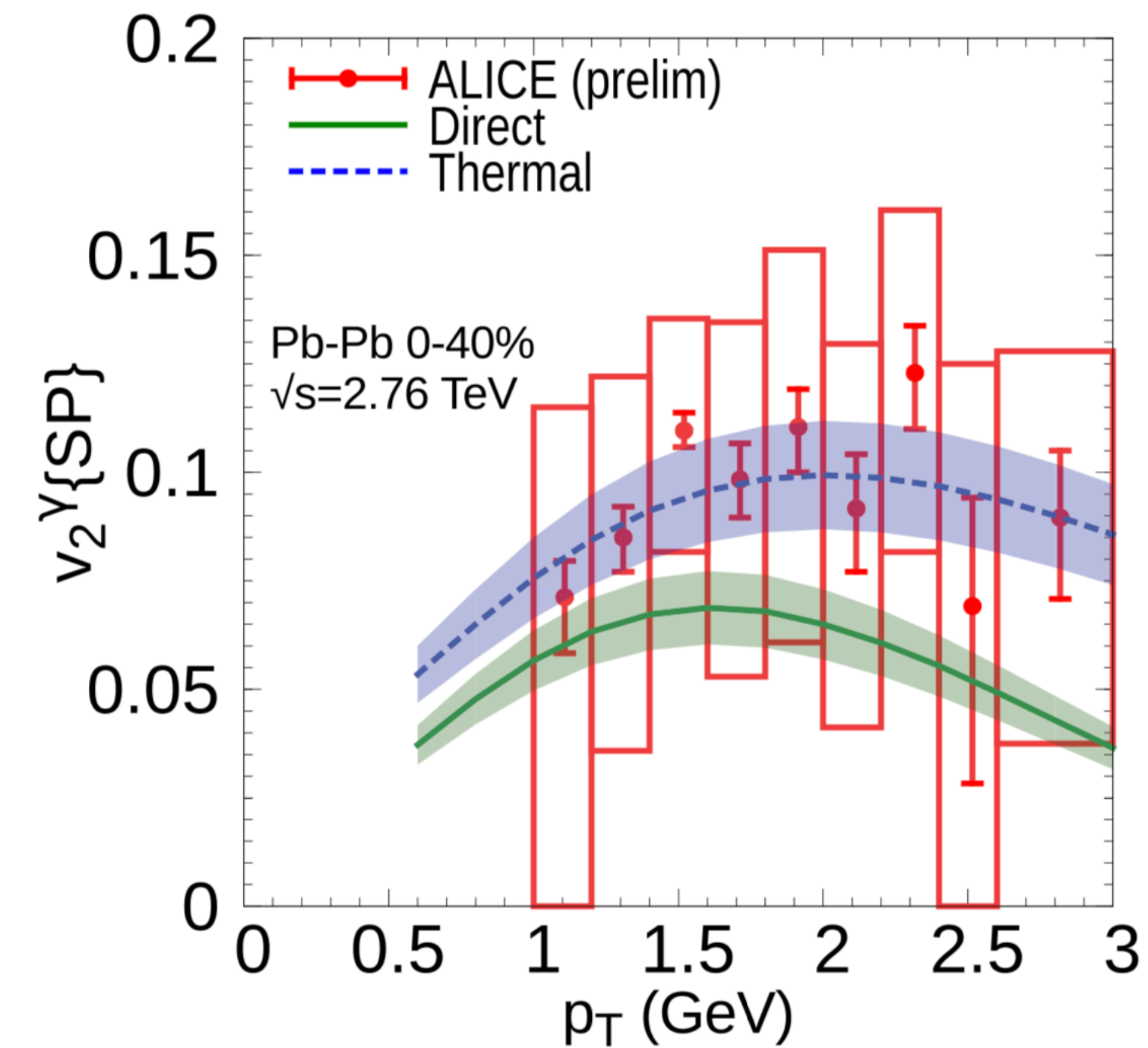
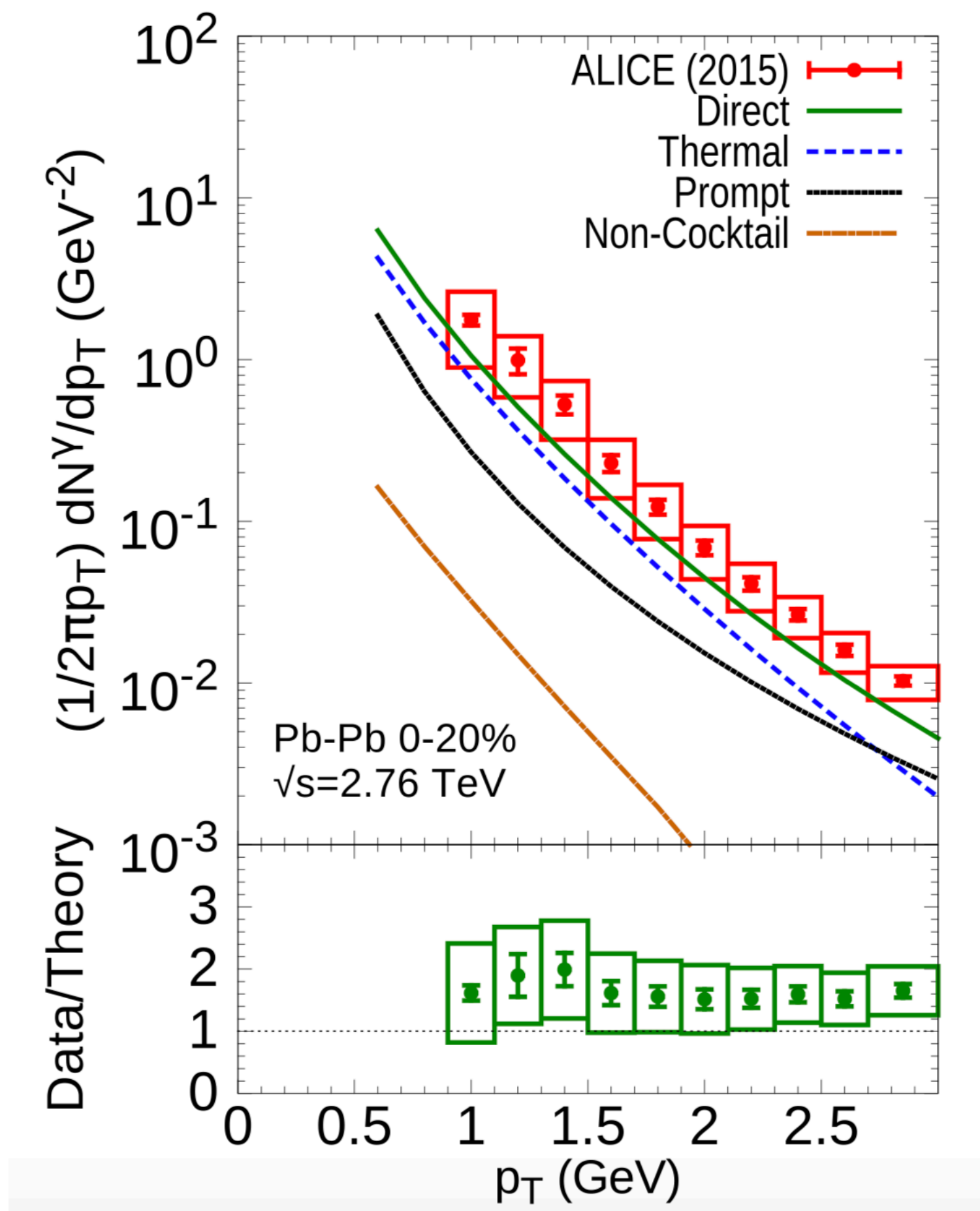


Figure from Paquet *et al*,
Phys.Rev. C93 (2016) no.4, 044906

Direct Photon Puzzle

"The inability to simultaneously describe both the photon yield and anisotropy."

Solution

- We are lacking a source of photons.
- Source can act as an extra knob for tuning.



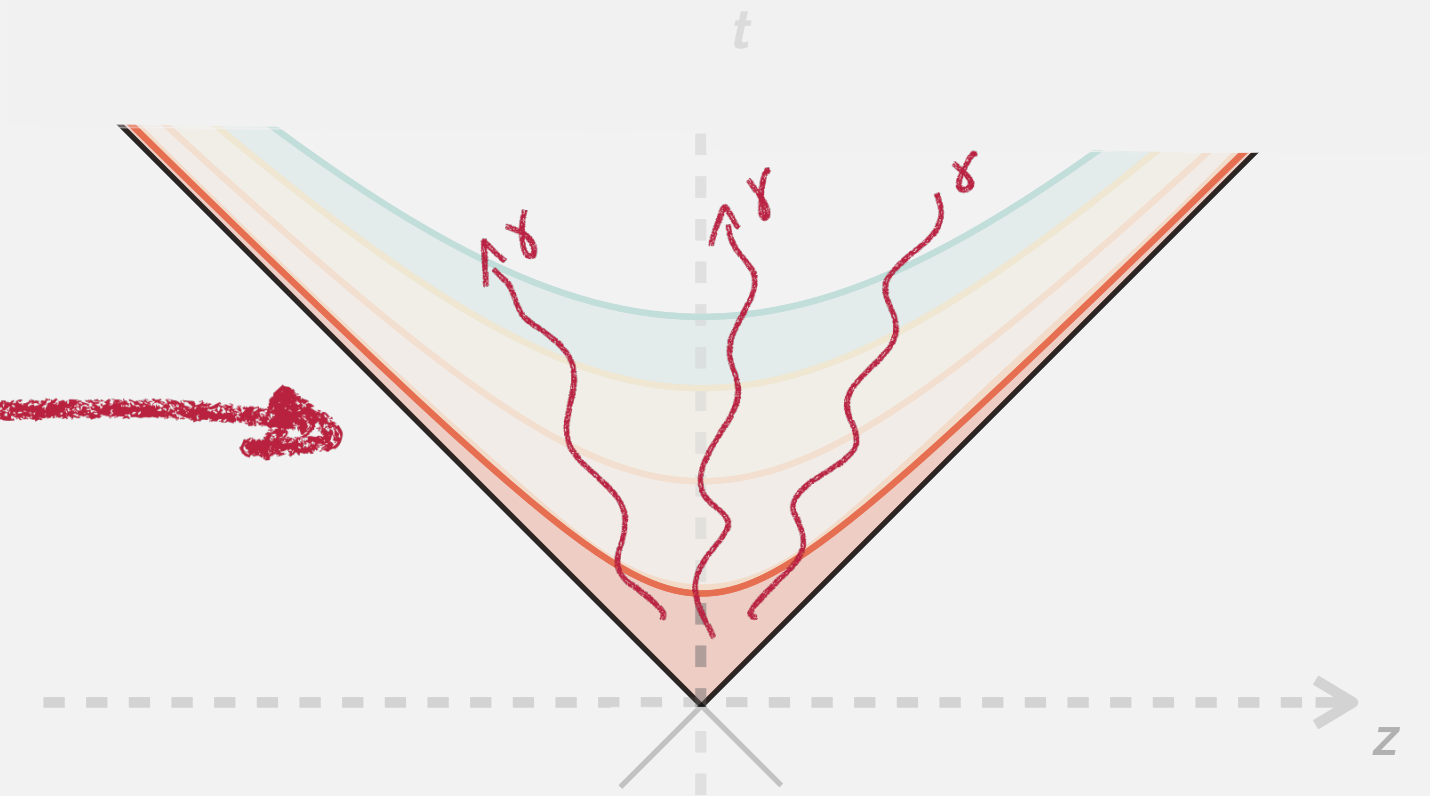


RENÉ MAGRITTE
EMPIRE OF LIGHT



RENÉ MAGRITTE
EMPIRE OF LIGHT

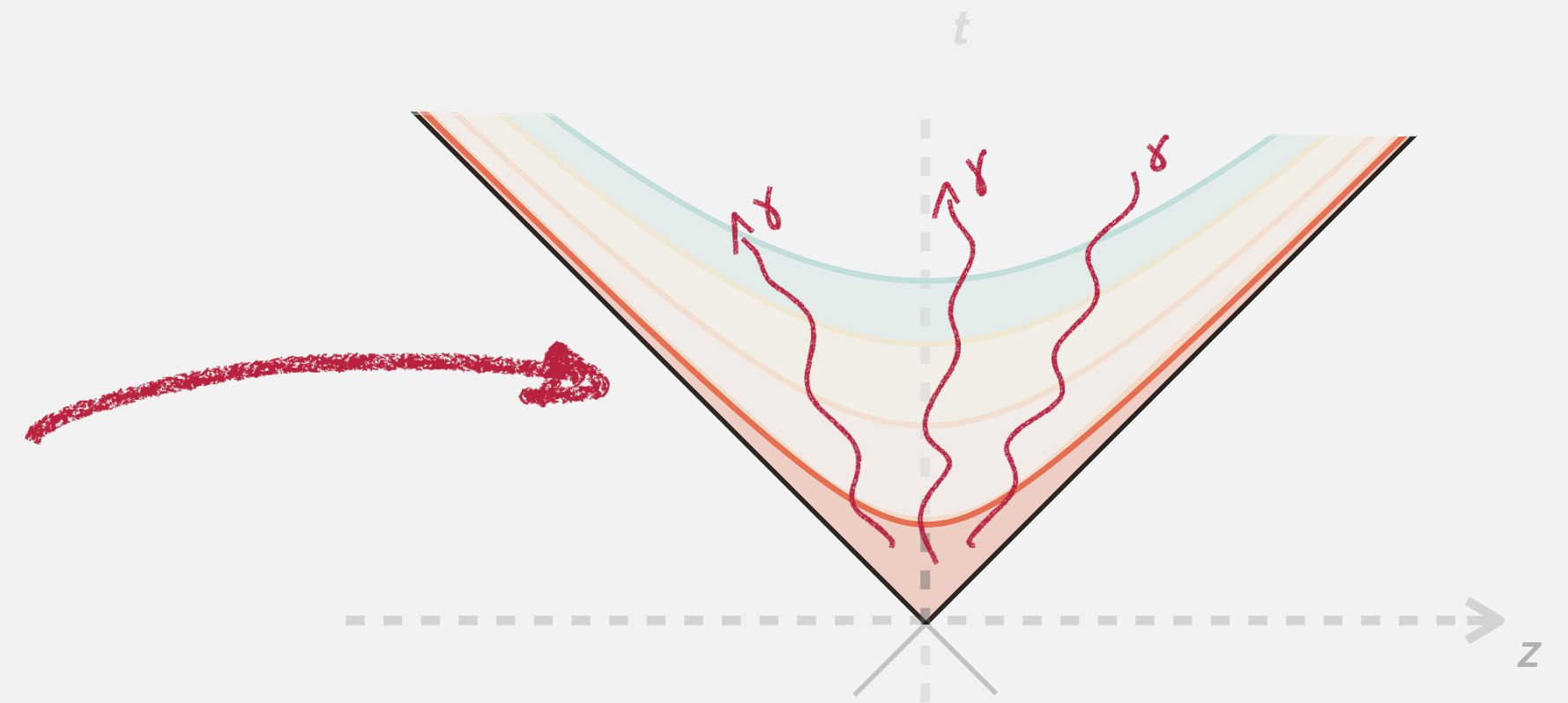
Early time
enhancement



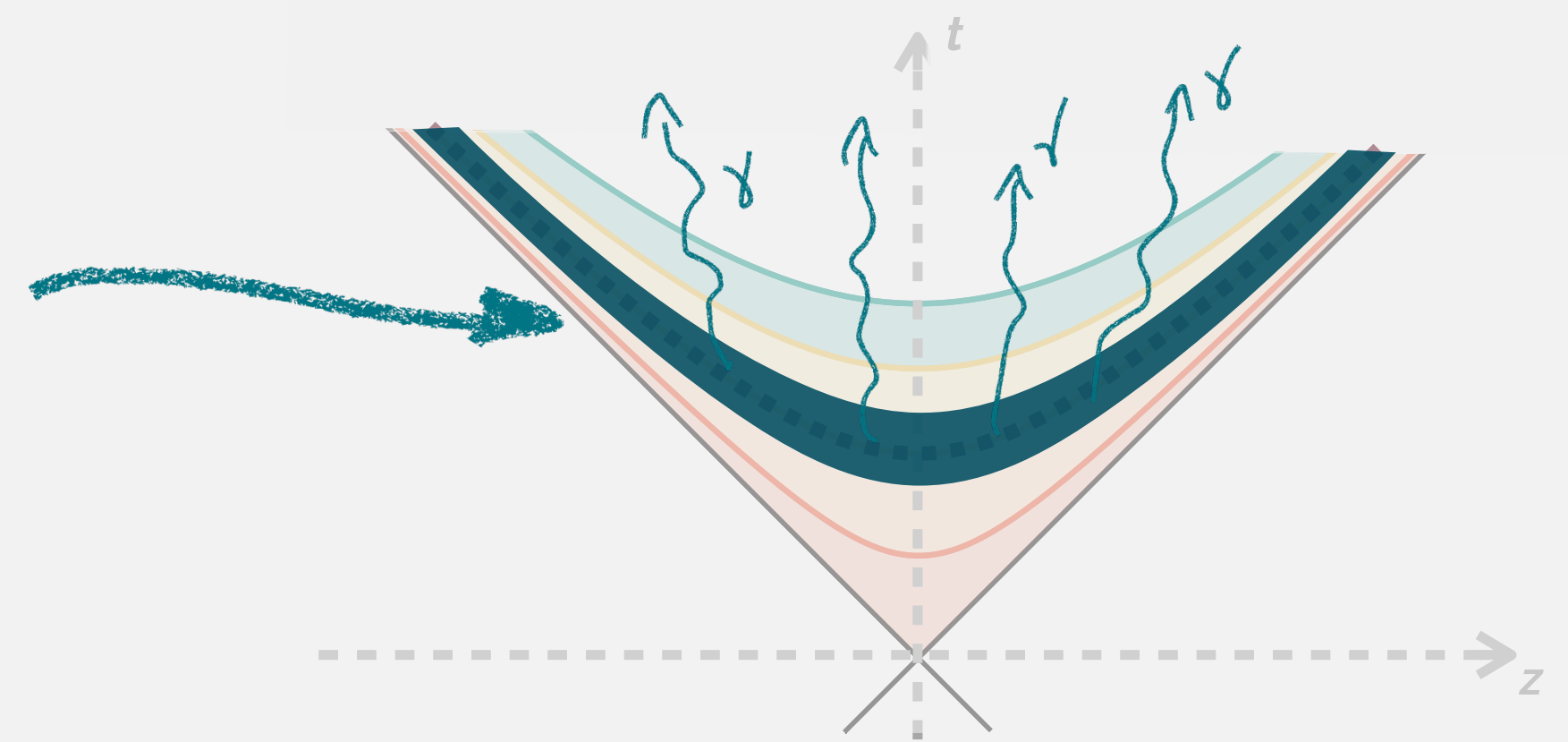


RENÉ MAGRITTE
EMPIRE OF LIGHT

Early time
enhancement



Late time
enhancement



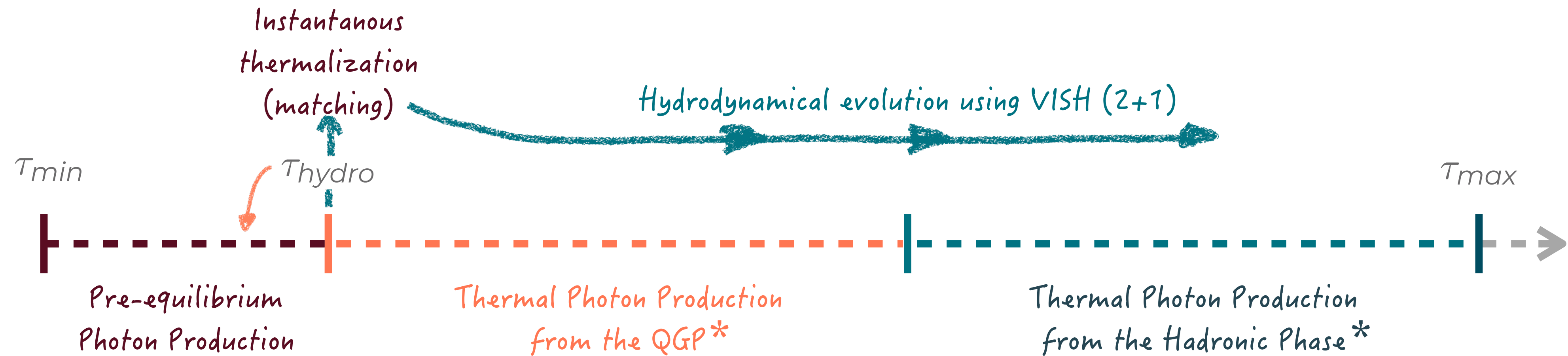
Two Models

(Early vs Late time production)

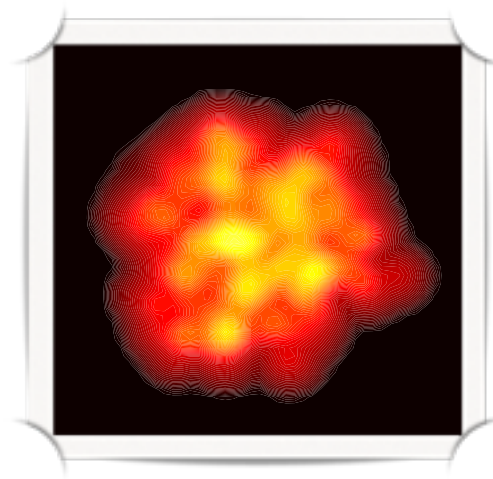
Photons from the “bottom-up” scenario

(QCD Kinetic Thermalization)

The model



MC-Glauber

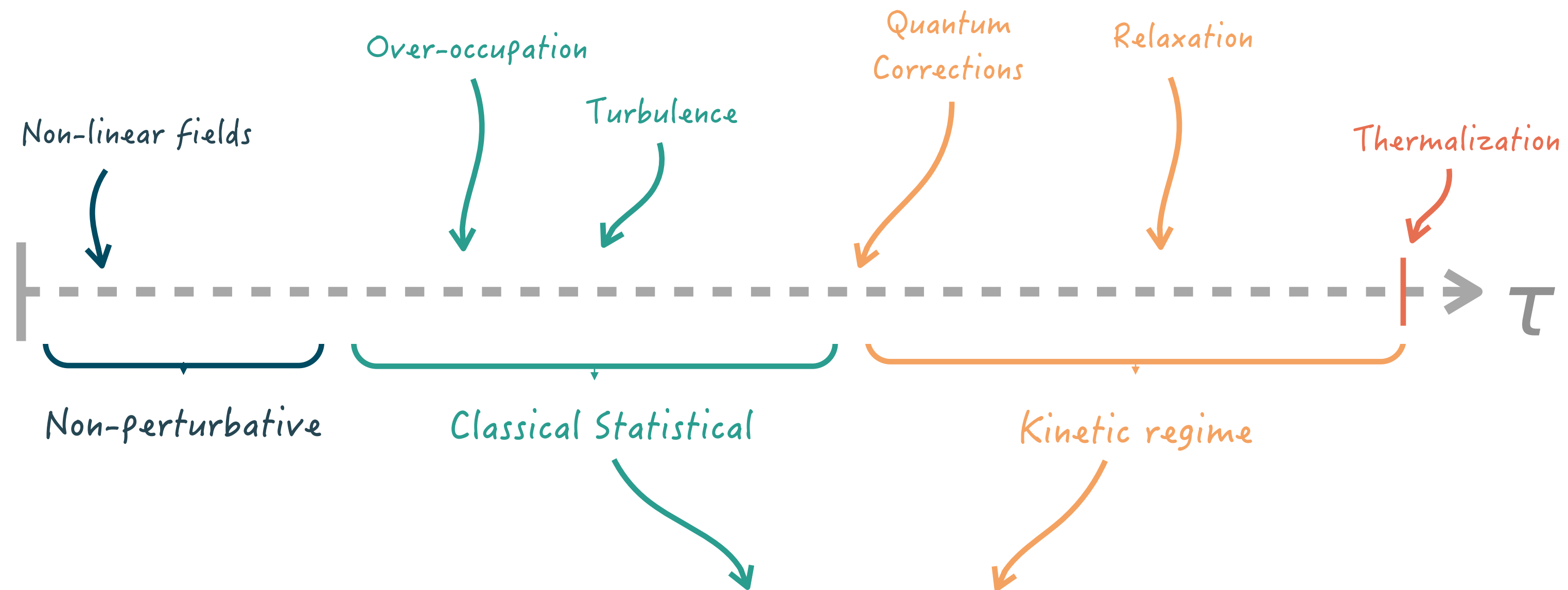


$\tau_{min} = 0.1\text{fm}$
 $\tau_{hydro} = 0.6\text{fm}$
 $\tau_{max} = 15\text{fm}$

* Using the thermal rates in

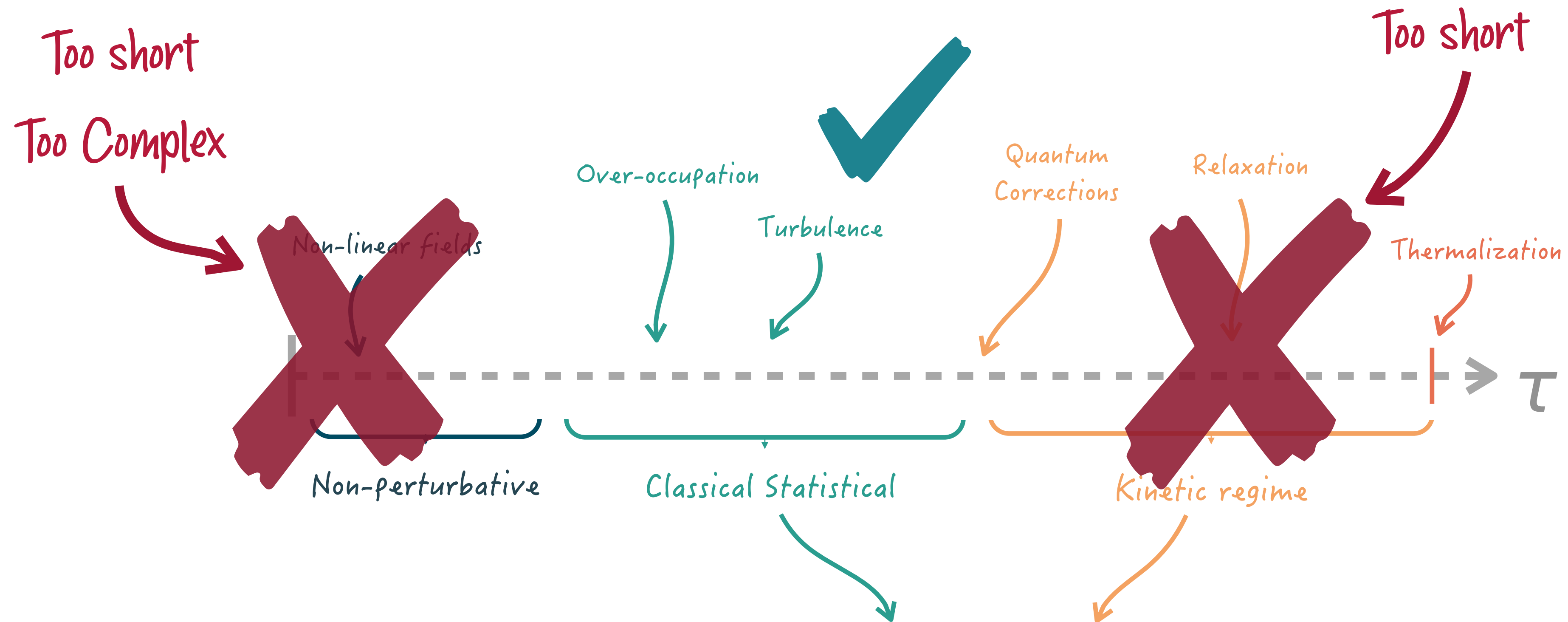
P. B. Arnold, *et al*, JHEP 12, 009 (2001)
S. Turbide, *et al*, Phys. Rev. C69, 014903 (2004)
M. Heffernan, *et al*, Phys. Rev. C91, 027902 (2015)

Far-from equilibrium QCD matter



Direct photons from pre-equilibrium epochs! *

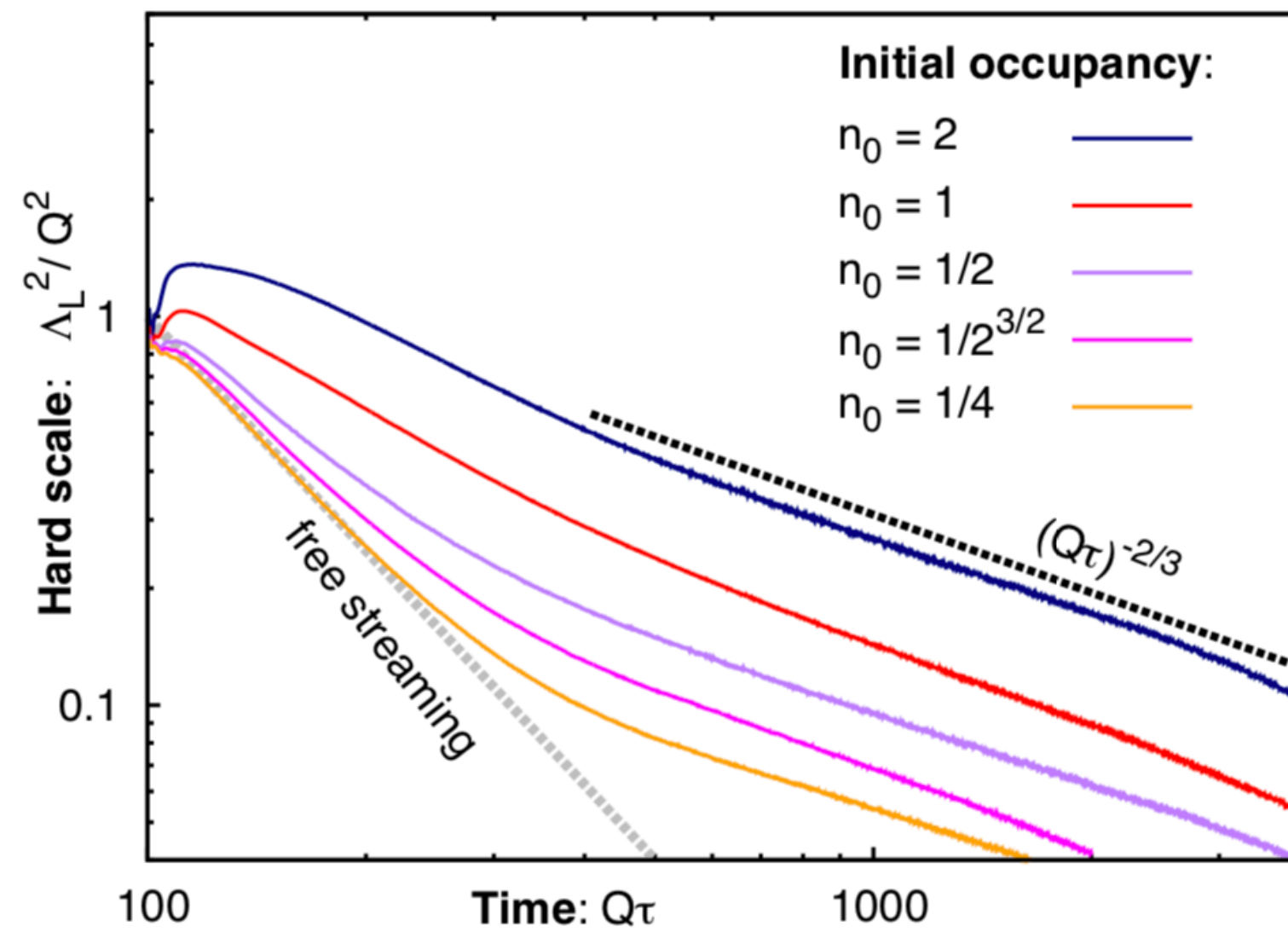
Far-from equilibrium QCD matter



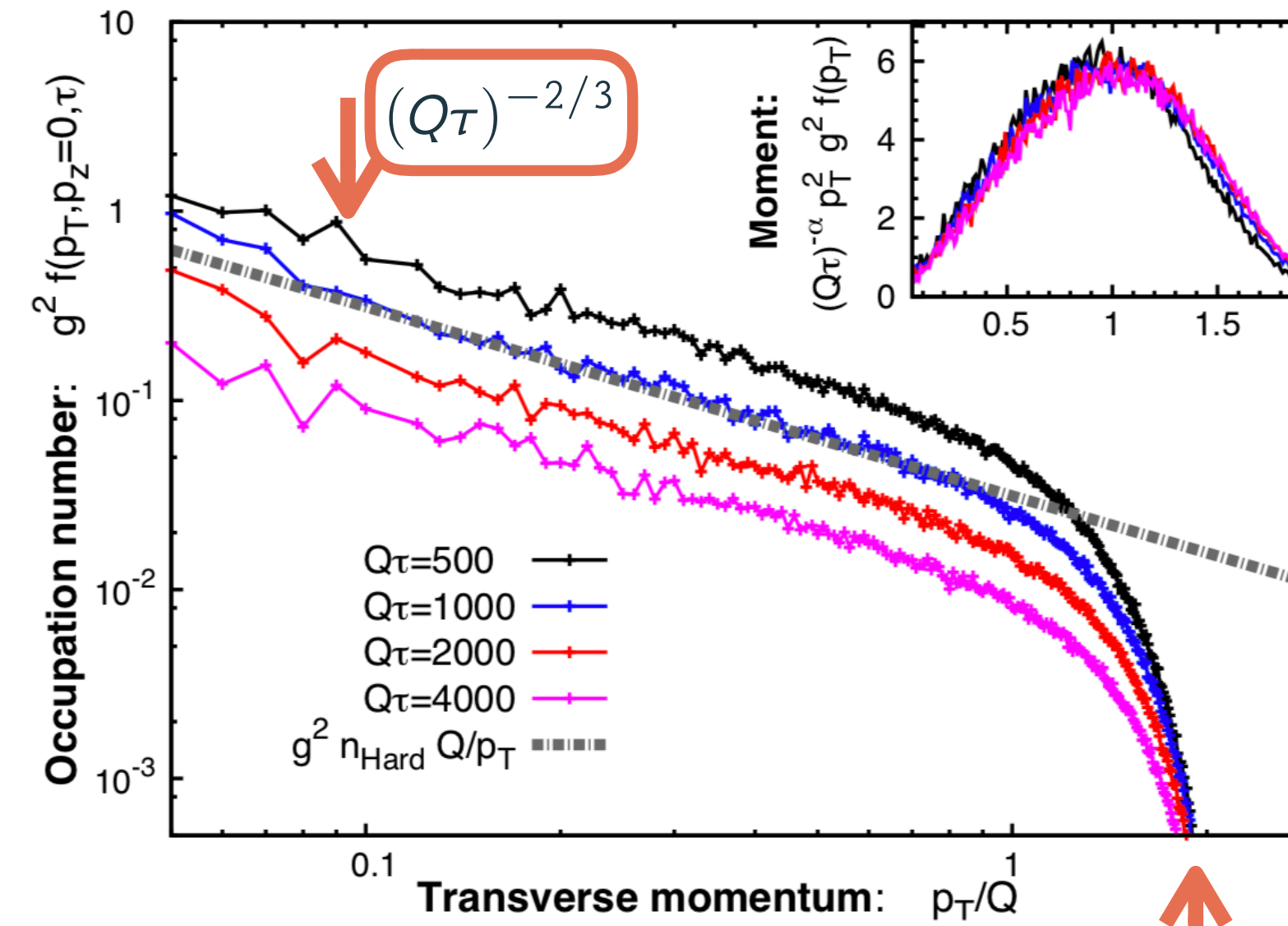
Direct photons from pre-equilibrium epochs! *

Gluon occupation

Hard Scale: $\Lambda_L^2 \sim \langle p_z \rangle^2$



Transverse p_\perp



$$\alpha = -2/3 \quad \beta = 0 \quad \gamma = 1/3$$

\downarrow
 $N(\tau)$

\downarrow
 $\langle p_\perp \rangle$

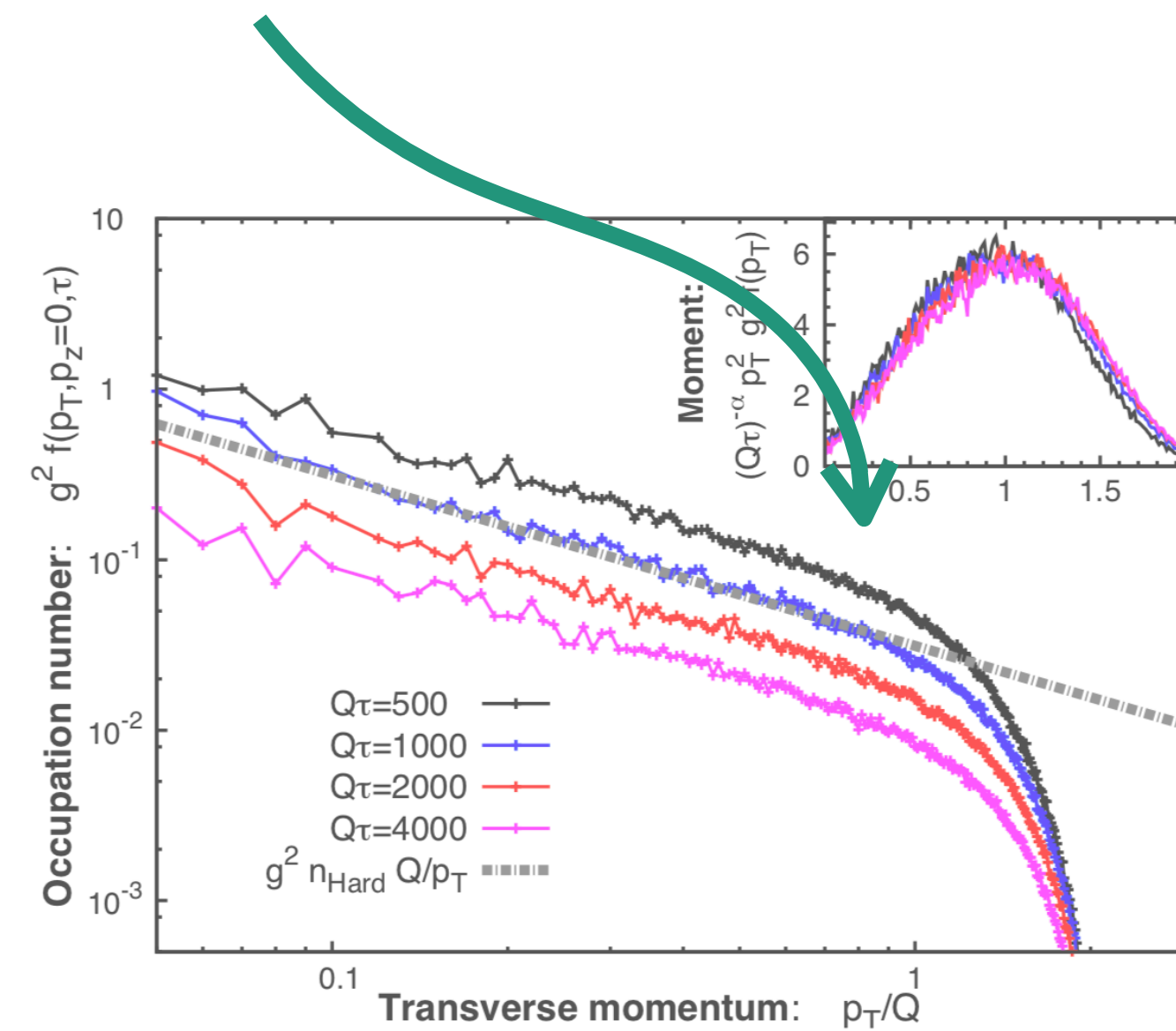
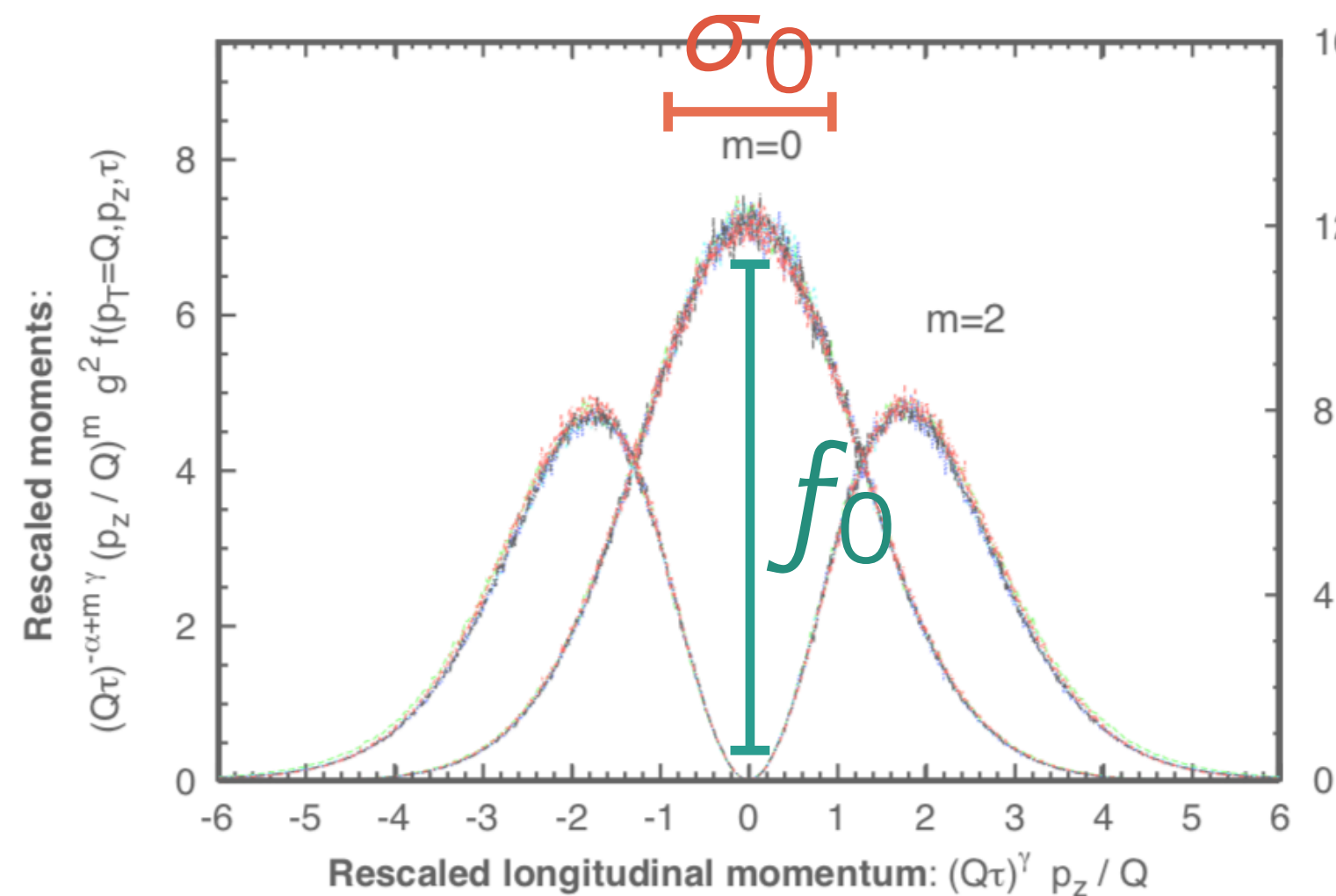
\downarrow
 $\langle p_z \rangle$

Fit the lattice results

Gluon Distribution $\rightarrow f_g(\tau, p_\perp, p_z) = \frac{1}{\alpha_s} (Q_s \tau)^\alpha f_S(p_\perp, (Q_s \tau)^\gamma p_z)$

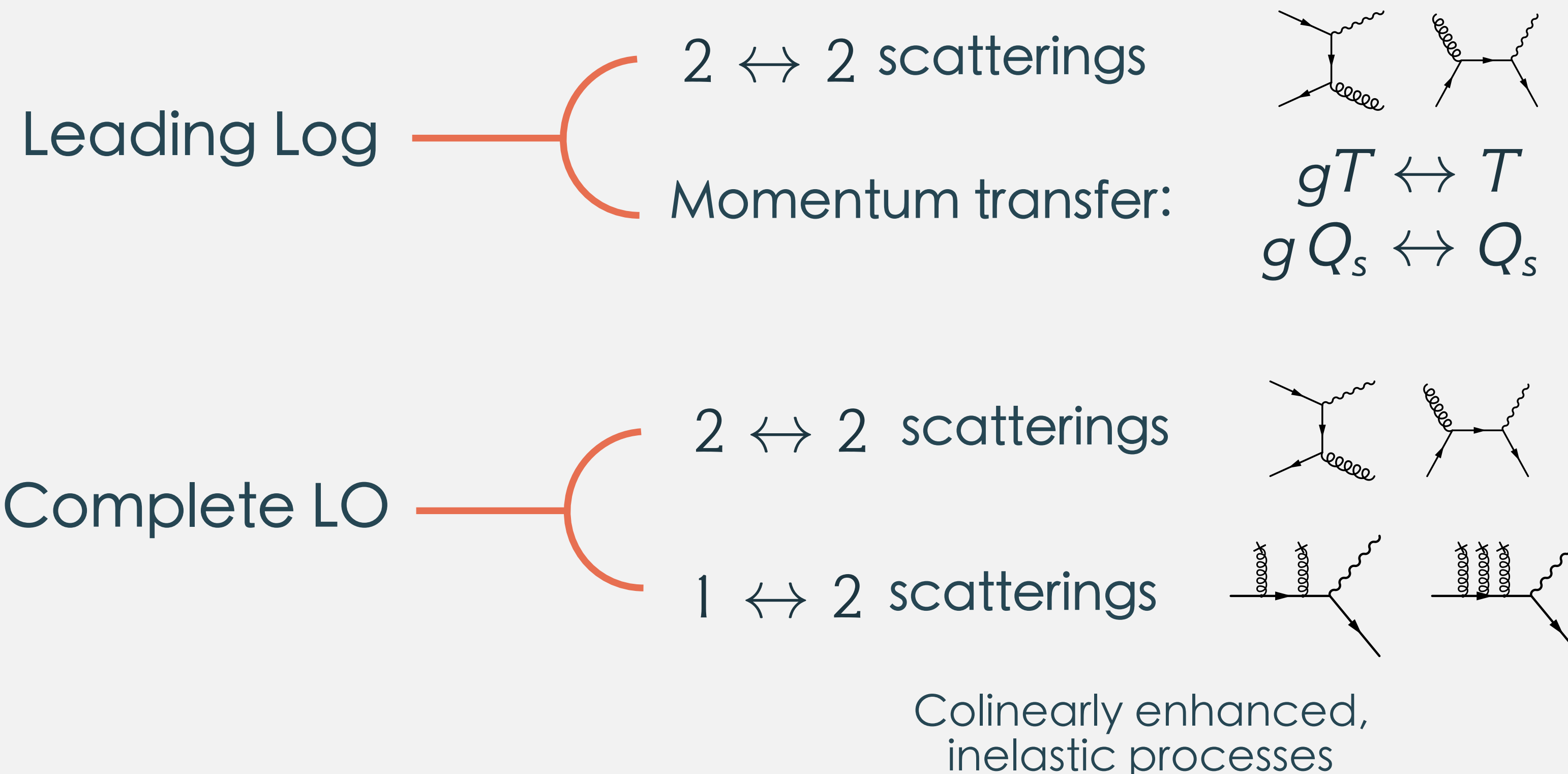
with $f_S(p_\perp, p_z) = f_0 \frac{Q_s}{p_\perp} e^{-\frac{1}{2} \left(\frac{p_z}{\sigma_0}\right)^2} W_r(p_\perp - Q_s, r)$

and $W_r(p_\perp - Q_s, r) = \theta(Q_s - p_\perp) + \theta(p_\perp - Q_s) e^{-\frac{1}{2} \left(\frac{p_\perp - Q_s}{r Q_s}\right)^2}$



Kinetic Rates

$$E \frac{dN_\gamma}{d^4X d^3p} = |\mathcal{M}|^2 \otimes F[f_i] \otimes \delta(p_{in} - p_{out})$$



LPM

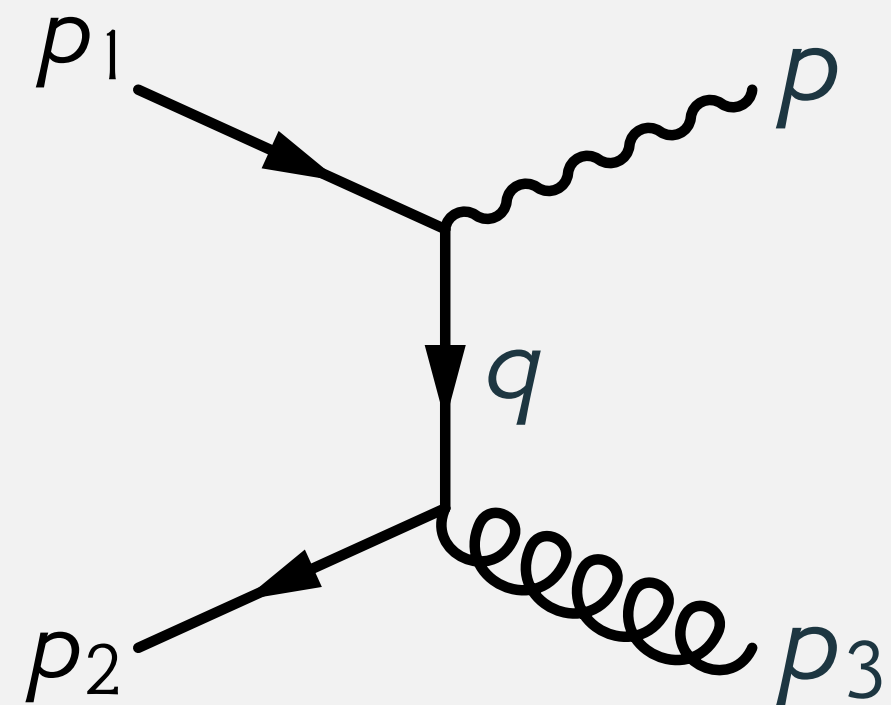
Kinetic Rates

$$E \frac{dN_\gamma}{d^4X d^3p} = \frac{1}{2 (2\pi)^{12}} \int \frac{d^3p_3}{2E_3} \frac{d^3p_2}{2E_2} \frac{d^3p_1}{2E_1} |\mathcal{M}|^2$$

$$\times (2\pi)^4 \delta^4(P_1 + P_2 - P_3 - P)$$

$$\times f_1(p_1) f_2(p_2) [1 \pm f_3(p_3)]$$

Small angle approximation \longrightarrow Expansion on momentum exchange



$$|\mathbf{p}| = \sqrt{(\mathbf{p}_1 + \mathbf{q})^2} \sim |\mathbf{p}_1| + \mathbf{q} \cdot \mathbf{p}_1 / |\mathbf{p}_1|$$

$$|\mathbf{p}_3| = \sqrt{(\mathbf{p}_2 - \mathbf{q})^2} \sim |\mathbf{p}_2| - \mathbf{q} \cdot \mathbf{p}_2 / |\mathbf{p}_2|$$

Kinetic Rates

$$E \frac{dN_\gamma}{d^4 X d^3 p} = \frac{1}{2 (2\pi)^{12}} \int \frac{d^3 p_3}{2E_3} \frac{d^3 p_2}{2E_2} \frac{d^3 p_1}{2E_1} |\mathcal{M}|^2$$
$$\times (2\pi)^4 \delta^4(P_1 + P_2 - P_3 - P)$$
$$\times f_1(p_1) f_2(p_2) [1 \pm f_3(p_3)]$$

Small angle approximation

$$E \frac{dN}{d^4 X d^3 p} = \frac{40}{9\pi^2} \alpha \alpha_s \mathcal{L} f_q(\mathbf{p}) (l_g + l_q)$$

Amplitude

Kinetic Rates

$$E \frac{dN_\gamma}{d^4 X d^3 p} = \frac{1}{2 (2\pi)^{12}} \int \frac{d^3 p_3}{2E_3} \frac{d^3 p_2}{2E_2} \frac{d^3 p_1}{2E_1} |\mathcal{M}|^2 \\ \times (2\pi)^4 \delta^4(P_1 + P_2 - P_3 - P) \\ \times f_1(p_1) f_2(p_2) [1 \pm f_3(p_3)]$$

Small angle approximation

$$E \frac{dN}{d^4 X d^3 p} = \frac{40}{9\pi^2} \alpha \alpha_s \mathcal{L} j_q(\mathbf{p}) (l_g + l_q)$$

Regulator

with

$$\mathcal{L} = 2 \log \left(\frac{2.912 E}{g_s^2 T} \right) \rightarrow 2 \log \left(1 + \frac{2.912}{g_s^2} \right)$$

Kinetic Rates

$$E \frac{dN_\gamma}{d^4 X d^3 p} = \frac{1}{2 (2\pi)^{12}} \int \frac{d^3 p_3}{2E_3} \frac{d^3 p_2}{2E_2} \frac{d^3 p_1}{2E_1} |\mathcal{M}|^2$$

$$\times (2\pi)^4 \delta^4(P_1 + P_2 - P_3 - P)$$

$$\times \underbrace{f_1(p_1) f_2(p_2) [1 \pm f_3(p_3)]}$$

Small angle approximation

$$E \frac{dN}{d^4 X d^3 p} = \frac{40}{9\pi^2} \alpha \alpha_S \mathcal{L} f_q(\mathbf{p}) \underbrace{(I_g + I_q)}_{\text{Screening Masses}}$$

$$I_{q,g}(\tau) = \int \frac{d^3 p}{(2\pi)^3} \frac{1}{p} f_{q,g}(\tau, p)$$

Kinetic Rates

$$E \frac{dN_\gamma}{d^4 X d^3 p} = \frac{1}{2 (2\pi)^{12}} \int \frac{d^3 p_3}{2E_3} \frac{d^3 p_2}{2E_2} \frac{d^3 p_1}{2E_1} |\mathcal{M}|^2 \\ \times (2\pi)^4 \delta^4(P_1 + P_2 - P_3 - P) \\ \times \underbrace{f_1(p_1) f_2(p_2) [1 \pm f_3(p_3)]}$$

Small angle approximation

$$E \frac{dN}{d^4 X d^3 p} = \frac{40}{9\pi^2} \alpha \alpha_s \mathcal{L} \underbrace{f_q(\mathbf{p})}_{\text{Quark Distribution}} (I_g + I_q)$$

Quark
Distribution

Kinetic Rates

$$E \frac{dN_\gamma}{d^4 X d^3 p} = \frac{1}{2 (2\pi)^{12}} \int \frac{d^3 p_3}{2E_3} \frac{d^3 p_2}{2E_2} \frac{d^3 p_1}{2E_1} |\mathcal{M}|^2 \\ \times (2\pi)^4 \delta^4(P_1 + P_2 - P_3 - P) \\ \times f_1(p_1) f_2(p_2) [1 \pm f_3(p_3)]$$

Small angle approximation

$$E \frac{dN}{d^4 X d^3 p} = \frac{40}{9\pi^2} \alpha \alpha_s \mathcal{L} \left(f_q(\mathbf{p}) \right) (I_g + I_q)$$

Quark \longrightarrow BMSS Scenario
Distribution \longrightarrow Dynamical Lattice Simulations

Occupation in the lattice

Gluon Distribution $\rightarrow f_g(\tau, p_\perp, p_z) = \frac{1}{\alpha_s} (Q_s \tau)^\alpha f_S(p_\perp, (Q_s \tau)^\gamma p_z)$

with $f_S(p_\perp, p_z) = f_0 \frac{Q_s}{p_\perp} e^{-\frac{1}{2} \left(\frac{p_z}{\sigma_0} \right)^2} W_r(p_\perp - Q_s, r)$

and $W_r(p_\perp - Q_s, r) = \theta(Q_s - p_\perp) + \theta(p_\perp - Q_s) e^{-\frac{1}{2} \left(\frac{p_\perp - Q_s}{r Q_s} \right)^2}$

Extension

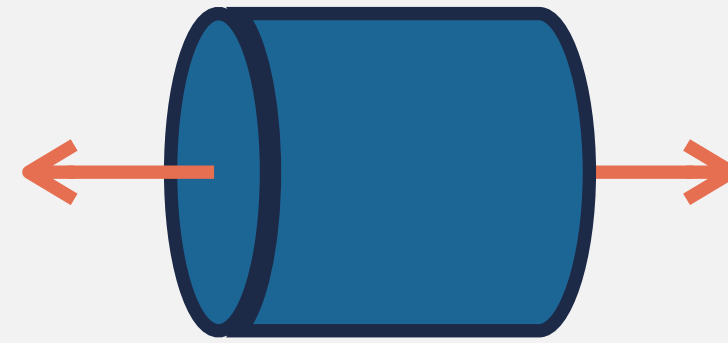
Quark Distribution

Hard dipole approximation $\rightarrow f_q(\tau, p_\perp, p_z) = \alpha_s f_g(\tau, p_\perp, p_z)$

* Q.Stat. kick in outside the region of interest.

ASSUMPTIONS

Bjorken Expansion



$$u = (\cosh \eta, u_x, u_y, \sinh \eta)$$



Transverse Translation
Invariance

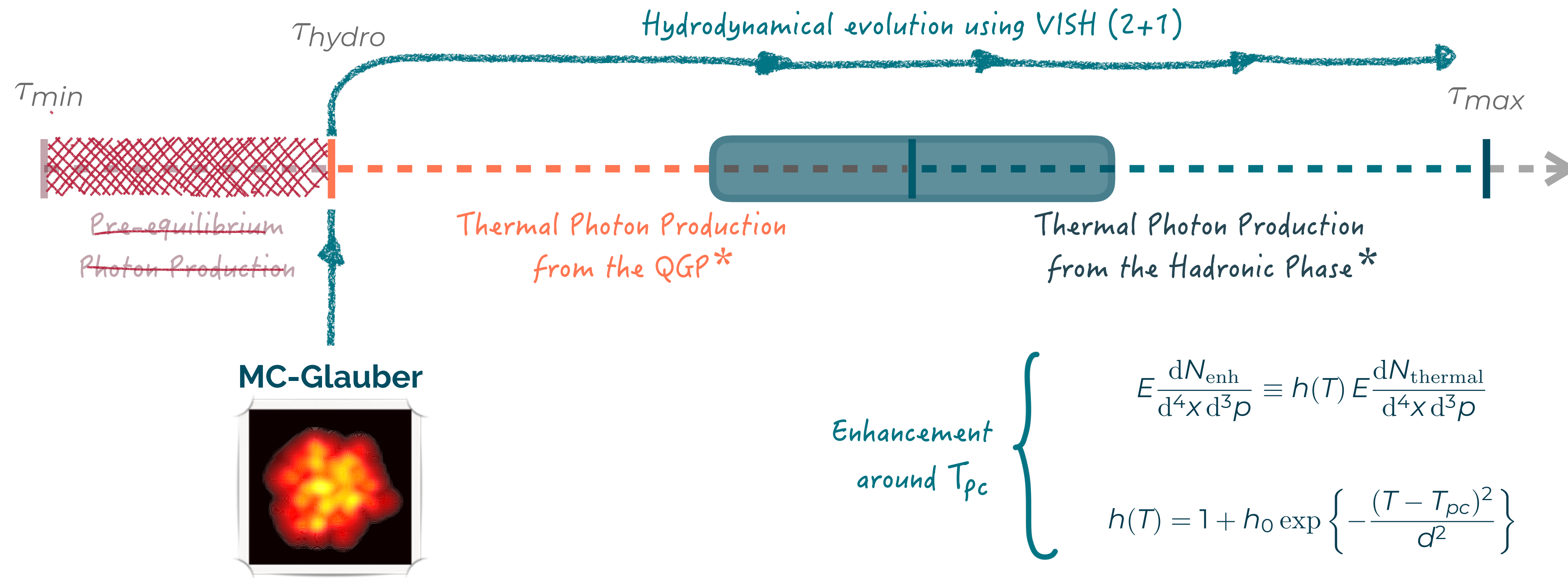
Weak coupling limit

$$\alpha_s \rightarrow 0$$

Photons from pseudocritical enhancement

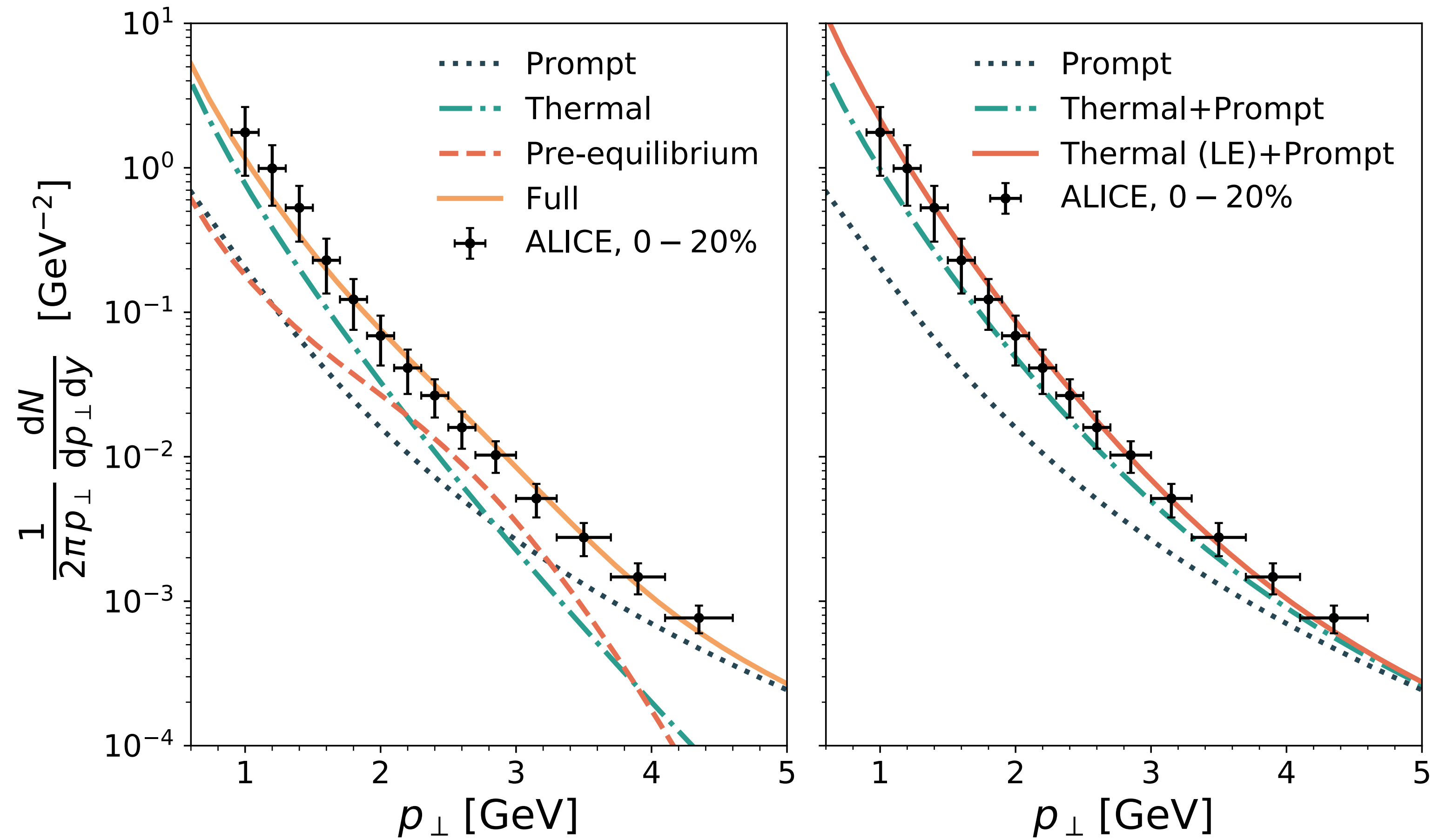
(Non-perturbative partonic enhancement)

The model

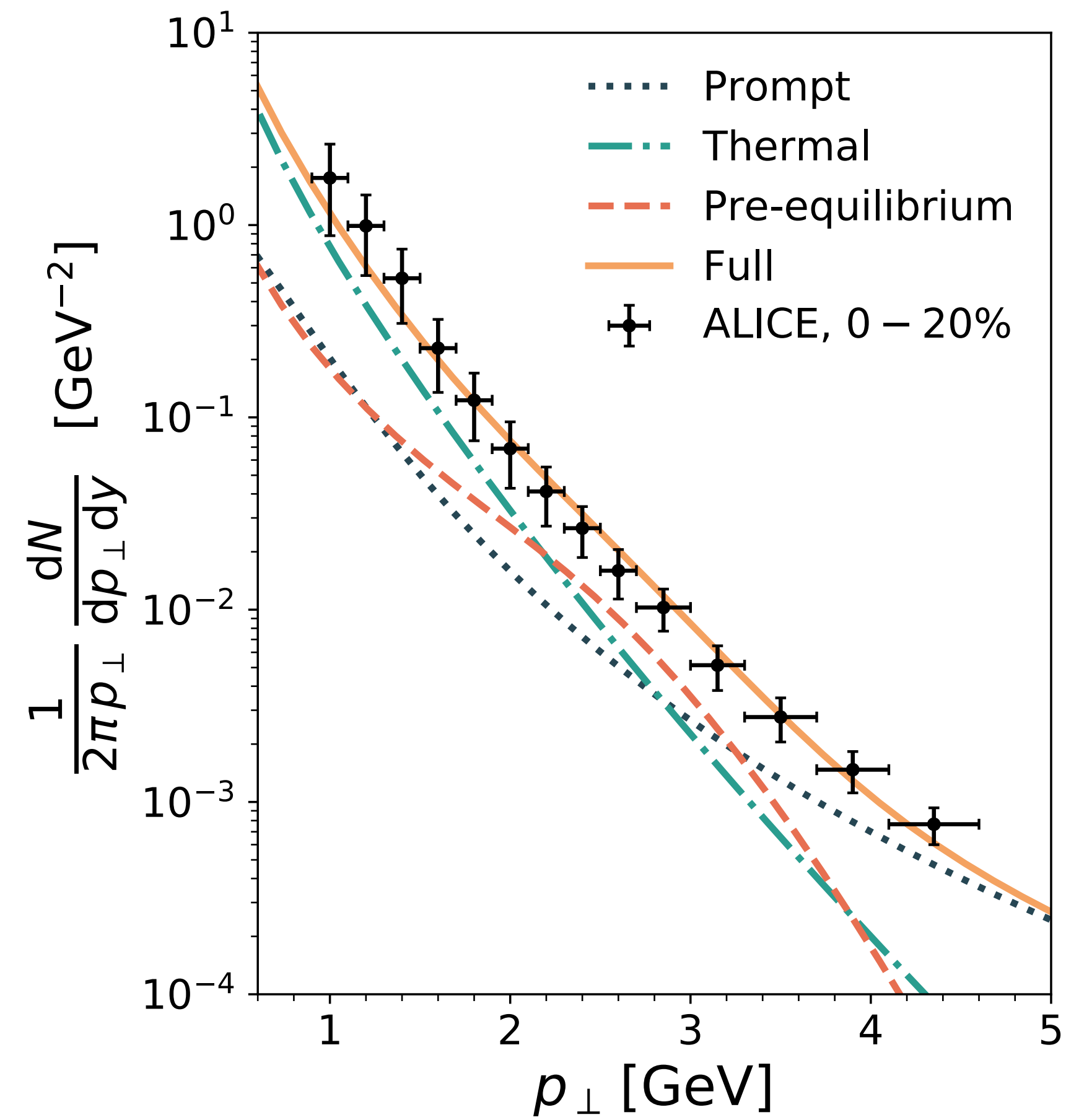


* Inspired by: H. van Hees, *et al*, Nucl. Phys. A933, 256 (2015)

Direct photon yield



How to disentangle a long exposure picture?



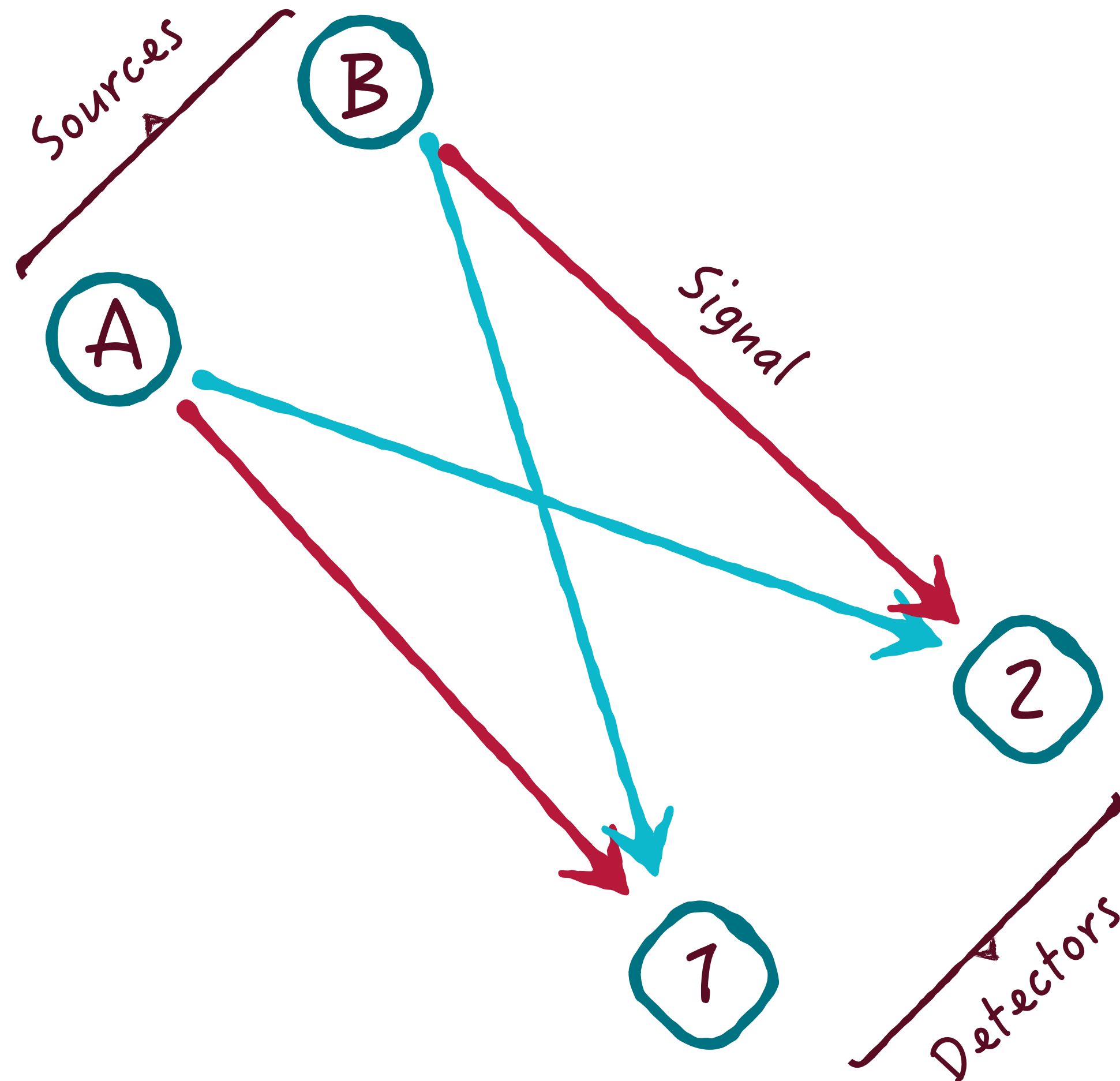
HBT

(Hanbury Brown-Twiss correlations)

“IF THE RADIATION RECEIVED AT TWO PLACES IS MUTUALLY COHERENT, THEN THE FLUCTUATION IN THE INTENSITY OF THE SIGNALS RECEIVED AT THOSE TWO PLACES IS ALSO CORRELATED”

→ *Robert Hanbury Brown*

HBT - What are they?



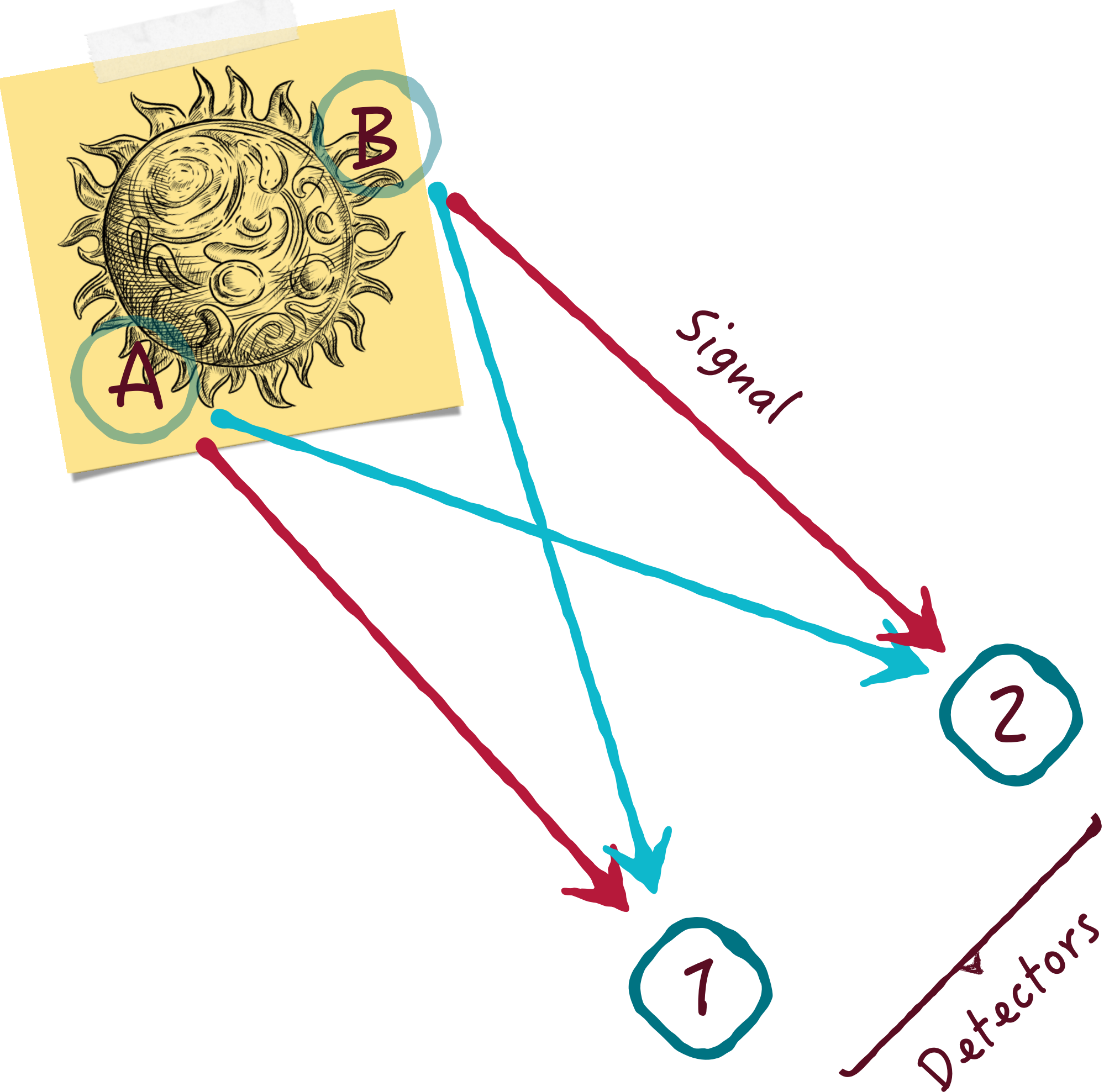
Intensity Interferometry

The distance between two sources using interference at the level of intensity

Main Object : HBT Correlator

$$C \sim \frac{\langle I_1 I_2 \rangle}{\langle I_1 \rangle \langle I_2 \rangle} \rightarrow \frac{\langle N_1 N_2 \rangle}{\langle N_1 \rangle \langle N_2 \rangle}$$

HBT - What are they?



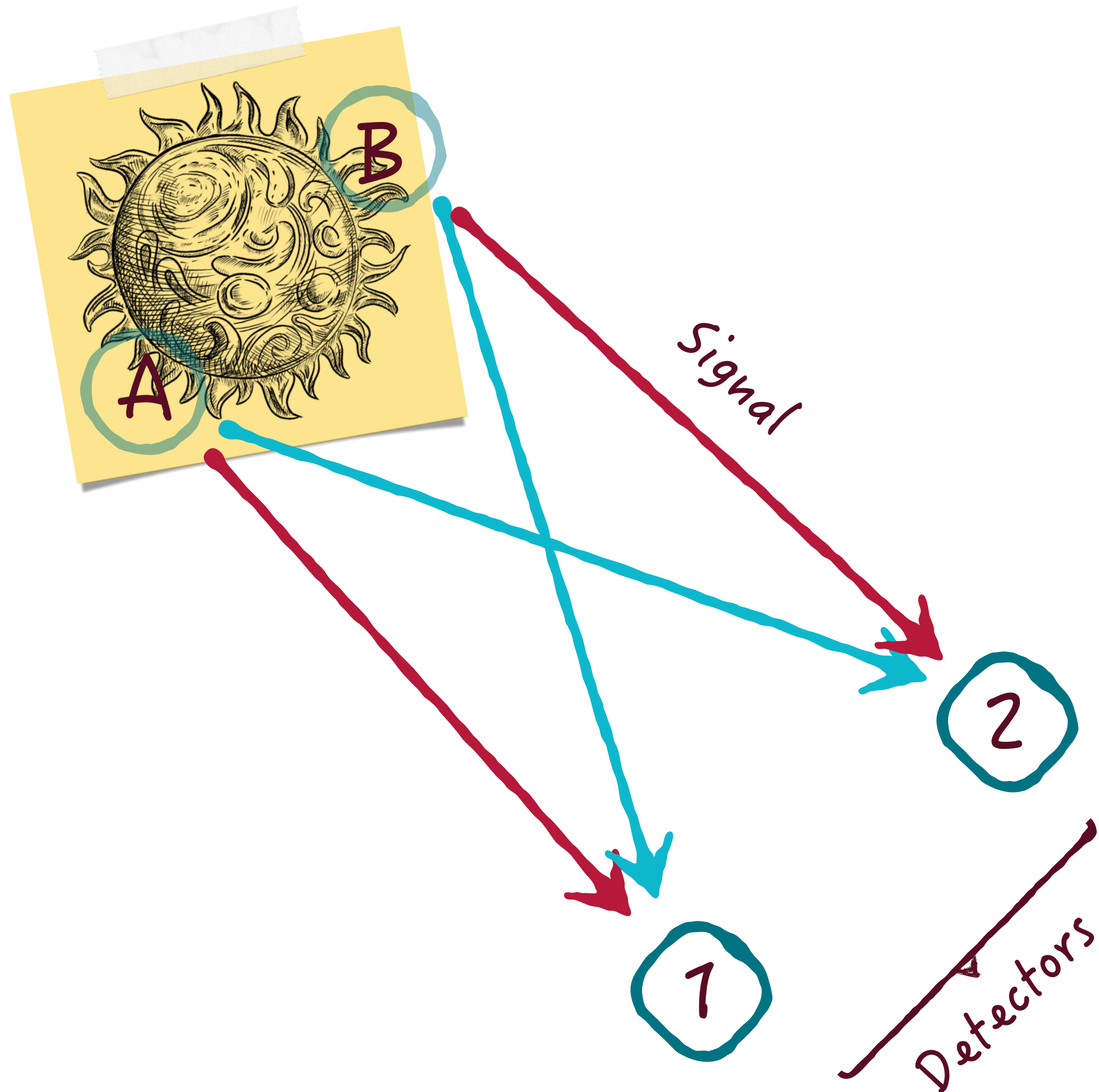
A little bit of History

Used to measure the size of astronomical light sources.

→ Cassiopeia A and Cygnus A

How?

HBT - What are they?



A little bit of History

Used to measure the size of astronomical light sources.

Cassiopeia A and Cygnus A

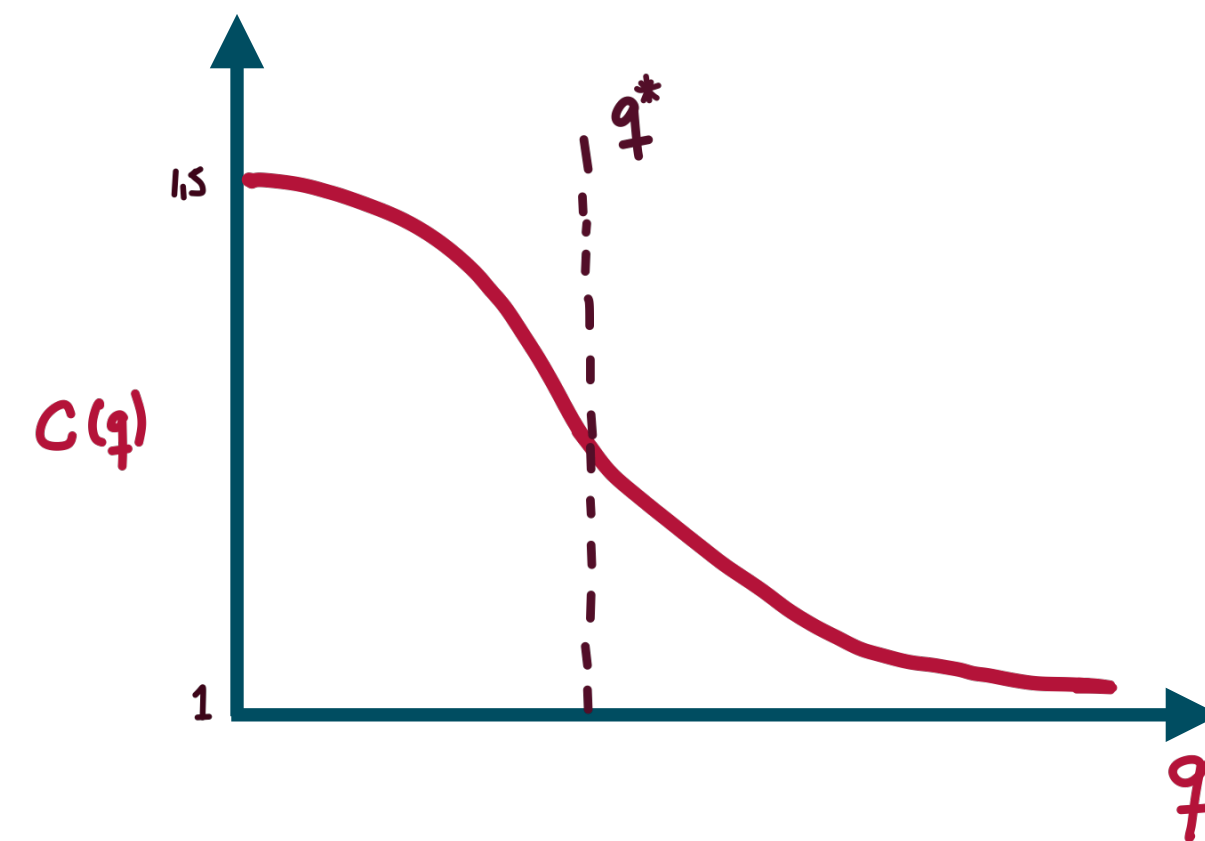
How?

$$\delta x \delta p \gg 2\pi\hbar$$

Photons behave classical

$$\delta x \delta p \lesssim 2\pi\hbar$$

Photons behave quantum



$$\delta x_{max} \sim 2R$$

Quantum effects start at

$$q^* = \frac{\pi\hbar}{R}$$

HBT - What are they (for us)?

Main Object : HBT Correlator

$$C \sim \frac{\langle I_1 I_2 \rangle}{\langle I_1 \rangle \langle I_2 \rangle} \rightarrow \frac{\langle N_1 N_2 \rangle}{\langle N_1 \rangle \langle N_2 \rangle}$$

In the context of Particle Physics

Two-particle correlations

Fermions	Anticorrelate
Bosons	Correlate

$$C(p_1, p_2) = \frac{E_{p_1} E_{p_2} \frac{dN}{d^3 p_1 d^3 p_2}}{E_{p_1} \frac{dN}{d^3 p_1} E_{p_2} \frac{dN}{d^3 p_2}} = 1 + \frac{1}{d} \frac{|S(q, K)|^2}{S(0, p_1) S(0, p_2)}$$

$S(q, K)$

Fourier Transform of
 d (Spin) Degeneracy

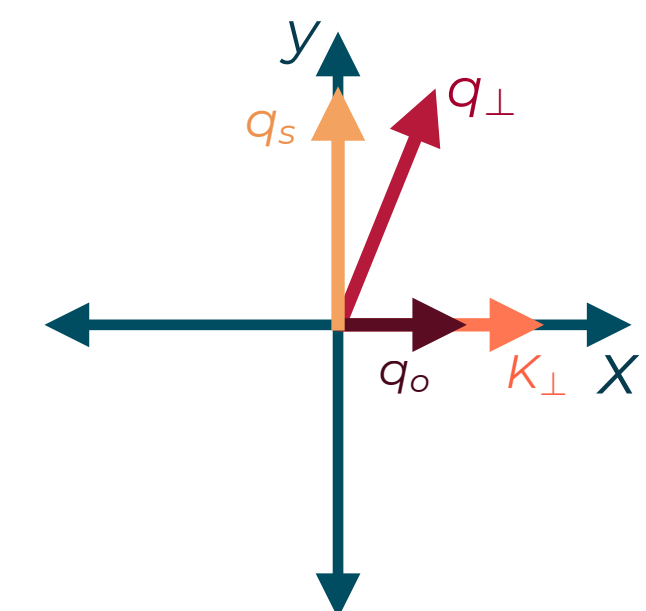
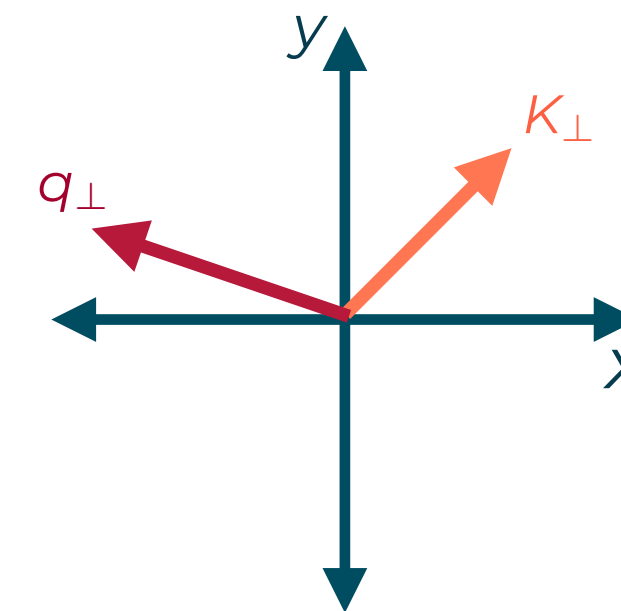
$S(x, K)$



Emission
Function
(rate)

$$K^\mu = (K^0, K_\perp, 0, K^z)$$

$$q^\mu = (q^0, q_o, q_s, q_l),$$



HBT - What are they (for us)?

Main Object : HBT Correlator

$$C \sim \frac{\langle I_1 I_2 \rangle}{\langle I_1 \rangle \langle I_2 \rangle} \rightarrow \frac{\langle N_1 N_2 \rangle}{\langle N_1 \rangle \langle N_2 \rangle}$$

In the context of Particle Physics

Two-particle correlations

Fermions	Anticorrelate
Bosons	Correlate

$$C(p_1, p_2) = 1 + \frac{1}{d} \frac{|S(q, K)|^2}{S(0, p_1) S(0, p_2)} \sim 1 + \frac{1}{2} \exp \left[-q_i R^{ij} q_j \right],$$

where $R_{ij}(K) = \begin{bmatrix} R_{o}^2 & R_{os}^2 & R_{ol}^2 \\ R_{os}^2 & R_s^2 & R_{sl}^2 \\ R_{ol}^2 & R_{sl}^2 & R_l^2 \end{bmatrix}$ are the HBT Radii

$$\langle\langle q_i q_j \rangle\rangle = \int d^3q q_i q_j g(q; K) \equiv \frac{1}{2} (R^{-1})_{ij}$$

$$g(q; K) \equiv \frac{C(q, K) - 1}{\int d^3q [C(q, K) - 1]}$$

In this talk, we will only be interested in the diagonal!

A little but important caveat

Stars are relatively close to being "Static Sources"

BUT

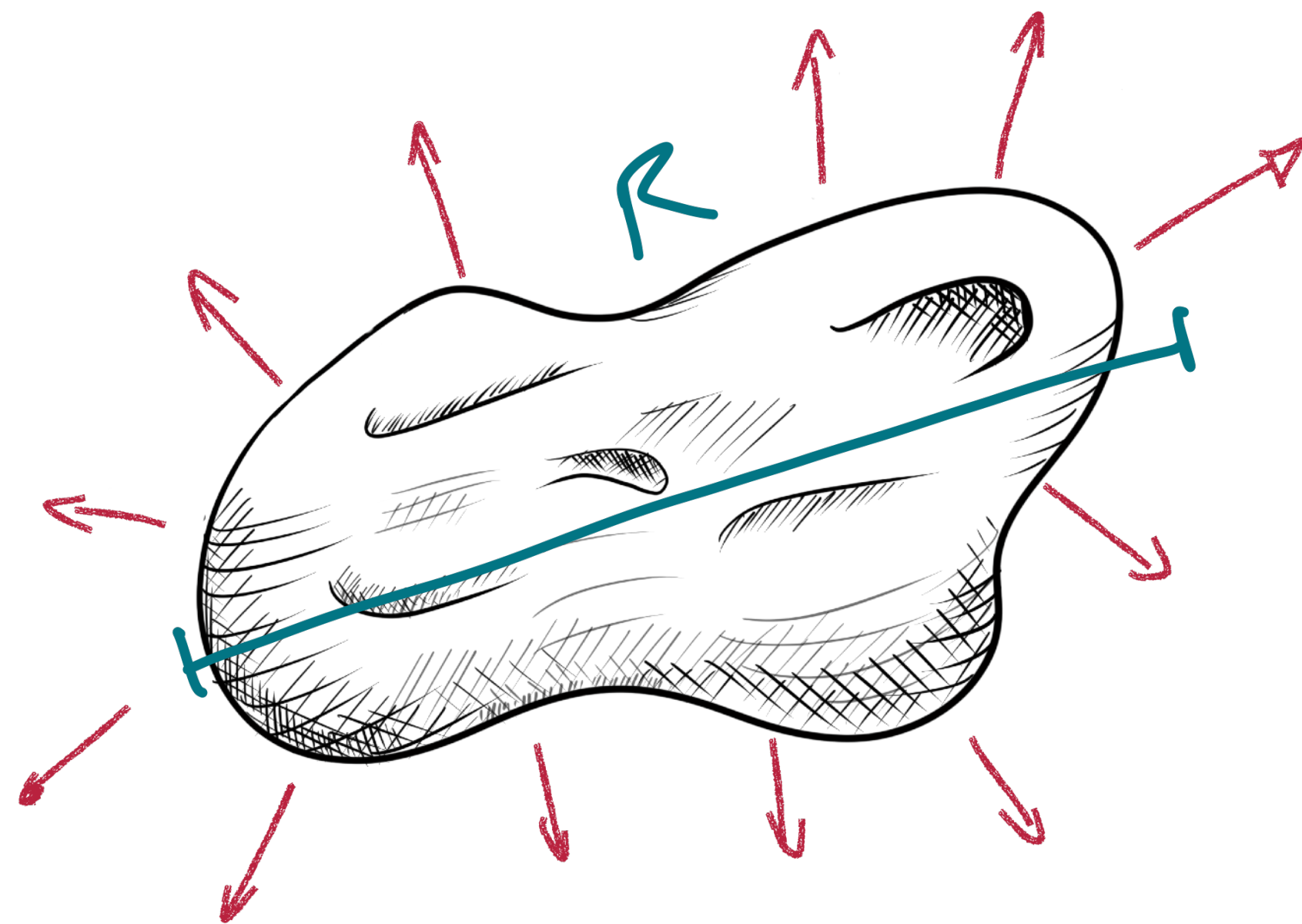
In the context of Particle Physics

Pion Interferometry

Photon Interferometry

Dynamical sources

Radii are more like weighted averages, in fact.



**MAIN POINT
OF MY TALK**

**Use photon HBT not to
extract source sizes, but
to cross-compare models**

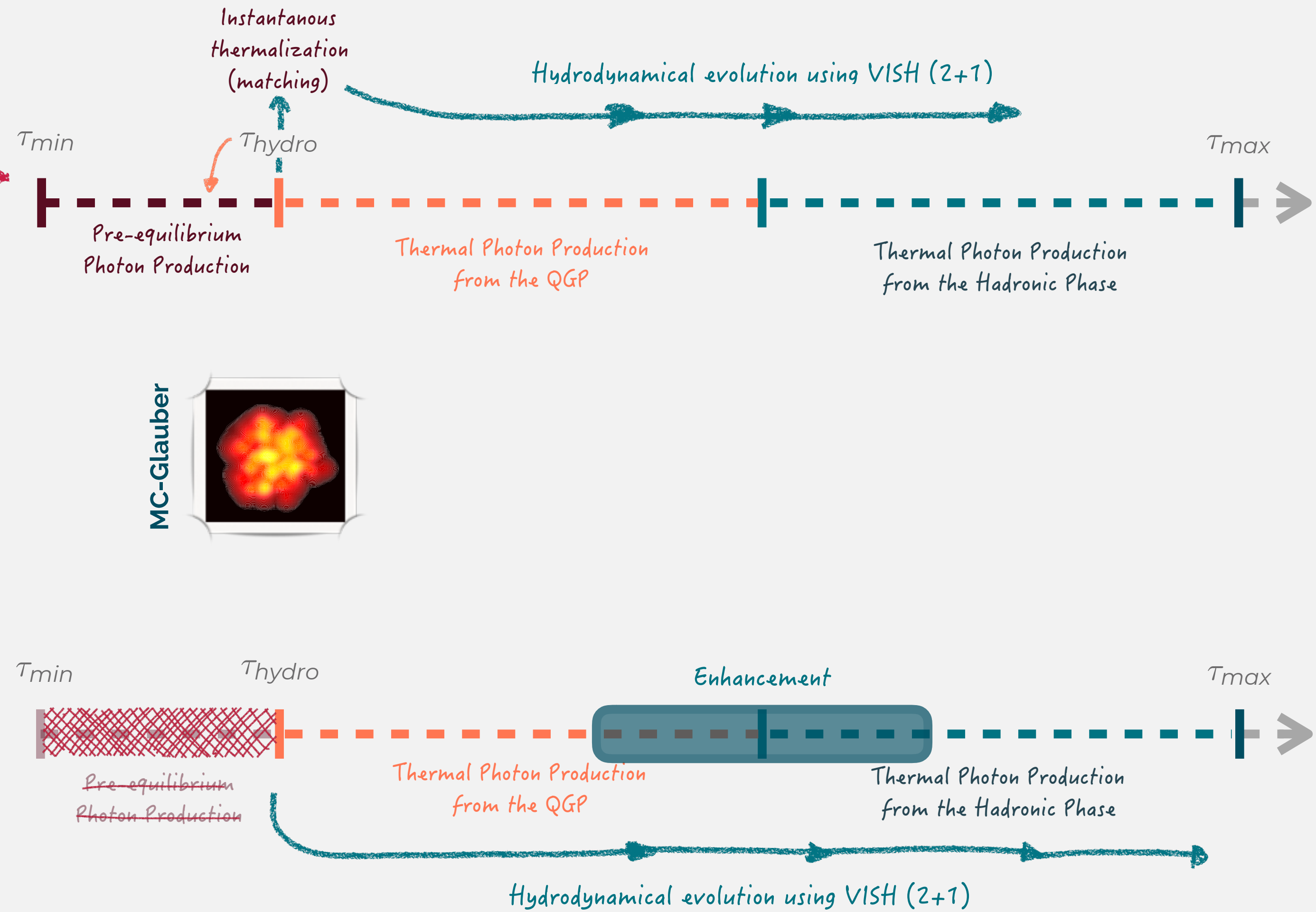
Some Results

(Hanbury Brown-Twiss correlations)

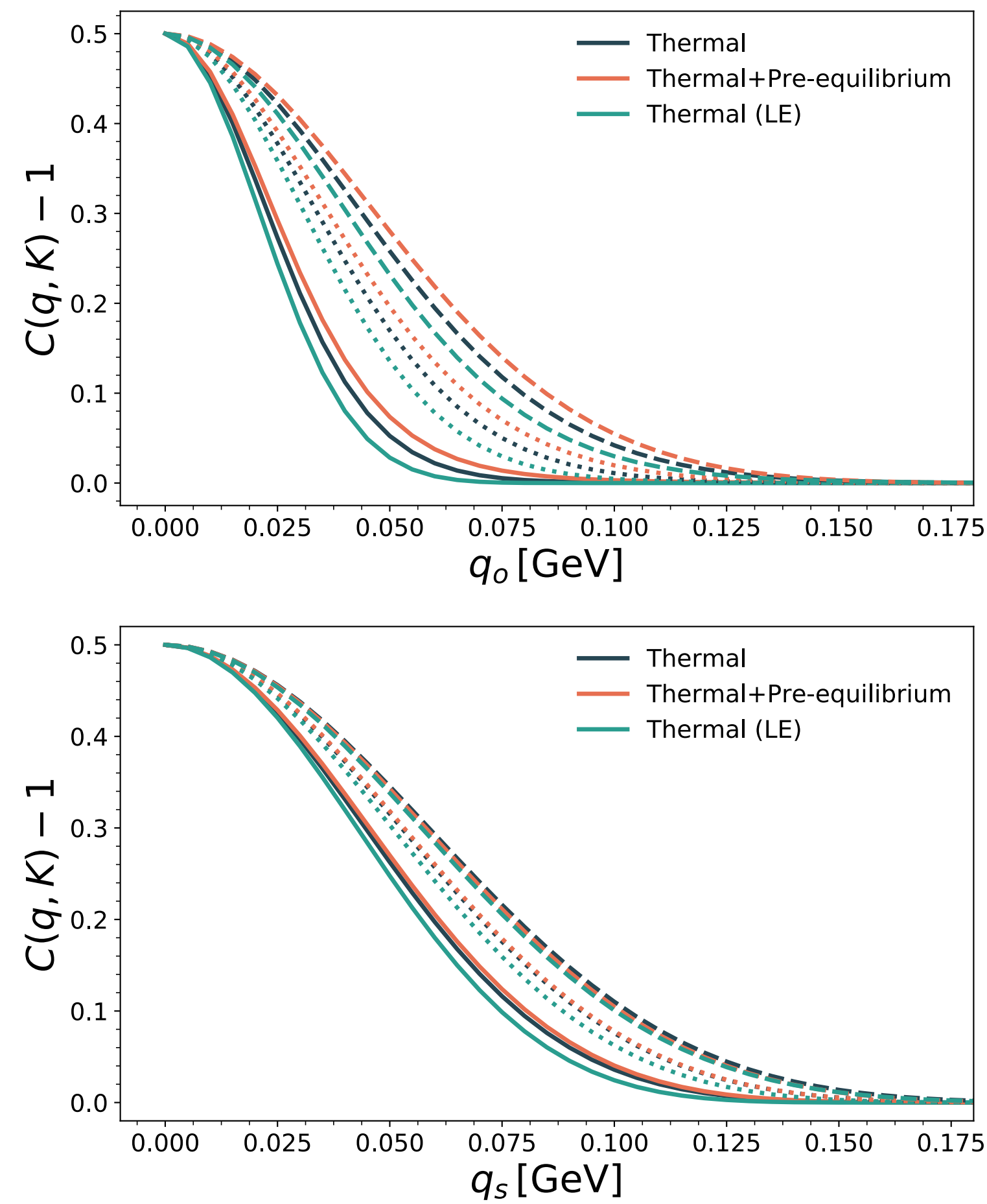
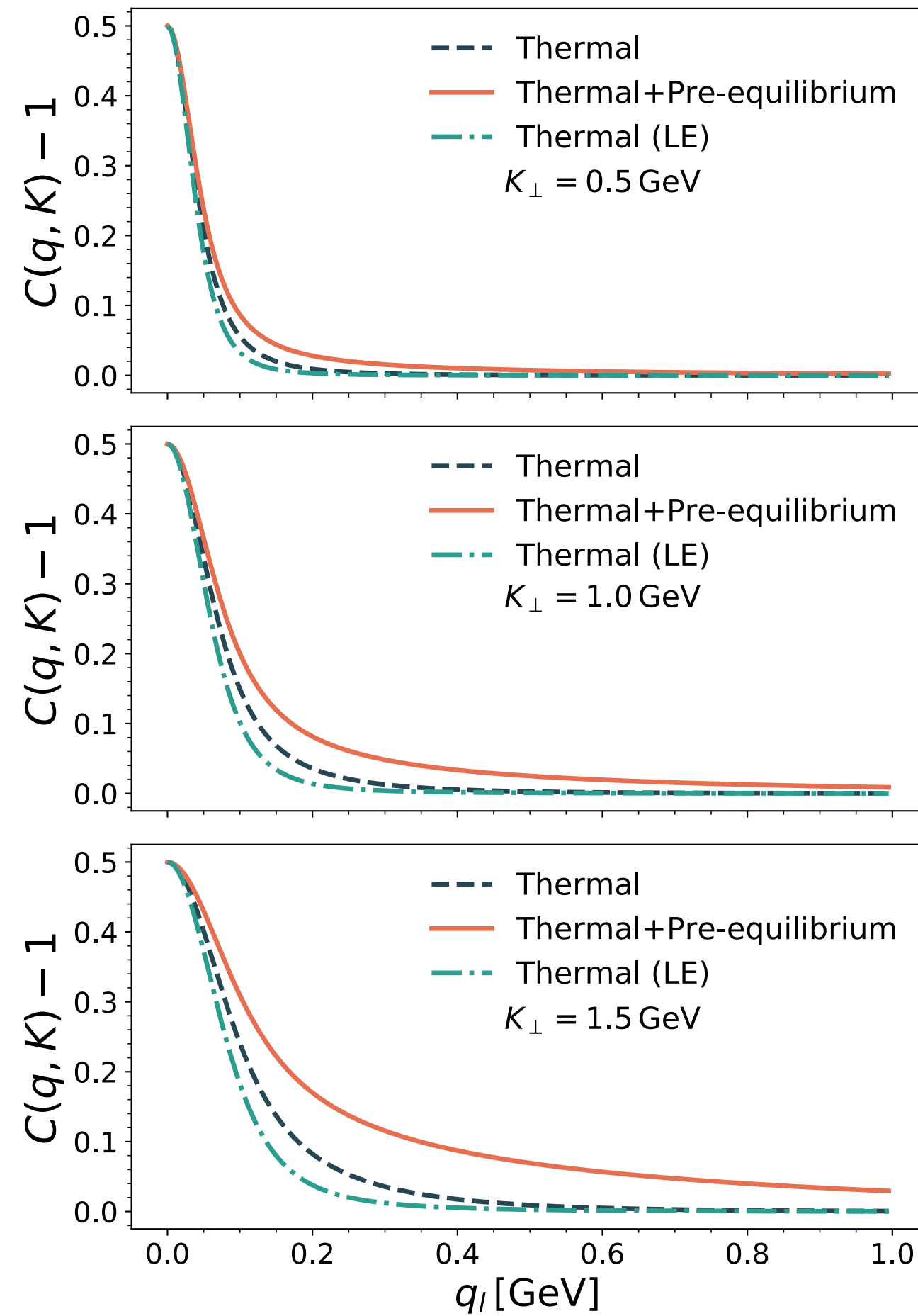
The Models



RENÉ MAGRITTE
EMPIRE OF LIGHT



The HBT-Correlator



$$C(p_1, p_2) = 1 + \frac{1}{d} \frac{|S(q, K)|^2}{S(0, p_1)S(0, p_2)}$$

Longitudinal direction affected the most by the inclusion of the sources.

Non-gaussianities strong at early times, thanks to Bjorken expansion

Early-times production reduces effective radii, while late times increase them.

The HBT-Radii

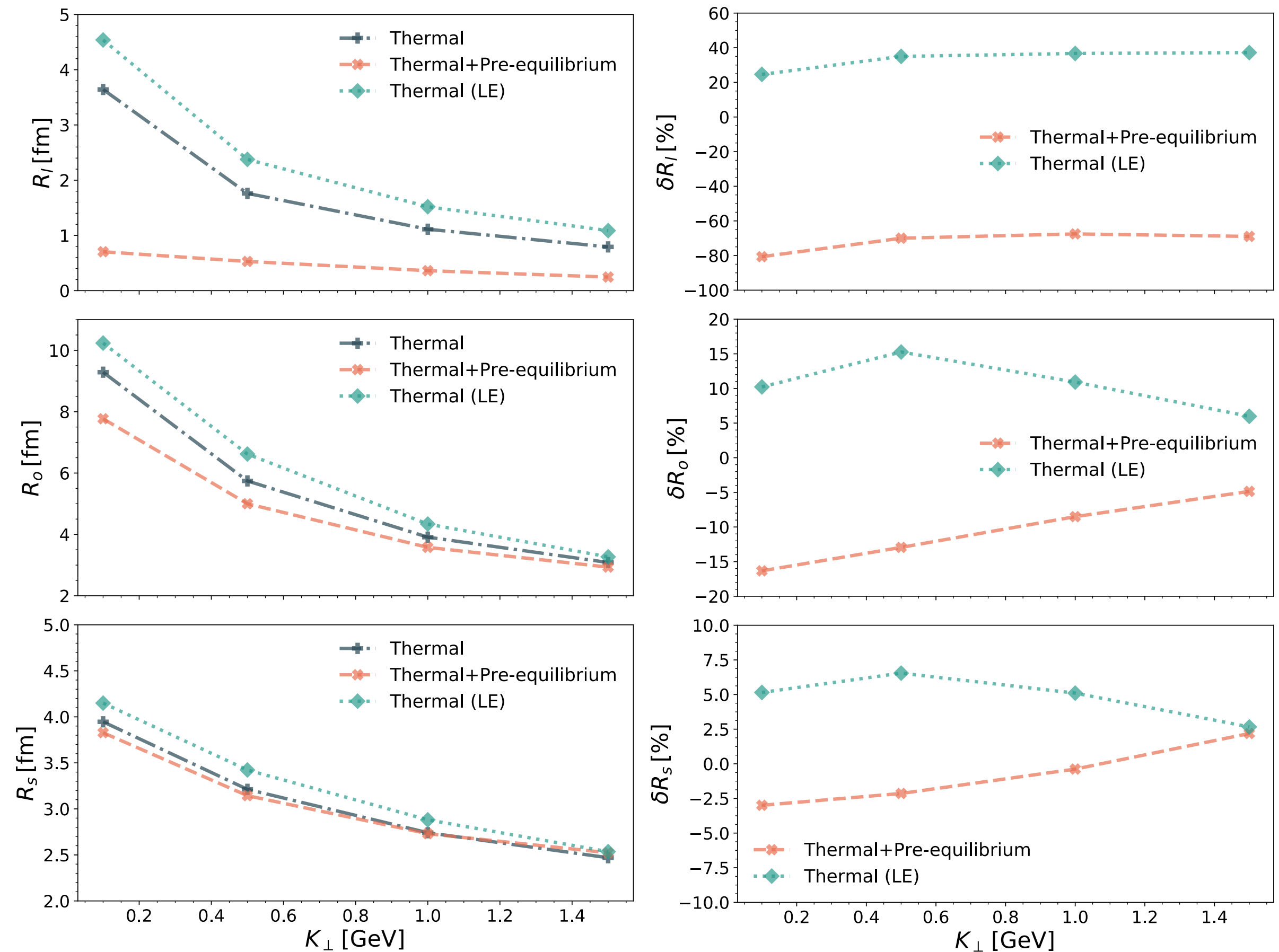
$$\langle\langle q_i q_j \rangle\rangle = \int d^3q q_i q_j g(q; K) \equiv \frac{1}{2} (R^{-1})_{ij}$$

$$g(q; K) \equiv \frac{C(q, K) - 1}{\int d^3q [C(q, K) - 1]}$$

Longitudinal direction affected the most by the inclusion of the sources.

Early-times production reduces effective radii, while late times increase it.

Are these differences enough to measure it?



But wait, there is more!

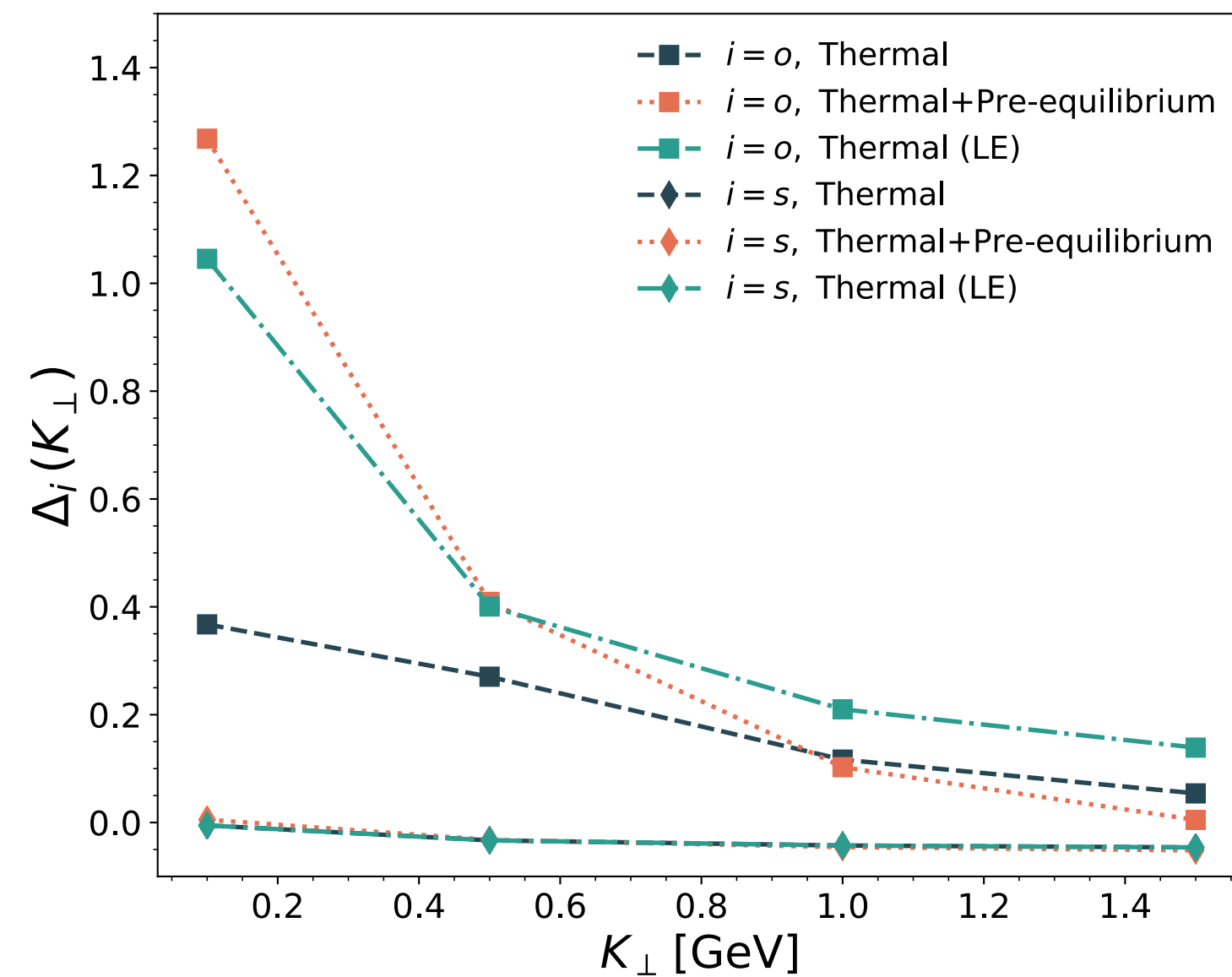
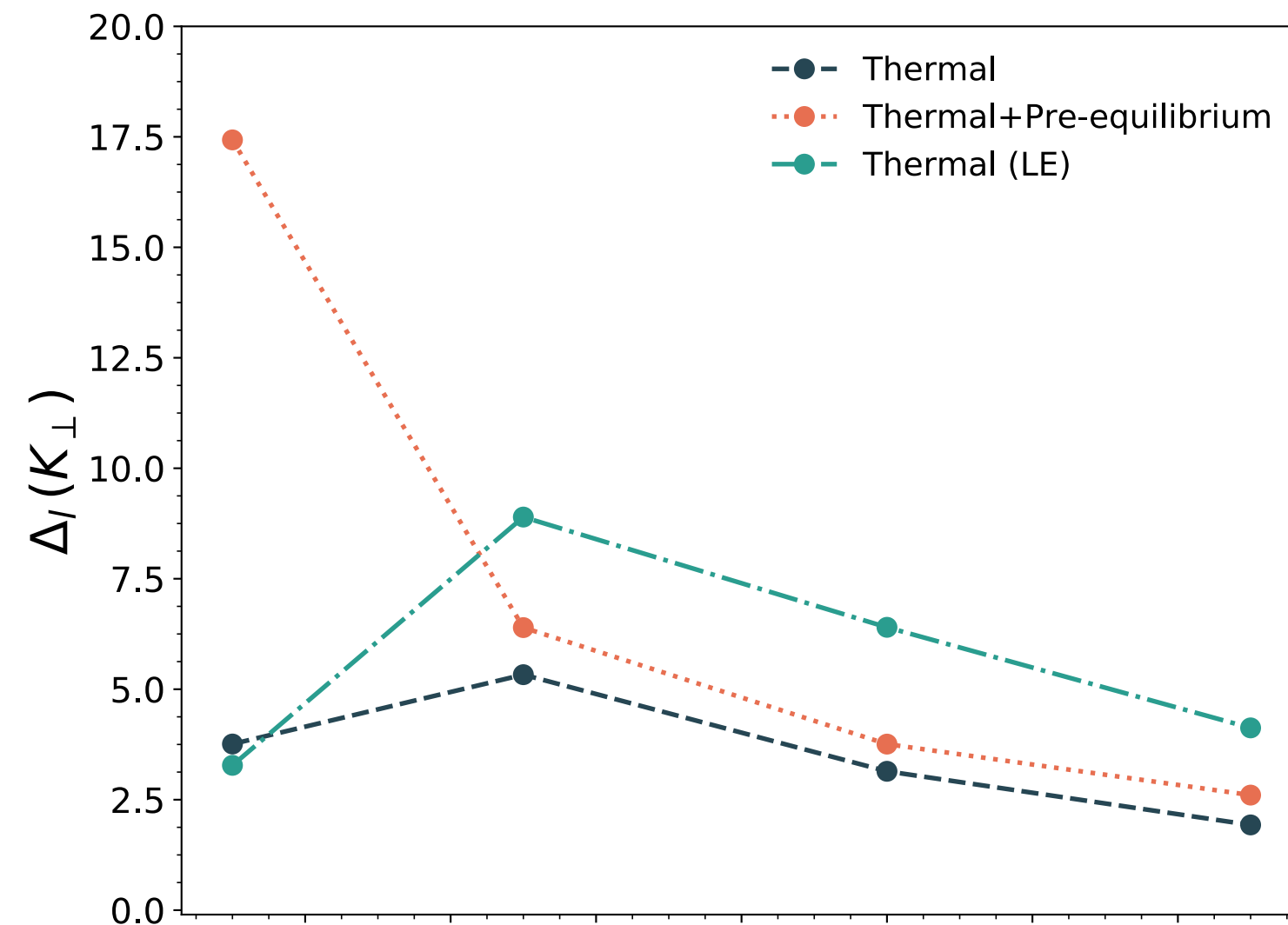
The Normalized Excess Kurtosis

$$\Delta_i = \frac{\langle\langle q_i^4 \rangle\rangle}{3\langle\langle q_i^2 \rangle\rangle^2} - 1$$

Measures “how much non-Gaussian is a distribution

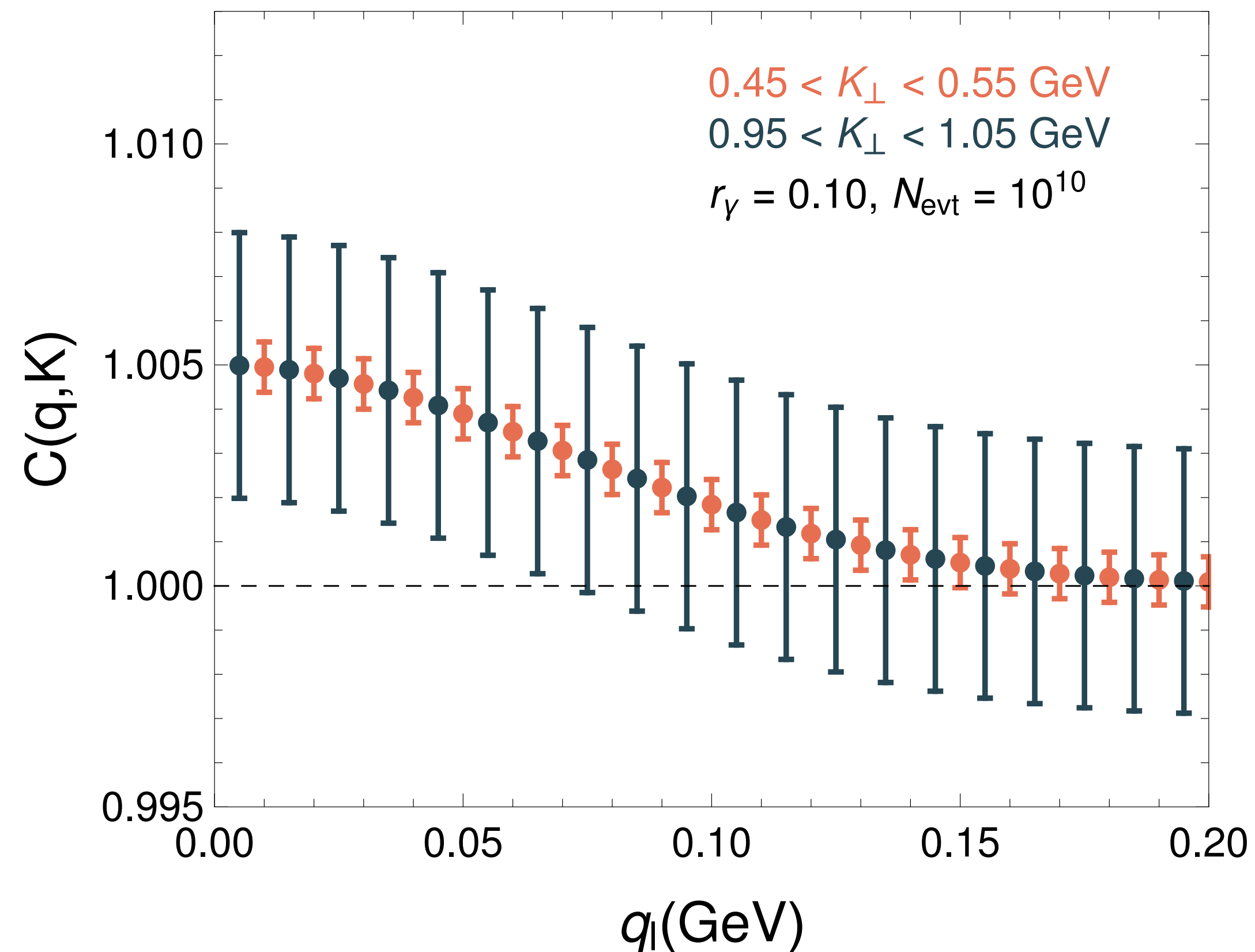
Early-times non-Gaussianities strong, particularly at small pair momenta.

Very interesting observable, hard to measure.

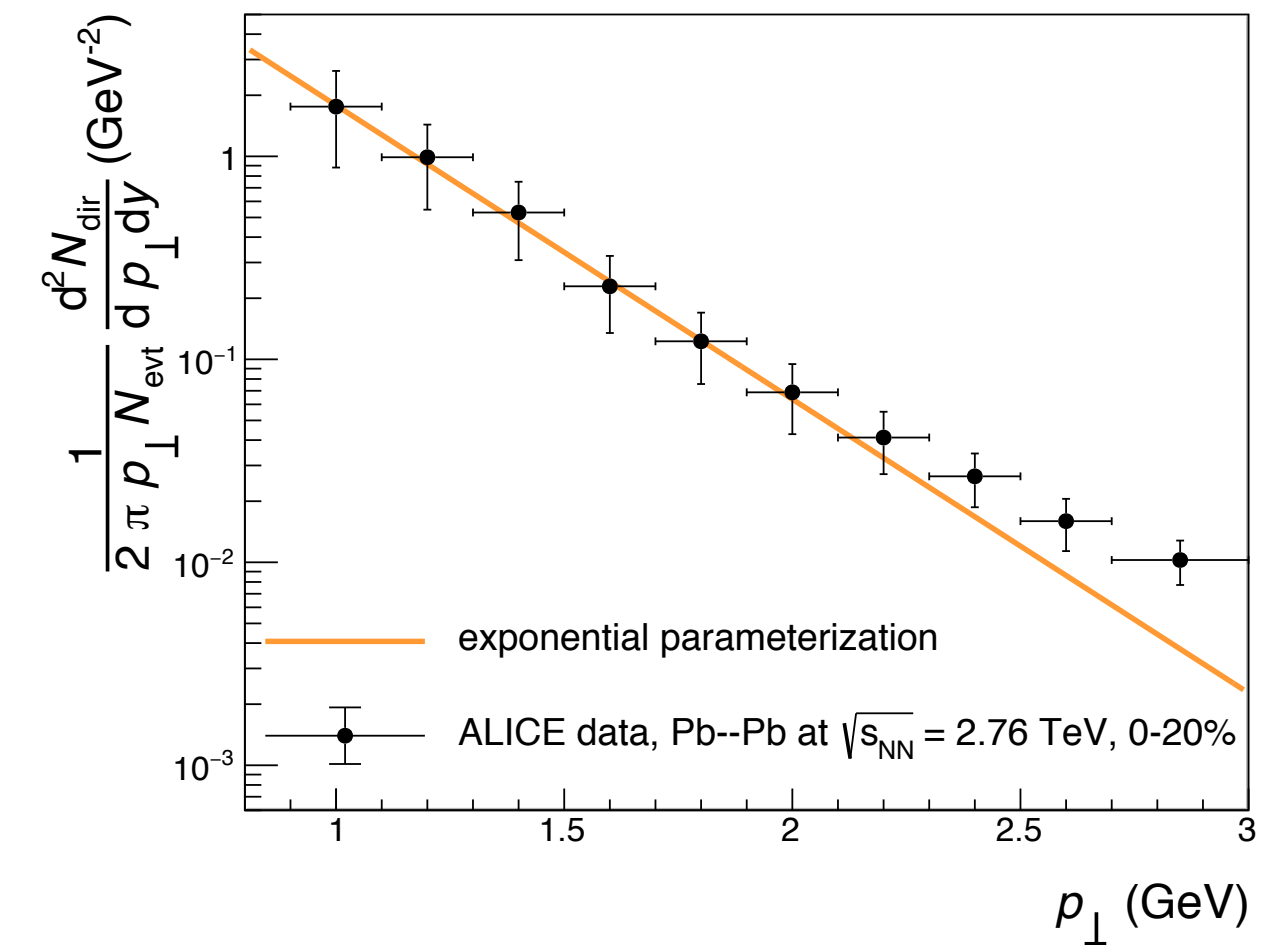


Statistical Model

Effective dilution of the signal!



Exponential distribution



Exponential-like source used as test model.

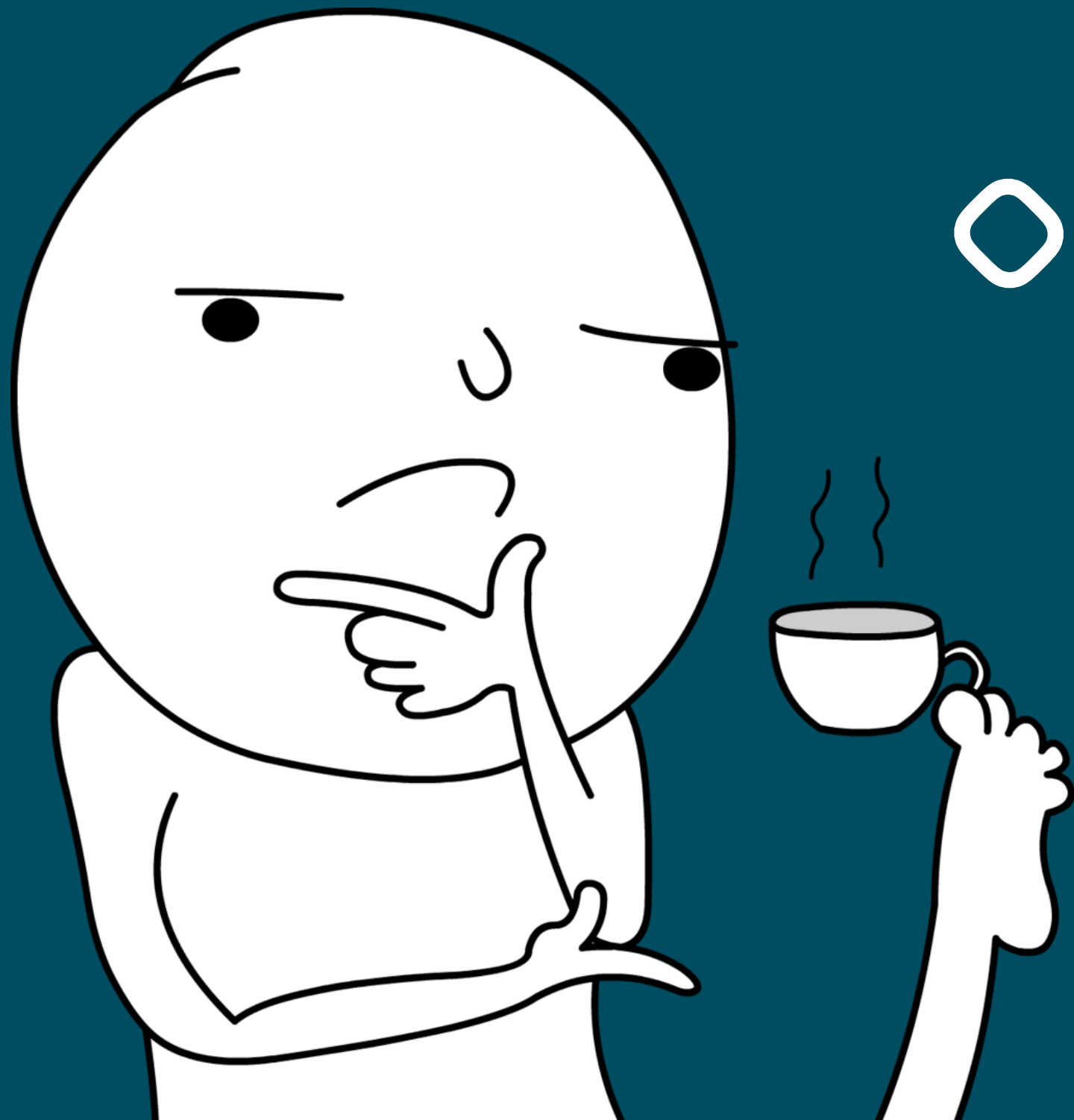
Correlator simulated sampling from source.

Error depends strongly on the average momentum of the pair.

Low momentum pairs statistically significant.

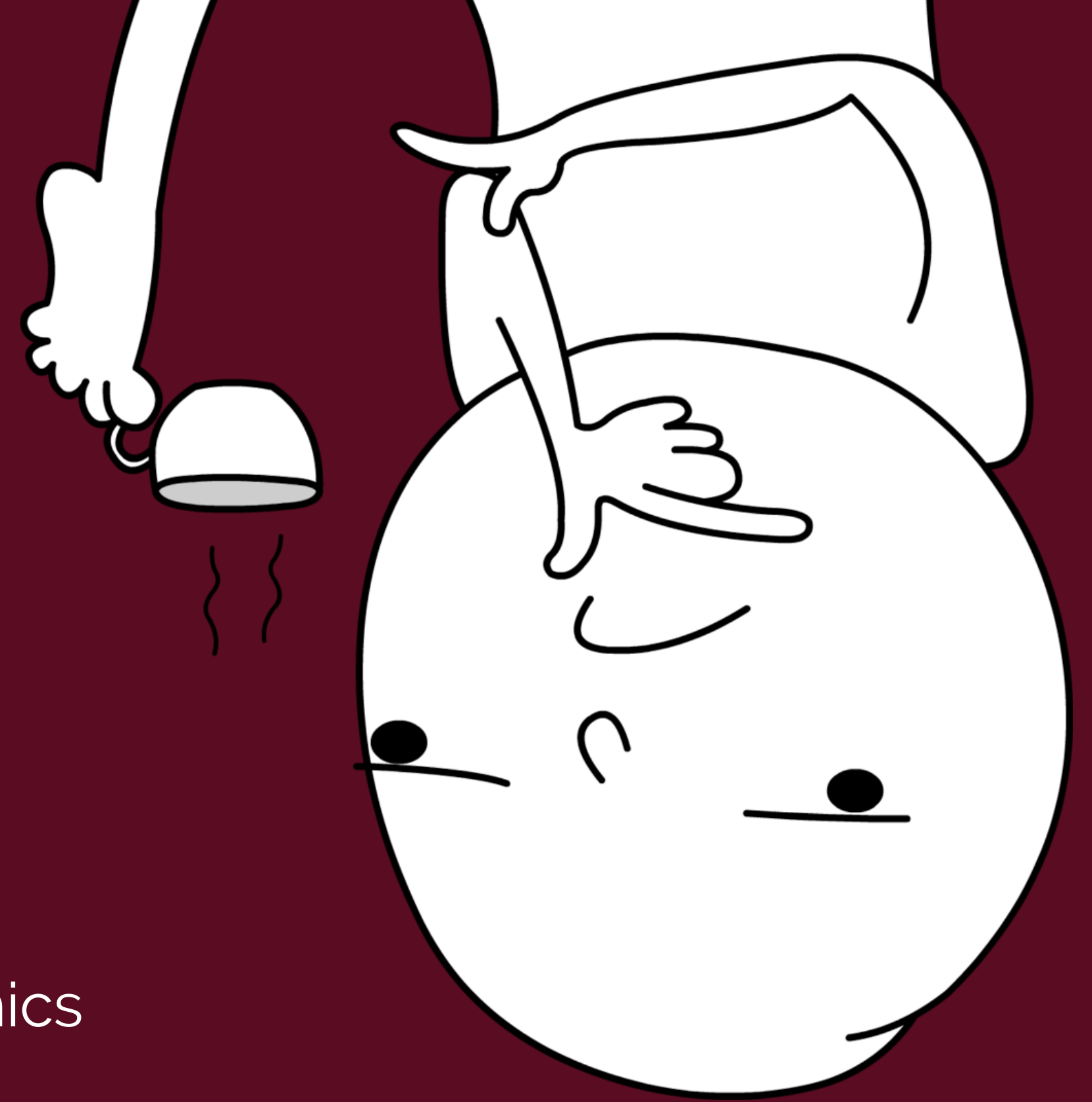
Summary

- Even when the discrepancies of the *Direct Photon Puzzle* are solved, we need to differentiate between the different models in the market.
- Photon correlations are the tool that we need to do so.
- Yes, they are very hard measurements, but not impossible anymore. We should start walking before we run.
- Remember that the endgame here is to untangle the space-time evolution of the medium created in a Heavy Ion Collision.



Outlook

- ◇ Refine existing models to be able to compare against experimental results.
 - ◇ Get better grasp of the enhancement at
 - ◇ Model/Simulate the pre-eq time expansion dynamics
- ◇ Compare new ideas in the HBT framework



Back-up Slides

Bottom-up
thermalization

Bottom-up thermalization

Three
Stages

I. Early Times. 2-2 broadening

$$1 \ll Q\tau \ll \alpha_s^{-3/2}$$

II. Onset of thermalization

$$\alpha_s^{-3/2} \ll Q\tau \ll \alpha_s^{-5/2}$$

III. Mini-jet quenching

$$\alpha_s^{-5/2} \ll Q\tau \ll \alpha_s^{-13/5}$$

I. Early Times: $2 \leftrightarrow 2$ broadening

$$1 \ll Q\tau \ll \alpha_s^{-3/2}$$

$$\frac{dN_g}{d^2p_\perp dy} = \frac{1}{\alpha_s} f\left(\frac{p_\perp}{Q_s}\right)$$

Dominated by
hard gluons

$$\langle p_\perp \rangle \sim Q_s$$

Instabilities freed hard Gluons

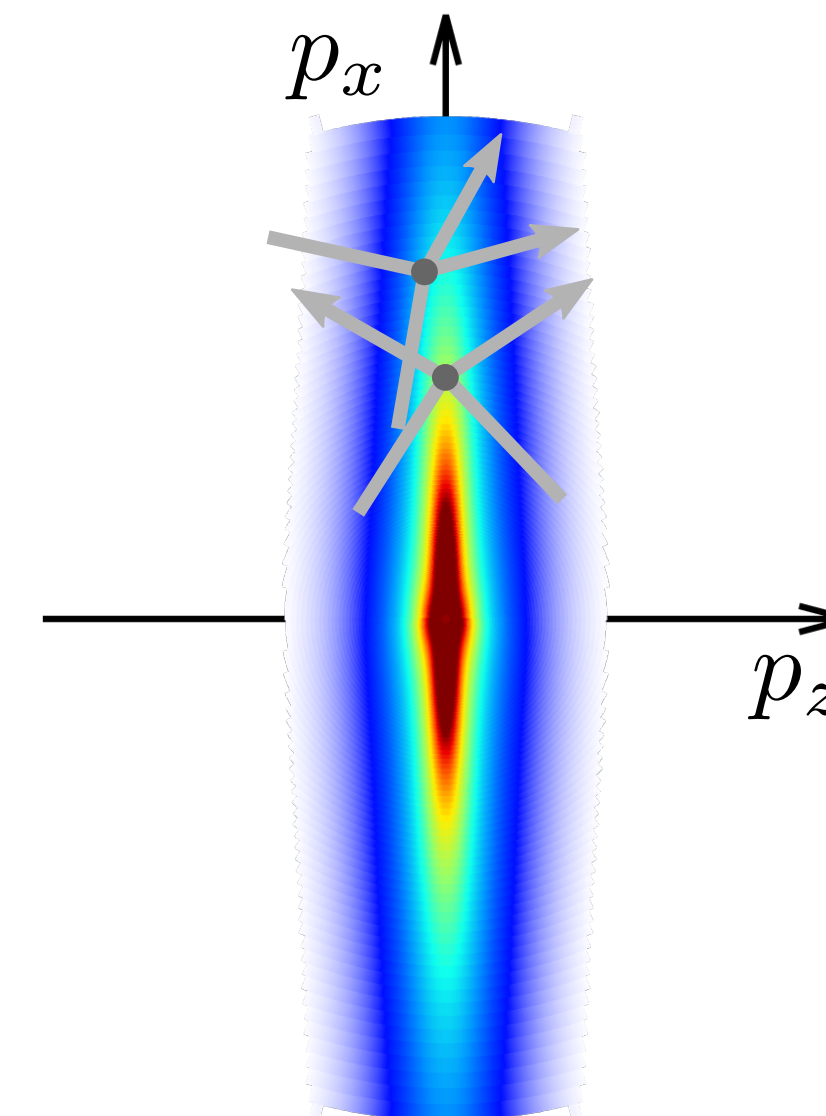
$$n_h \sim \frac{1}{\alpha_s} \frac{Q^3}{Q_s \tau}$$

Hard-hard interactions
dominated by soft exchange

$$m_D^2 \sim \alpha_s \int \frac{d^3p}{p} f_g \sim \frac{Q_s^2}{Q_s \tau}$$

Longitudinal broadening

$$\langle p_z \rangle \sim Q_s (Q_s \tau)^{-1/3}$$



From Kurkela et al.
Phys.Rev. C99 (2019) no.3, 034910

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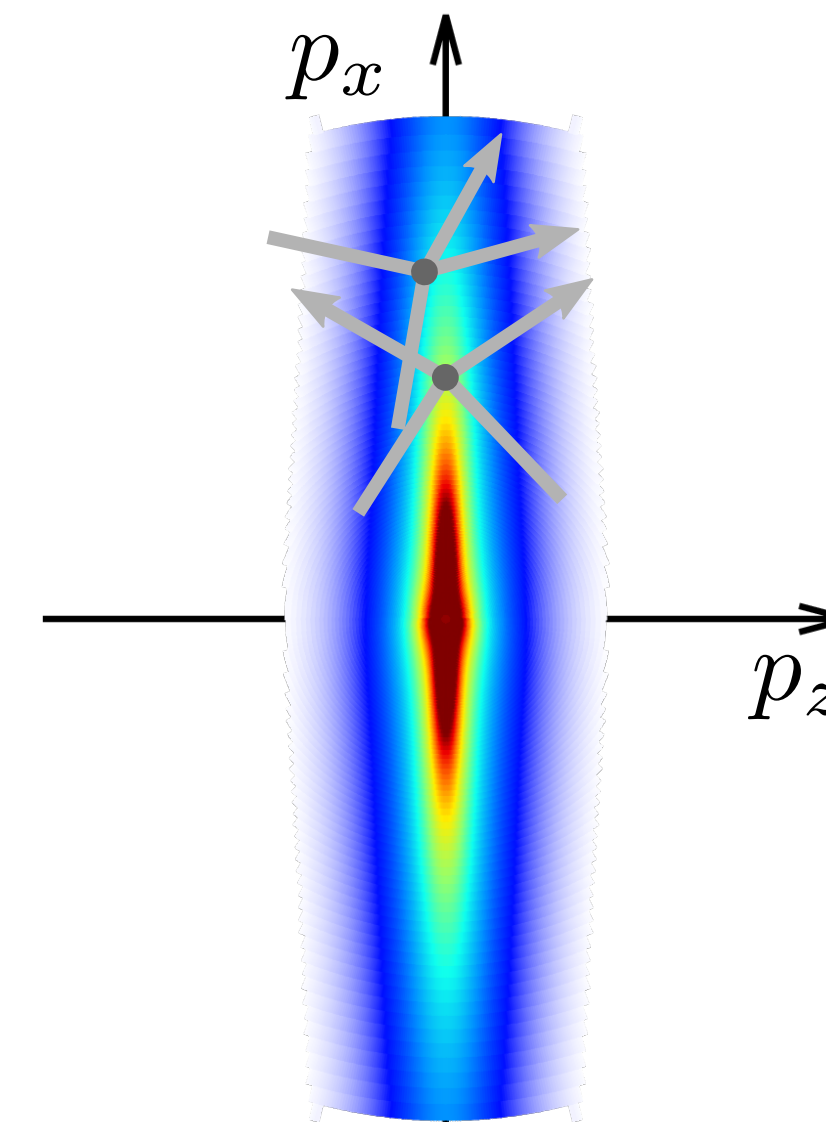
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II. Onset of thermalization

$$\alpha_s^{-3/2} \ll Q\tau \ll \alpha_s^{-5/2}$$

Occupation of Hard Gluon drops below unity

$$\frac{dN_g}{d^2p_\perp dy} \sim (Q_s\tau)^{-2/3}$$

Dominated by hard gluons

$$\langle p_\perp \rangle \sim Q_s$$

Soft gluons dominate the screening mass

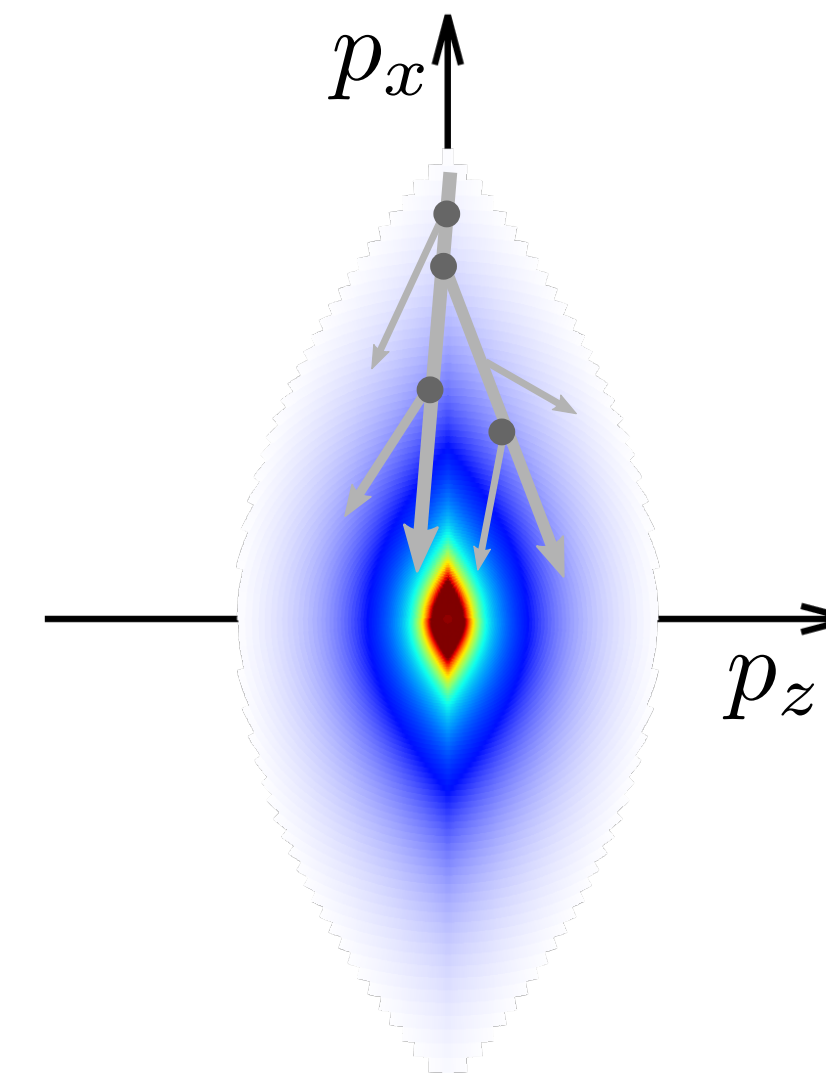
$$n_s \sim \frac{\alpha_s^{1/4} Q_s^3}{(Q_s\tau)^{1/2}}$$

Longitudinal momentum

$$\langle p_z \rangle \sim \sqrt{\alpha_s} Q_s$$

Screening mass is dominated by soft sector

$$m_D^2 \sim \frac{\alpha_s^{3/4} Q_s}{(Q_s\tau)^{1/2}}$$



From Kurkela et al.
Phys.Rev. C99 (2019) no.3, 034910

III. Mini-jet Quenching

$$\alpha_s^{-5/2} \ll Q\tau \ll \alpha_s^{-13/5}$$

Soft sector thermalizes

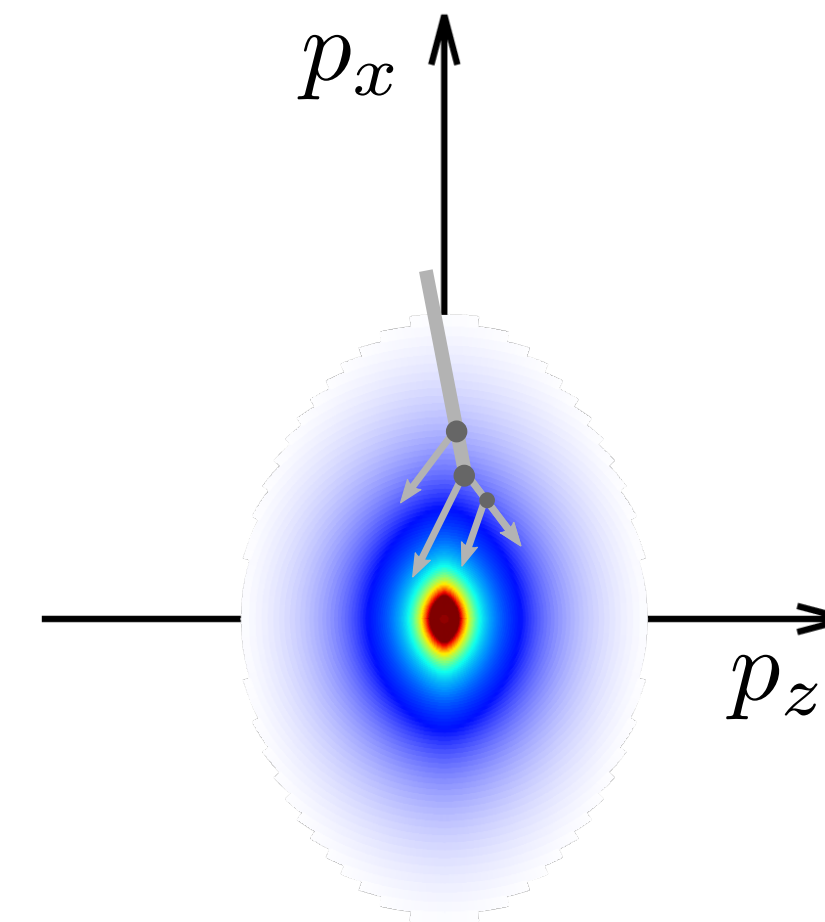
Acts like a bath

Hard sector loses energy to soft bath

Temperature rises as
 $T = c_T \alpha_s^3 Q_s(Q_s \tau)$

Dominated by soft gluons
 $\langle p_\perp \rangle \sim m_D$

$$\tau_{th} \sim c_{eq} \alpha_s^{-13/5} Q_s^{-1}$$
$$T_{th} \sim c_T c_{eq} \alpha_s^{2/5} Q_s$$



From Kurkela et al.
Phys.Rev. C99 (2019) no.3, 034910

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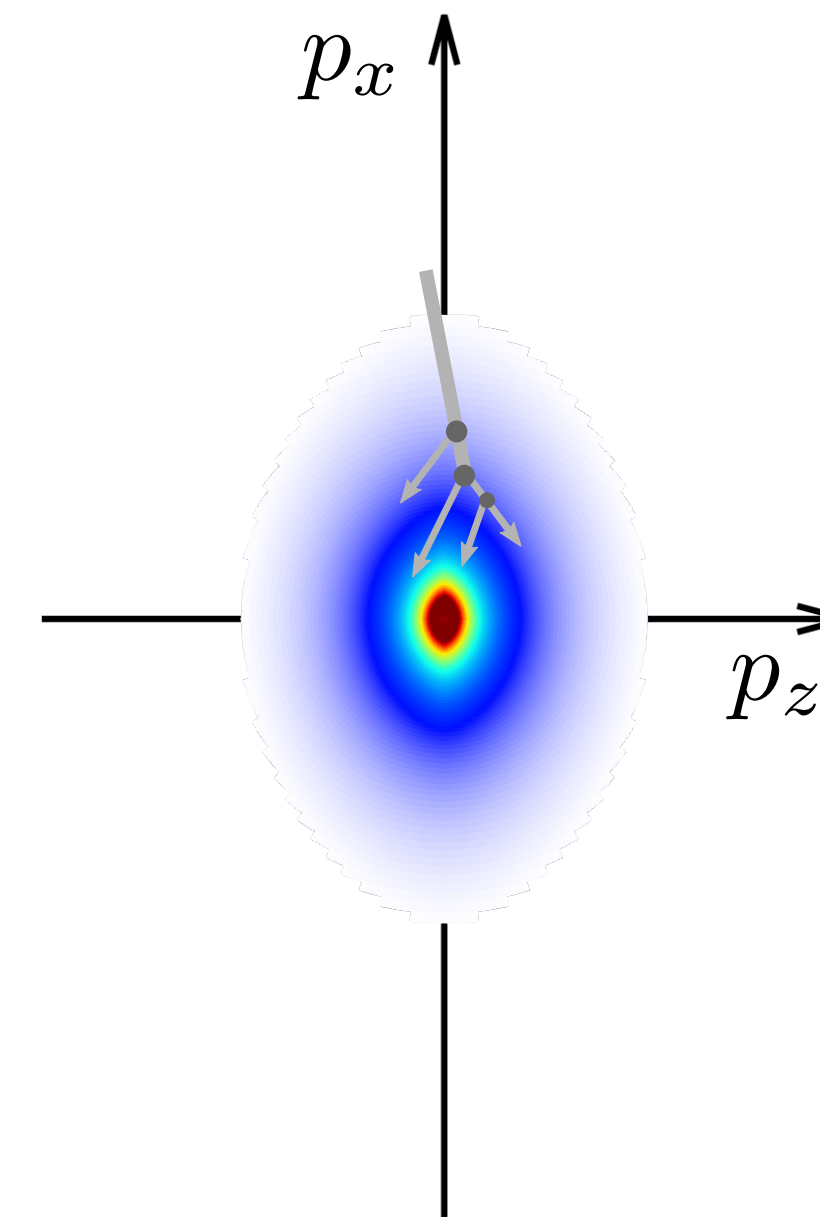
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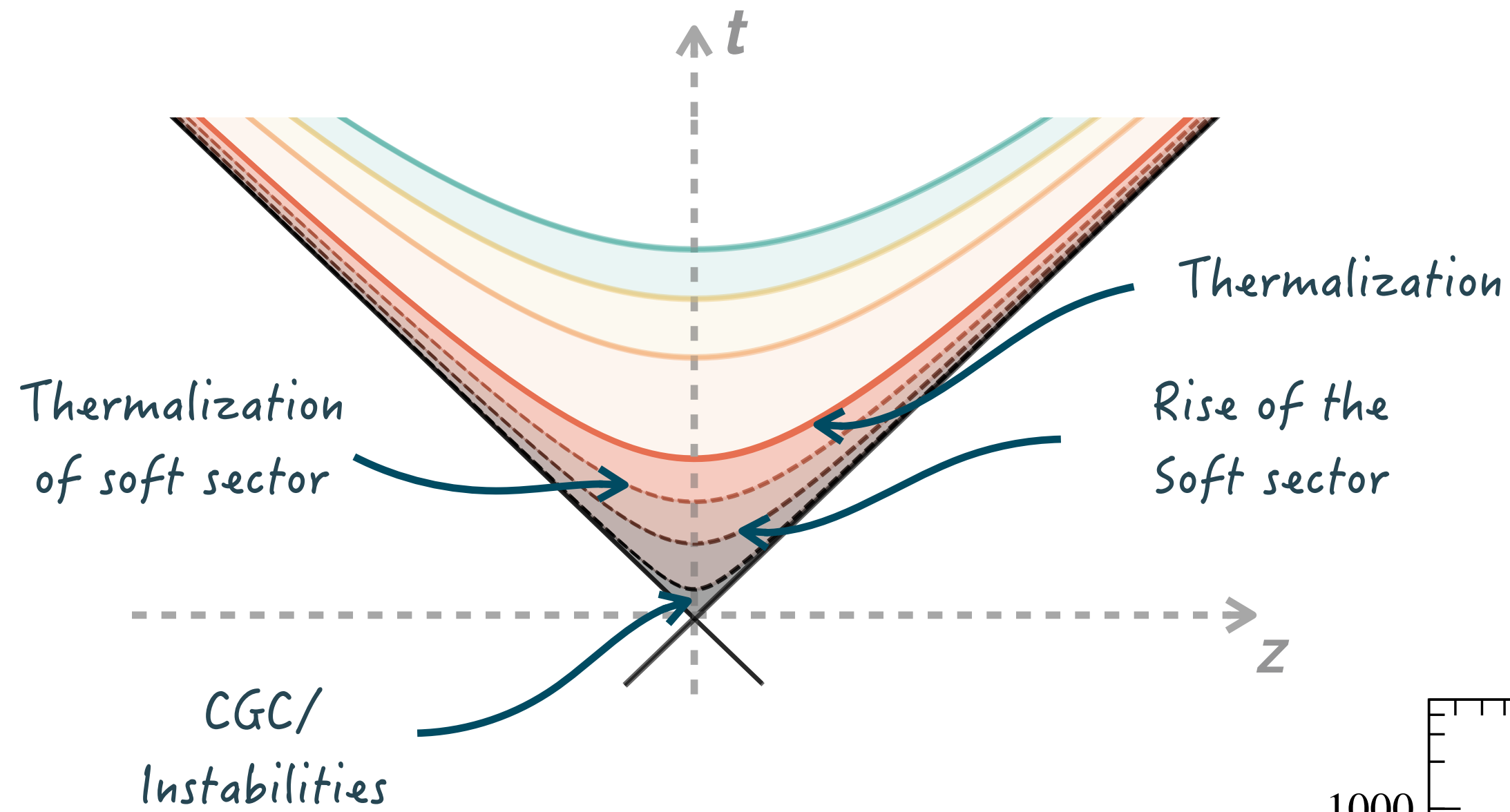
$$T_{th} \sim c_T c_{eq} \alpha_s^{2/5} Q_S$$



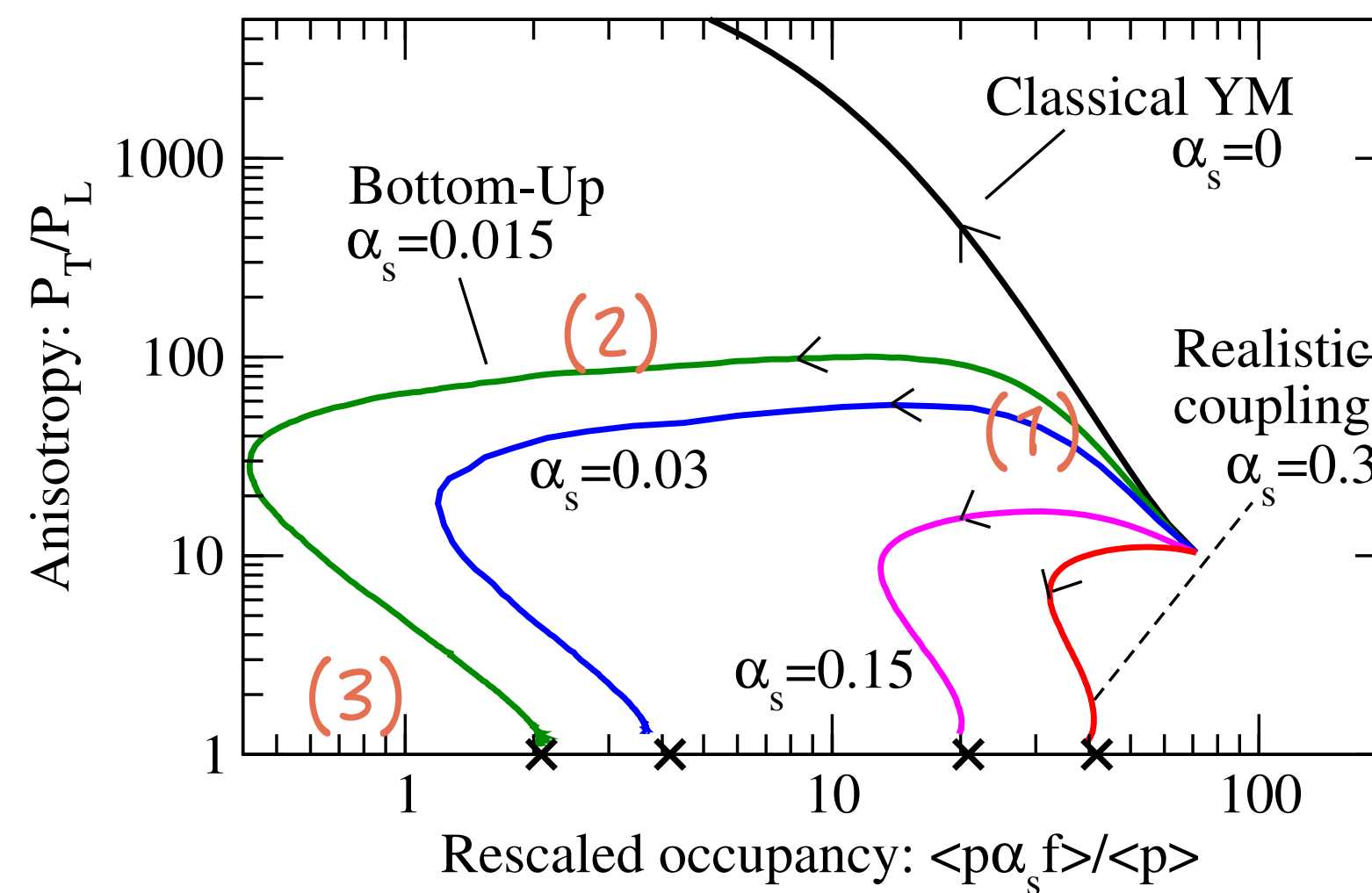
From Kurkela et al.
 Phys.Rev. C99 (2019) no.3, 034910

The Standard Model of Heavy Ion Collisions

(revisited)

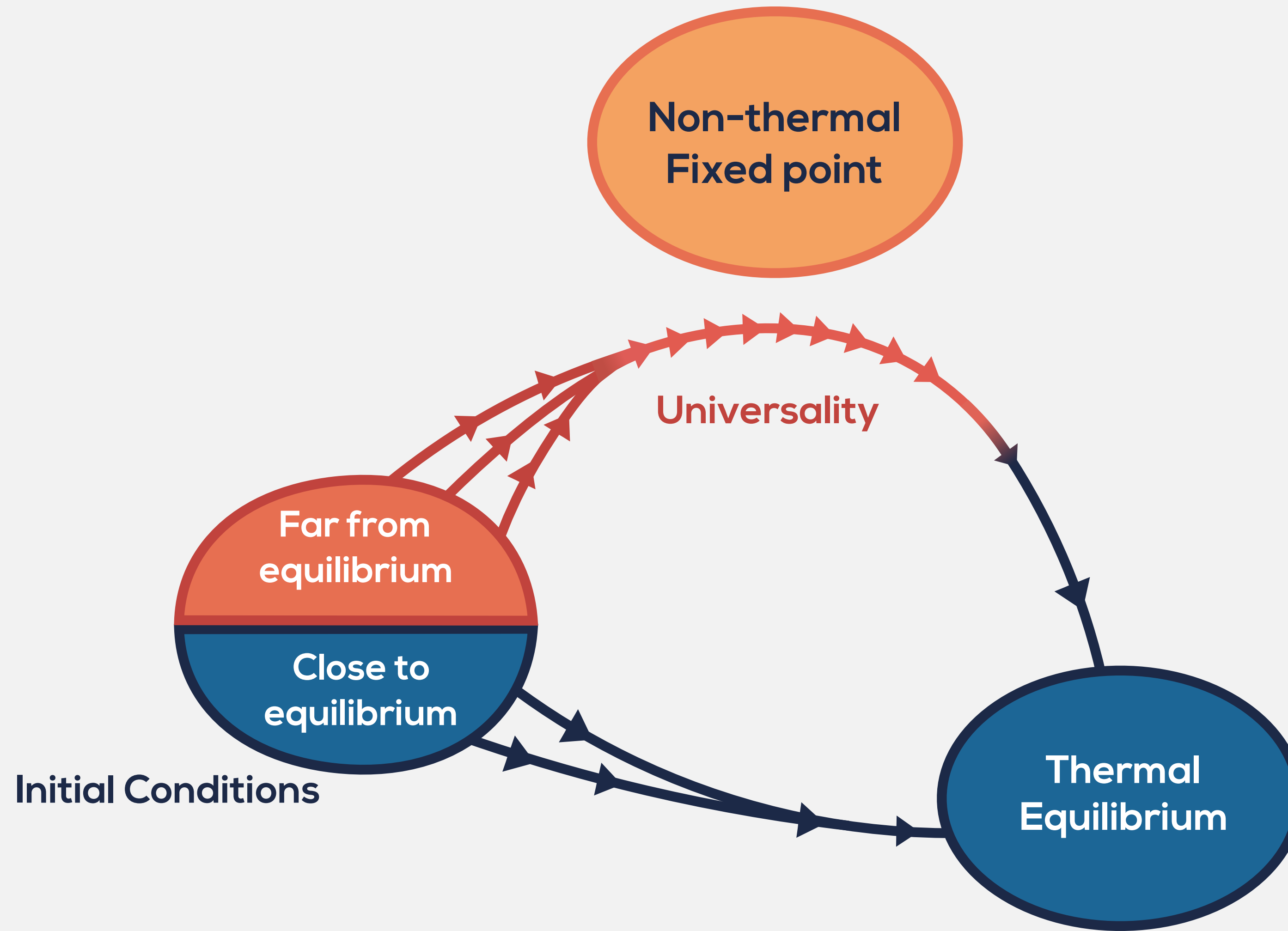


From Kurkela and Zhu
Phys.Rev.Lett. 115 (2015) no.18, 182301



Turbulent thermalization

Of **highly occupied**
non-abelian plasmas
very far from equilibrium



Non-thermal fixed point

def. Parametrically long self-similar regime quantum fields under go in their way to Thermal Equilibrium

Self - Similarity

def. distribution function depends on a Universal, time-independent function

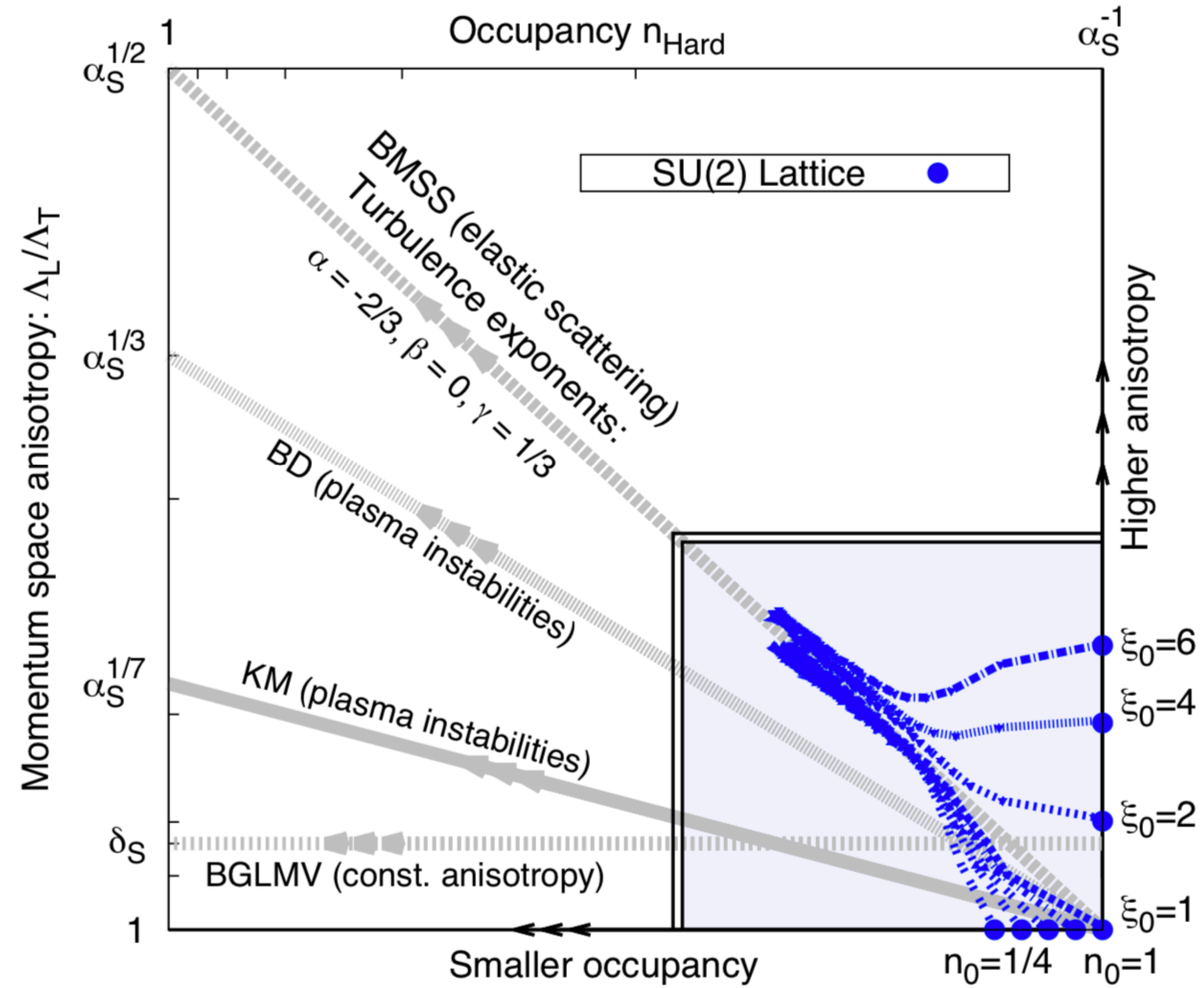
$$f(t, p) = t^\alpha f_S(p_\perp t^\beta, p_z t^\gamma)$$



Transport and Turbulence

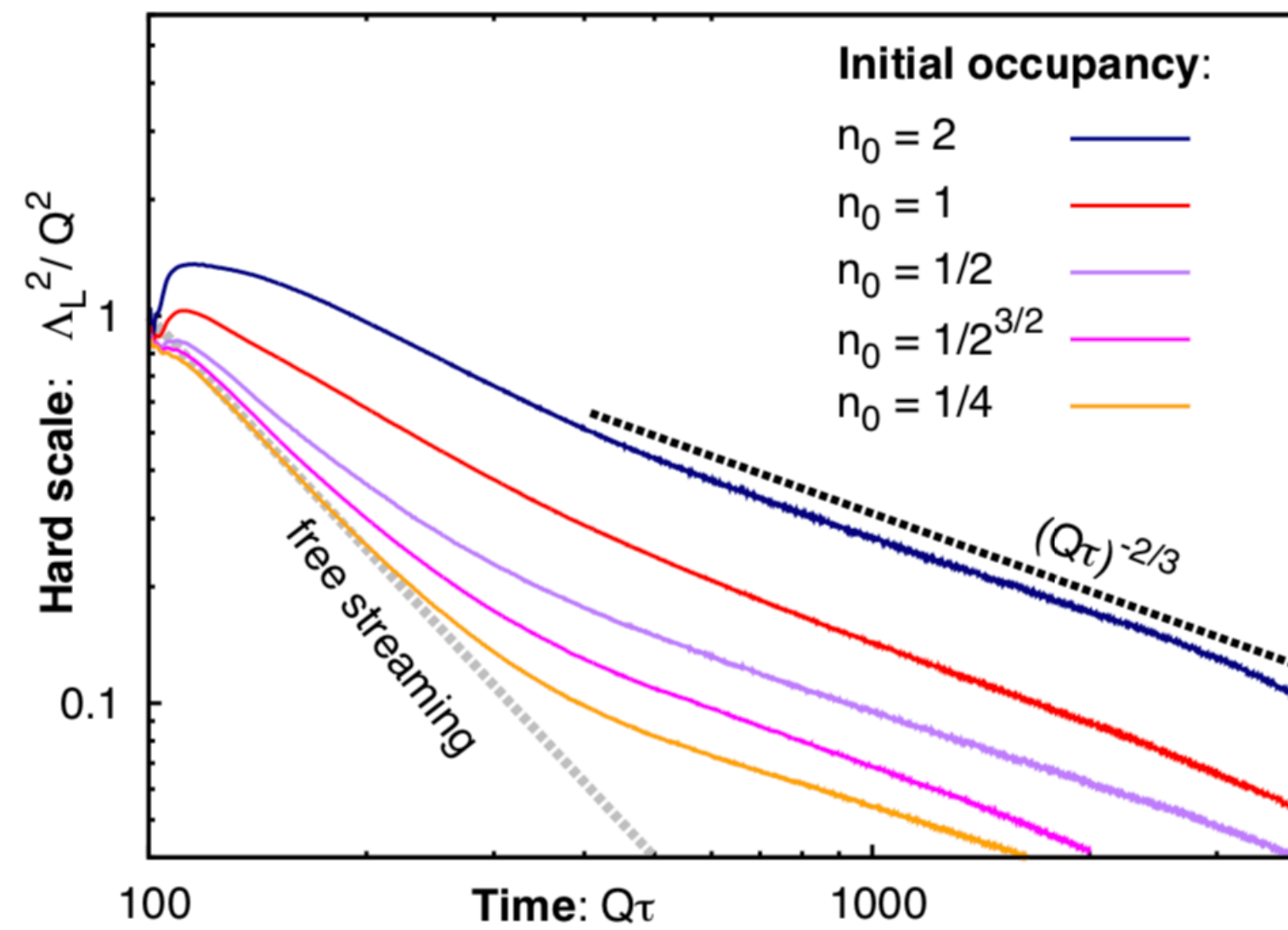
def. Local flow of conserved charges to accommodate better the total corresponding charge. The flow is turbulent when is self-similar

Finding the right scenario

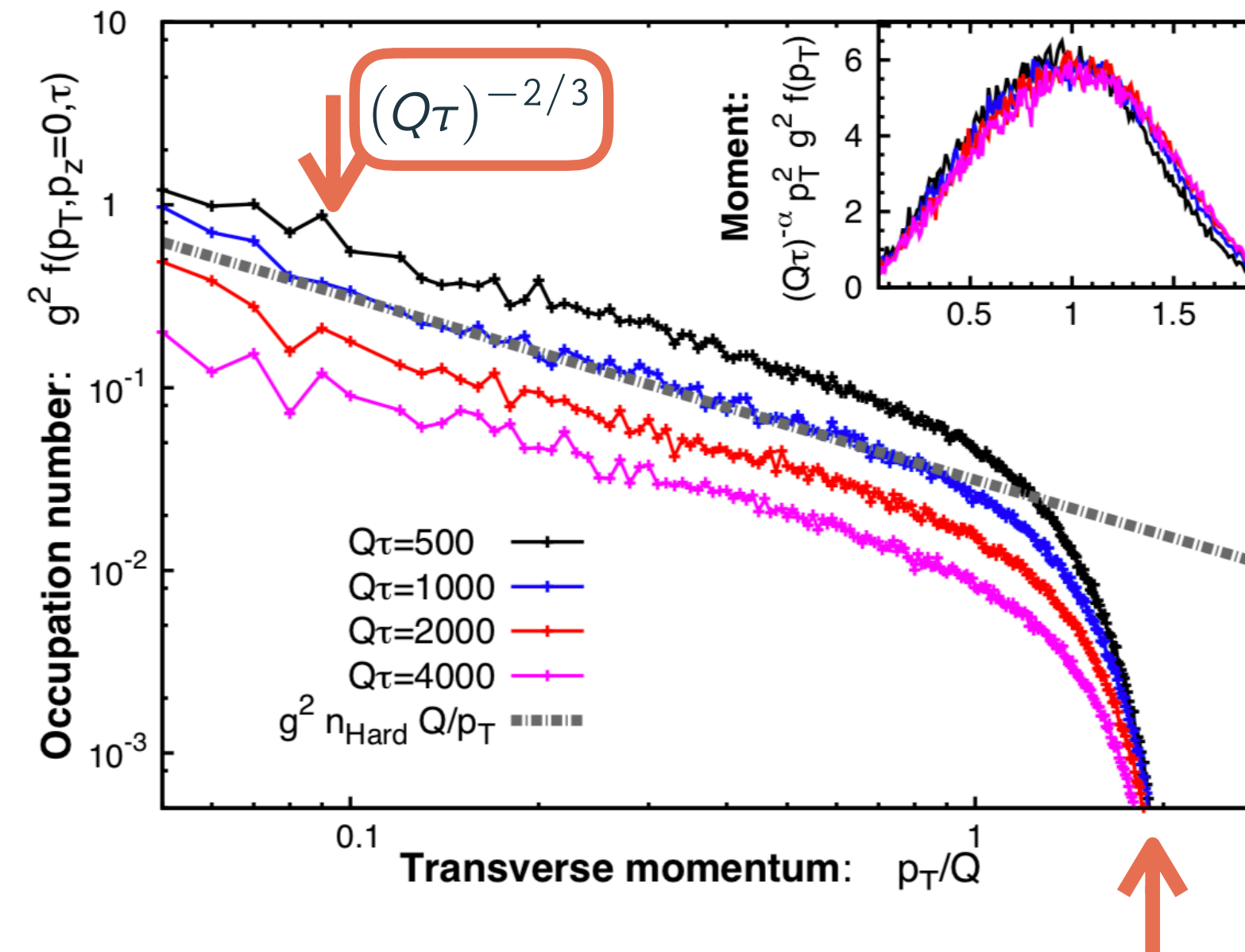


Gluon occupation: High

Hard Scale: $\Lambda_L^2 \sim \langle p_z \rangle^2$

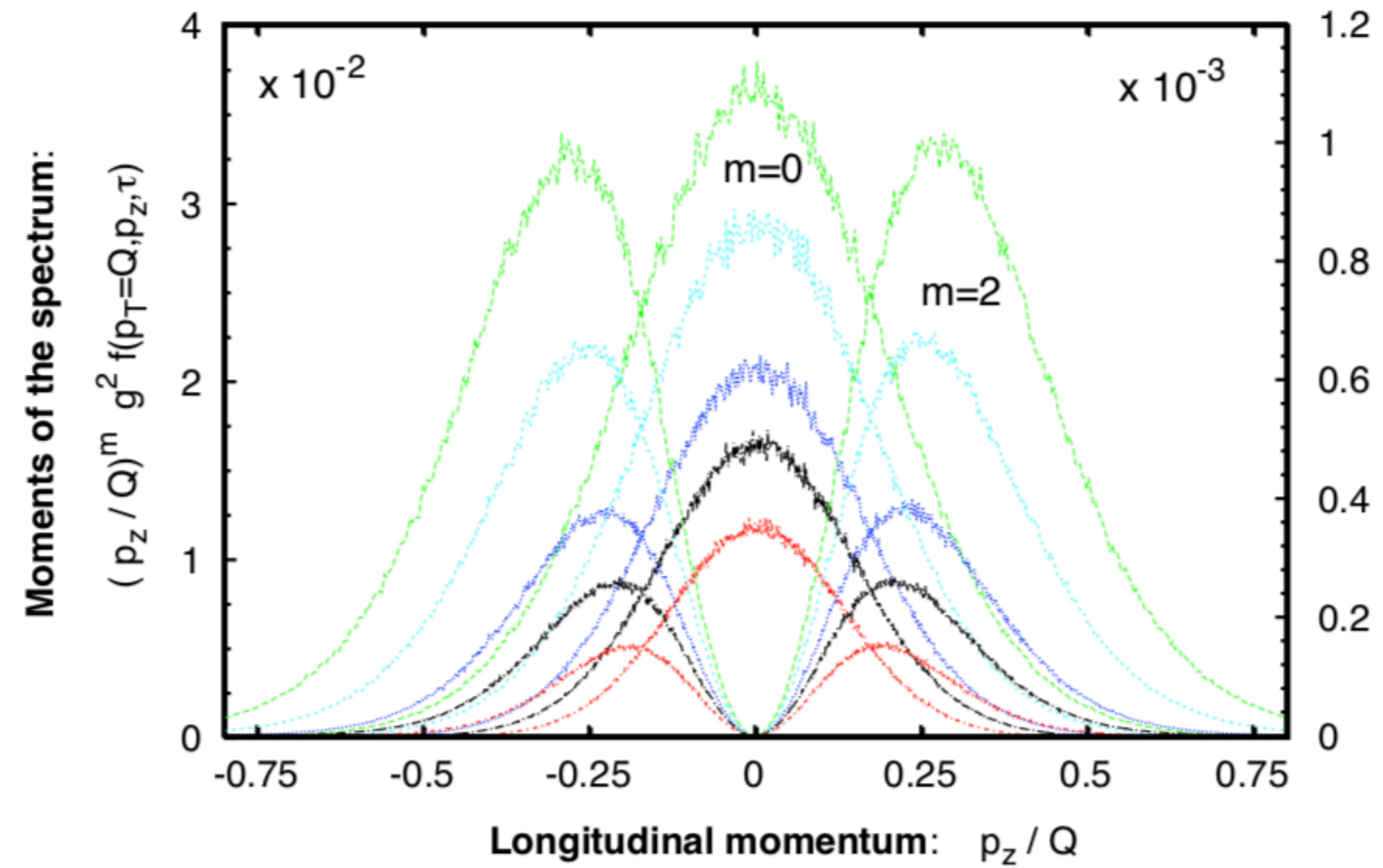


Transverse p_\perp

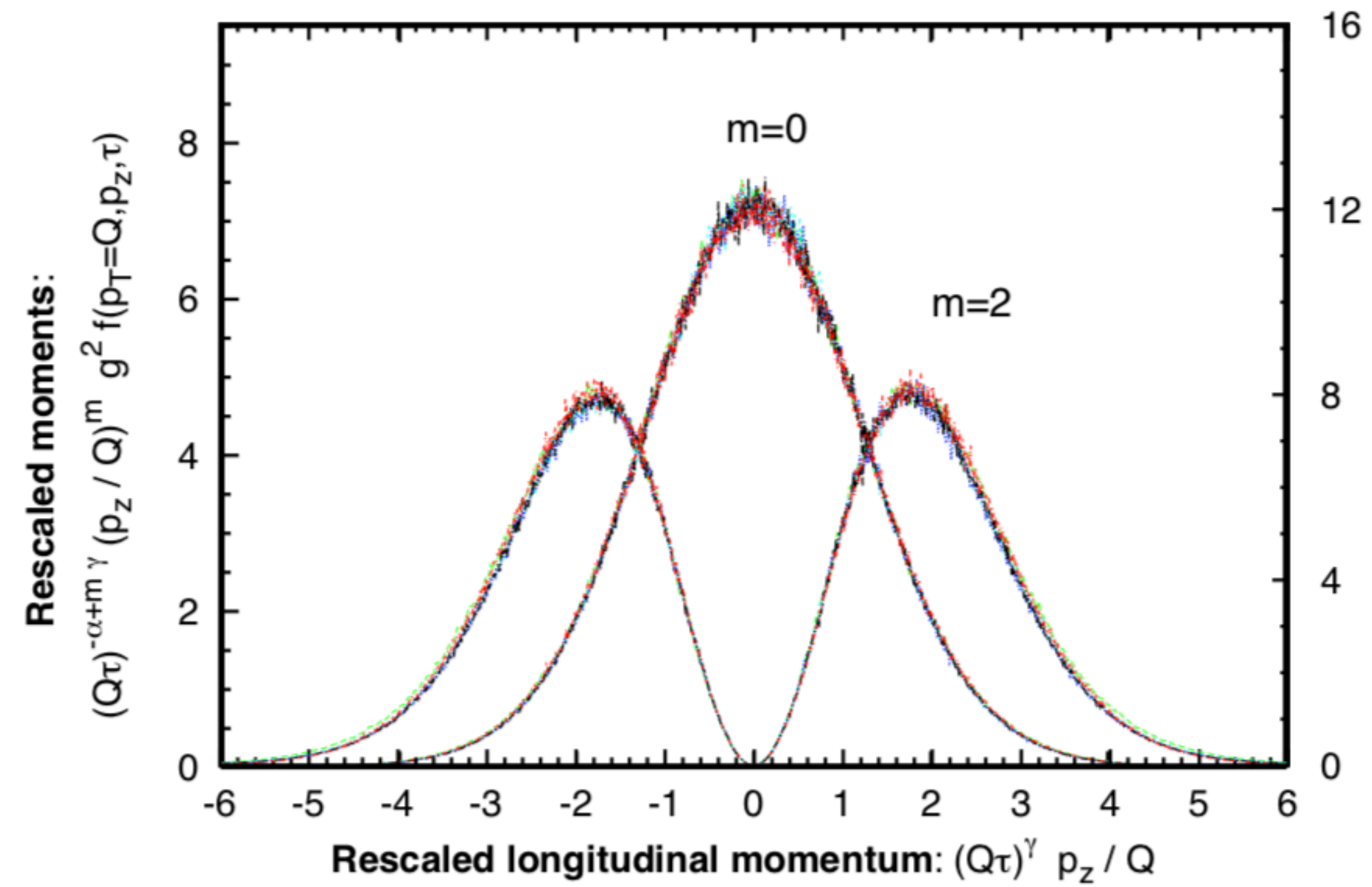


$$\alpha = -2/3 \quad \beta = 0 \quad \gamma = 1/3$$

Occupancy: p_z



Occupancy: p_z



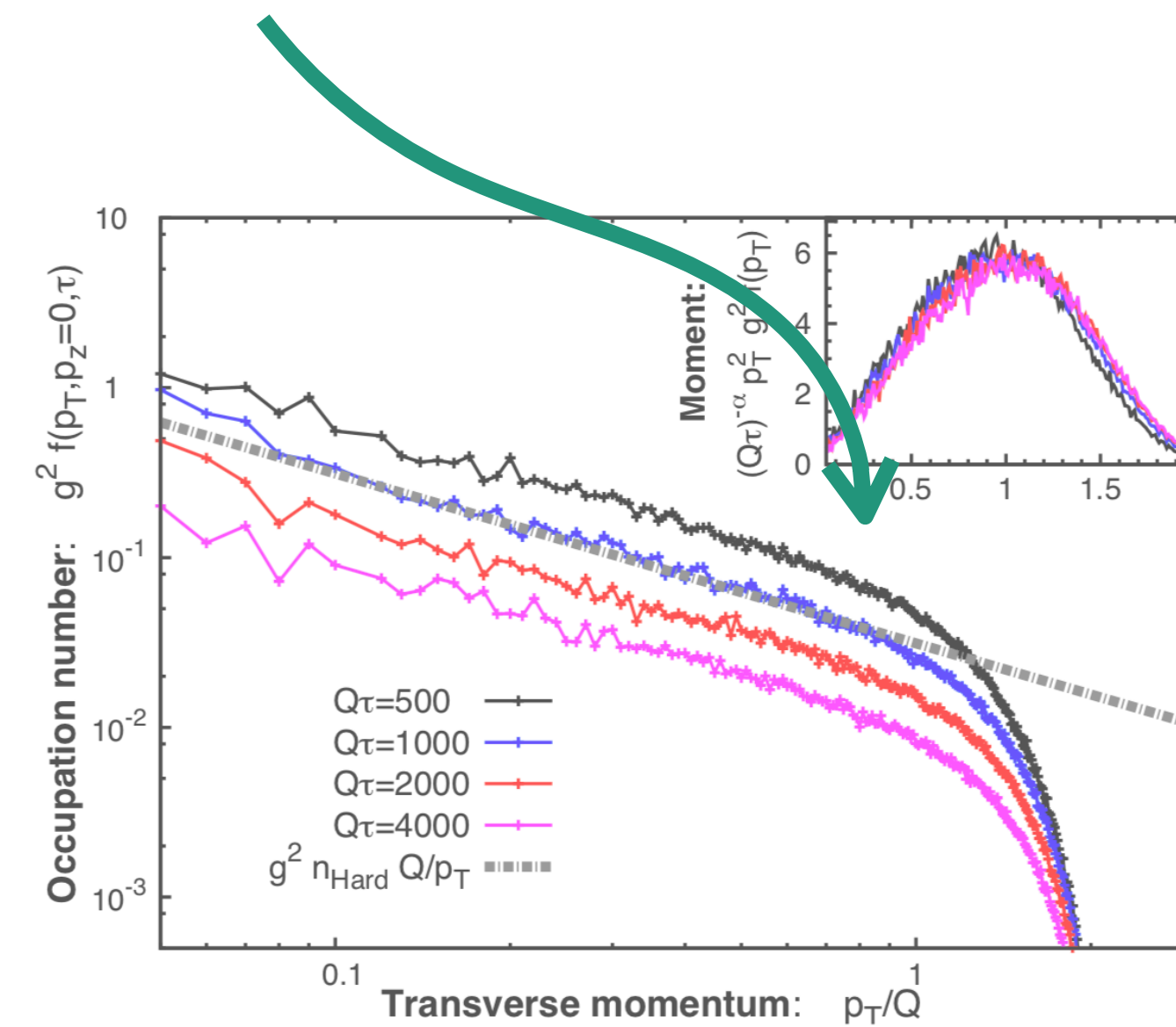
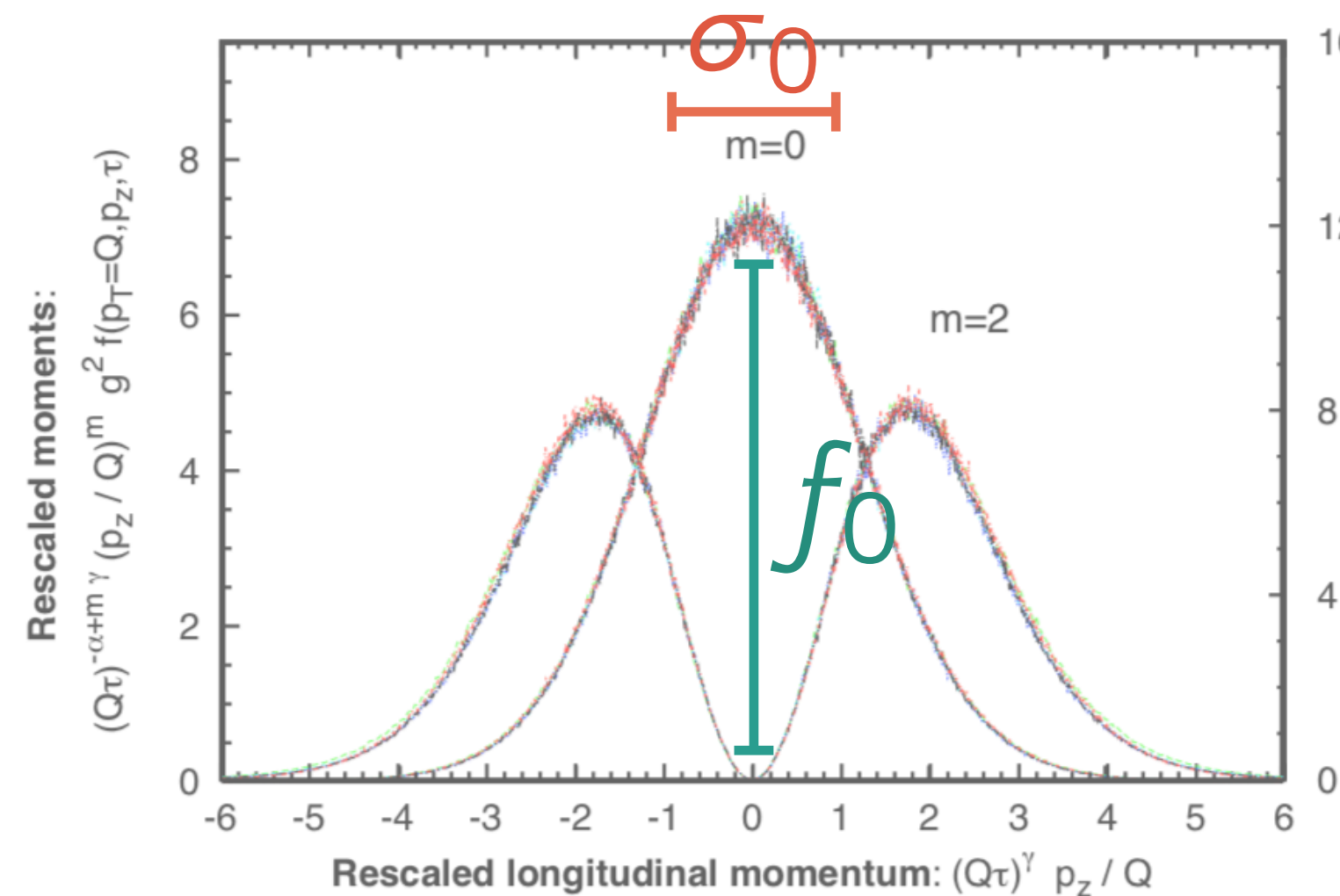
$$\gamma = 1/3$$

Fit the lattice results

Gluon Distribution $\rightarrow f_g(\tau, p_\perp, p_z) = \frac{1}{\alpha_s} (Q_s \tau)^\alpha f_S(p_\perp, (Q_s \tau)^\gamma p_z)$

with $f_S(p_\perp, p_z) = f_0 \frac{Q_s}{p_\perp} e^{-\frac{1}{2} \left(\frac{p_z}{\sigma_0} \right)^2} W_r(p_\perp - Q_s, r)$

and $W_r(p_\perp - Q_s, r) = \theta(Q_s - p_\perp) + \theta(p_\perp - Q_s) e^{-\frac{1}{2} \left(\frac{p_\perp - Q_s}{r Q_s} \right)^2}$



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Extension

Quark Distribution

Hard dipole approximation $\rightarrow f_q(\tau, p_\perp, p_z) = \alpha_s f_g(\tau, p_\perp, p_z)$

* Q.Stat. kick in outside the region of interest.

Phenomenological Matching

$$\langle Q_s^2 \rangle = \frac{\int d^2 x_{\perp} Q_s^2(x_{\perp})}{\langle S_{\perp} \rangle}$$

↓
IP-Glasma

$\langle Q_s^2 \rangle = 2 \text{ GeV}^2$ → RHIC, 200GeV, 0-5% (Reference)

$\langle Q_s^2 \rangle = 1.67 \text{ GeV}^2$ → RHIC, 200GeV, 0-20%

$\langle Q_s^2 \rangle = 2.97 \text{ GeV}^2$ → ALICE, 2.76TeV, 0-20%

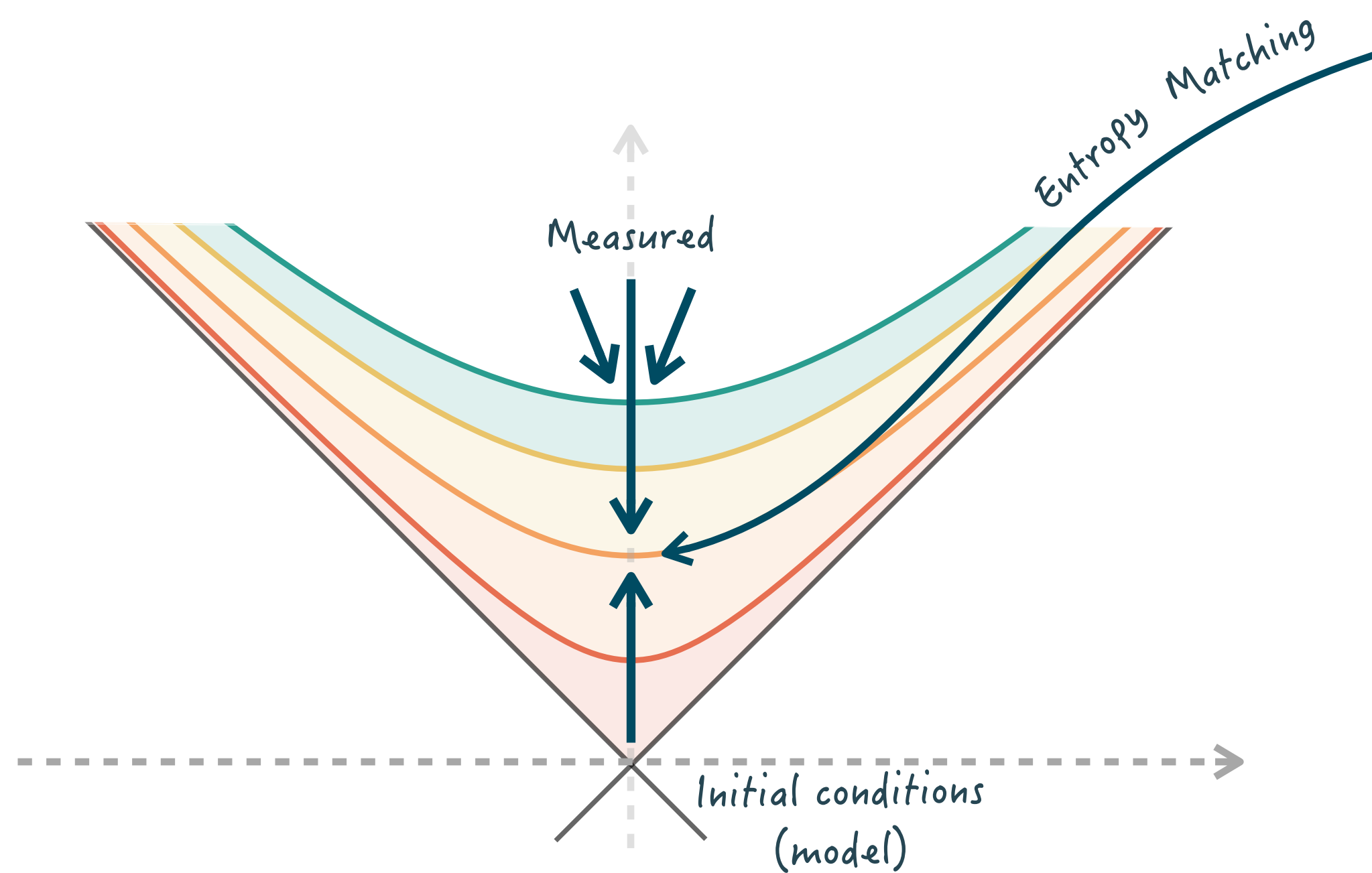
Phenomenological Matching

$$c_{eq} c_T^{3/4} = \left[\frac{45}{148\pi^2} k_{S/N} \alpha^{7/5} \frac{N_{part}}{Q_S^2 S_{\perp}} \frac{2}{N_{part}} \frac{dN_{ch}}{dy} \right]^{1/4}$$

Entropy is transported via

$$\frac{dS_{QGP}}{d\eta} = k_{S/N} \frac{dN_{ch}}{d\eta}$$

Kinetic Freeze-out

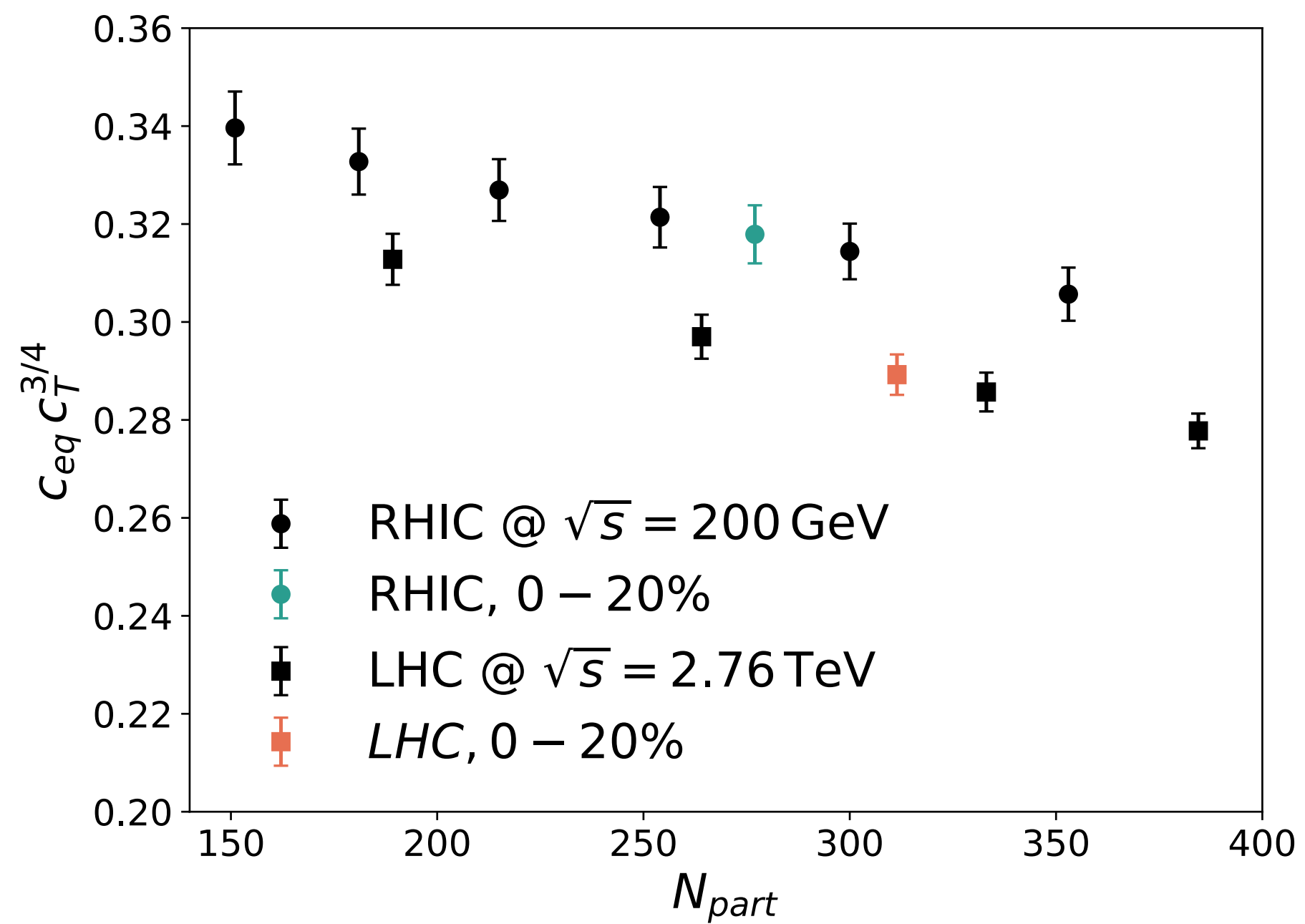


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Entropy is transported via

$$\frac{dS_{QGP}}{d\eta} = k_{S/N} \frac{dN_{ch}}{d\eta}$$



c_T known to logarithmic precision

$$c_T = 0.18$$

Phenomenological Matching

$$c_{eq} c_T^{3/4} = \left[\frac{45}{148\pi^2} k_{S/N} \alpha^{7/5} \frac{N_{part}}{Q_S^2 S_{\perp}} \frac{2}{N_{part}} \frac{dN_{ch}}{dy} \right]^{1/4}$$

Entropy is transported via

$$\frac{dS_{QGP}}{d\eta} = k_{S/N} \frac{dN_{ch}}{d\eta}$$

τ_c is independent of match

$$\tau_{th} \sim 2 \text{ fm}$$

$$T_{th} \sim 0.25 \text{ GeV}$$

