Mass Correction to CKT

Chiral Phase Transition in an Expanding System

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Chiral Phase Transition in Expanding System

BACKGROUND

QCD topology & magnetic field

Chiral Magnetic Effect

Quantum Transport

QCD topology & magnetic field

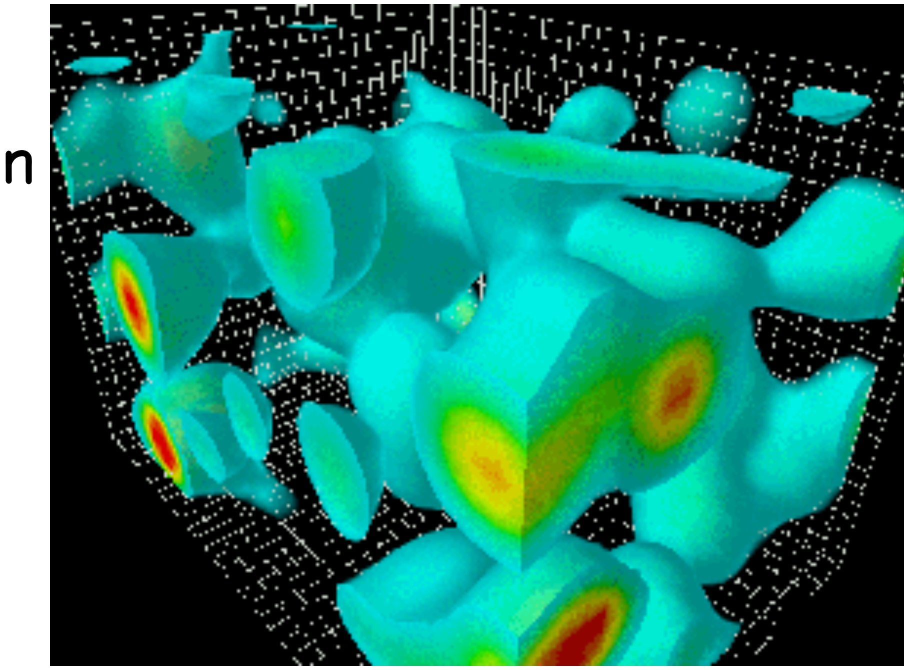
- **Mysterious QCD**

  Confinement, Chiral symmetry breaking, Hadronization

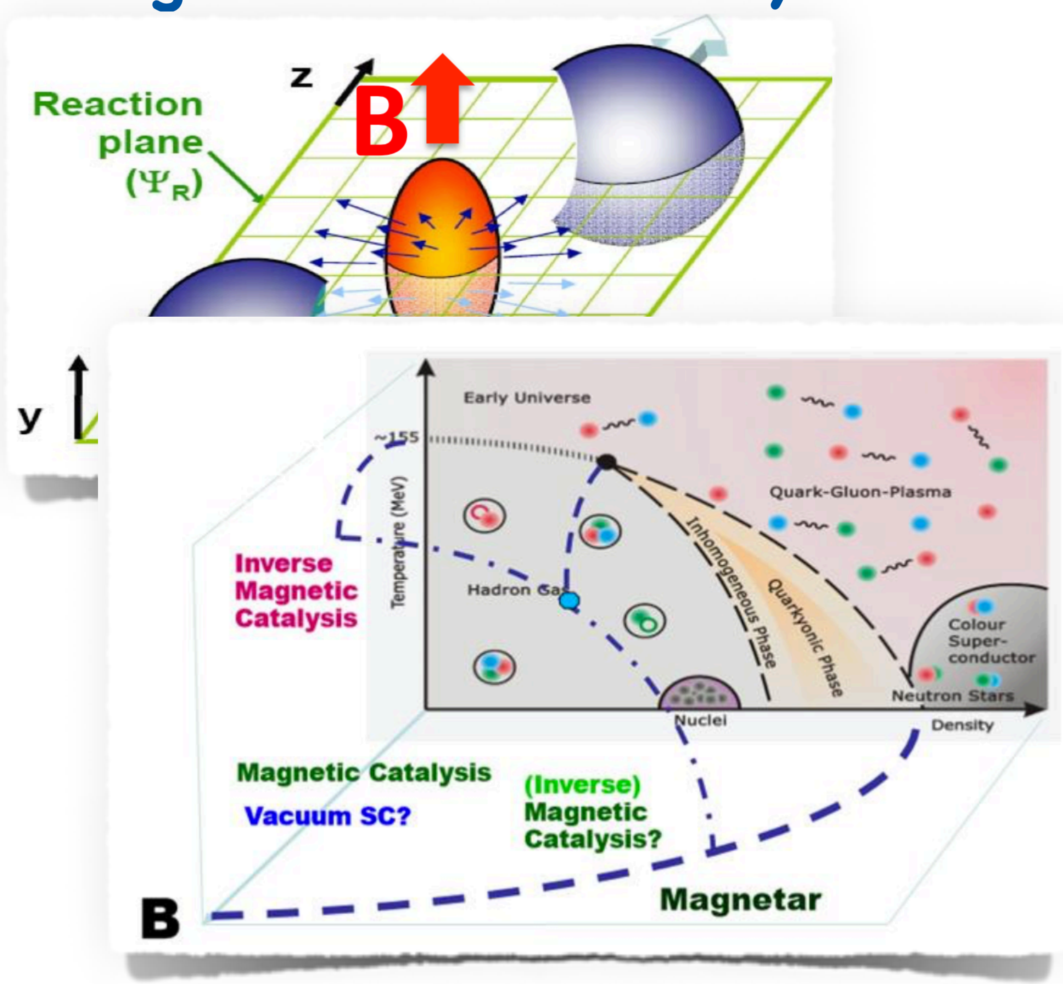
  Topologically nontrivial gluon field configurations

  θ -vacuum , non-perturbative

  UA(1) anomaly, Strong CP problem



- **Magnetic field in heavy ion collision**



- Graphene, Dirac (Weyl) semimental 10^5 Gauss

  Compact stars $10^{10} \sim 10^{16}$ Gauss

  Heavy ion collision $10^{18} \sim 10^{19}$ Gauss ($eB \sim 6m_\pi^2$)

- Particle production

  Phase structure (MC, IMC)

  Transport property (CME, CMW, CSE...)

Actress and stage, what performance?

Chiral Magnetic Effect

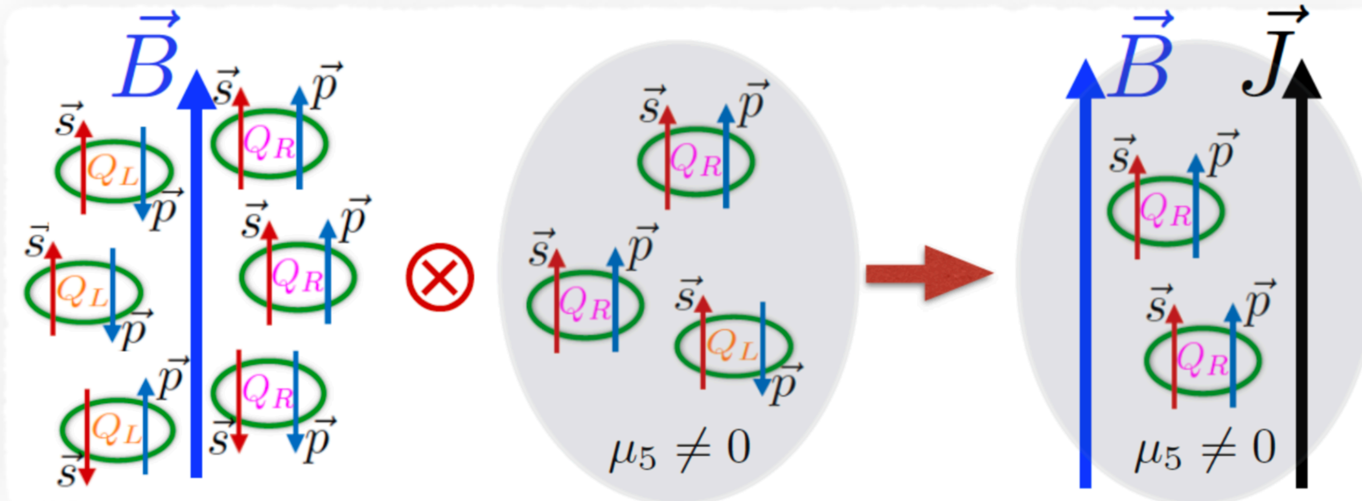
- **Chiral Magnetic Effect (CME)**

$$\vec{J}_{\rm CME} = \frac{e^2}{2\pi^2}\mu_A \vec{B}$$

Electric current along B

Chiral charge density

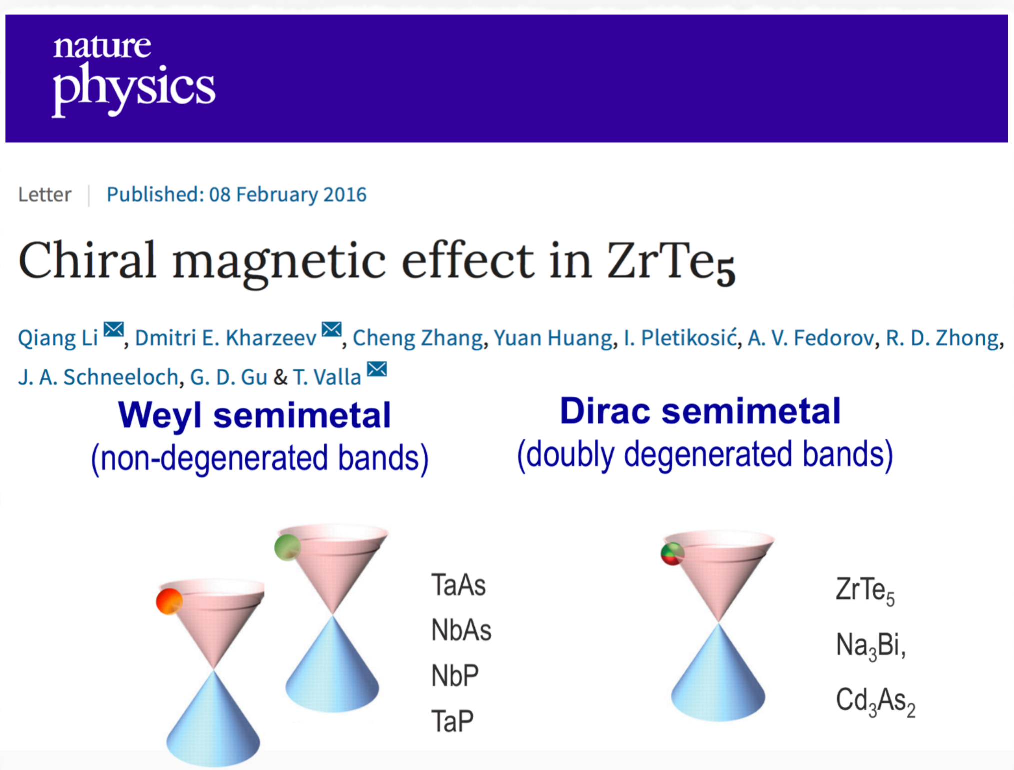
T-even: non-dissipative, topological protected



- **Realize in systems with chiral fermion and B field**

Condensed mater system $\vec{E}\cdot\vec{B}$

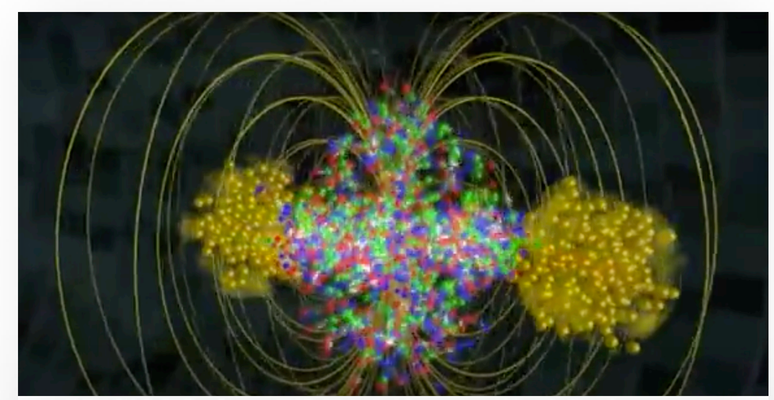
Heavy Ion Collision $QCD \times QED$

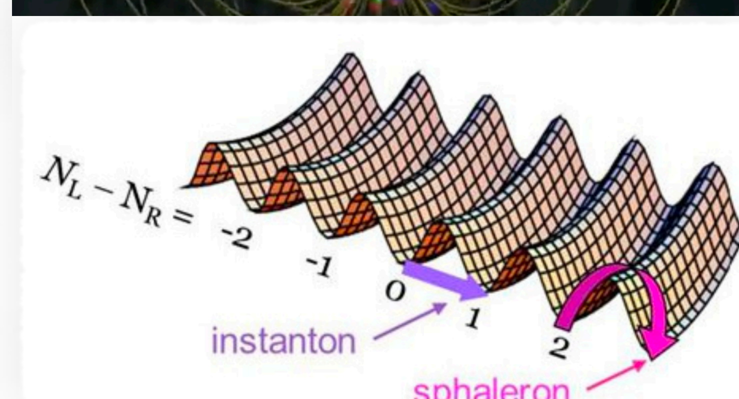

nature physics

Letter | Published: 08 February 2016

Chiral magnetic effect in $ZrTe_5$

Qiang Li, Dmitri E. Kharzeev, Cheng Zhang, Yuan Huang, I. Pletikosić, A. V. Fedorov, R. D. Zhong, J. A. Schneeloch, G. D. Gu & T. Valla





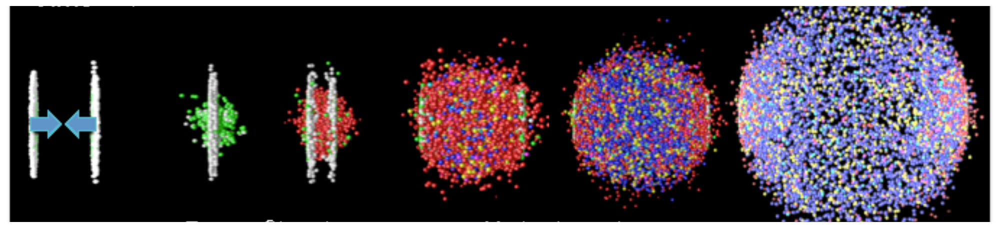
Perfect actress and stage, perfect performance… But?

Quantum Transportation

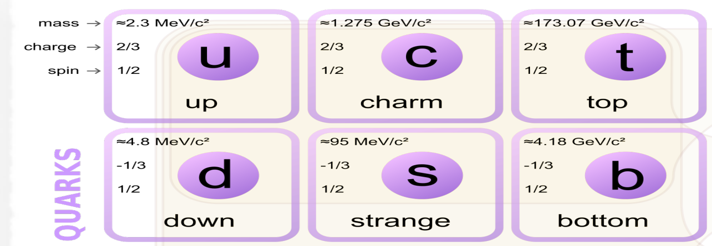
- **CME in heavy ion collision**

$$\mu_A = \partial_t \theta$$

non-equilibrium nature



Highly dynamical, inhomogeneous



Finite mass

- **Non-equilibrium**

local equilibrium — relativistic hydrodynamics + triangle anomaly

Non-equilibrium — quantum transport theory (Wigner function)

$$W(x,\vec{p}) = \int d^3\vec{y}\, e^{i\vec{p}\cdot\vec{y}} \left\langle \psi\left(t, \vec{x}+\frac{\vec{y}}{2}\right) e^{ie\int_{-1/2}^{1/2} ds A(x+sy)\cdot y} \psi^\dagger\left(t, \vec{x}-\frac{\vec{y}}{2}\right) \right\rangle = \int dp_0 W(x,p)\gamma^0$$

P.Zhuang and U.Heinz, Ann.Phys.245, 311(1996); PRD57, 6525(1998)

Chiral Kinetic Theory & Berry curvature

- **Finite mass:** **Almost all consider chiral case**

From massless to massive, not a trivial problem

Dissipation rate — for hydrodynamical modeling D.f.Hou and S.Lin, 1712.08429 [hep-ph].

Modify CKT framework — non-Abel berry curvature

J.W.Chen, J.y.Pang, S.Pu and Q.Wang, PRD 89, 094003 (2014)

FRAMEWORK

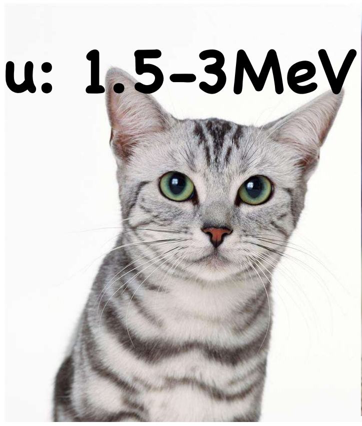
Covariant Kinetic Equation

Equal-time equation

Semiclassical expansion

Covariant Kinetic Equation

- u, d quark moving in external EM field



$$\mathcal{L} = \bar{\psi}\Big(i\gamma^{\mu}(\partial_{\mu} + ieA_{\mu}) - m\Big)\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$

- Covariant equation of Wigner operator

$$\left[\gamma^{\mu}\left(\Pi_{\mu} + \frac{1}{2}i\hbar D_{\mu}\right) - m\right]\hat{W}_4(x,p) = 0$$

$$\begin{aligned} D_{\mu}(x,p) &= \partial_{\mu} - e\int_{-1/2}^{1/2} ds F_{\mu\nu}(x - i\hbar s\partial_p)\partial_p^{\nu} \\ \Pi_{\mu}(x,p) &= p_{\mu} - ie\hbar\int_{-1/2}^{1/2} ds s F_{\mu\nu}(x - i\hbar s\partial_p)\partial_p^{\nu} \end{aligned}$$

- Po integrating the covariant equation

$$f(x,\vec{p}) \rightarrow W(x,\vec{p})$$

$$W(x,\vec{p}) = \int d^3\vec{y}e^{i\vec{p}\cdot\vec{y}}\left\langle\psi\left(t,\vec{x}+\frac{\vec{y}}{2}\right)e^{ie\int_{-1/2}^{1/2} dsA(x+sy)\cdot y}\psi^{\dagger}\left(t,\vec{x}-\frac{\vec{y}}{2}\right)\right\rangle = \int dp_0 W(x,p)\gamma^0$$

Equal-time transport equation $\quad W(x,\vec{p}) = \int dp_0 W(x,p)\gamma^0$

Equal-time constraint equation $\quad W^{(1)}(x,\vec{p}) = \int dp_0\, p_0 W(x,p)\gamma^0$

Equal-time transport & constraint equations

- **Spin decomposition — 16 components**

$$W(x,\vec{p}) = \frac{1}{4}\Big[f_0 + \gamma_5 f_1 - i\gamma_0\gamma_5 f_2 + \gamma_0 f_3 + \gamma_5\gamma_0\vec{\gamma}\cdot\vec{g}_0 + \gamma_0\vec{\gamma}\cdot\vec{g}_1 - i\vec{\gamma}\cdot\vec{g}_2 - \gamma_5\vec{\gamma}\cdot\vec{g}_3\Big]$$

f_0 : number density
$\vec{g}_1$: number current

f_1 : helicity density
$\vec{g}_0$: spin density

f_3 : mass density
$\vec{g}_3$: magnetic moment

- **16 transport equations** + **16 constraint equations**

$$\begin{aligned}
\hbar(D_t f_0 + \vec{D}\cdot\vec{g}_1) &= 0\\
\hbar(D_t f_1 + \vec{D}\cdot\vec{g}_0) &= -2m f_2\\
\hbar(D_t\vec{g}_0 + \vec{D} f_1) - 2\vec{\Pi}\times\vec{g}_1 &= 0\\
\hbar(D_t\vec{g}_1 + \vec{D} f_0) - 2\vec{\Pi}\times\vec{g}_0 &= -2m\vec{g}_2\\
\hbar D_t f_2 - 2\vec{\Pi}\cdot\vec{g}_3 &= 2m f_1\\
\hbar D_t f_3 - 2\vec{\Pi}\cdot\vec{g}_2 &= 0\\
\hbar(D_t\vec{g}_2 - \vec{D}\times\vec{g}_3) + 2\vec{\Pi} f_3 &= 2m\vec{g}_1\\
\hbar(D_t\vec{g}_3 + \vec{D}\times\vec{g}_2) + 2\vec{\Pi} f_2 &= 0
\end{aligned}$$

$$\begin{aligned}
D_t &= \partial_t + \int_{-1/2}^{1/2} ds\vec{E}(\vec{x} + is\hbar\nabla_p)\cdot\nabla_p,\\
\vec{D} &= \nabla + \int_{-1/2}^{1/2} ds\vec{B}(\vec{x} + is\hbar\nabla_p)\times\nabla_p,\\
\widetilde{\Pi}_0 &= i\hbar\int_{-1/2}^{1/2} dss\vec{E}(\vec{x} + is\hbar\nabla_p)\cdot\nabla_p,\\
\vec{\Pi} &= \vec{p} - i\hbar\int_{-1/2}^{1/2} dss\vec{B}(\vec{x} + is\hbar\nabla_p)\times\nabla_p,
\end{aligned}$$

+

$$\begin{aligned}
\int dp_0 p_0 V_0 - \vec{\Pi}\cdot\vec{g}_1 + \widetilde{\Pi}_0 f_0 &= m f_3\\
\int dp_0 p_0 A_0 + \vec{\Pi}\cdot\vec{g}_0 - \widetilde{\Pi}_0 f_1 &= 0\\
\int dp_0 p_0 \vec{A} + \frac{1}{2}\hbar\vec{D}\times\vec{g}_1 + \vec{\Pi} f_1 - \widetilde{\Pi}_0\vec{g}_0 &= -m\vec{g}_3\\
\int dp_0 p_0 \vec{V} - \frac{1}{2}\hbar\vec{D}\times\vec{g}_0 + \vec{\Pi} f_0 - \widetilde{\Pi}_0\vec{g}_1 &= 0\\
\int dp_0 p_0 P + \frac{1}{2}\hbar\vec{D}\cdot\vec{g}_3 + \widetilde{\Pi}_0 f_2 &= 0\\
\int dp_0 p_0 F - \frac{1}{2}\hbar\vec{D}\cdot\vec{g}_2 + \widetilde{\Pi}_0 f_3 &= m f_0\\
\int dp_0 p_0 S^{0i}\vec{e}_i - \frac{1}{2}\hbar\vec{D} f_3 + \vec{\Pi}\times\vec{g}_3 - \widetilde{\Pi}_0\vec{g}_2 &= 0\\
\int dp_0 p_0 S_{jk}\epsilon^{jki}\vec{e}_i - \hbar\vec{D} f_2 + 2\vec{\Pi}\times\vec{g}_2 + 2\widetilde{\Pi}_0\vec{g}_3 &= 2m\vec{g}_0
\end{aligned}$$

Massless vs Massive

$$\begin{aligned}
\hbar(D_t f_0 + \vec{D}\cdot\vec{g}_1) &= \\
\hbar(D_t f_1 + \vec{D}\cdot\vec{g}_0) &= \\
\hbar(D_t \vec{g}_0 + \vec{D} f_1) - 2\vec{\Pi}\times\vec{g}_1 &= \\
\hbar(D_t \vec{g}_1 + \vec{D} f_0) - 2\vec{\Pi}\times\vec{g}_0 &= \quad \mathbf{0}\\
\hbar D_t f_2 - 2\vec{\Pi}\cdot\vec{g}_3 &= \\
\hbar D_t f_3 - 2\vec{\Pi}\cdot\vec{g}_2 &= \\
\hbar(D_t \vec{g}_2 - \vec{D}\times\vec{g}_3) + 2\vec{\Pi} f_3 &= \\
\hbar(D_t \vec{g}_3 + \vec{D}\times\vec{g}_2) + 2\vec{\Pi} f_2 &=
\end{aligned}$$

$$\begin{aligned}
f_0(x,\mathbf{p}) &= \int dp_0 V_0(x,p)\\
f_1(x,\mathbf{p}) &= -\int dp_0 A_0(x,p)\\
g_{0i}(x,\mathbf{p}) &= -\int dp_0 A_i(x,p)\\
g_{1i}(x,\mathbf{p}) &= \int dp_0 V_i(x,p)
\end{aligned}$$

$$\begin{aligned}
\int dp_0 p_0 V_0 - \vec{\Pi}\cdot\vec{g}_1 + \widetilde{\Pi}_0 f_0 &= \\
\int dp_0 p_0 A_0 + \vec{\Pi}\cdot\vec{g}_0 - \widetilde{\Pi}_0 f_1 &= \\
\int dp_0 p_0 \vec{A} + \frac{1}{2}\hbar\vec{D}\times\vec{g}_1 + \vec{\Pi} f_1 - \widetilde{\Pi}_0 \vec{g}_0 &= \\
\int dp_0 p_0 \vec{V} - \frac{1}{2}\hbar\vec{D}\times\vec{g}_0 + \vec{\Pi} f_0 - \widetilde{\Pi}_0 \vec{g}_1 &= \quad \mathbf{0}\\
\int dp_0 p_0 P + \frac{1}{2}\hbar\vec{D}\cdot\vec{g}_3 + \widetilde{\Pi}_0 f_2 &= \\
\int dp_0 p_0 F - \frac{1}{2}\hbar\vec{D}\cdot\vec{g}_2 + \widetilde{\Pi}_0 f_3 &= \\
\int dp_0 p_0 S^{0i}\vec{e}_i - \frac{1}{2}\hbar\vec{D} f_3 + \vec{\Pi}\times\vec{g}_3 - \widetilde{\Pi}_0 \vec{g}_2 &= \\
\int dp_0 p_0 S_{jk}\epsilon^{jki}\vec{e}_i - \hbar\vec{D} f_2 + 2\vec{\Pi}\times\vec{g}_2 + 2\widetilde{\Pi}_0 \vec{g}_3 &=
\end{aligned}$$

- Expand all 16 constraint + 16 transport equation by order of $\hbar$

On-shell (quasi-particle) — Classical transport equation @ 0th

Quantum effects — Quantum transport equation @ 0th+1st

Massless vs Massive

$$
\begin{aligned}
\hbar(D_t f_0 + \vec{D}\cdot\vec{g}_1) &= 0\\
\hbar(D_t f_1 + \vec{D}\cdot\vec{g}_0) &= -2mf_2\\
\hbar(D_t \vec{g}_0 + \vec{D}f_1) - 2\vec{\Pi}\times\vec{g}_1 &= 0\\
\hbar(D_t \vec{g}_1 + \vec{D}f_0) - 2\vec{\Pi}\times\vec{g}_0 &= -2m\vec{g}_2\\
\hbar D_t f_2 - 2\vec{\Pi}\cdot\vec{g}_3 &= 2mf_1\\
\hbar D_t f_3 - 2\vec{\Pi}\cdot\vec{g}_2 &= 0\\
\hbar(D_t \vec{g}_2 - \vec{D}\times\vec{g}_3) + 2\vec{\Pi}f_3 &= 2m\vec{g}_1\\
\hbar(D_t \vec{g}_3 + \vec{D}\times\vec{g}_2) + 2\vec{\Pi}f_2 &= 0
\end{aligned}
$$

$$
\begin{aligned}
f_0(x,\mathbf{p}) &= \int dp_0 V_0(x,p)\\
f_1(x,\mathbf{p}) &= -\int dp_0 A_0(x,p)\\
g_{0i}(x,\mathbf{p}) &= -\int dp_0 A_i(x,p)\\
g_{1i}(x,\mathbf{p}) &= \int dp_0 V_i(x,p)
\end{aligned}
$$

$$
\begin{aligned}
\int dp_0 p_0 V_0 - \vec{\Pi}\cdot\vec{g}_1 + \widetilde{\Pi}_0 f_0 &= mf_3\\
\int dp_0 p_0 A_0 + \vec{\Pi}\cdot\vec{g}_0 - \widetilde{\Pi}_0 f_1 &= 0\\
\int dp_0 p_0 \vec{A} + \frac{1}{2}\hbar\vec{D}\times\vec{g}_1 + \vec{\Pi}f_1 - \widetilde{\Pi}_0\vec{g}_0 &= -m\vec{g}_3\\
\int dp_0 p_0 \vec{V} - \frac{1}{2}\hbar\vec{D}\times\vec{g}_0 + \vec{\Pi}f_0 - \widetilde{\Pi}_0\vec{g}_1 &= 0\\
\int dp_0 p_0 P + \frac{1}{2}\hbar\vec{D}\cdot\vec{g}_3 + \widetilde{\Pi}_0 f_2 &= 0\\
\int dp_0 p_0 F - \frac{1}{2}\hbar\vec{D}\cdot\vec{g}_2 + \widetilde{\Pi}_0 f_3 &= mf_0\\
\int dp_0 p_0 S^{0i}\vec{e}_i - \frac{1}{2}\hbar\vec{D}f_3 + \vec{\Pi}\times\vec{g}_3 - \widetilde{\Pi}_0\vec{g}_2 &= 0\\
\int dp_0 p_0 S_{jk}\epsilon^{jki}\vec{e}_i - \hbar\vec{D}f_2 + 2\vec{\Pi}\times\vec{g}_2 + 2\widetilde{\Pi}_0\vec{g}_3 &= 2m\vec{g}_0
\end{aligned}
$$

- **Expand all 16 constraint + 16 transport equation by order of $\hbar$**

On-shell (quasi-particle) — Classical transport equation @ 0th

Quantum effects — Quantum transport equation @ 0th+1st

RESULTS

1 Chiral fermion

2 Massive fermion

3 Solution for CME

1. Chiral fermion

- **8 transport equations** + **8 constraint equations**

$$
\begin{aligned}
\hbar(D_t f_0 + \vec{D}\cdot\vec{g}_1) &= 0\\
\hbar(D_t f_1 + \vec{D}\cdot\vec{g}_0) &= 0\\
\hbar(D_t \vec{g}_0 + \vec{D} f_1) - 2\vec{\Pi}\times\vec{g}_1 &= 0\\
\hbar(D_t \vec{g}_1 + \vec{D} f_0) - 2\vec{\Pi}\times\vec{g}_0 &= 0
\end{aligned}
$$

+

$$
\begin{aligned}
\int dp_0 p_0 V_0 - \vec{\Pi}\cdot\vec{g}_1 + \tilde{\Pi}_0 f_0 &= 0\\
\int dp_0 p_0 A_0 + \vec{\Pi}\cdot\vec{g}_0 - \tilde{\Pi}_0 f_1 &= 0\\
\int dp_0 p_0 \vec{A} + \frac{1}{2}\hbar\vec{D}\times\vec{g}_1 + \vec{\Pi} f_1 - \tilde{\Pi}_0\vec{g}_0 &= 0\\
\int dp_0 p_0 \vec{V} - \frac{1}{2}\hbar\vec{D}\times\vec{g}_0 + \vec{\Pi} f_0 - \tilde{\Pi}_0\vec{g}_1 &= 0
\end{aligned}
$$

- **m=0: Axial and Vector still coupled! Decouple by introducing chiral component!**

$$
J^\mu_\chi = \frac{1}{2}(V^\mu - \chi A^\mu) \qquad f_\chi = f_0 + \chi f_1, \quad \vec{g}_\chi = \vec{g}_1 + \chi\vec{g}_0 = G[f_\chi]
$$

$\hbar$ expansion

Equations of 0th, 1st order

Combine $f_\chi = f_\chi^{(0)} + \hbar f_\chi^{(1)}$

- **m=0: Chiral kinetic theory & Berry curvature from hbar order**

$$
\begin{aligned}
\partial_t f^\pm_\chi + \dot{\vec{x}}\cdot\nabla f^\pm_\chi + \dot{\vec{p}}\cdot\nabla_p f^\pm_\chi &= 0\\
\dot{\vec{x}} &= \frac{1}{\sqrt{G}}\left(\vec{v}_p + \hbar(\vec{v}_p\cdot\vec{b})\vec{B} + \hbar\vec{E}\times\vec{b}\right)\\
\dot{\vec{p}} &= \frac{1}{\sqrt{G}}\left(\vec{v}_p\times\vec{B} + \vec{E} + \hbar(\vec{E}\cdot\vec{B})\vec{b}\right)
\end{aligned}
$$

$$
\begin{aligned}
\vec{b} &= \chi\frac{\hat{p}}{2p^2}\\
\epsilon_p &= p(1 - \hbar\vec{B}\cdot\vec{b})\\
\vec{v}_p &= \nabla_p\epsilon_p = \hat{p}(1 + 2\hbar\vec{b}\cdot\vec{B}) - \hbar(\hat{p}\cdot\vec{b})\vec{B}
\end{aligned}
$$

2. massive fermion?

- 16 transport equations
- 16 constraint equations

$$
\begin{aligned}
\hbar(D_t f_0 + \vec{D}\cdot\vec{g}_1) &= 0\\
\hbar(D_t f_1 + \vec{D}\cdot\vec{g}_0) &= -2mf_2\\
\hbar(D_t \vec{g}_0 + \vec{D}f_1) - 2\vec{\Pi}\times\vec{g}_1 &= 0\\
\hbar(D_t \vec{g}_1 + \vec{D}f_0) - 2\vec{\Pi}\times\vec{g}_0 &= -2m\vec{g}_2\\
\hbar D_t f_2 - 2\vec{\Pi}\cdot\vec{g}_3 &= 2mf_1\\
\hbar D_t f_3 - 2\vec{\Pi}\cdot\vec{g}_2 &= 0\\
\hbar(D_t \vec{g}_2 - \vec{D}\times\vec{g}_3) + 2\vec{\Pi}f_3 &= 2m\vec{g}_1\\
\hbar(D_t \vec{g}_3 + \vec{D}\times\vec{g}_2) + 2\vec{\Pi}f_2 &= 0
\end{aligned}
$$

$$
\begin{aligned}
\int dp_0 p_0 V_0 - \vec{\Pi}\cdot\vec{g}_1 + \tilde{\Pi}_0 f_0 &= mf_3\\
\int dp_0 p_0 A_0 + \vec{\Pi}\cdot\vec{g}_0 - \tilde{\Pi}_0 f_1 &= 0\\
\int dp_0 p_0 \vec{A} + \frac{1}{2}\hbar\vec{D}\times\vec{g}_1 + \vec{\Pi}f_1 - \tilde{\Pi}_0\vec{g}_0 &= -m\vec{g}_3\\
\int dp_0 p_0 \vec{V} - \frac{1}{2}\hbar\vec{D}\times\vec{g}_0 + \vec{\Pi}f_0 - \tilde{\Pi}_0\vec{g}_1 &= 0\\
\int dp_0 p_0 P + \frac{1}{2}\hbar\vec{D}\cdot\vec{g}_3 + \tilde{\Pi}_0 f_2 &= 0\\
\int dp_0 p_0 F - \frac{1}{2}\hbar\vec{D}\cdot\vec{g}_2 + \tilde{\Pi}_0 f_3 &= mf_0\\
\int dp_0 p_0 S^{0i}\vec{e}_i - \frac{1}{2}\hbar\vec{D}f_3 + \vec{\Pi}\times\vec{g}_3 - \tilde{\Pi}_0\vec{g}_2 &= 0\\
\int dp_0 p_0 S_{jk}\epsilon^{jki}\vec{e}_i - \hbar\vec{D}f_2 + 2\vec{\Pi}\times\vec{g}_2 + 2\tilde{\Pi}_0\vec{g}_3 &= 2m\vec{g}_0
\end{aligned}
$$

- m: All coupled! Yet still

$$
f_\chi = f_0 + \chi f_1
$$

$$
\vec{g}_\chi = \vec{g}_1 + \chi\vec{g}_0 = G[f_\chi, \vec{g}_3]
$$

$\hbar$ expansion

Equations of 0th, 1st order

Combine $f_\chi = f_\chi^{(0)} + \hbar f_\chi^{(1)}$

- Small m limit: keep to the first order of m

$$
\partial_t f_\chi^\pm + \dot{\vec{x}}\cdot\nabla f_\chi^\pm + \dot{\vec{p}}\cdot\nabla_p f_\chi^\pm = -m\chi\frac{\vec{E}\cdot\vec{g}_3^{(0)\pm}}{\sqrt{G}p^2} - m\hbar\chi\frac{\vec{E}\cdot\vec{g}_3^{(1)\pm}}{\sqrt{G}p^2} + m\hbar\frac{F_2\left[\vec{g}_3^{(0)\pm}\right]}{\sqrt{G}}
$$

$$
F_2\left[\vec{g}_3^{(0)\pm}\right] = \frac{1}{2p^4}(\vec{p}\cdot\vec{D})(\vec{B}\cdot\vec{g}_3^{(0)\pm}) \pm \frac{1}{2p^3}\vec{D}\cdot(\vec{E}\times\vec{g}_3^{(0)\pm}) \mp \frac{3}{2p^5}(\vec{B}\times\vec{p})\cdot(\vec{E}\times\vec{g}_3^{(0)\pm})
$$

2. massive fermion

$$\partial_t f_\chi^\pm + \dot{\vec{x}} \cdot \nabla f_\chi^\pm + \dot{\vec{p}} \cdot \nabla_p f_\chi^\pm = -m\chi \frac{\vec{E} \cdot \vec{g}_3^{(0)\pm}}{\sqrt{G} p^2} - m\hbar\chi \frac{\vec{E} \cdot \vec{g}_3^{(1)\pm}}{\sqrt{G} p^2} + m\hbar \frac{F_2 \left[\vec{g}_3^{(0)\pm}\right]}{\sqrt{G}}$$

$$F_2 \left[\vec{g}_3^{(0)\pm}\right] = \frac{1}{2p^4} (\vec{p} \cdot \vec{D})(\vec{B} \cdot \vec{g}_3^{(0)\pm}) \pm \frac{1}{2p^3} \vec{D} \cdot (\vec{E} \times \vec{g}_3^{(0)\pm}) \mp \frac{3}{2p^5} (\vec{B} \times \vec{p}) \cdot (\vec{E} \times \vec{g}_3^{(0)\pm})$$

- First oder of mass — Framework of CKT with small mass
- Dissipation terms — related to : EM field, magnetic moment g3, inhomogeneous
- Same phase space factor, berry curvature, structure — comparable with m=0
- Kinetic theory — pre-thermal evolution

  — offer initial condition for hydrodynamics
- Compare to other framework — solvable as initial value problem
- Application to different physical condition — CME, CSE…

3. CME & Solution

Transport equation $\partial_t f_\chi^\pm + \dot{\vec{x}} \cdot \nabla f_\chi^\pm + \dot{\vec{p}} \cdot \nabla_p f_\chi^\pm = \frac{m\hbar}{\sqrt{G}} (\vec{p} \cdot \nabla) \beta^\pm (\vec{x}, \vec{p}, t)$

EoM

$$\dot{\vec{x}} = \frac{\hat{p}}{\sqrt{G}} \left(1 + 2Q\hbar \vec{b} \cdot \vec{B}\right)$$

$$\dot{\vec{p}} = \frac{1}{\sqrt{G}} Q\, \hat{p} \times \vec{B}$$

Berry curvature $\vec{b} = \chi \frac{\hat{p}}{2p^2}$

Phase space factor $\sqrt{G} = 1 + Q\hbar \vec{b} \cdot \vec{B}$

- Berry curvature — s follows p, $\hbar$ order — semiclassical expansion

- Non-equilibrium evolution is dissipative

  Local-equilibrim : hydrodynamics $\Gamma \propto m^2$ **Modeling**

  Pre-equilibrim : Quantum transport $\frac{m\hbar}{\sqrt{G}} (\vec{p} \cdot \nabla) \beta^\pm (\vec{x}, \vec{p}, t)$ **Theory**

  1st order, inhomogeneous

- Analytically Solvable !

BACKUP

Constraint equations

Transport equations

Analytical solution for CME

Backup I. Constraint equations @ 0th and 1st order

- **0th Constraint equations**

$$f_1^{(0)\pm} = \pm\frac{\vec{p}}{E_p}\cdot\vec{g}_0^{(0)\pm},$$

$$f_2^{(0)\pm} = 0,$$

$$f_3^{(0)\pm} = \pm\frac{m}{E_p}f_0^{(0)\pm},$$

$$\vec{g}_1^{(0)\pm} = \pm\frac{\vec{p}}{E_p}f_0^{(0)\pm},$$

$$\vec{g}_2^{(0)\pm} = \frac{\vec{p}\times\vec{g}_0^{(0)\pm}}{m},$$

$$\vec{g}_3^{(0)\pm} = \mp\frac{E_p^2\vec{g}_0^{(0)\pm} - (\vec{p}\cdot\vec{g}_0^{(0)\pm})\vec{p}}{E_p m}.$$

- **1st Constraint equations**

$$f_1^{(1)\pm} = \pm\frac{\vec{p}\cdot\vec{g}_0^{(1)\pm}}{E_p} \pm \frac{\vec{p}\cdot\vec{B}}{2E_p^3}f_0^{(0)\pm}$$

$$\vec{g}_1^{(1)\pm} = \pm\frac{\vec{p}}{E_p}f_0^{(1)} + \left(\frac{\vec{E}}{2E_p^2}\times\vec{g}_0^{(0)\pm} \pm \frac{\vec{B}(\vec{p}\cdot\vec{g}_0^{(0)\pm})}{2E_p^3}\right) \pm \frac{1}{2E_p}\vec{D}\times\vec{g}_0^{(0)}$$

$$\vec{g}_2^{(1)\pm} = \frac{\vec{p}\times\vec{g}_0^{(1)\pm}}{m} \pm \left(\frac{\vec{p}(\vec{p}\cdot\vec{E})}{2mE_p^3} - \frac{\vec{E}}{2mE_p}\right)f_0^{(0)\pm} + \frac{\vec{p}}{2mE_p^2}\vec{p}\cdot\vec{D}f_0^{(0)\pm} - \frac{1}{2m}\vec{D}f_0^{(0)\pm}$$

$$\vec{g}_3^{(1)\pm} = \mp\left(\frac{E_p}{m}\vec{g}_0^{(1)\pm} - \frac{\vec{p}\cdot\vec{g}_0^{(1)\pm}}{mE_p}\vec{p}\right) + \left(\frac{\vec{E}\times\vec{p}}{2mE_p^2} \mp \frac{m\vec{B}}{2E_p^3}\right)f_0^{(0)\pm} \mp \frac{1}{2mE_p}\vec{p}\times\vec{D}f_0^{(0)\pm}$$

Backup II. Transport equations @ 0th and 1st order

- **Transport equation of the classical components**

$$\left(D_t \pm \frac{\vec{p}}{E_p}\cdot\vec{D}\right) f_0^{(0)\pm} = 0$$

$$\left(D_t \pm \frac{\vec{p}}{E_p}\cdot\vec{D}\right) \vec{g}_0^{(0)\pm} - \frac{1}{E_p^2}\left[\vec{p}\times\left(\vec{E}\times\vec{g}_0^{(0)\pm}\right) \mp E_p\vec{B}\times\vec{g}_0^{(0)\pm}\right] = 0$$

$$\frac{\vec{p}}{m}\vec{p}\cdot\left(D_t \pm \frac{\vec{p}}{E_p}\cdot\vec{D}\right)\vec{g}_3^{(0)\pm} + m\left(D_t \pm \frac{\vec{p}}{E_p}\cdot\vec{D}\right)\vec{g}_3^{(0)\pm} + \frac{\vec{p}}{m}\vec{g}_3^{(0)\pm}\cdot\vec{E} \pm m\frac{\vec{B}\times\vec{g}_3^{(0)\pm}}{E_p} \pm \frac{\vec{p}\cdot(\vec{B}\times\vec{g}_3^{(0)\pm})}{mE_p}\vec{p} = 0$$

- **Transport equation of the first order components**

$$\left(D_t \pm \frac{\vec{p}}{E_p}\cdot\vec{D}\right) f_0^{(1)\pm} = \frac{\vec{E}}{2E_p^2}\cdot\vec{D}\times\vec{g}_0^{(0)\pm} \mp \frac{1}{2E_p^3}\vec{B}\cdot(\vec{p}\cdot\vec{D})\vec{g}_0^{(0)\pm} + \frac{(\vec{B}\times\vec{p})}{E_p^4}\cdot(\vec{E}\times\vec{g}_0^{(0)\pm})$$

$$\begin{aligned}\left(D_t \pm \frac{\vec{p}}{E_p}\cdot\vec{D}\right) \vec{g}_0^{(1)\pm} - \frac{1}{E_p^2}\left[\vec{p}\times\left(\vec{E}\times\vec{g}_0^{(1)\pm}\right) \mp E_p\vec{B}\times\vec{g}_0^{(1)\pm}\right] \\ = \mp\left(\frac{\vec{B}}{2E_p^3} \pm \frac{\vec{E}\times\vec{p}}{2E_p^4}\right)\vec{p}\cdot\vec{D}f_0^{(0)\pm} \mp \left(\frac{(\vec{p}\cdot\vec{E})(\vec{E}\times\vec{p})}{E_p^5} \pm \frac{\vec{p}\times(\vec{B}\times\vec{E})}{2E_p^4}\right) f_0^{(0)\pm}\end{aligned}$$

$$\begin{aligned}\frac{\vec{p}}{m}\vec{p}\cdot\left(D_t \pm \frac{\vec{p}}{E_p}\cdot\vec{D}\right)\vec{g}_3^{(1)\pm} + m\left(D_t \pm \frac{\vec{p}}{E_p}\cdot\vec{D}\right)\vec{g}_3^{(1)\pm} + \frac{\vec{p}}{m}\left(\vec{E} \pm \frac{\vec{p}}{E_p}\times\vec{B}\right)\cdot\vec{g}_3^{(1)\pm} \pm \frac{m}{E_p}\vec{B}\times\vec{g}_3^{(1)\pm} \\ = \left(\frac{\vec{p}(\vec{p}\cdot\vec{B})}{2E_p^4} + \frac{m^2\vec{B}}{2E_p^4}\right)(\vec{p}\cdot\vec{D}f_0^{(0)\pm}) \mp \left(\frac{\vec{p}\vec{p}\cdot(\vec{E}\times\vec{D}f_0^{(0)\pm})}{2E_p^3} + \frac{m^2\vec{E}\times\vec{D}f_0^{(0)\pm}}{2E_p^3}\right) \\ \pm \frac{3(m^2\vec{B} + \vec{p}\,\vec{p}\cdot\vec{B})}{2E_p^5}(\vec{p}\cdot\vec{E})f_0^{(0)\pm} \pm \frac{\vec{p}(\vec{B}\cdot\vec{E})}{2E_p^3}f_0^{(0)\pm}\end{aligned}$$

Backup III. Solution for CME

- Solution for g0

$$
\begin{aligned}
\vec{g}^{(0)\pm}_{0z}(x,\vec{p}) &= \sum_n \int \frac{dk^t dk^z k_T dk_T}{2\pi} c_{z,n} e^{i(k^t x^t - k^z x^z)} e^{i\frac{(k^t p^t - k^z p^z)\phi_p}{B}} \\
&\times e^{in\arctan\left(\frac{p^y + Bx^x}{p^x - Bx^y}\right)} J_n\left[k_T \sqrt{\left(x^x + \frac{p^y}{B}\right)^2 + \left(x^y - \frac{p^x}{B}\right)^2}\ \right] \\
\vec{g}^{(0)\pm}_{0x}(x,\vec{p}) &= \sum_n \int \frac{dk^t dk^z k_T dk_T}{2\pi} c_{z,n} e^{i(k^t x^t - k^z x^z)} e^{i\frac{(k^t p^t - k^z p^z)\phi_p}{B}} \cos(\phi_p - \phi_c) \\
&\times e^{in\arctan\left(\frac{p^y + Bx^x}{p^x - Bx^y}\right)} J_n\left[k_T \sqrt{\left(x^x + \frac{p^y}{B}\right)^2 + \left(x^y - \frac{p^x}{B}\right)^2}\right] \\
\vec{g}^{(0)\pm}_{0y}(x,\vec{p}) &= \sum_n \int \frac{dk^t dk^z k_T dk_T}{2\pi} c_{z,n} e^{i(k^t x^t - k^z x^z)} e^{i\frac{(k^t p^t - k^z p^z)\phi_p}{B}} \sin(\phi_p - \phi_c) \\
&\times e^{in\arctan\left(\frac{p^y + Bx^x}{p^x - Bx^y}\right)} J_n\left[k_T \sqrt{\left(x^x + \frac{p^y}{B}\right)^2 + \left(x^y - \frac{p^x}{B}\right)^2}\right]
\end{aligned}
$$

- Method of characteristics.

ELSEVIER

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PHYSICS LETTERS B

Out-of-equilibrium chiral magnetic effect from chiral kinetic theory

Anping Huang [a, b], Yin Jiang [c], Shuzhe Shi [b], Jinfeng Liao [b], Pengfei Zhuang [a]

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https://doi.org/10.1016/j.physletb.2017.12.025

Mass Correction to CKT

Chiral Phase Transition in an Expanding System

BACKGROUND

1. Phase transition signals
2. Solve Vlasov equation
3. Shell-like structure

Vlasov equation & gap equation

$$\frac{\partial f}{\partial t} + \frac{\mathbf{p}}{E}.\boldsymbol{\nabla}_{\boldsymbol{r}} f - \boldsymbol{\nabla}_{\boldsymbol{r}} E.\boldsymbol{\nabla}_{\boldsymbol{p}} f = 0$$

$$M_c(\mathbf{r},t) = m + g^2 M_c(\mathbf{r},t) \int_0^{\Lambda} \frac{\mathrm{d}^3 p}{\sqrt{\mathbf{p}^2 + M_c^2(\mathbf{r},t)}} \left(\frac{2N_c N_f}{(2\pi)^3} - f(\mathbf{r},\mathbf{p},t) - \tilde{f}(\mathbf{r},\mathbf{p},t) \right)$$

Solution — analytical solution, constant mass

C. Greiner and D. Rischke, Phys.Rev. C54 (1996)

— numerical solution, test particle method

A. Abada and J. Aichelin, Phys. Rev. Lett. 74 (1995)

H. van Hees, C. Wesp, A. Meistrenko and C. Greiner, Acta Phys.Polon.Supp. 7 (2014)

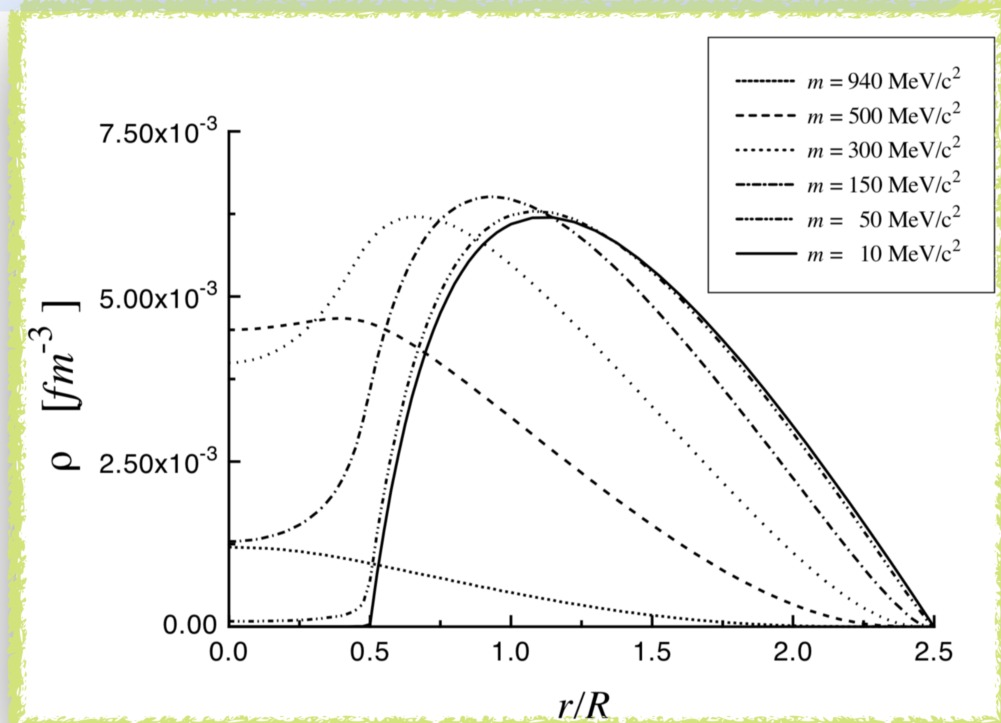
Physical content

— shell-like structure in 3D expansion

Relativistic, sensitive to mass

— thermal anomalous at phase transition

From hydrodynamics & TN, TA equation of state



- **Vlasov equation & gap equation**

$$\frac{\partial f}{\partial t} + \frac{\mathbf{p}}{E}.\boldsymbol{\nabla}_{\boldsymbol{r}} f - \boldsymbol{\nabla}_{\boldsymbol{r}} E.\boldsymbol{\nabla}_{\boldsymbol{p}} f = 0$$

$$M_c(\mathbf{r},t) = m + g^2 M_c(\mathbf{r},t) \int_0^{\Lambda} \frac{\mathrm{d}^3 p}{\sqrt{\mathbf{p}^2 + M_c^2(\mathbf{r},t)}} \left(\frac{2N_c N_f}{(2\pi)^3} - f(\mathbf{r},\mathbf{p},t) - \tilde{f}(\mathbf{r},\mathbf{p},t) \right)$$

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- **Physical content**

  — shell-like structure in 3D expansion

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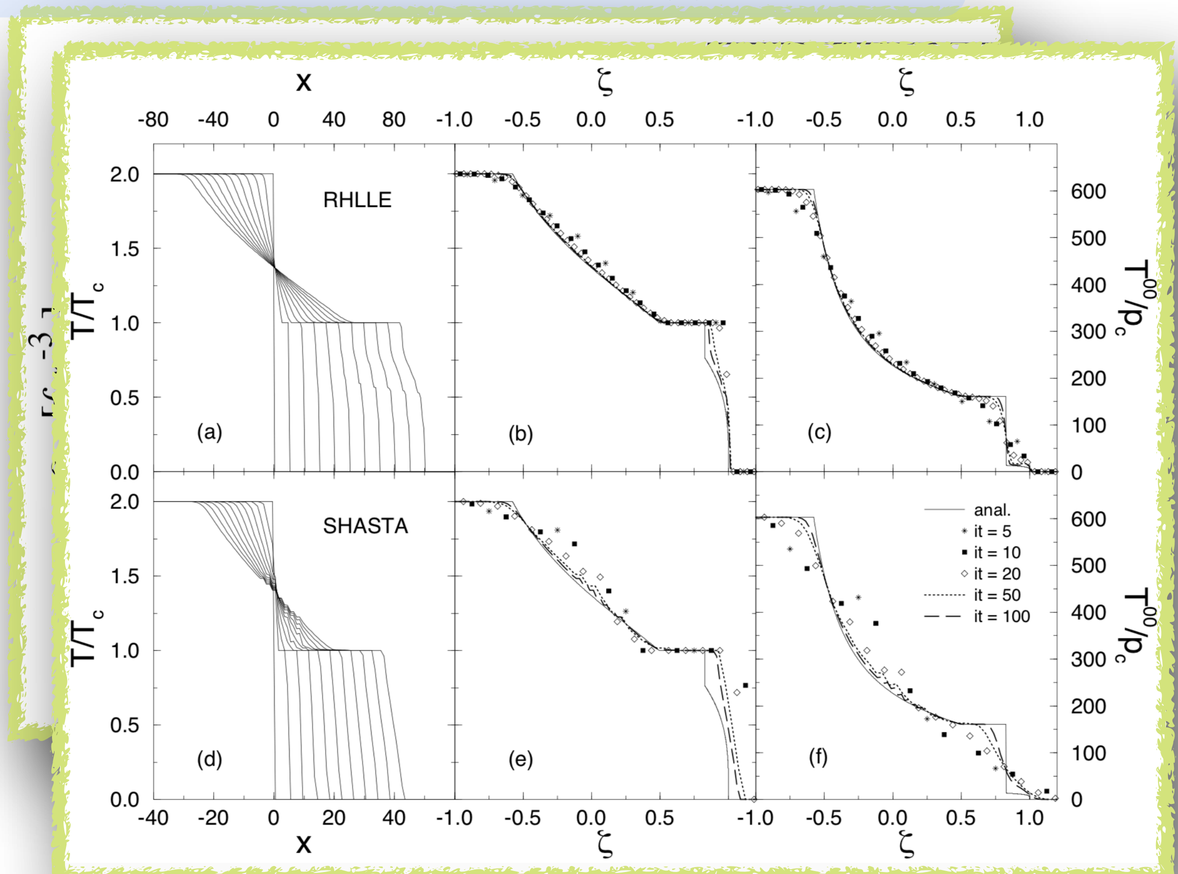

- Vlasov equation & gap equation

$$\frac{\partial f}{\partial t} + \frac{\mathbf{p}}{E}.\boldsymbol{\nabla}_{\boldsymbol{r}} f - \boldsymbol{\nabla}_{\boldsymbol{r}} E.\boldsymbol{\nabla}_{\boldsymbol{p}} f = 0$$

$$M_c(\mathbf{r},t) = m + g^2 M_c(\mathbf{r},t) \int_0^{\Lambda} \frac{\mathrm{d}^3 p}{\sqrt{\mathbf{p}^2 + M_c^2(\mathbf{r},t)}} \left(\frac{2N_c N_f}{(2\pi)^3} - f(\mathbf{r},\mathbf{p},t) - \tilde{f}(\mathbf{r},\mathbf{p},t) \right)$$

- Solution — analytical solution, constant mass

  C. Greiner and D. Rischke, Phys.Rev. C54 (1996)

  — numerical solution, simultaneously, test particle method

  A. Abada and J. Aichelin, Phys. Rev. Lett. 74 (1995)

  H. van Hees, C. Wesp, A. Meistrenko and C. Greiner, Acta Phys.Polon.Supp. 7 (2014)

- Physical content

  — shell-like structure in 3D expansion

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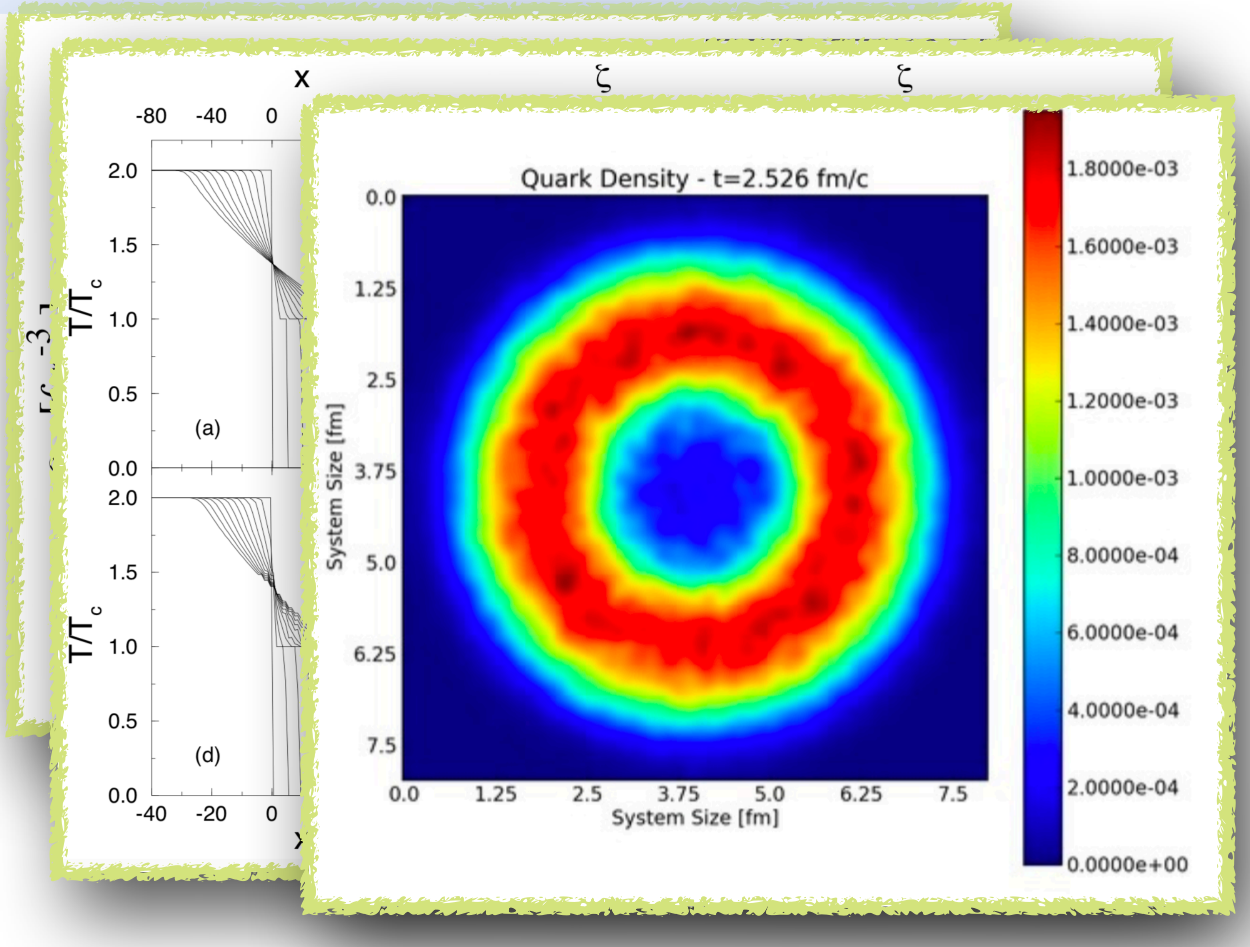


FRAMEWORK

1. Longitudinal expansion – 1D
2. Numerical solution
3. Thermal quantities

Transport Equation

- Simultaneously solve transport equation & gap equation

Force term

$$\partial_t f^{\pm} \boxed{\mp \frac{\boldsymbol{\nabla}_{\boldsymbol{r}} m^2}{2E_p} \cdot \boldsymbol{\nabla}_{\boldsymbol{p}} f^{\pm}} \pm \frac{\boldsymbol{p}}{\boxed{E_p}} \cdot \boldsymbol{\nabla}_{\boldsymbol{r}} f^{\pm} = 0$$

$$m\left(1 + 2G \int \frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^3} \frac{f^+(x,\boldsymbol{p}) - f^-(x,\boldsymbol{p})}{E_p}\right) = m_0$$

Mass involves in (1) force term, (2) energy

- Simplify: 1D Longitudinal expansion

$$\frac{\boldsymbol{\nabla}_{\boldsymbol{r}} m^2}{2E_p} \cdot \boldsymbol{\nabla}_{\boldsymbol{p}} f^{\pm} \quad \rightarrow \quad \frac{\partial_z m^2}{2E_p} \frac{\partial}{\partial p_z} f^{\pm}$$

$$\frac{\boldsymbol{p}}{E_p} \cdot \boldsymbol{\nabla}_{\boldsymbol{r}} f^{\pm} \quad \rightarrow \quad \frac{p_z}{E_p} \frac{\partial}{\partial z} f^{\pm}$$

f(t, x, y, z, px, py, pz) reduces to f(t, z, pz, pT)

- Self-consistent numerical solution

Finite difference & f(t, z, pz, pT) discrete on a fixed grid (z, pz, pT)

Initial Condition & Thermal Quantities

- **Initial condition**

(1) $f(t_0, z, p_z, p_T) = (e^{E_p/T(z)} + 1)^{-1}$ $\quad T(z) = T_0 \exp\left(-z^2/z_0^2\right)$ $\quad E_p = \sqrt{p_z^2 + p_T^2 + m_0^2}$

(2) $f(t_0, z, p_z, p_T) = (e^{E_p/T_0} + 1)^{-1}$ when $z \le r_0$

$f(t_0, z, p_z, p_T) = 0$ when $z > r_0$

- **Thermal quantities**

Current & energy-momentum tensor

$$\partial_\mu J^\mu = \int \frac{\mathrm{d}^3\mathbf{p}}{E_p} p^\mu \partial_\mu f(\mathbf{p}) = 0$$

$$\partial_\nu T^{\mu\nu} = \int \frac{\mathrm{d}^3\mathbf{p}}{E_p} p^\mu p^\nu \partial_\nu f(\mathbf{p}) = 0$$

$$J^\mu(t, \mathbf{x}) = \int f(t, \mathbf{x}, \mathbf{p})(p^\mu/E_p)\, \mathrm{d}^3\mathbf{p}$$

$$T^{\mu\nu}(t, \mathbf{x}) = \int f(t, \mathbf{x}, \mathbf{p}) p^\mu p^\nu \, \mathrm{d}^3\mathbf{p}/E_p = \begin{bmatrix} T^{tt} & -T^{tz} & 0 & 0 \\ T^{tz} & -T^{zz} & 0 & 0 \\ 0 & 0 & -T^{xx} & 0 \\ 0 & 0 & 0 & -T^{yy} \end{bmatrix}$$

Velocity & energy density

$$u^\mu = \gamma_v\{1, v\} \equiv \gamma_v \left\{1, \frac{2T^{tz}}{T^{tt} + T^{zz} + \sqrt{(T^{tt} + T^{zz})^2 - 4(T^{tz})^2}}\right\}$$

$$\epsilon = \frac{T^{tt} - T^{zz} + \sqrt{(T^{tt} + T^{zz})^2 - 4(T^{tz})^2}}{2}$$

RESULTS

1 Gaussian, real case

2 Step function, real case

3 Step function, chiral limit

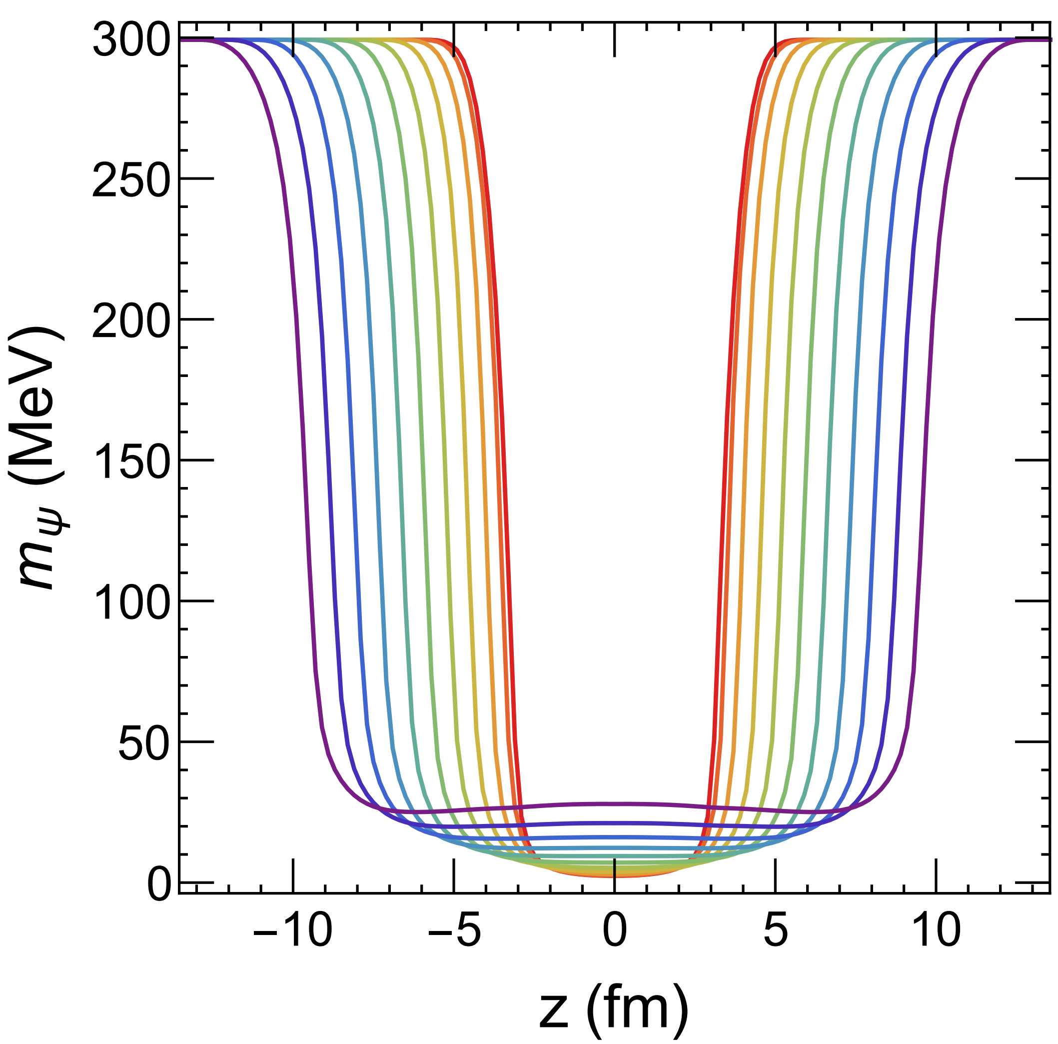
Result 1 — Gaussian, real case

$$f(t_0, z, p_z, p_T) = (e^{E_p/T(z)} + 1)^{-1} \qquad T(z) = T_0 \exp\left(-z^2/z_0^2\right) \qquad E_p = \sqrt{p_z^2 + p_T^2 + m_0^2}$$

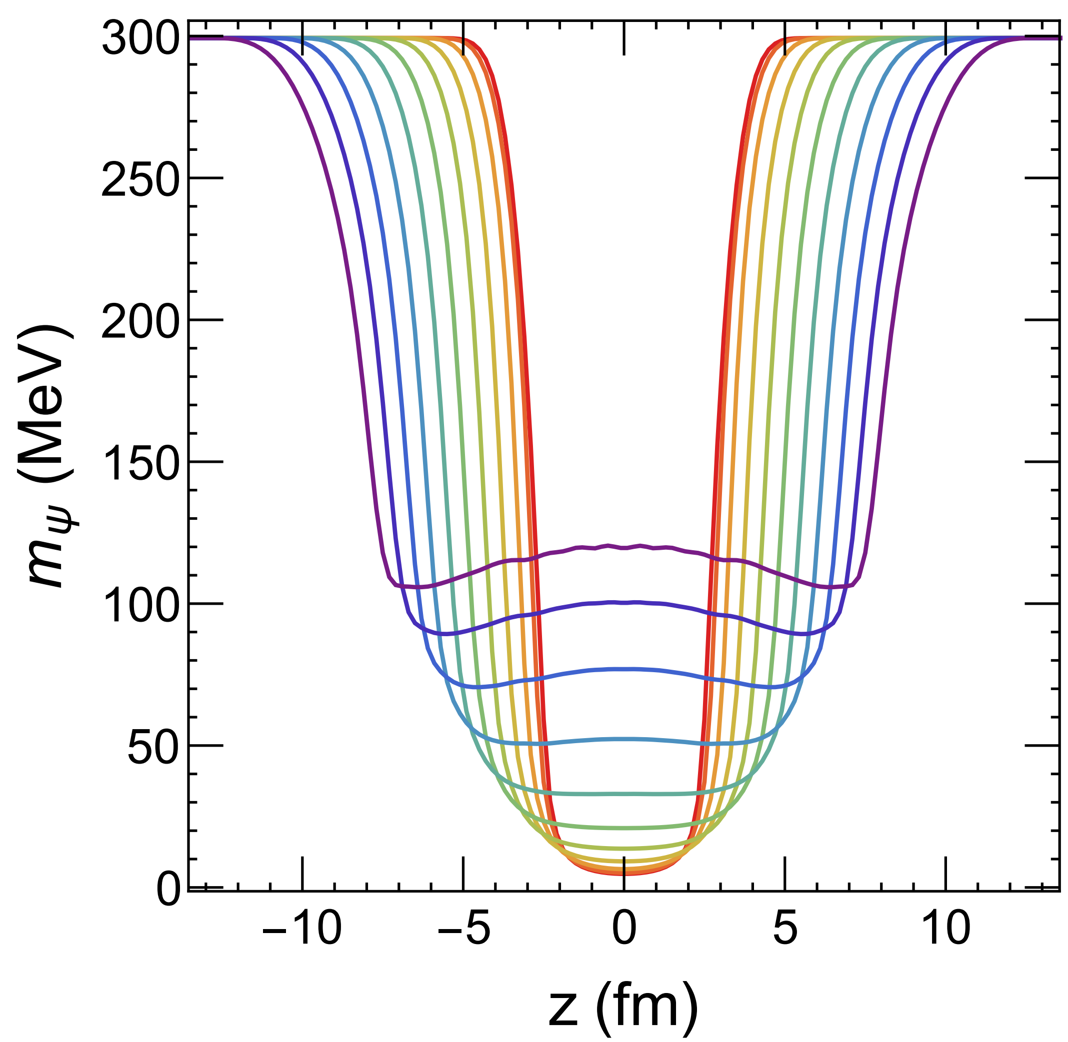
$T_0 = 300$ MeV, $T_0 = 240$ MeV with $z_0 = 4$fm **t from t=0 to t=10fm**

- **Constituent mass m(t, z)**

T0 = 300 MeV

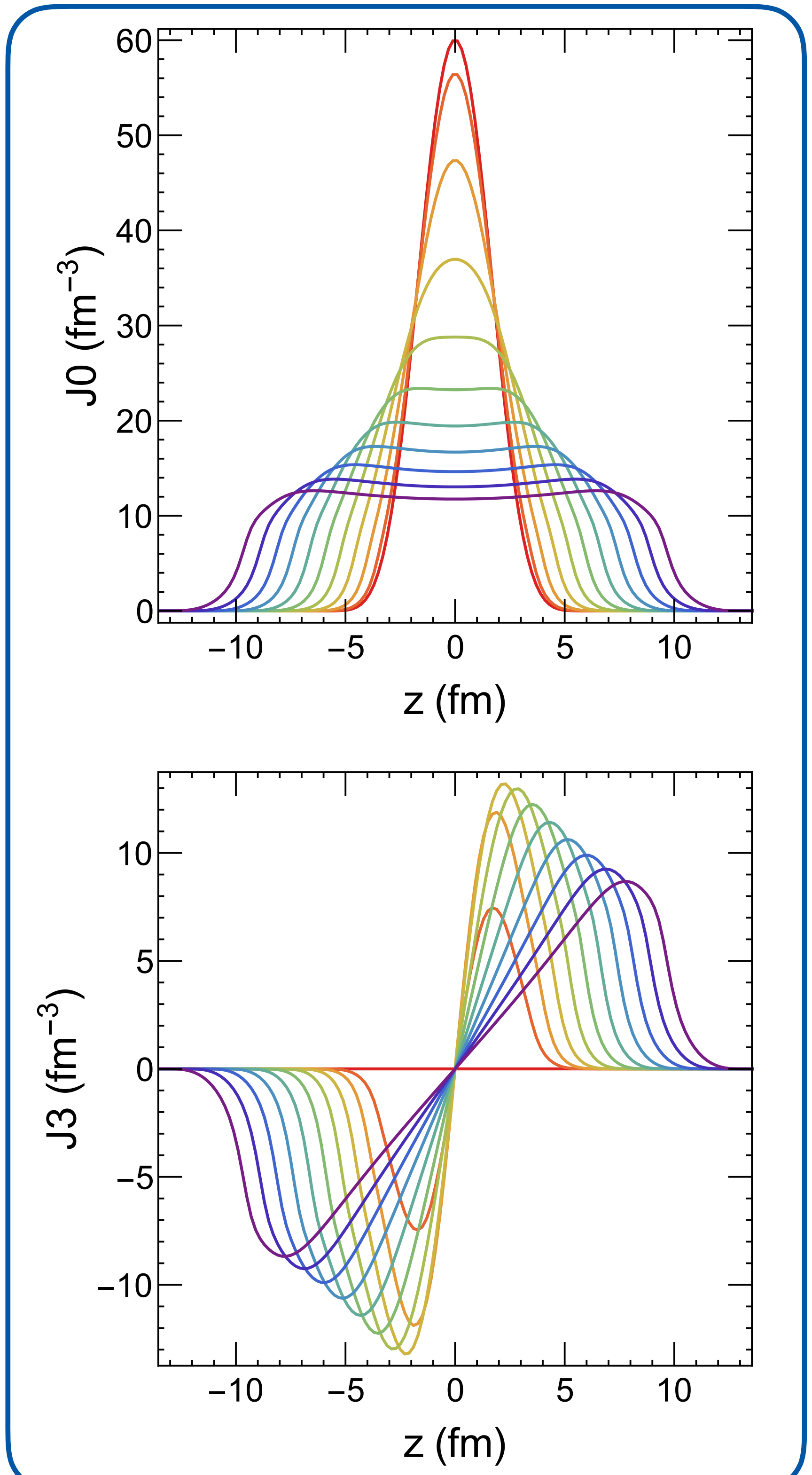


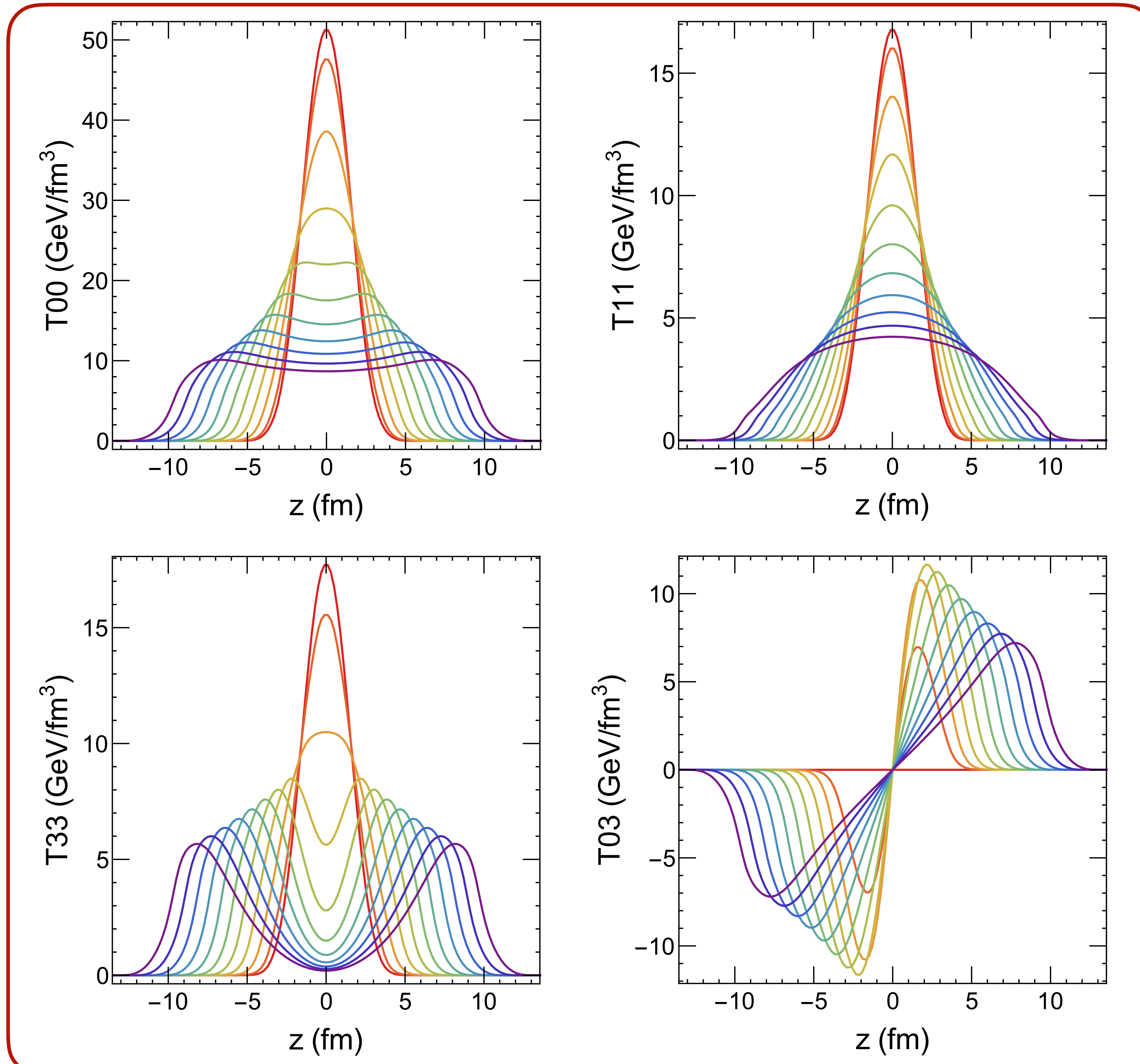
T0 = 240 MeV



Result 1 — Gaussian, real case

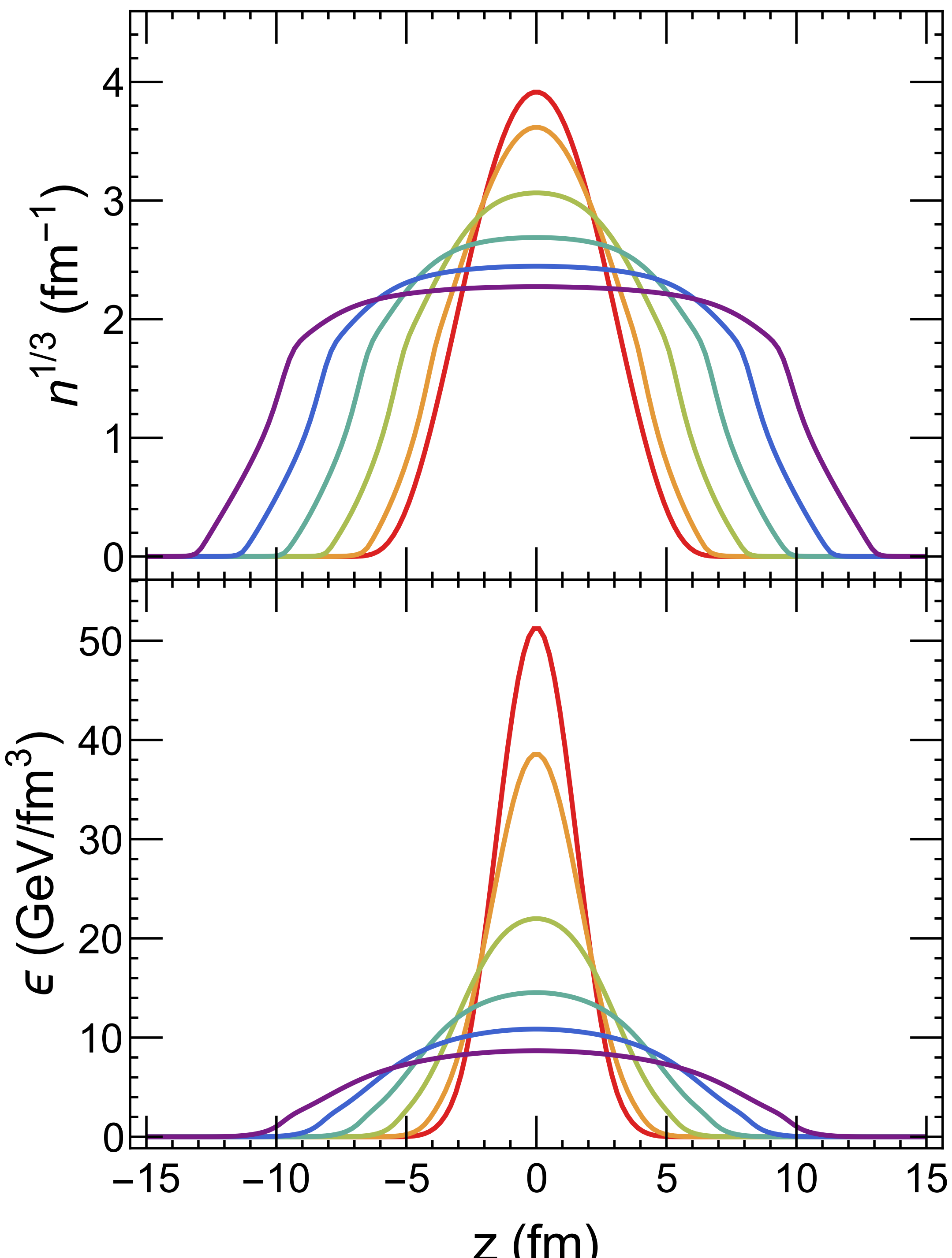
- Current and Energy-momentum tensor **Conservation checked!**



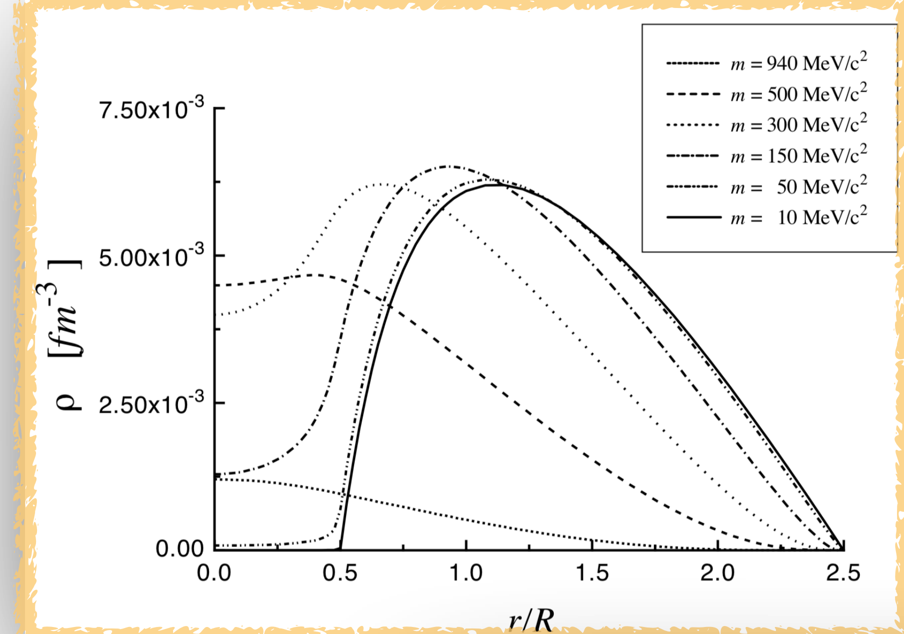


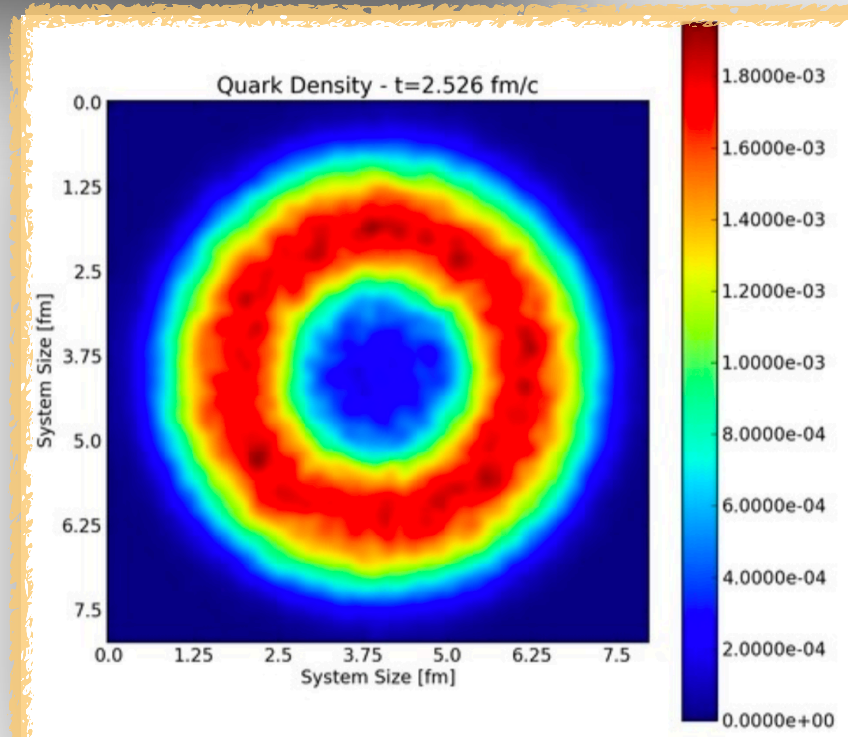
Result 1 — Gaussian, real case

- Number density & energy density



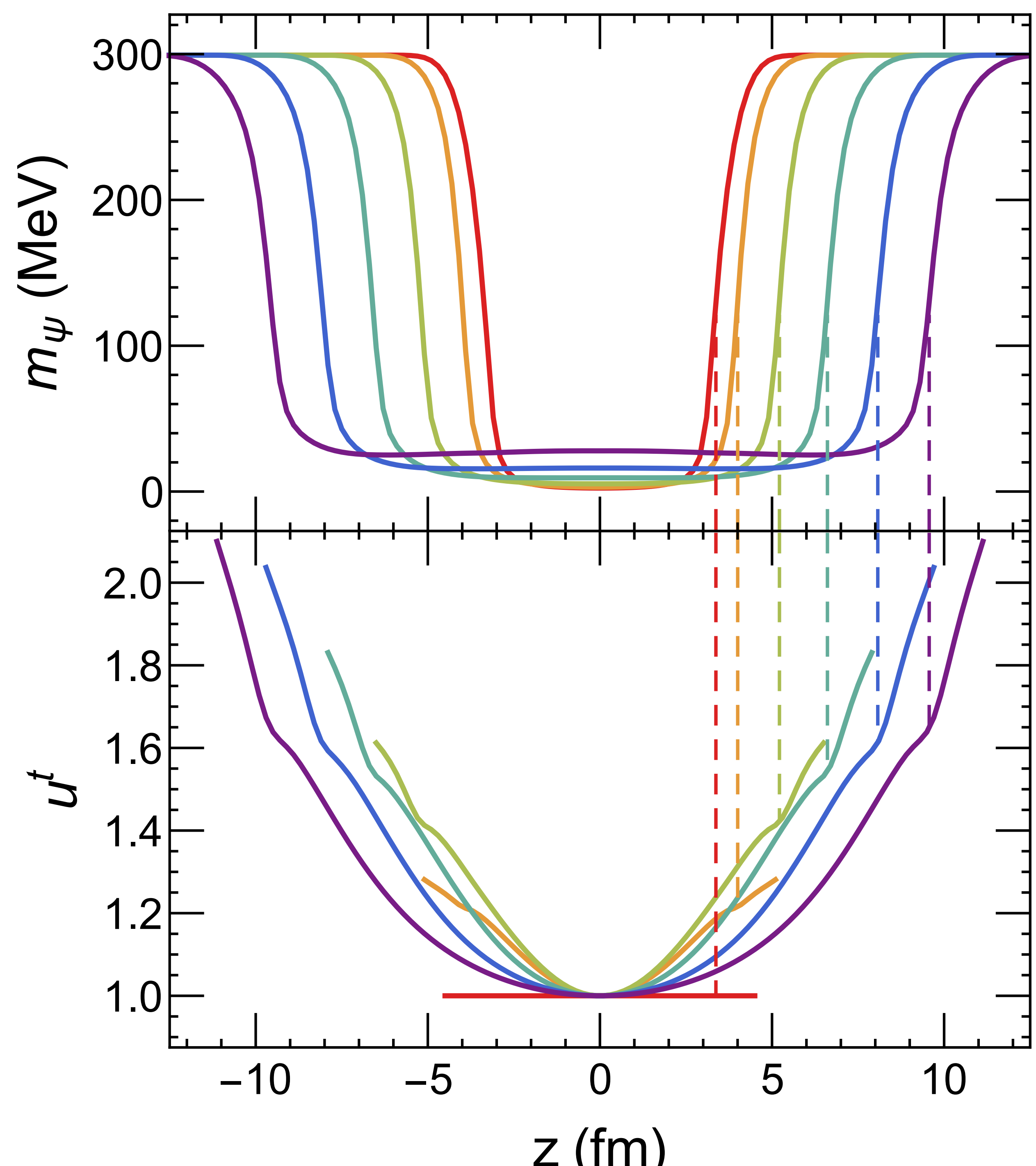
There is no shell-like structure in the 1D expansion





Result 1 — Gaussian, real case

- Velocity and the "fold"

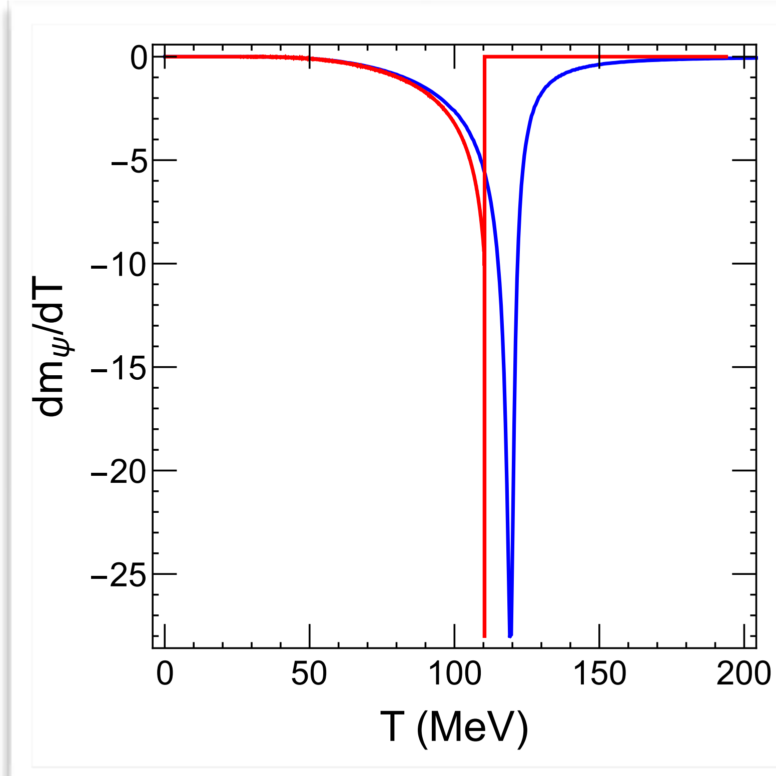


A "fold" in the velocity at the phase transition

$$\partial_t f^{\pm} \mp \boxed{\frac{\boldsymbol{\nabla}_{\boldsymbol{r}} m^2}{2E_p} \cdot \boldsymbol{\nabla}_{\boldsymbol{p}} f^{\pm}} \pm \frac{\boldsymbol{p}}{E_p} \cdot \boldsymbol{\nabla}_{\boldsymbol{r}} f^{\pm} = 0$$

A strong force at the phase transition

$$\frac{dm}{dz} = \boxed{\frac{dm}{dT}} \frac{dT}{dz}$$

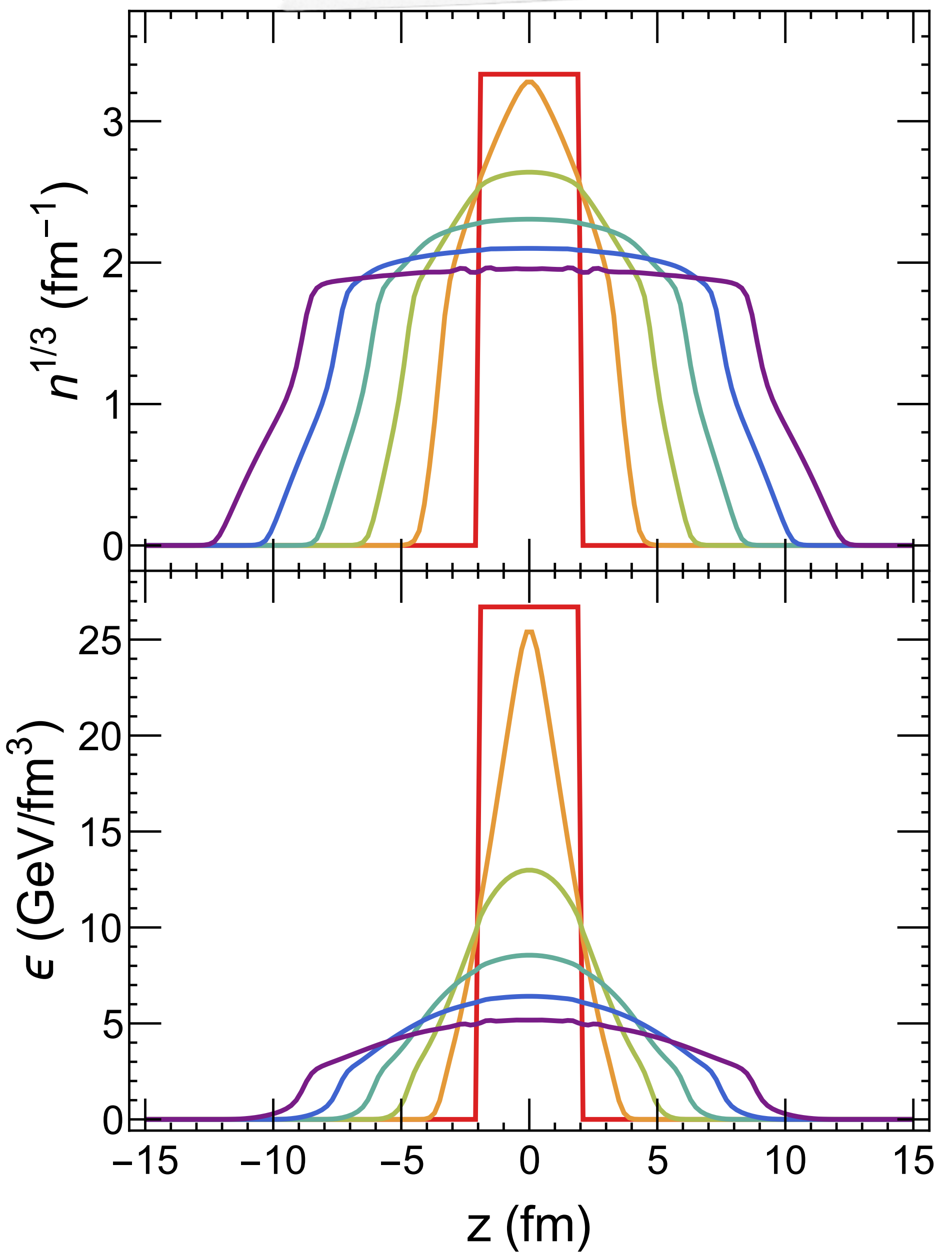


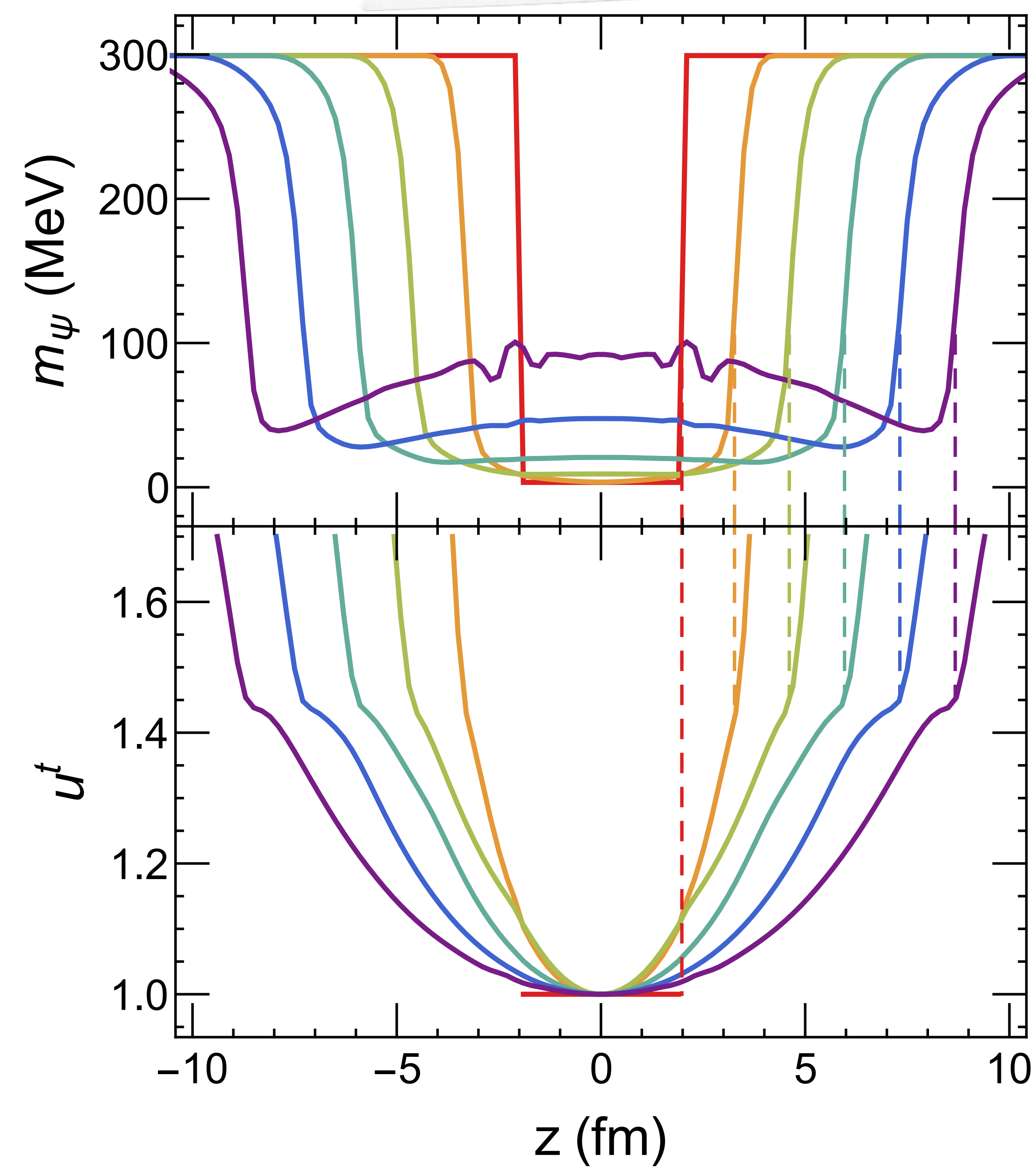
Result 2 — Step function, real case

$f(t_0, z, p_z, p_T) = (e^{E_p/T_0} + 1)^{-1}$ when $z \le r_0$ $\quad f(t_0, z, p_z, p_T) = 0$ when $z > r_0$

$T_0 = 240$ MeV, $z_0 = 2$ fm

t from t=0 to t=10fm



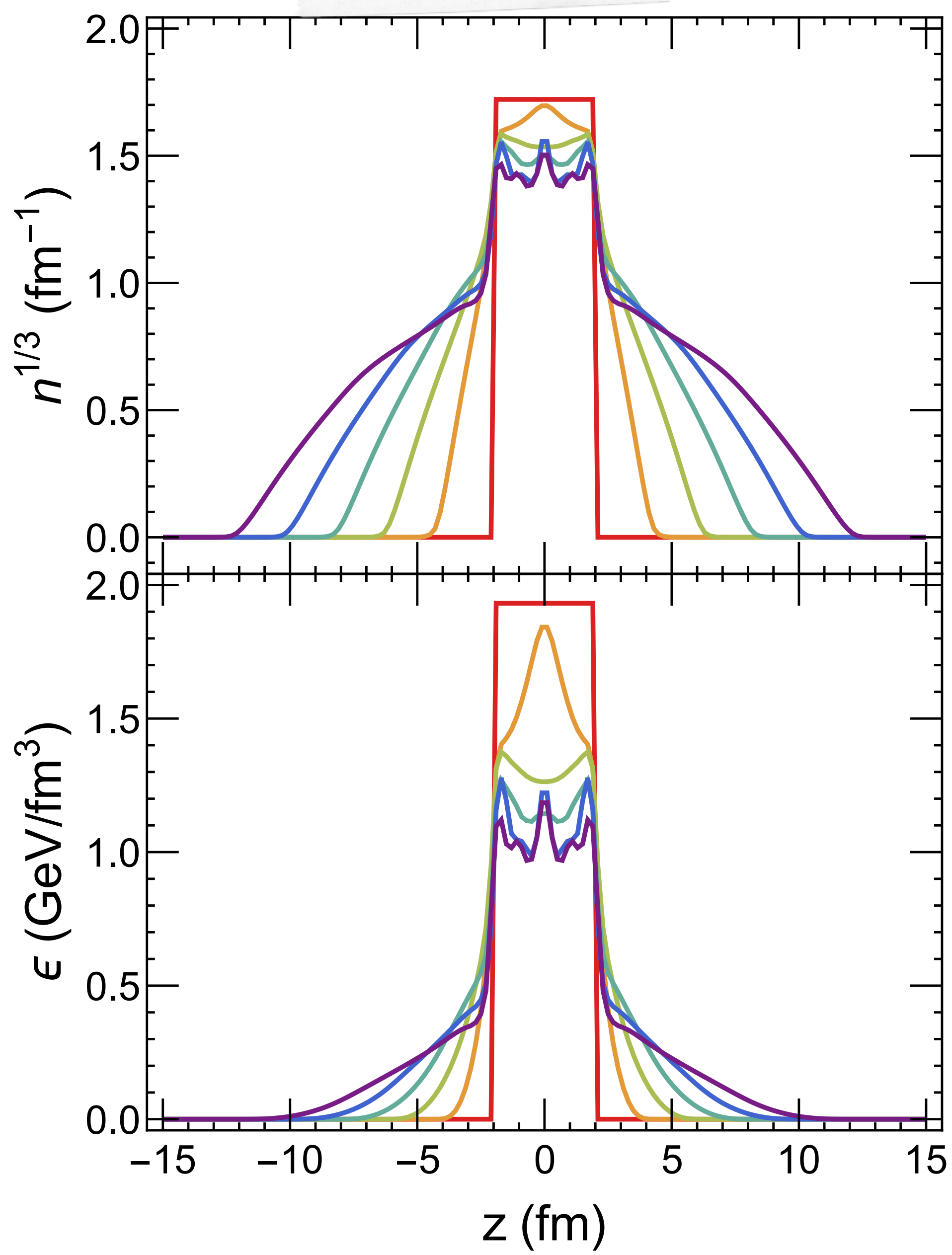


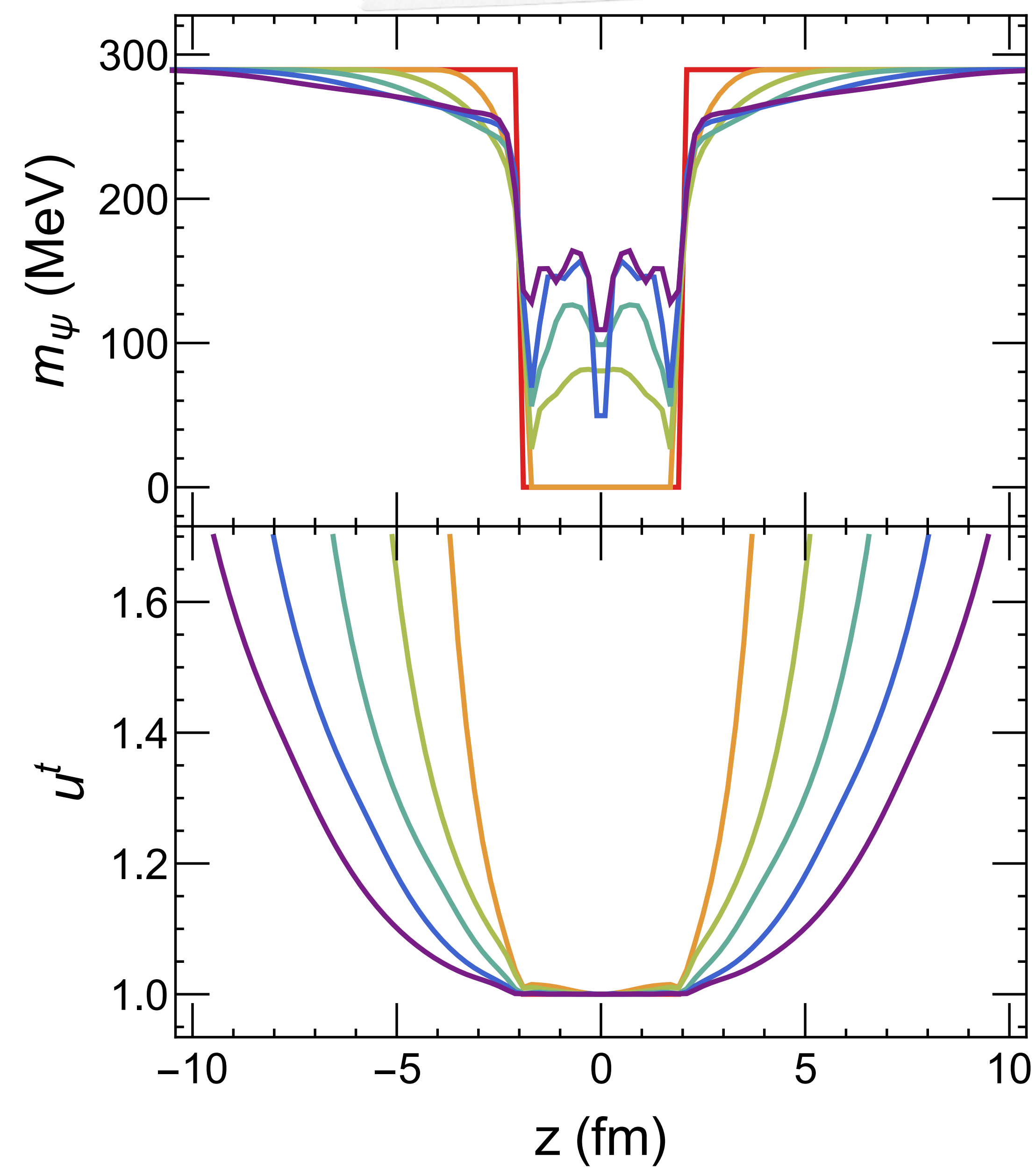
Result 3 — Step function, chiral limit

$f(t_0, z, p_z, p_T) = (e^{E_p/T_0} + 1)^{-1}$ when $z \leq r_0$ $\quad f(t_0, z, p_z, p_T) = 0$ when $z > r_0$

$T_0 = 120$ MeV, $z_0 = 2$ fm

t from t=0 to t=10fm





NEXT STEP

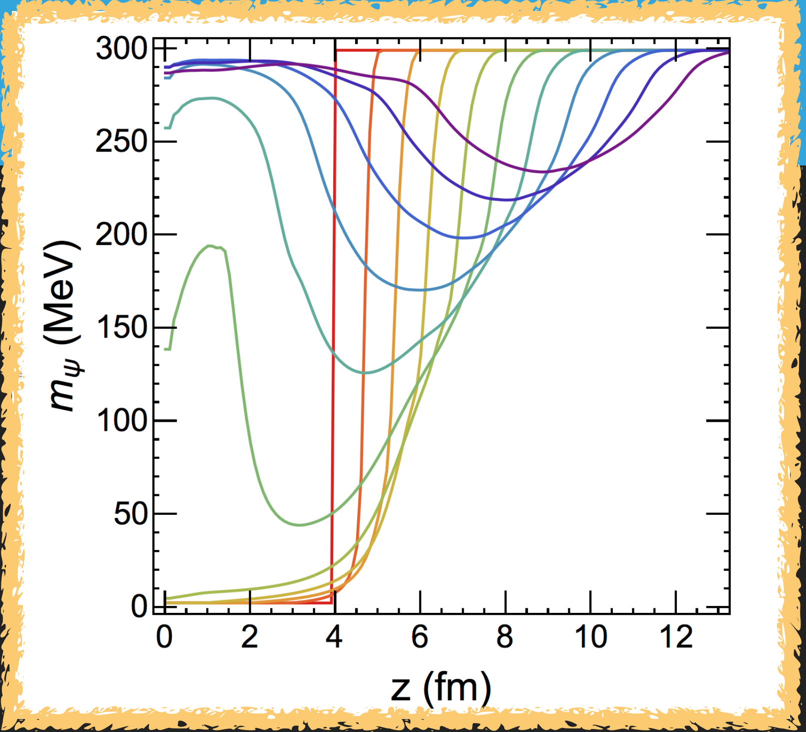
1) Spherical expansion — shell-like structure

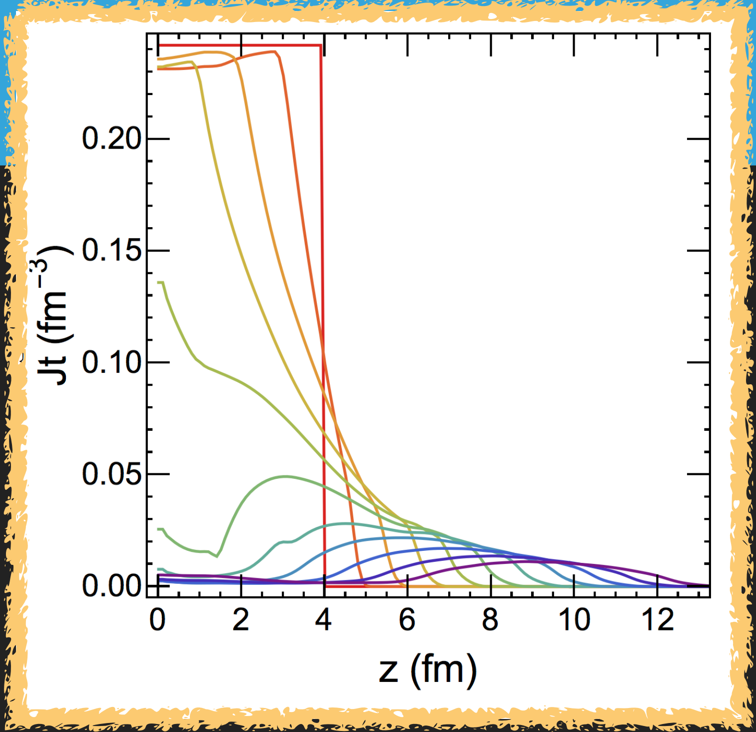
Longitudinal-boost-invariant Transverse-rotational-symmetric Expansion

2) Collisions — relaxation time approximation

3) Together with transport equation of sigma and pion

FORECAST





Mass Correction to CKT

Thank you!

Chiral Phase Transition in Expanding System