





Moving Towards Investigations of Multi-Charge Diffusion in Heavy Ion Collisions

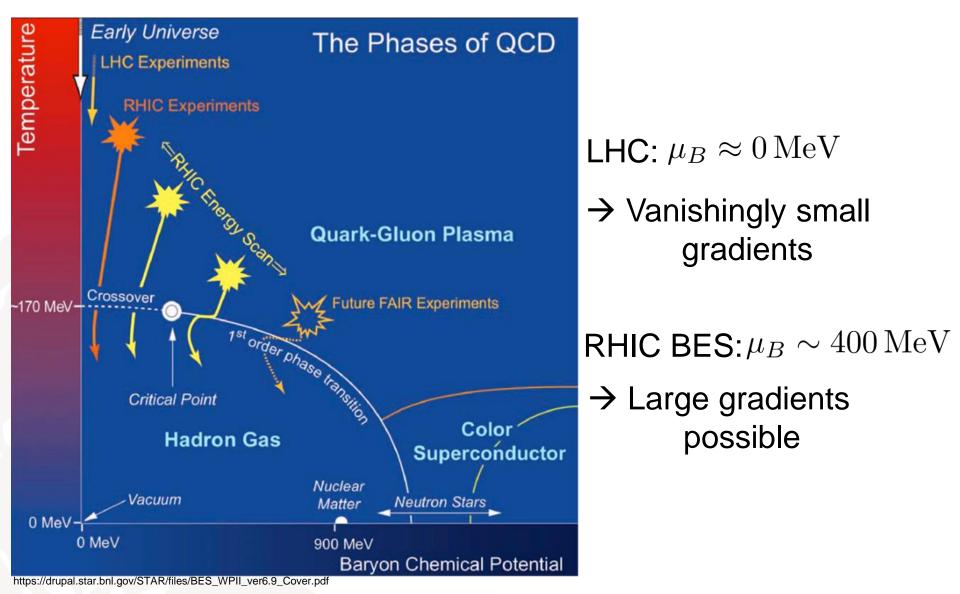
Presented by Jan Fotakis

Collaborators Moritz Greif, Gabriel Denicol, Carsten Greiner and Harri Niemi

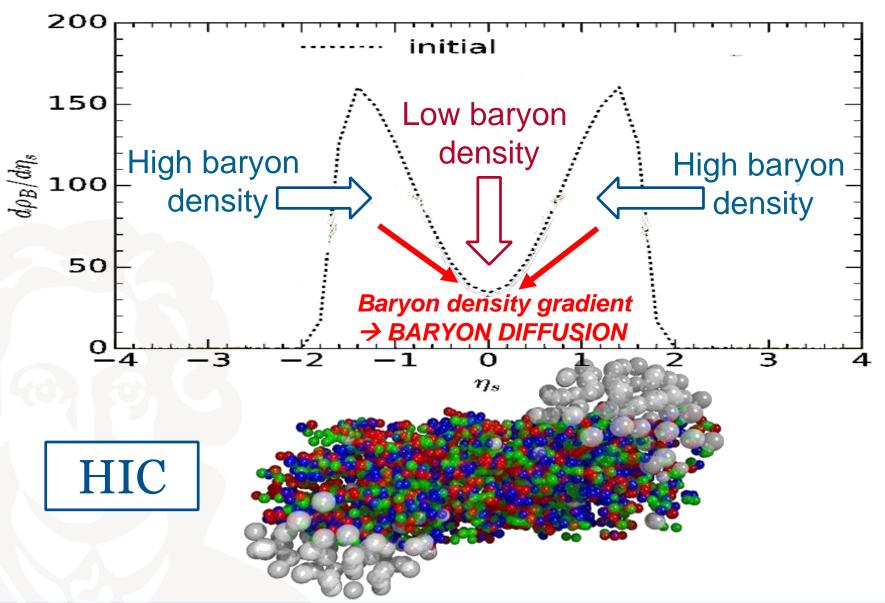
Greif, Fotakis, Denicol, Greiner, Phys. Rev. Lett. 120, 242301 (2018)

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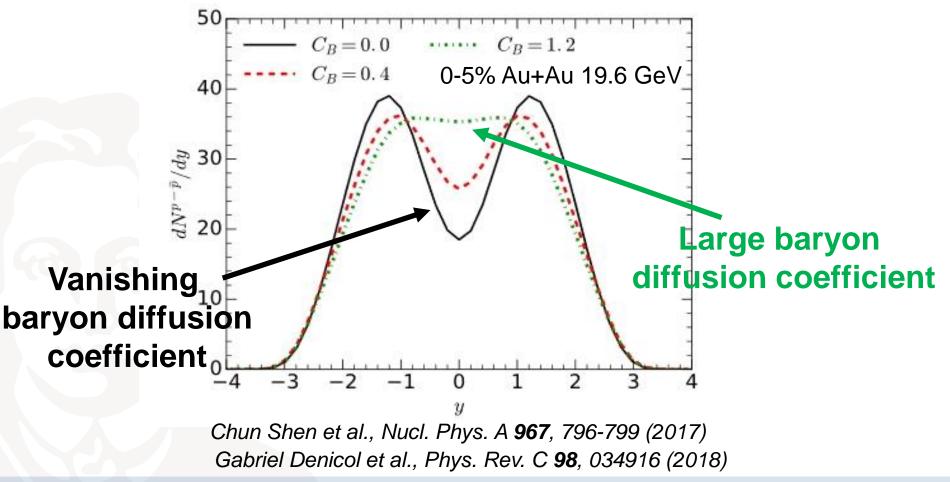


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Why could diffusion be important?



 During low-energy HIC (e.g. RHIC BES, FAIR): diffusion could have great impact on dynamic evolution





- Dynamic evolution of HIC modeled in relativistic dissipative fluid dynamics
- In Navier-Stokes theory for one conserved charge (q):





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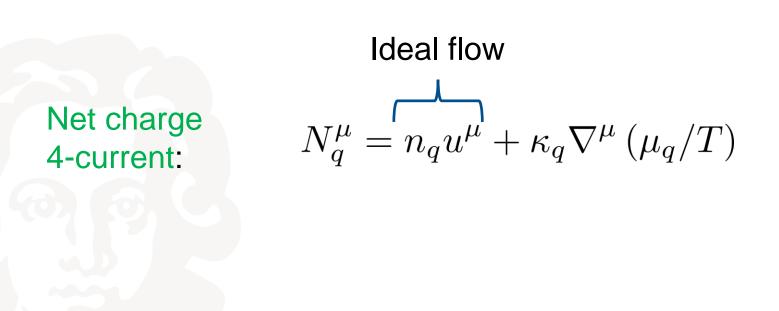
Net charge 4-current:

$$N_q^{\mu} = n_q u^{\mu} + \kappa_q \nabla^{\mu} \left(\mu_q / T \right)$$

 u^{μ} : flow velocity n_q : net charge density



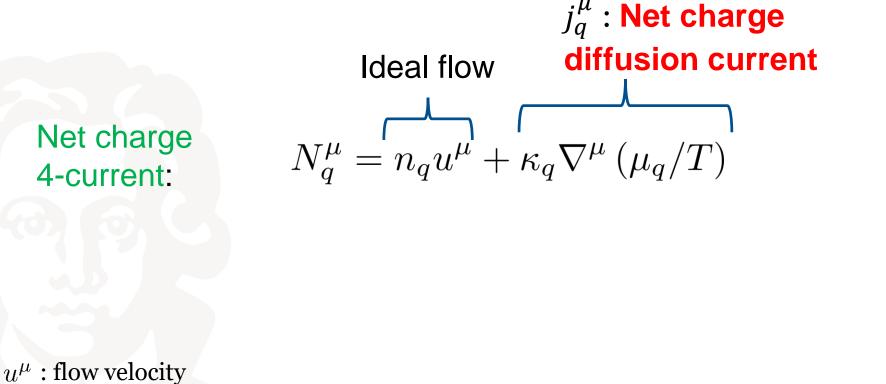
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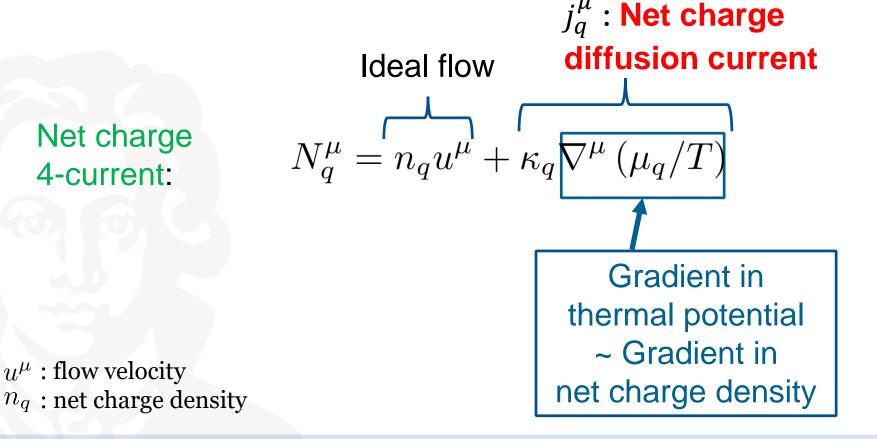
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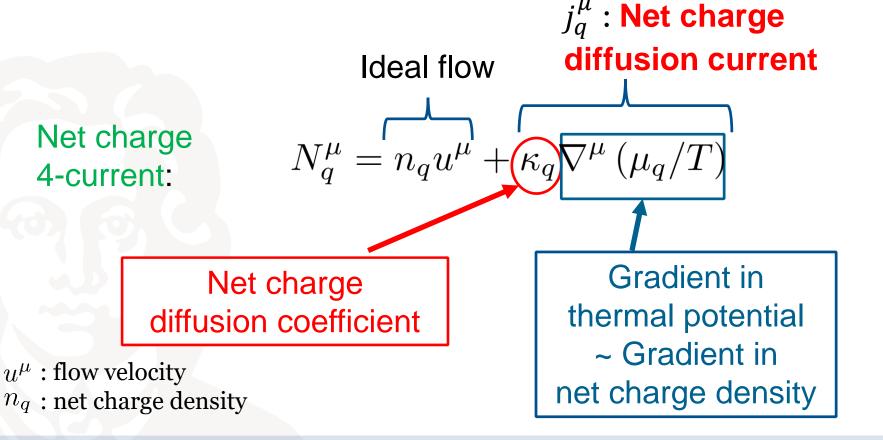


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- In multi-component system with multiple conserved charges: particles can have any combination of charges (e.g. proton: electric and baryon charge)
- Net charge diffusion currents effect each other



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$$\begin{pmatrix} j^{\mu}_{B} \\ j^{\mu}_{Q} \\ j^{\mu}_{S} \end{pmatrix} = \begin{pmatrix} \kappa_{BB} & \kappa_{BQ} & \kappa_{BS} \\ \kappa_{QB} & \kappa_{QQ} & \kappa_{QS} \\ \kappa_{SB} & \kappa_{SQ} & \kappa_{SS} \end{pmatrix} \cdot \begin{pmatrix} \nabla^{\mu} \alpha_{B} \\ \nabla^{\mu} \alpha_{Q} \\ \nabla^{\mu} \alpha_{S} \end{pmatrix}$$

Off-diagonal coefficients: gradients of given charge can Are the offeffect diffusion currents of other charges diagonal coefficients

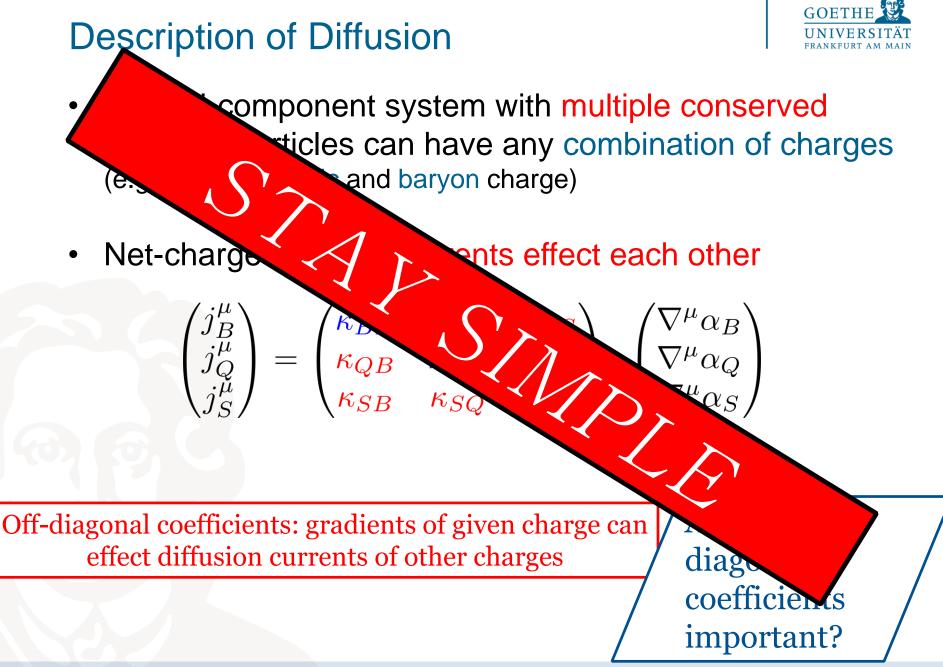
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important?

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Consider massless, conformal QGP $(u, \bar{u}, d, \bar{d}, s, \bar{s}, g)$ with conserved baryon (B) and strangeness (S) charge only







Consider massless, conformal QGP ($u, \bar{u}, d, d, s, \bar{s}, g$) with conserved baryon (B) and strangeness (S) charge only

Particle	Mass	В	S	Degeneracy
Gluon	0	0	0	16
Up	0	+1/3	0	6
Anti-Up	0	-1/3	0	6
Down	0	+1/3	0	6
Anti-Down	0	-1/3	0	6
Strange	0	+1/3	-1	6
Anti-Strange	0	-1/3	+1	6



Consider massless, conformal QGP $(u, \bar{u}, d, \bar{d}, s, \bar{s}, g)$ with conserved baryon (B) and strangeness (S) charge only

$$\epsilon_0 = 3P_0 = 3n_{\rm tot}T$$

$$n_{\rm B} \sim T^3 \left(2 \sinh\left(\frac{1}{3}\frac{\mu_B}{T}\right) + \sinh\left(\frac{1}{3}\frac{\mu_B}{T} - \frac{\mu_S}{T}\right) \right)$$
$$n_{\rm S} \sim -T^3 \sinh\left(\frac{1}{3}\frac{\mu_B}{T} - \frac{\mu_S}{T}\right)$$
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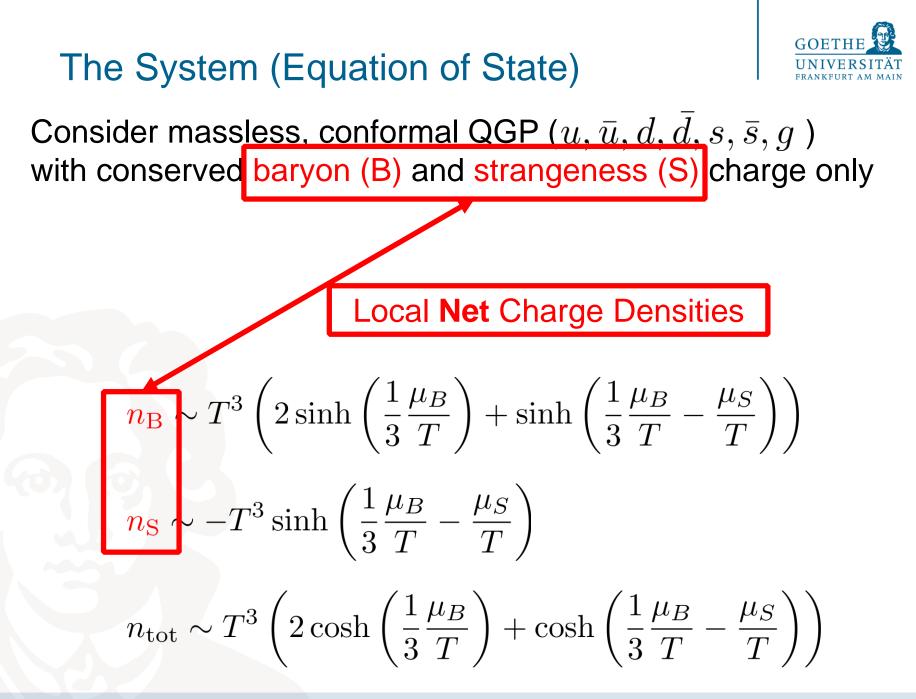
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$$\text{Local Total Number Density}$$

$$n_{\text{S}} \sim -T^{3} \sinh\left(\frac{1}{3}\frac{\mu_{B}}{T} - \frac{\mu_{S}}{T}\right)$$

$$n_{\text{tot}} \sim T^{3} \left(2 \cosh\left(\frac{1}{3}\frac{\mu_{B}}{T}\right) + \cosh\left(\frac{1}{3}\frac{\mu_{B}}{T} - \frac{\mu_{S}}{T}\right)\right)$$





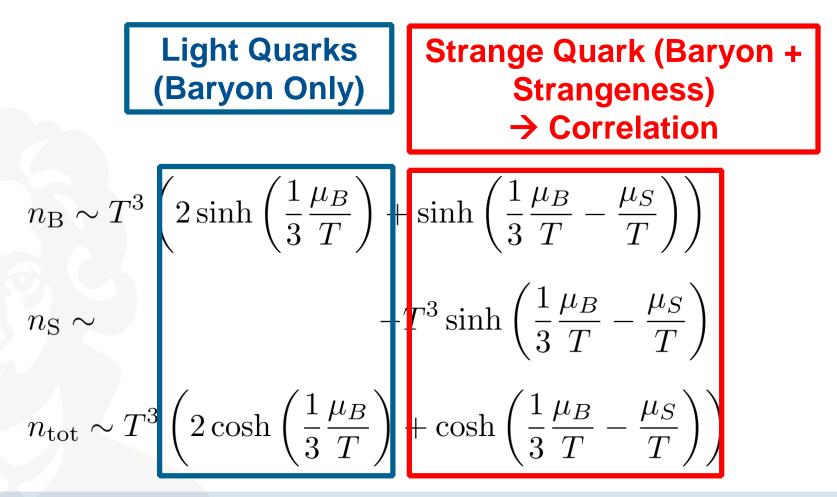
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Light Quarks (Baryon Only)

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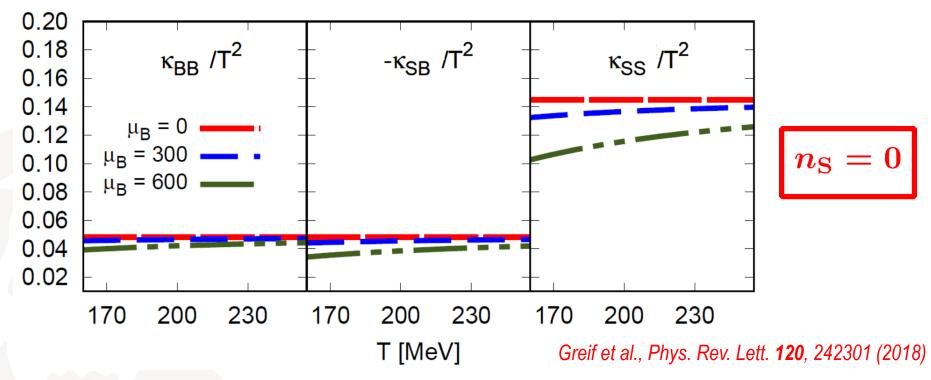
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The System (Diffusion Coefficients)

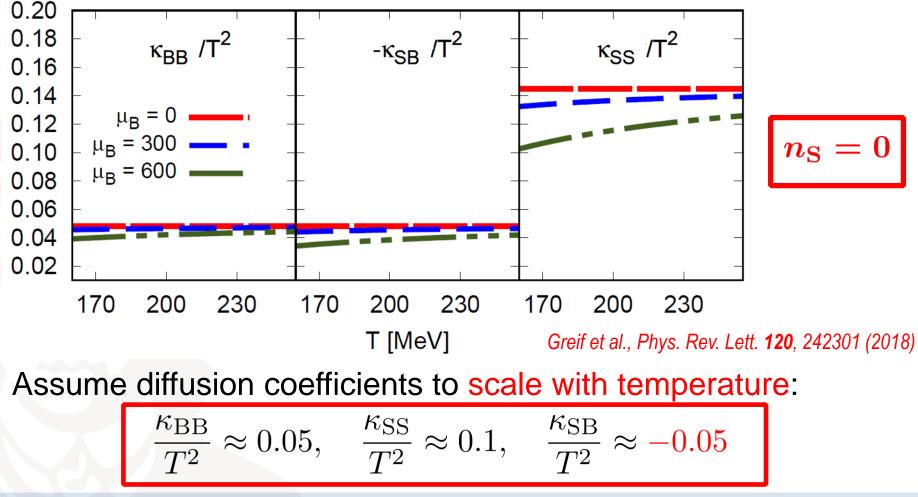
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Assume diffusion coefficients to scale with temperature:

$$\frac{\kappa_{\rm BB}}{T^2} \approx 0.05, \quad \frac{\kappa_{\rm SS}}{T^2} \approx 0.1, \quad \frac{\kappa_{\rm SB}}{T^2} \approx -0.05$$

Strangeness and Baryon Diffusion Currents = Anti-Correlated

Is there Baryon-Strangeness (Anti-)Correlation in a dynamic setting? → Use Fluid Dynamics!

Greif et al., Phys. Rev. Lett. 120, 242301 (2018)



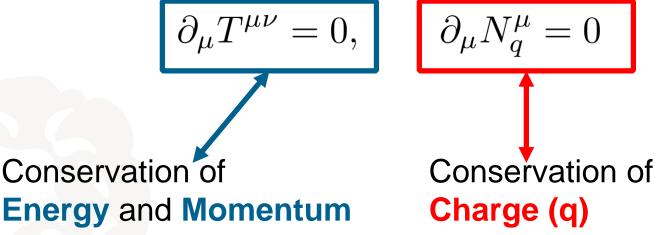
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→ Apply Dissipative Relativistic Fluid Dynamics

$$\partial_{\mu}T^{\mu\nu} = 0, \qquad \partial_{\mu}N^{\mu}_{q} = 0$$

with

$$T^{\mu
u} = \epsilon_0 u^{\mu} u^{
u} - P_0 \left(g^{\mu
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u} \right)$$
 Energy-Momentum Tensor

$$N_q^\mu = n_q u^\mu + j_q^\mu$$

Net Charge Flow



Assume system to be close to local equilibrium with multiple conserved charges

→ Apply Dissipative Relativistic Fluid Dynamics

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 Net Charge Flow
Net Charge Diffusion Current

Here: We do not consider any viscous corrections!



- Conservation of Energy-Momentum and Charge = exact!
- Extract dissipative currents from Boltzmann equation

Denicol et al., Phys.Rev. D85 (2012) 114047, Erratum: Phys.Rev. D91 (2015) no.3, 039902





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Source term of dissipative currents ~ Expansion in Knudsen Number and inverse Reynolds Number

 ${\rm Kn} \sim {\rm microscopic\ scale}\over {\rm macroscopic\ scale}}$, ${\rm Rn}^{-1} \sim {\rm dissipative\ field}\over {\rm equilibrium\ field}}$



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 ${\rm Kn} \sim {{\rm microscopic\ scale}\over {\rm macroscopic\ scale}}$, ${\rm Rn}^{-1} \sim {{\rm dissipative\ field}\over {\rm equilibrium\ field}}$

Fluid Dynamics: close local equilibrium + mean free path smaller fluid cell

$$\Rightarrow$$
 Kn, Rn⁻¹ $\ll 1$



Impose transient equation of first order in Knudsen number for diffusion currents (neglect higher orders)

$$\begin{pmatrix} \tau_{\rm B} \frac{\mathrm{d}}{\mathrm{d}\tau} j_{\rm B}^{\langle\mu\rangle} + j_{\rm B}^{\mu} \\ \tau_{\rm S} \frac{\mathrm{d}}{\mathrm{d}\tau} j_{\rm S}^{\langle\mu\rangle} + j_{\rm S}^{\mu} \end{pmatrix} = \begin{pmatrix} \kappa_{\rm BB} & \kappa_{\rm SB} \\ & & \\ \kappa_{\rm SB} & \kappa_{\rm SS} \end{pmatrix} \begin{pmatrix} \nabla^{\mu} \alpha_{\rm B} \\ \nabla^{\mu} \alpha_{\rm S} \end{pmatrix} + \mathcal{O}(\mathrm{Kn}^2)$$



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$$\mathbf{Relaxation Times} \qquad \mathbf{Navier-Stokes term}$$

Use vSHASTA solver to solve fluid dynamics numerically Molnar, Niemi, Rischke, Eur. Phys. J. C65, 615-635 (2010)

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The Setup

- (1+1)-hydrodynamic evolution in longitudinal setup
- In hyperbolic coordinates (proper time $\tau \equiv \sqrt{t^2 - z^2}$ and spacetime rapidity $\eta_s = \operatorname{arctanh}(z/t)$)

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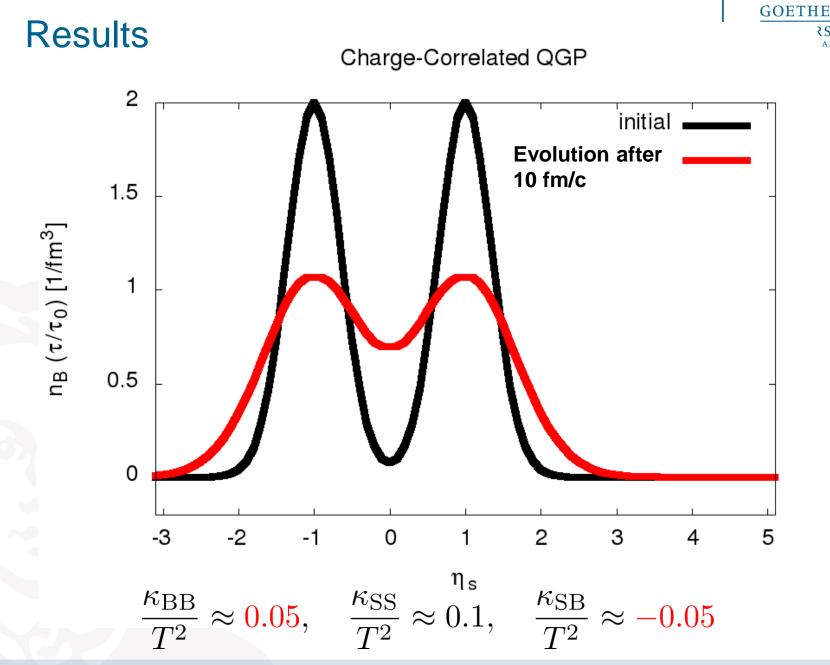
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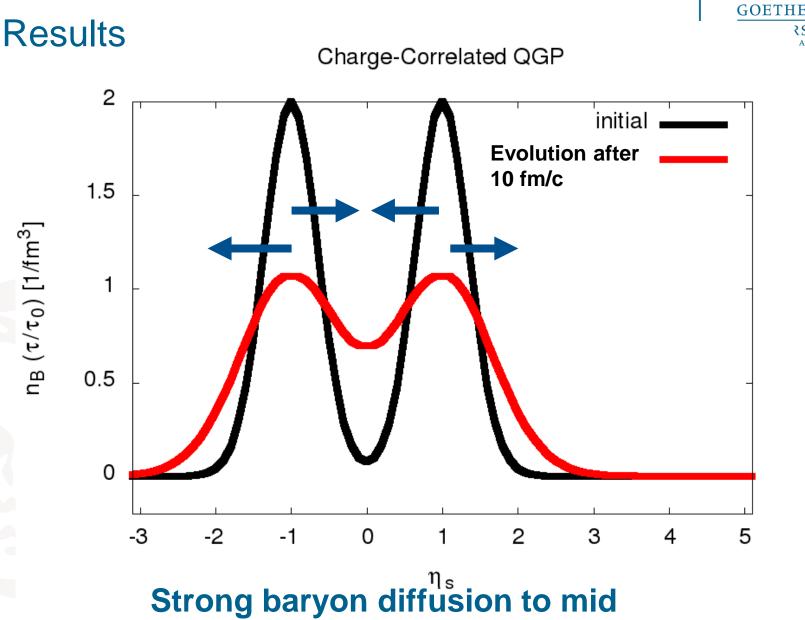
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 \rightarrow Fluid velocity is always zero during the evolution



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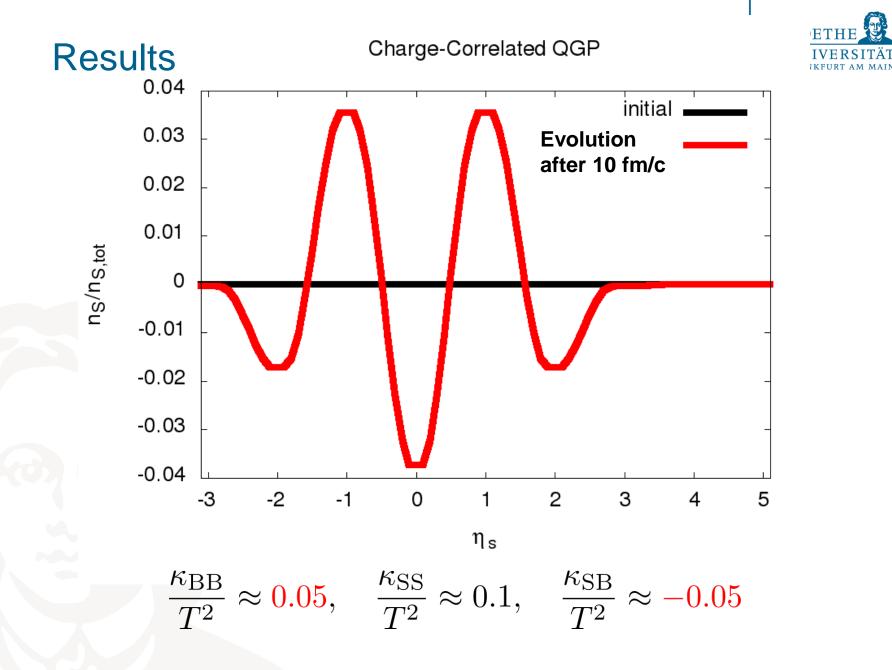


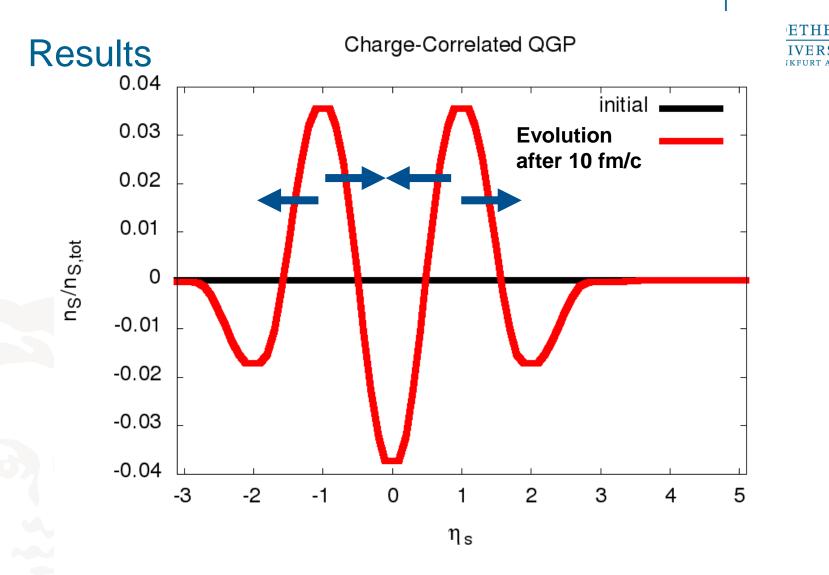
and outward spacetime rapidities

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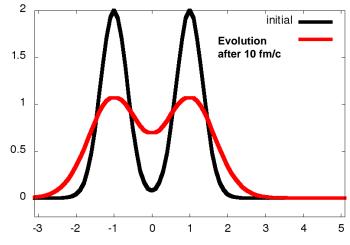
There is induced strangeness separation through Baryon-Strangeness anti-correlation!?!

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n_B (τ/τ_0) [1/fm³]



Charge-Correlated QGP

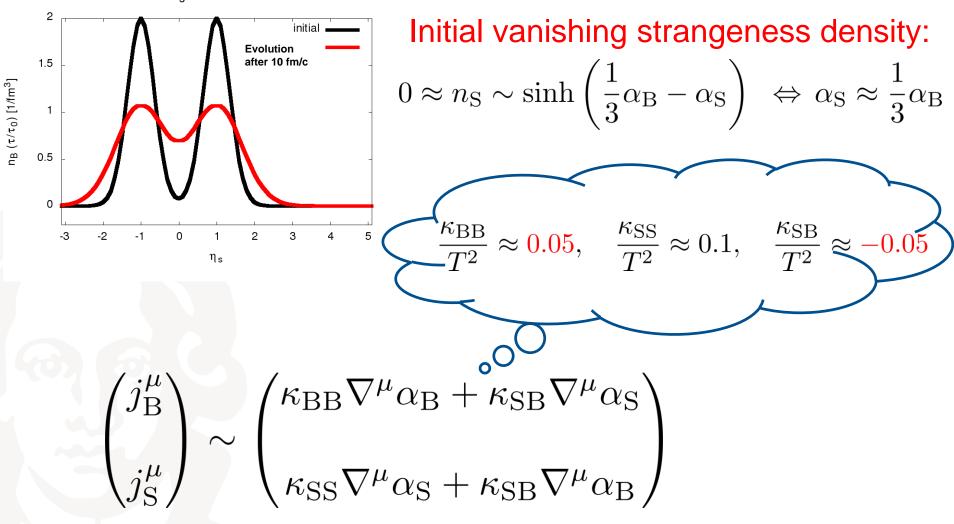
Initial vanishing strangeness density:

$$0 \approx n_{\rm S} \sim \sinh\left(\frac{1}{3}\alpha_{\rm B} - \alpha_{\rm S}\right) \iff \alpha_{\rm S} \approx \frac{1}{3}\alpha_{\rm B}$$

 η_{s}

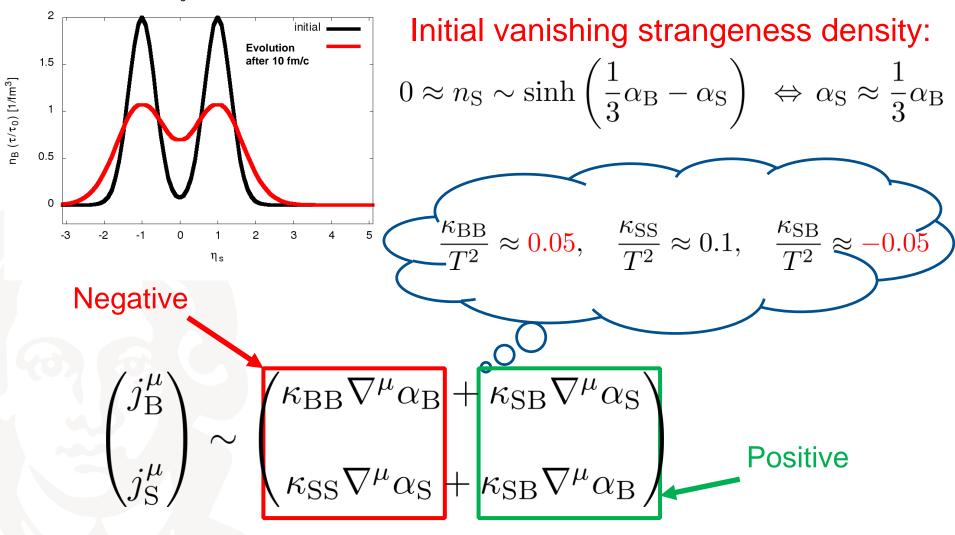


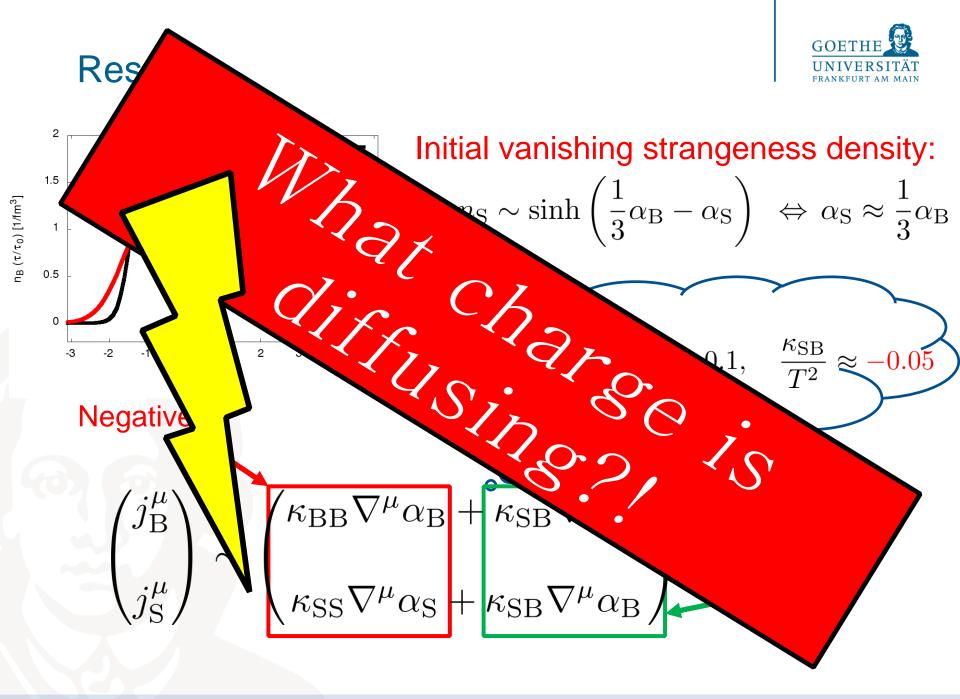
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- Initially vanishing net strangeness density → as many s-quarks as anti-s-quarks everywhere!
- Current of light quarks carrying baryon charge towards mid rapidities and outwards





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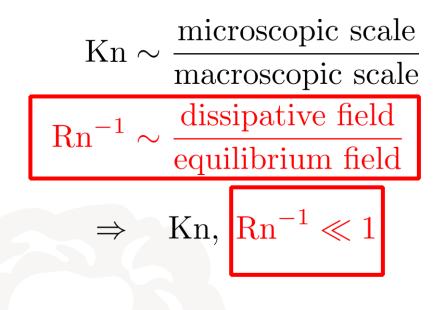
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→ Expectation: no induced strangeness separation in this case!



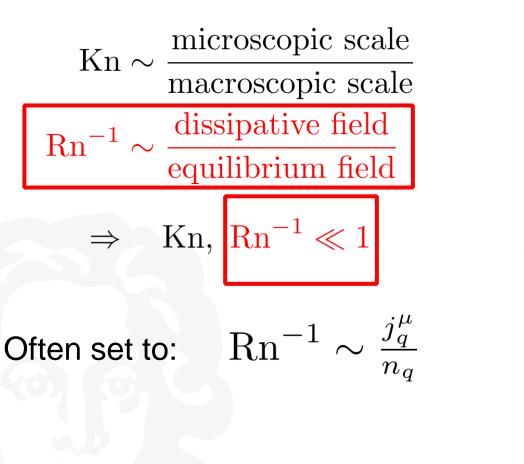








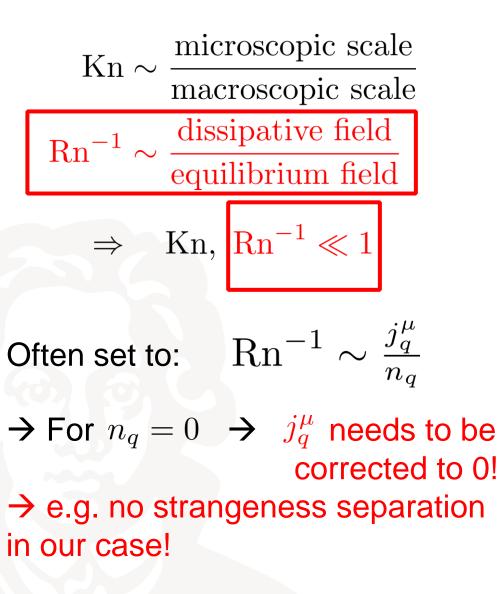


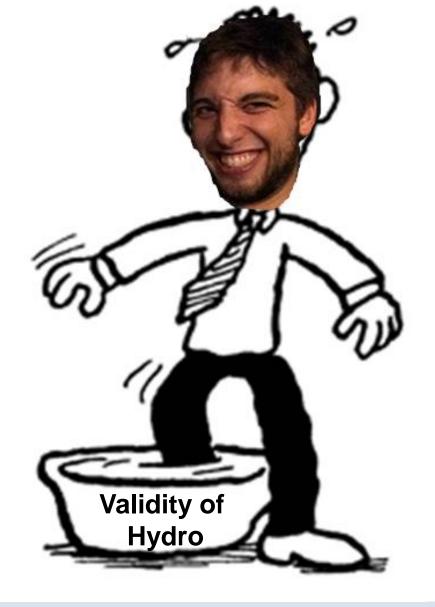






My View on the Problem







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However several problems:

1. The choice of Reynolds number is ambiguous $\operatorname{Rn}^{-1} \sim \frac{j_q^{\mu}}{n_{q,\text{tot}}}$??





My View on the Problem

However several problems:

- 1. The choice of Reynolds number is ambiguous $\operatorname{Rn}^{-1} \sim \frac{j_q^{\mu}}{n_{q,\text{tot}}}$??
- 2. No natural way of limiting the diffusion current!
- → Is hydro the appropriate model to use to do the investigations in the case of HIC (low density regions in strangeness)?!?

Validity of

Hydro

Conclusion



- First investigations of the (fluid) dynamic effects of the diffusion matrix
- Diffusive evolution of a massless, conformal QGP with conserved baryon number and strangeness and constant diffusion coefficients was examined in a dissipative fluid dynamic framework without viscous corrections

Conclusion



- First investigations of the (fluid) dynamic effects of the diffusion matrix
- Diffusive evolution of a massless, conformal QGP with conserved baryon number and strangeness and constant diffusion coefficients was examined in a dissipative fluid dynamic framework without viscous corrections

- We found signals of strong baryon diffusion and separation of strangeness
- Baryon diffusion could be important in describing the evolution of heavy ion collisions at low collisional energies
- More realistic investigations are needed
- However: hydro does seems to lead to misleading results

Outlook



- Compare to kinetic models: SMASH? BAMPS?
- Improve investigations with dissipative hydro:
 - Use more realistic initial state with fluctuating strangeness
 - Allow viscous corrections in evolution
 - Include higher-order terms in source term in diffusion equations
 - Use more realistic equation of state (IQCD+HRG)
 - Include freeze-out stage in order to compare to experiments
- Extend investigations to (3+1)-fluid dynamics
 - Initial state correlations (flow harmonics?)