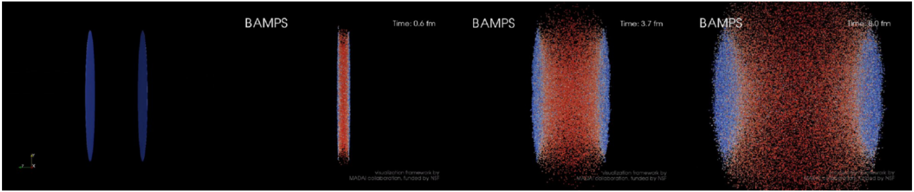
The AMY emission kernel in a partonic transport approach

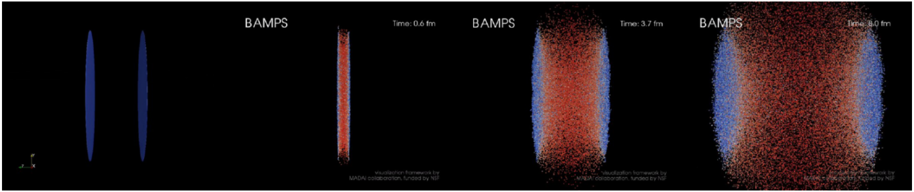
Florian Senzel
with M. Greif and C. Greiner



Transport meeting, 01.02.2018

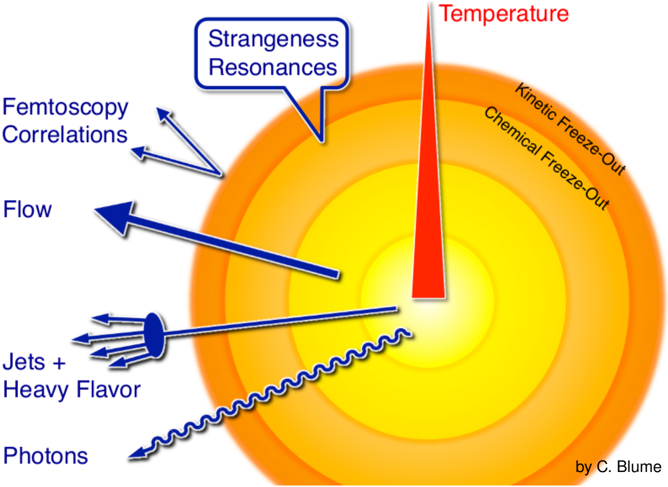
Outline

1. What is jet quenching?
2. The partonic transport model BAMPS
3. AMY formalism for gluon emissions
4. Energy loss ΔE in a static medium

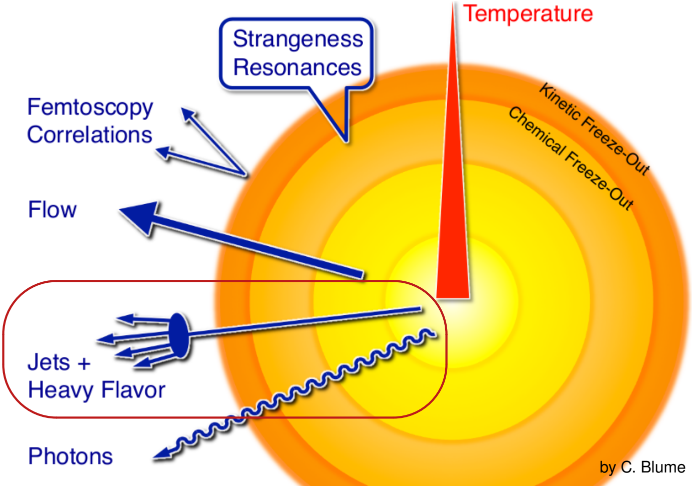


Visualization by Jan Uphoff
Visualization framework courtesy MADAI collaboration
funded by the NSF under grant NSF-PHY-09-41373

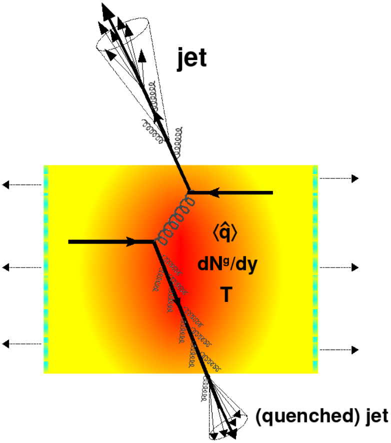
Tools for probing QCD matter: Ultra-relativistic heavy-ion collisions



Tools for probing QCD matter: Ultra-relativistic heavy-ion collisions



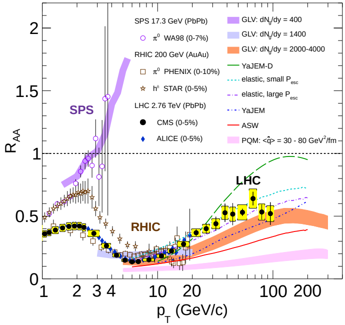
Jet quenching in the 90s: Energy loss of leading particles



by D'Enterria, Nucl.Phys. A827 (2009)

Nuclear modification factor

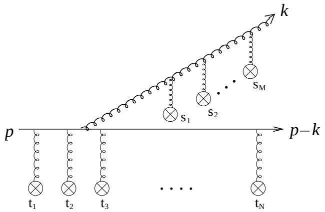
$$R_{\mathrm{AA}} = \frac{\mathrm{d}^2 N_{\mathrm{AA}}/\mathrm{d}p_t \mathrm{d}y}{N_{\mathrm{bin}}\, \mathrm{d}^2 N_{\mathrm{pp}}/\mathrm{d}p_t \mathrm{d}y}$$



by CMS Collaboration, Eur. Phys. J. C (2012)

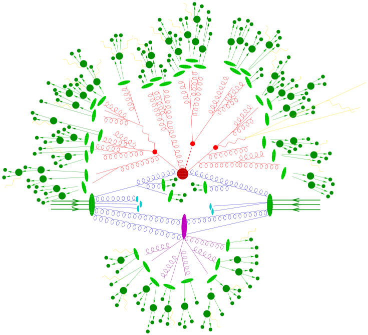
Radiative energy loss ΔE in perturbative QCD

- Analytic formulation of medium-induced gluon spectrum $\omega \frac{\mathrm{d}I}{\mathrm{d}\omega}$.
- Integration gives radiative E-loss $\Delta E^{\mathrm{rad}} \sim \int \mathrm{d}\omega\, \omega \frac{\mathrm{d}I}{\mathrm{d}\omega}$.
- Possible approximations:
  - Eikonal limit: $E \gg \omega \gg k_t, q_t$
  - Only static scattering centers
  - Multiple soft scatterings / single hard scattering
- Coherence between diagrams leads to LPM effect in pQCD.



Radiative energy loss formalisms: BDMPS-Z, GLV, ASW, AMY, Higher Twist, AdS/CFT . . .

Nowadays: Monte-Carlo tools for heavy-ion collisions!



Limitations of pQCD energy loss calculations

- Eikonal limit violates energy-momentum conservation.
- Analytic calculations require kinematic approximations (static scattering centers, multiple soft/one hard scattering,. . .).
- Modern jet studies demand for single events, not event averages.

The partonic transport model BAMPS

BAMPS $\widehat{=}$ Boltzmann Approach to Multi-Parton Scattering

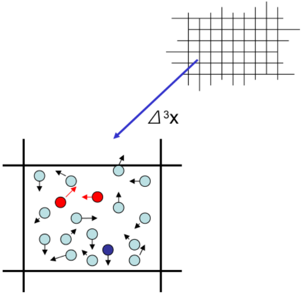
Numerically solving the (3+1)D Boltzmann transport equation for partons on the mass-shell:

$$\frac{\partial f}{\partial t} + \frac{\mathbf{p}}{E}\frac{\partial f}{\partial \mathbf{r}} = C_{2\to 2} + C_{2\leftrightarrow 3}$$

- Massless particles (gluons & quarks)
- Discretized space ΔV and time Δt:

$$P_{2\to 2} = v_{\mathrm{rel}}\sigma_{2\to 2}\frac{\Delta t}{\Delta V} \qquad P_{2\to 3} = v_{\mathrm{rel}}\sigma_{2\to 3}\frac{\Delta t}{\Delta V}$$

- Test-particles ansatz N_{test}

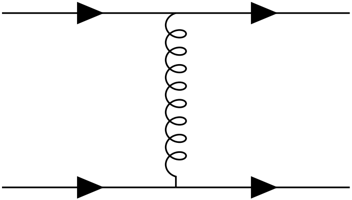


Xu and Greiner, Phys. Rev. C71 (2005); Xu and Greiner, Phys. Rev. C76 (2007)

Implemented processes

Screened leading-order pQCD

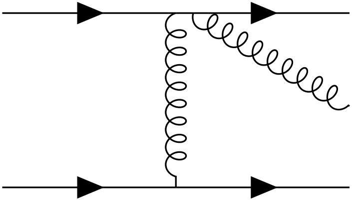
$$\left|\overline{\mathcal{M}}_{\mathrm{X}\to\mathrm{Y}}\right|^2 \sim \frac{\alpha_s^2}{\left[t - m_D^2(\alpha_s)\right]^2}$$



Uphoff, Fochler, Xu, Greiner: Phys. Rev. C84 (2011)

Improved Gunion-Bertsch approx.

$$\left|\overline{\mathcal{M}}_{\mathrm{X}\to\mathrm{Y+g}}\right|^2 \sim \left|\overline{\mathcal{M}}_{\mathrm{X}\to\mathrm{Y}}\right|^2 \times \alpha_s P_{\mathrm{g}}\left(q_t, k_t, y, \phi\right)$$

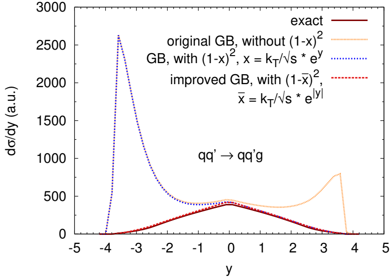


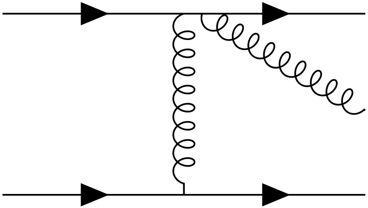
Gunion, Bertsch: Phys. Rev. D25 (1982)
Fochler, Uphoff, Xu, Greiner: Phys. Rev. D88 (2013)

Closer look on the radiative processes

Improved Gunion-Bertsch ME

$$\left|\overline{\mathcal{M}}_{\mathrm{X}\rightarrow\mathrm{Y+g}}\right|^2 = 48\pi\alpha_s \left|\overline{\mathcal{M}}_{X\rightarrow Y}\right|^2 (1-\bar{x})^2 \left[\frac{\mathbf{k}_\perp}{k_\perp^2} + \frac{\mathbf{q}_\perp - \mathbf{k}_\perp}{(\mathbf{q}_\perp - \mathbf{k}_\perp)^2 + m_D^2(\alpha_s)}\right]^2$$



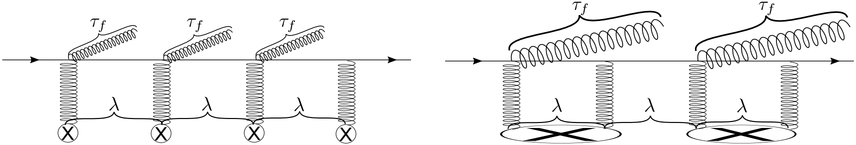


Gunion, Bertsch: Phys. Rev. D25 (1982)
Fochler, Uphoff, Xu, Greiner: Phys. Rev. D88 (2013)

with $\bar{x} = k_\perp e^{|y|}/\sqrt{s}$

What is the LPM effect?

- The Landau-Pomeranchuk-Migdal effect is a coherence effect caused by finite photon (QED) or gluon (QCD) formation time.



- Scattering centers act coherently during formation time τ_f when

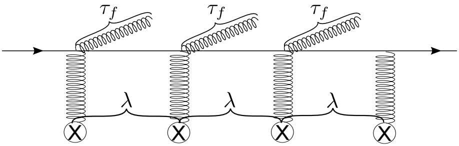
$$\tau_f > \lambda_{\text{MFP}}$$

- Coherent scatterings lead to suppression of emissions.
- In QCD: gluons may also interact with medium.

Issue

Coherence effects within Monte-Carlo approaches are not trivial.

Effective LPM effect in BAMPS



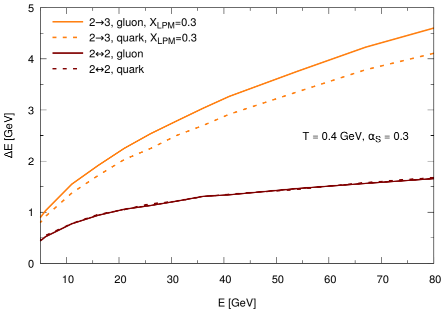
Ensuring incoherent Bethe-Heitler regime

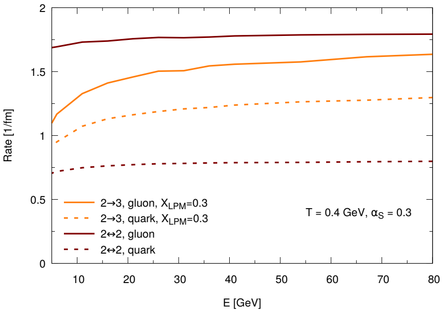
Parent parton is not allowed to scatter before emitted gluon is formed:

$$|\mathcal{M}_{2\to3}|^2 \to |\mathcal{M}_{2\to3}|^2 \, \Theta\left(\lambda - X_{\mathrm{LPM}}\, \tau_f\right)$$

$X_{\mathrm{LPM}} = 0$	No LPM suppression
$X_{\mathrm{LPM}} = 1$	Only independent scatterings (forbids too many emissions)
$X_{\mathrm{LPM}} \in (0;1)$	Allows effectively some collinear gluons.

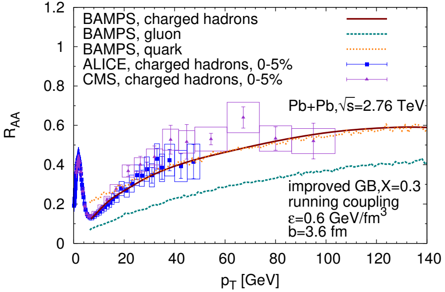
Energy loss of partons in a static medium





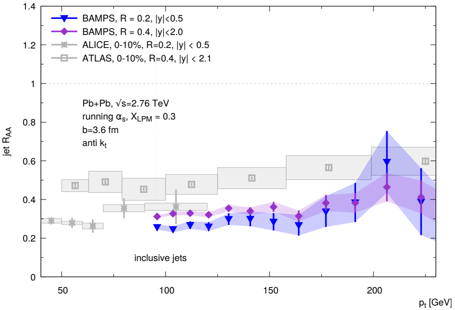
Previous jet quenching results with BAMPS

inclusive hadrons



Uphoff, FS, Fochler, Wesp, Xu, Greiner
Phys. Rev. Lett. 114 (2015) 112301

reconstructed jets

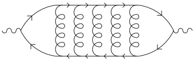
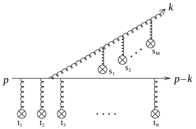
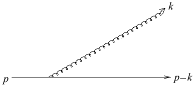


FS, Uphoff, Xu, Greiner
Phys.Lett. B773 (2017) 620-624

Open questions

- How can we determine the LPM parameter X_{LPM} theoretically?
- Why are the reconstructed jets so strongly suppressed?

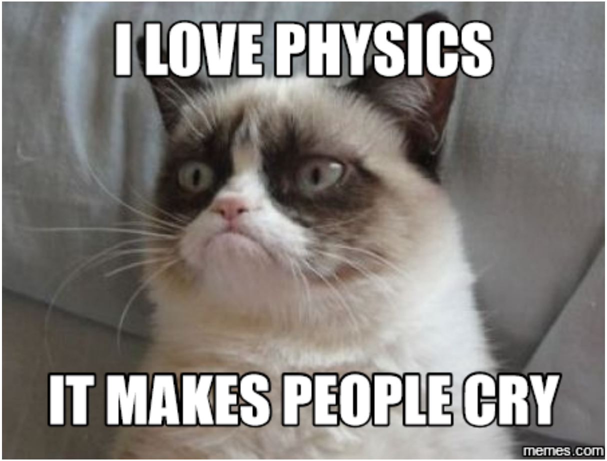
AMY formalism for photons and gluons

photon/gluon self-energy		$N \to N + 1$		effective "$1 \leftrightarrow 2$"
	$\Leftrightarrow$		$\Rightarrow$	

Assumptions by Arnold, Moore & Yaffe (AMY)

- Leading order hard-thermal-loop calculation
- LPM effect is calculated by resumming infinite ladder diagrams.
- Separation of scales: $T \gg gT \gg g^2T^2$
- Calculated in momentum space $\to$ "infinite medium"

Arnold, Moore & Yaffe: JHEP 0111 (2001) 057, JHEP 0206 (2002) 030, JHEP 0112 (2001) 009


I LOVE PHYSICS
IT MAKES PEOPLE CRY
memes.com

AMY formalism for gluons

Thermal gluon emission rate (only q $\leftrightarrow$ qg)

$$k\frac{\mathrm{d}R^g}{\mathrm{d}^3k} = \frac{g_s^2}{16(2\pi)^3k^4}\sum_f 12C_s\int_{-\infty}^{\infty}\frac{\mathrm{d}p}{2\pi}f_F(p)$$

$$\times\left[1-f_F(p-k)\right]\left[1+f_B(k)\right]\frac{(p-k)^2+p^2}{(p-k)^2p^2}\int\frac{\mathrm{d}^2\vec{h}}{(2\pi)^2}2\vec{h}\cdot\mathrm{Re}\vec{F}(\vec{h},p,k)$$

- p: emitting particle
- k: emitted gluon
- f_i: Fermi/Bose distributions
- C_s: gluon C_A, quark C_F

- $-\infty < p < 0$: bremsstrahlung of anti-quark
- $0 < p < k$: pair annihilation with anti-quark
- $k < p < \infty$: bremsstrahlung of quark

Arnold, Moore & Yaffe: JHEP 0111 (2001) 057, JHEP 0206 (2002) 030, JHEP 0112 (2001) 009

AMY formalism for gluons

Integral equation for function $\vec{F}(\vec{h}, p, k)$

$$
\begin{aligned}
2\vec{h} = \mathrm{i}\delta E(\vec{h}, p, k)\vec{F}(\vec{h}, p, k) + g_s^2 T \int \frac{\mathrm{d}^2\vec{q}_\perp}{(2\pi)^2}\mathcal{C}(q_\perp) \\
\times \Big\{ (C_s - C_A/2)\left[\vec{F}(\vec{h}) - \vec{F}(\vec{h} - k\vec{q}_\perp)\right] \\
+ (C_A/2)\left[\vec{F}(\vec{h}) - \vec{F}(\vec{h} + p\vec{q}_\perp)\right] \\
+ (C_A/2)\left[\vec{F}(\vec{h}) - \vec{F}(\vec{h} - (p-k)\vec{q}_\perp)\right] \Big\}
\end{aligned}
$$

Inverse formation time

$$
\delta E = \frac{\vec{h}^2}{2pk(p-k)} + \frac{m_k^2}{2k} + \frac{m_{p-k}^2}{2(p-k)} - \frac{m_p^2}{2p}
$$

Collision kernel

$$
\mathcal{C}(\vec{q}_\perp) = \frac{m_D^2}{\vec{q}_\perp^{\,2}(\vec{q}_\perp^{\,2} + m_D^2)}
$$

Arnold, Moore & Yaffe: JHEP 0111 (2001) 057, JHEP 0206 (2002) 030, JHEP 0112 (2001) 009

Extracting the transition rate $\frac{\mathrm{d}\Gamma}{\mathrm{d}k}(p,k)$

Comparison with $k\frac{\mathrm{d}R^g}{\mathrm{d}^3k} \propto \int \frac{\mathrm{d}^3\vec{p}}{(2\pi)^3} f_F(p) k \frac{\mathrm{d}\Gamma^g(p,k)}{\mathrm{d}^3k}$ gives

Transition rates

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}k}(p,k) = \frac{C_s g_s^2}{16\pi p^7} \frac{1}{1 \pm e^{-k/T}} \frac{1}{1 \pm e^{-(p-k)/T}}$$
$$\times \left\{ \begin{array}{ll} \frac{1+(1-x)^2}{x^3(1-x)^2} & q \leftrightarrow qg \\ N_\mathrm{f} \frac{x^2+(1-x)^2}{x^2(1-x)^2} & g \leftrightarrow q\bar{q} \\ \frac{1+x^4+(1-x)^4}{x^3(1-x)^3} & g \leftrightarrow gg \end{array} \right\} \times \int \frac{d^2\vec{h}}{(2\pi)^2} 2\vec{h} \cdot \mathrm{Re}\, \vec{F}(\vec{h},p,k)$$

with momentum fraction $x = k/p$.

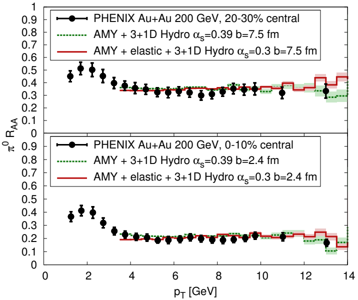
Jeon et al.: Phys.Rev. C71 (2005) 034901
Turbide et al.: Phys.Rev. C72 (2005) 014906

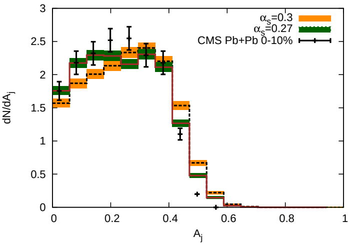
MARTINI: AMY jet energy loss in dynamic background

- Solves set of coupled Fokker-Planck type rate equations

$$\frac{\mathrm{d}P(p)}{\mathrm{d}t} = \int_{-\infty}^{\infty} \mathrm{d}k \left(P(p+k) \frac{\mathrm{d}\Gamma(p+k,k)}{\mathrm{d}k} - P(p) \frac{\mathrm{d}\Gamma(p,k)}{\mathrm{d}k} \right) .$$

- Jets embedded in 3+1D hydrodynamic medium (MUSIC).





Schenke et al.: Phys.Rev. C80 (2009) 054913
Young et al.: Phys.Rev. C84 (2011) 024907

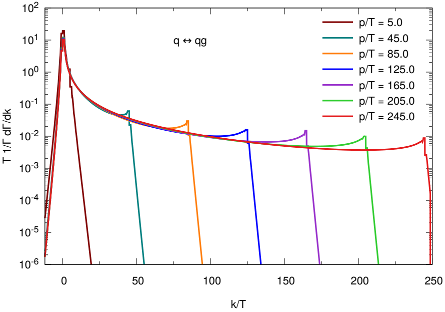
From MARTINI/AMY to BAMPS

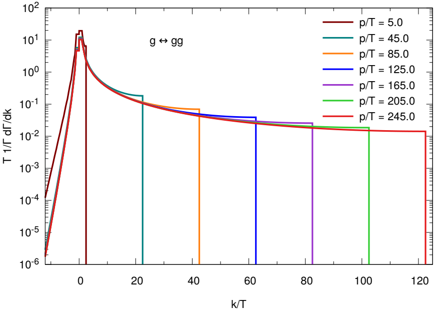
$\Rightarrow$

AMY emission in BAMPS

- Emission probability is $P_{12} = \Delta t \, \Gamma(p) = \Delta t \int \mathrm{d}k \, \frac{\mathrm{d}\Gamma}{\mathrm{d}k}(p,k)$
- Energy of emitted partons is sampled via $\frac{\mathrm{d}\Gamma}{\mathrm{d}k}(p,k)$.
- Emission is collinear ($\theta \propto gT^2$).

Transition rate $\frac{d\Gamma}{dk}(p,k)$

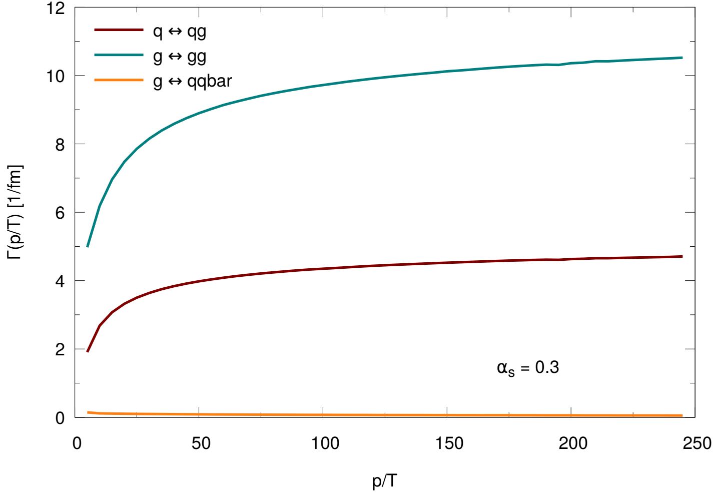




Characteristics of $\frac{d\Gamma}{dk}(p,k)$

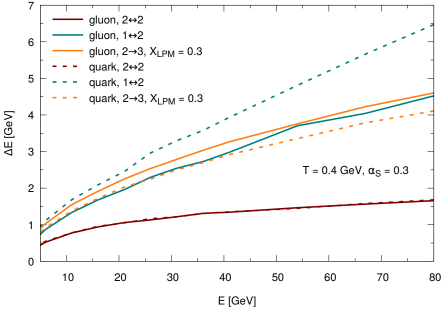
- Transition rate diverges for $k = 0$ and $k = p$.
- $k < 0$ corresponds to energy gain from medium.
- To avoid double counting $g \to gg$ and $g \to q\bar{q}$ only up to $k = p/2$.

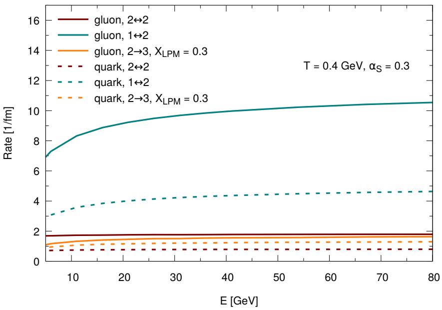
Integrated emission rate $\Gamma(p)$



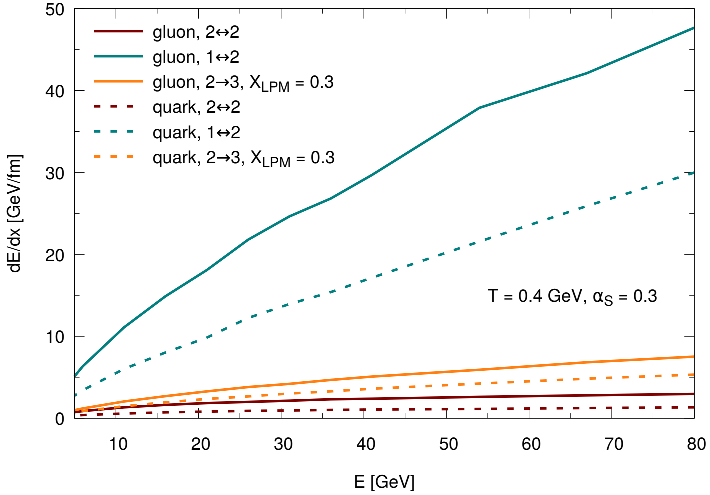

Research
in progress

Energy loss ΔE and rate in a static medium

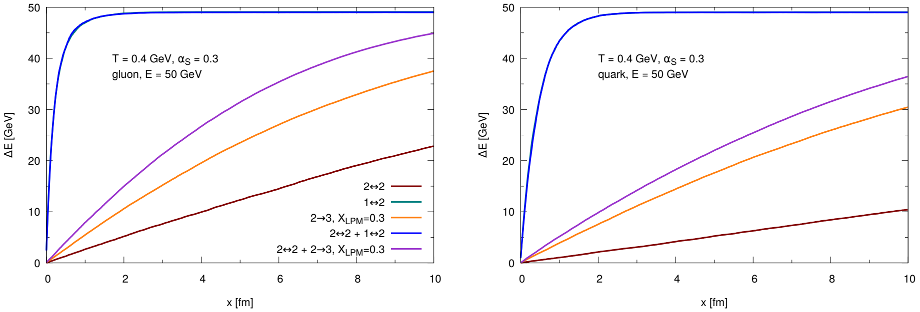




Differential energy loss dE/dx in a static medium



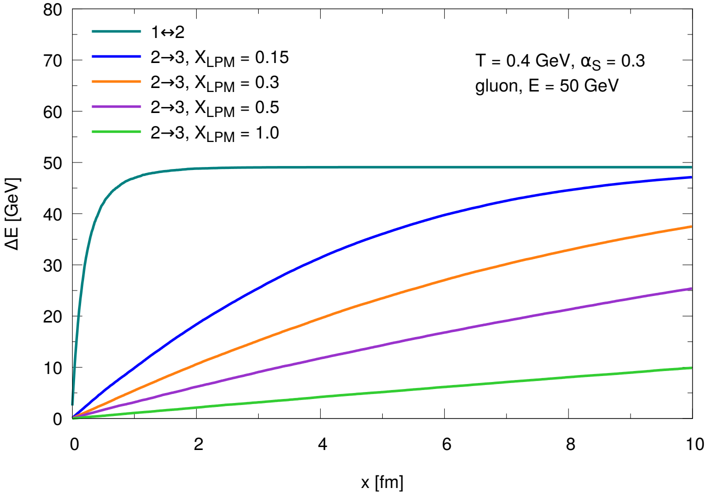
Evolution of parton in a static medium



Comparison to $2 \to 3$ processes

- Stronger energy loss than Gunion-Bertsch with θ function.
- Attention: saturation at $\Delta E = 45$ GeV due to limited numerical tables.

Comparison with θ LPM effect



Stochastic algorithm for BDMPS-Z formalism

Stochastic LPM suppression by K. Zapp et al.

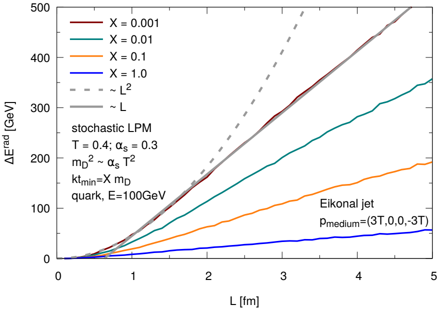
1. Determine radiative process by incoherent gluon emission rate.
2. During formation time τ_f gluon may scatter elastically (N_{coh}), accumulate traverse momentum and thereby modify its τ_f.
3. When gluon is formed, reject emission with probability $\sim \frac{1}{N_{\mathrm{coh}}}$ to account for coherent gluon emission.

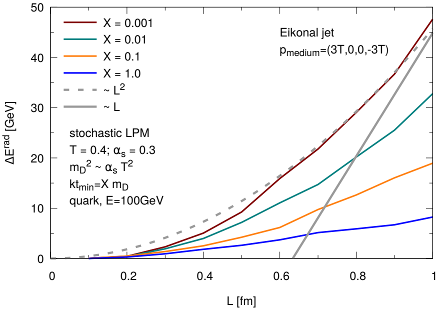
Comparison with analytic BDMPS-Z

✓ Monte-Carlo algorithm shows characteristic LPM features.

K.Zapp et al.: Phys.Rev.Lett. 103 (2009), JHEP 1107 (2011)

Length dependence of ΔE with stochastic LPM

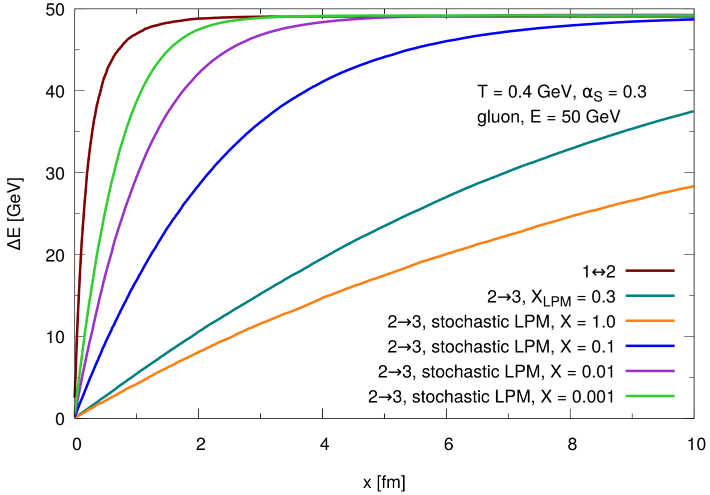




Comparison θ LPM vs. stochastic LPM

- $\Delta E(L)$ of stochastic LPM shows characteristic BDMPS-Z $\sim L^2$.
- However: $\Delta E(L)$ depends again on screening with X.

Comparison with stochastic LPM effect



Remark: AMY formalism originally for photons

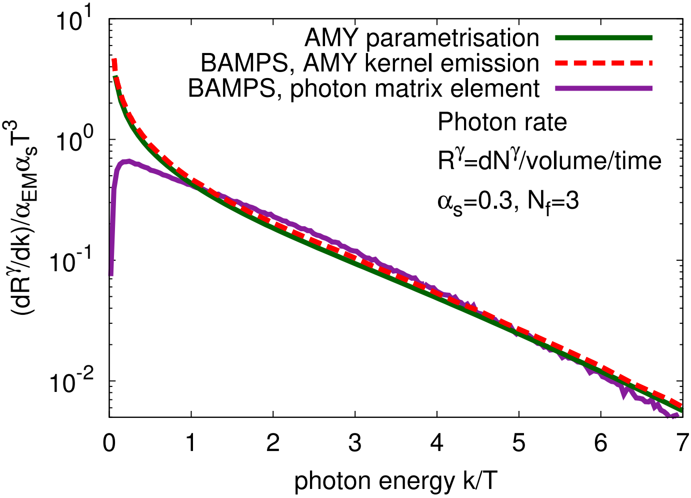
Thermal photon emission rate

$$k\frac{\mathrm{d}R^\gamma}{\mathrm{d}^3k} = \frac{3\alpha_{\mathrm{EM}}}{4\pi^2}\left(\sum_f \frac{q_f^2}{e^2}\right)\int_{-\infty}^{\infty}\frac{\mathrm{d}p_z}{(2\pi)} f_F(p_z+k)\left[1-f_F(p_z)\right]$$
$$\times\frac{p_z^2+(p_z+k)^2}{2p_z^2(p_z+k)^2}\int\frac{\mathrm{d}^2\vec{p}_\perp}{(2\pi)^2}2\vec{p}_\perp\cdot\text{Re}\,\vec{f}(\vec{p}_\perp,p_z,k)$$

Integral equation for function $\vec{f}(\vec{p}_\perp,p_z,k)$

$$2\vec{p}_\perp = \mathrm{i}\delta E\vec{f}(\vec{p}_\perp,p_z,k)$$
$$+g_s^2T\int\frac{\mathrm{d}^2\vec{q}_\perp}{(2\pi)^2}\mathcal{C}(\vec{q}_\perp)\left[\vec{f}(\vec{p}_\perp,p_z,k)-\vec{f}(\vec{p}_\perp-\vec{q}_\perp,p_z,k)\right]$$

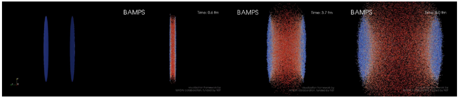
Remark: AMY formalism originally for photons



M. Greif, FS et. al.: Phys.Rev. C95 (2017) no.5, 054903

Conclusions

- Proof of concept:
  Implementation of AMY formalism into partonic transport
- First results for energy loss in static medium
- Stronger energy loss than previous $2 \to 3$ processes



Open questions:

- Why is energy loss so strong? Is this in agreement with MARTINI?
- How does the AMY formalism modify jets quenching in expanding BAMPS media? R_{AA}, reconstructed jets ...
- Is it possible to use AMY emissions also for the medium evolution?