Fictions, fluctuations and mean fields

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Uniwersytet Wrocławski

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in collaboration with Peter Petreczky, arXiv:1708.00879

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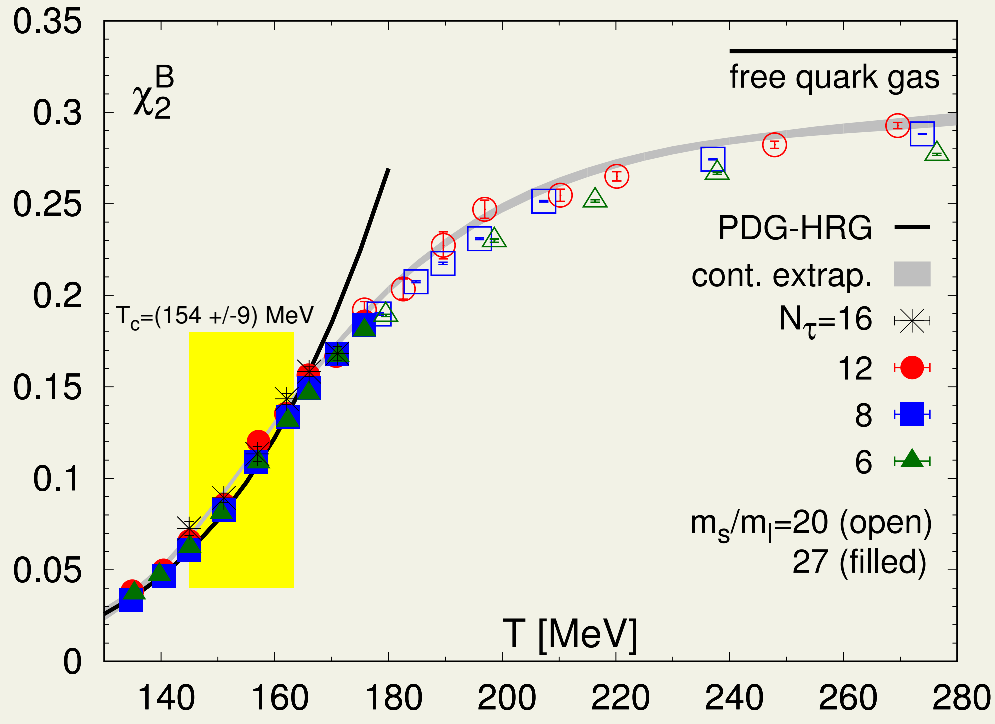
Fiction, `noun`

`A fictive particle, i.e. a particle predicted by some model without solid empirical evidence for its existence`

Fluctuations of conserved charges

$$\chi_n^X = T^n \left.\frac{\partial^n P/T^4}{\partial \mu_X^n}\right|_{\mu_X=0}$$

$$\chi_{nm}^{XY} = T^{n+m} \left.\frac{\partial^{n+m} P/T^4}{\partial \mu_X^n \partial_Y^m}\right|_{\mu_X=0,\,\mu_Y=0}$$



Bazavov et al., PRD95, 054504 (2017)

Hadron resonance gas

- EoS of interacting hadron gas well approximated by non-interacting gas of hadrons and resonances
- treat hadrons and resonances as free particles:

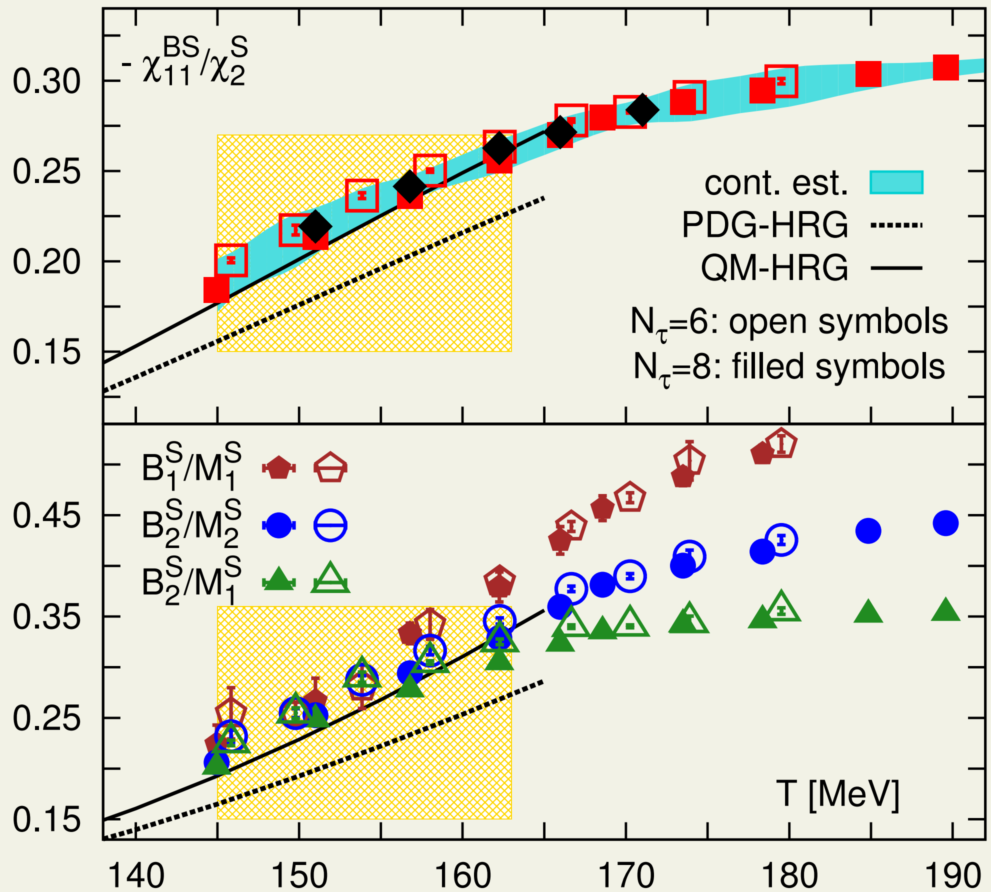
$$P(T,\mu) = \sum_i \frac{\pm g_i}{(2\pi)^3} T \int \mathrm{d}^3p \, \ln\left(1 \pm e^{-\frac{\sqrt{p^2+m^2}-\mu_i}{T}}\right)$$

- valid when
- interactions mediated by resonances
- resonances have zero width

- Prakash & Venugopalan, NPA546, 718 (1992): experimental phase shifts
- Gerber & Leutwyler, NPB321, 387 (1989): chiral perturbation theory

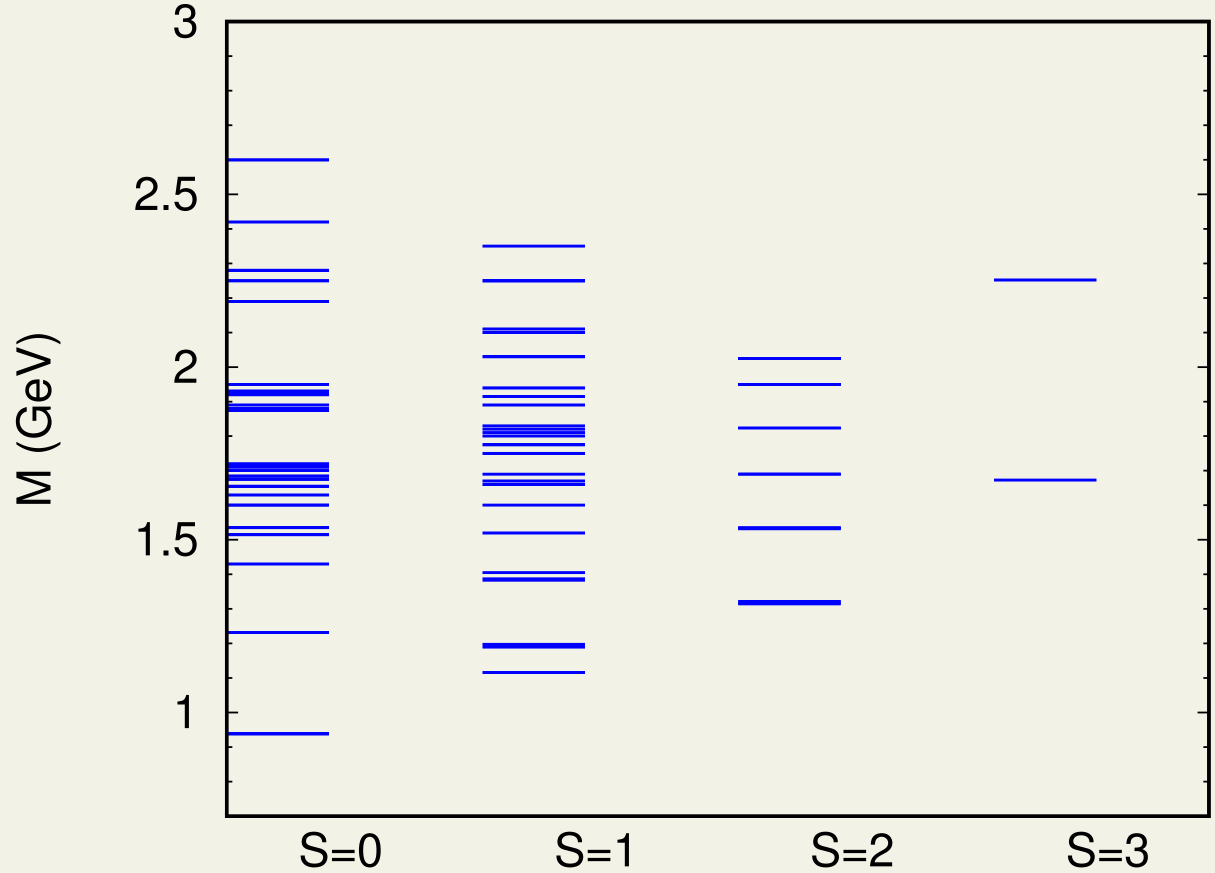
⇒ HRG good approximation at low temperatures

More resonances?



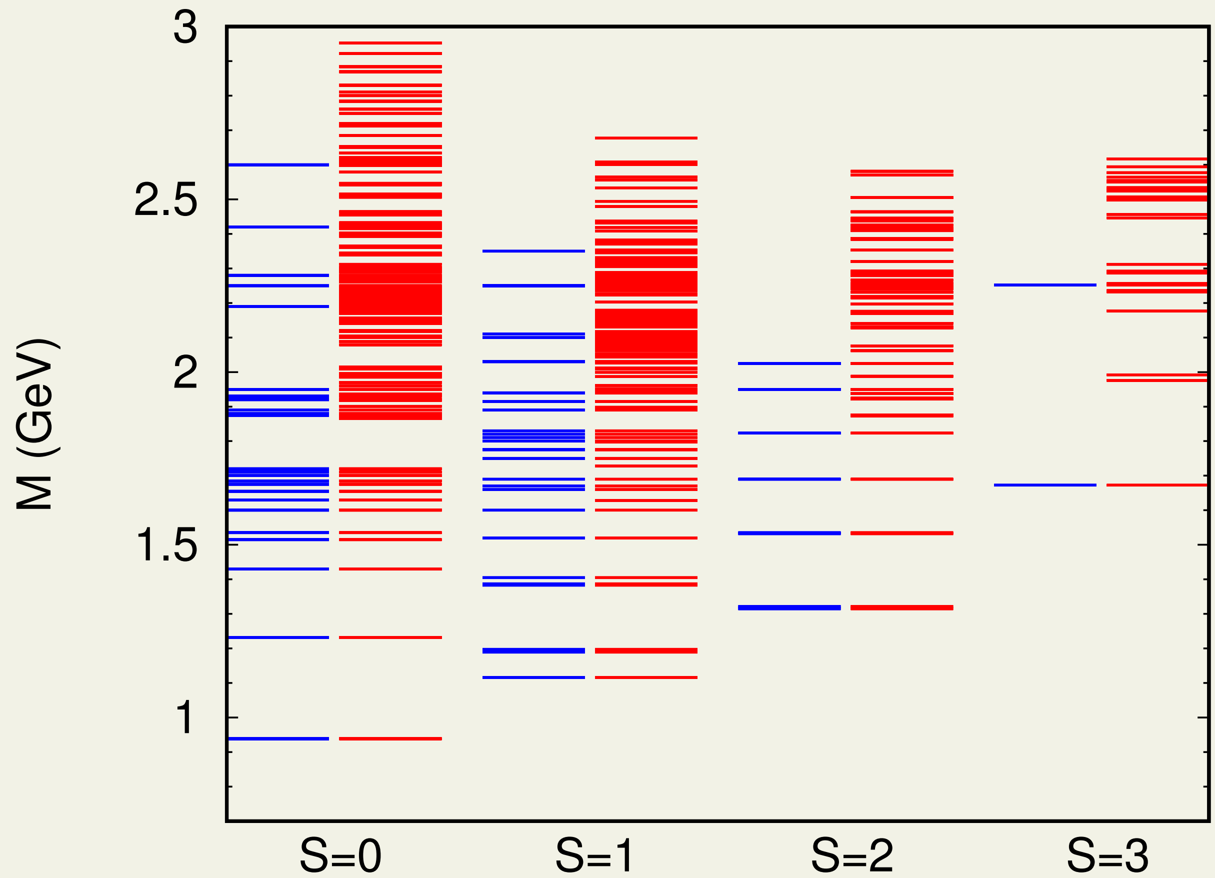
Bazavov et al., PRL113, 072001 (2014)

Baryon spectrum



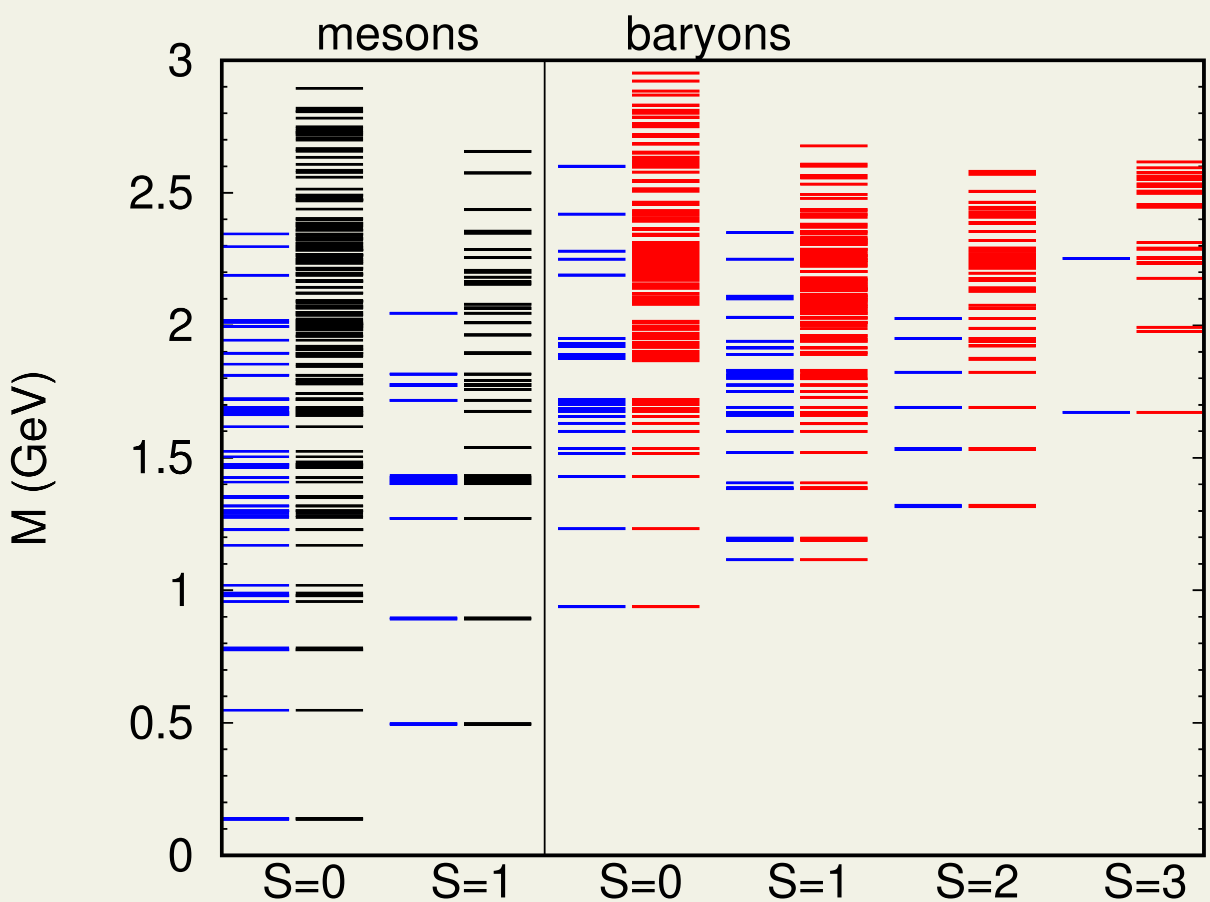
Blue: Particle Data Group

Baryon spectrum



Blue: Particle Data Group
Red: PDG + Löring et al., EPJA10, 395 (2001) & EPJA10, 447 (2001)

Hadron spectrum



Blue: **Particle Data Group**
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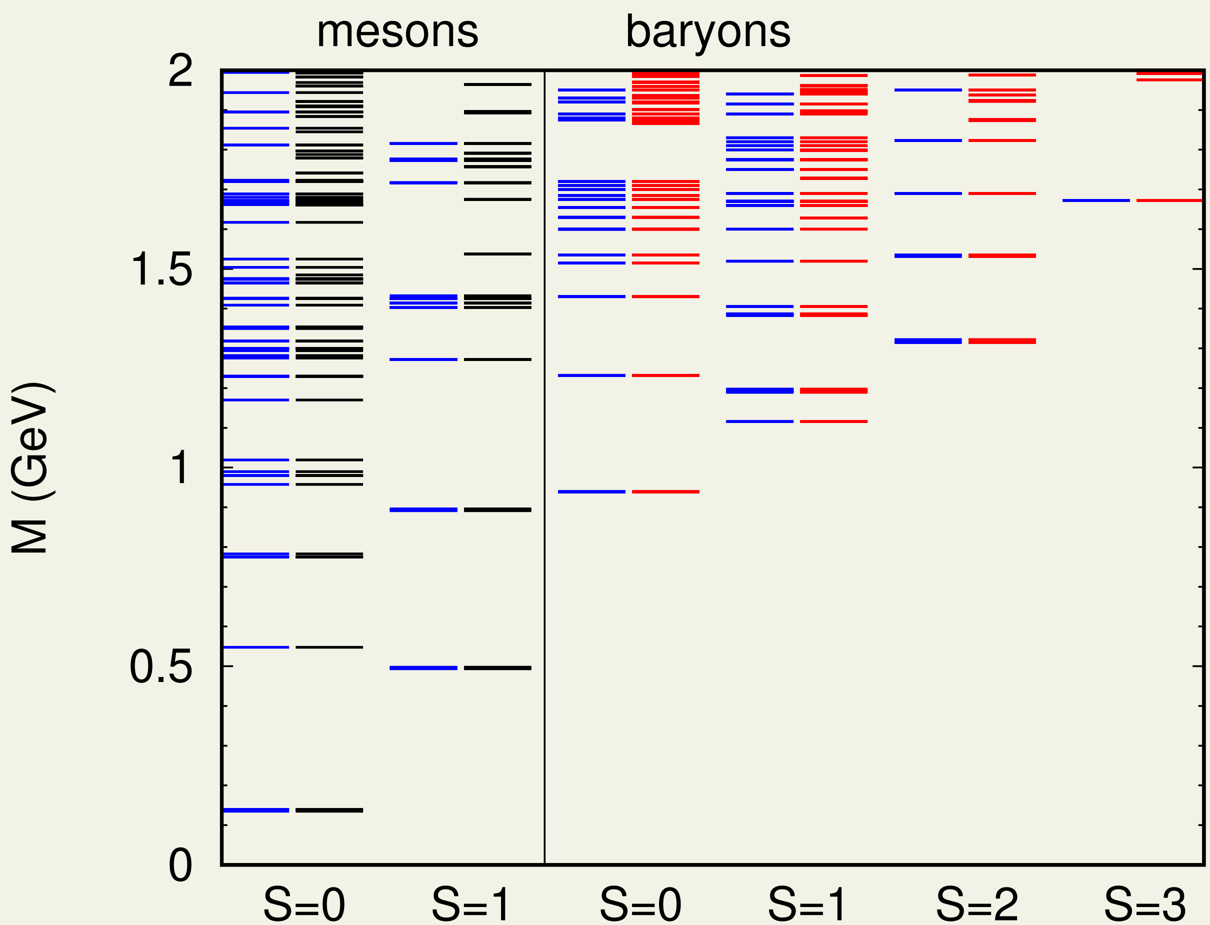
Black: **PDG + Ebert et al., PRD79, 114029 (2009)**

Hadron spectrum



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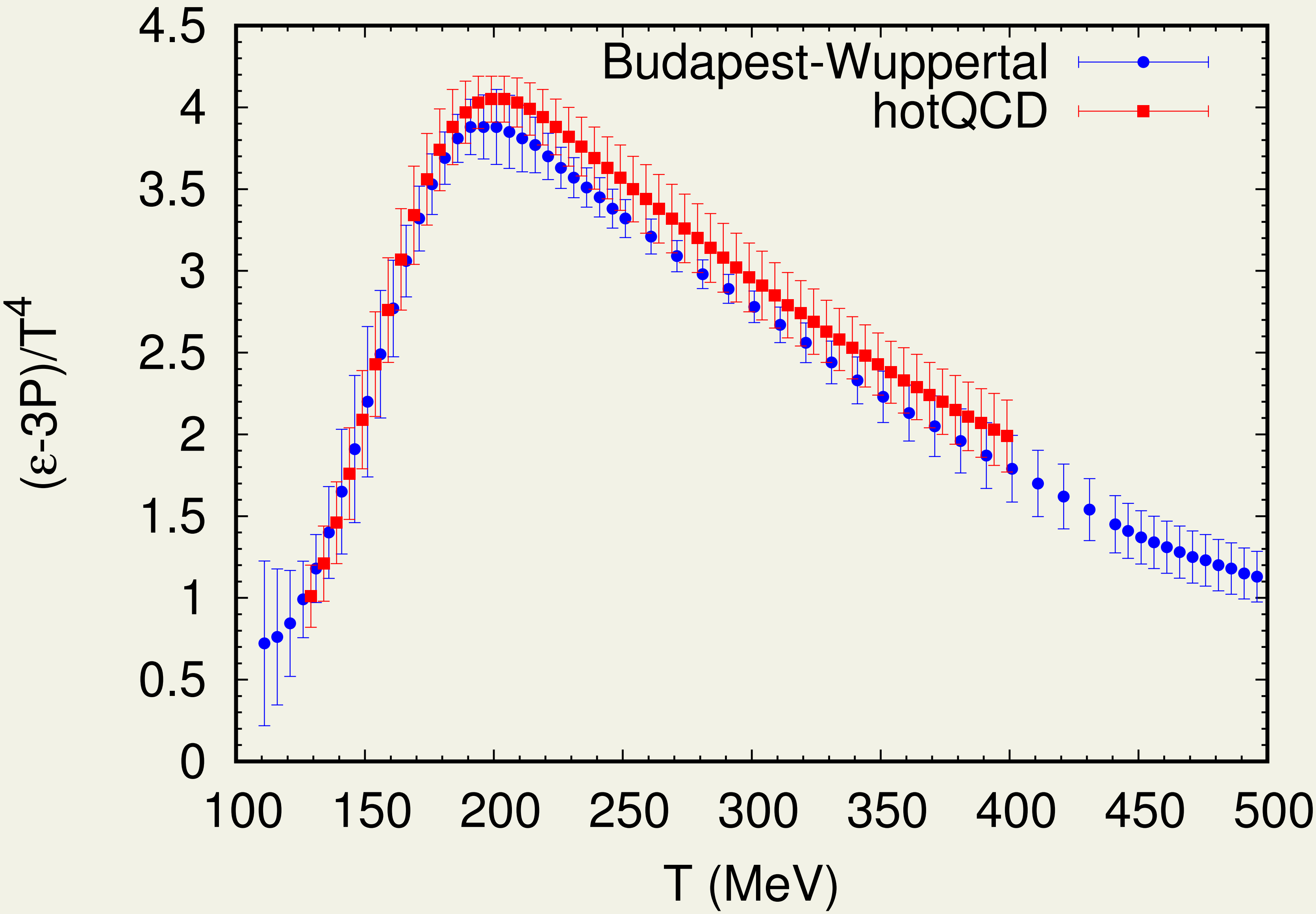
Trace anomaly



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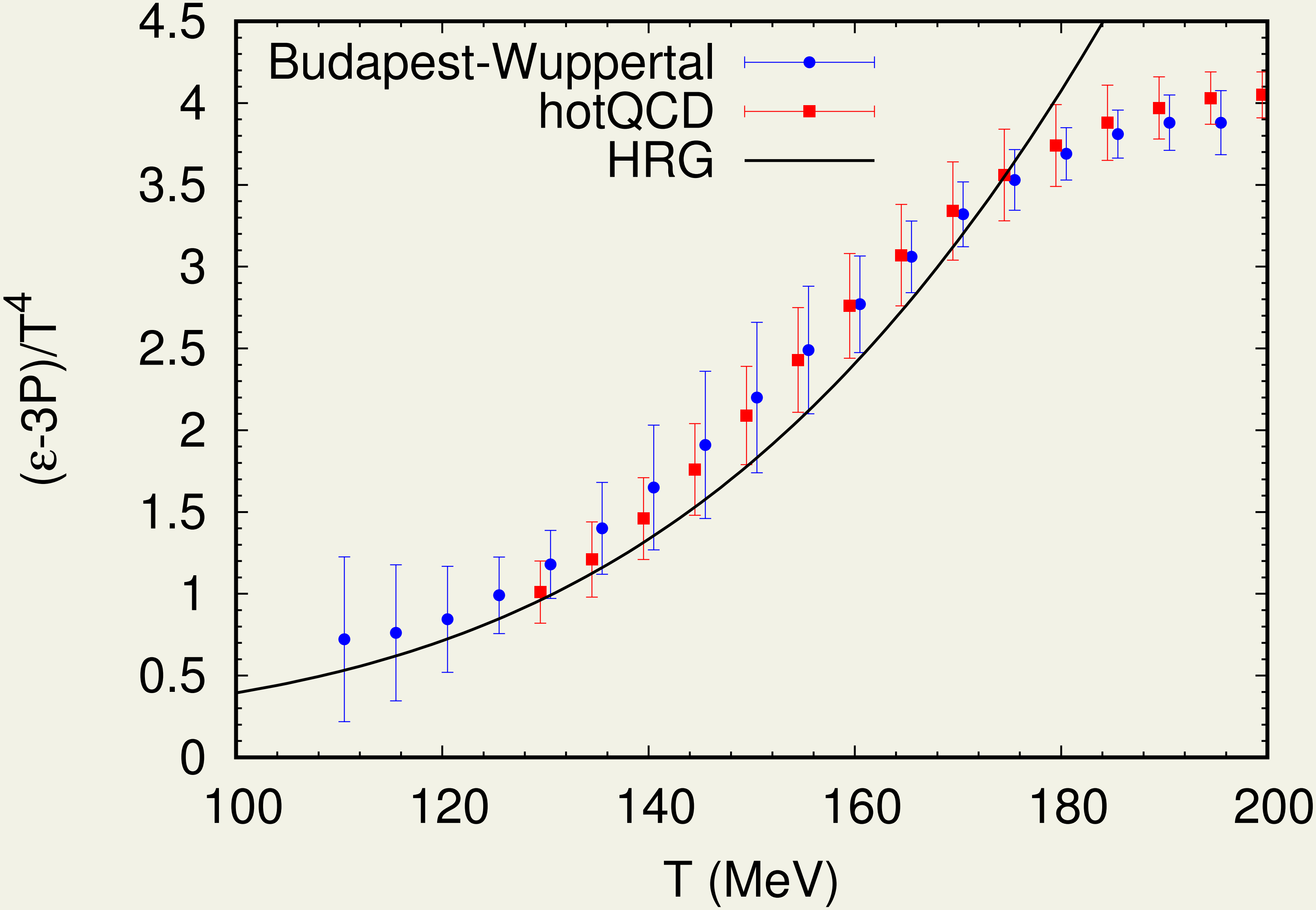

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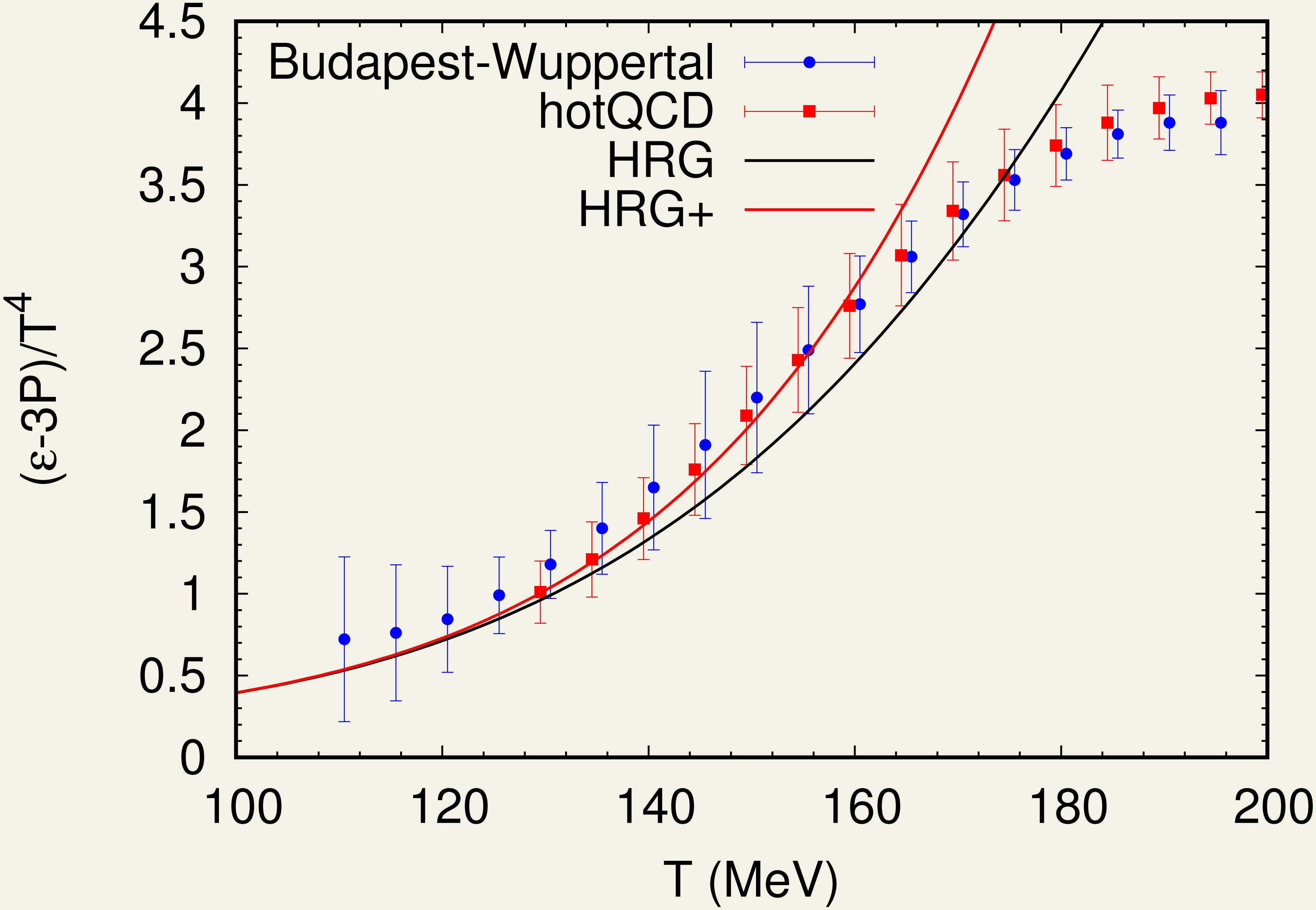
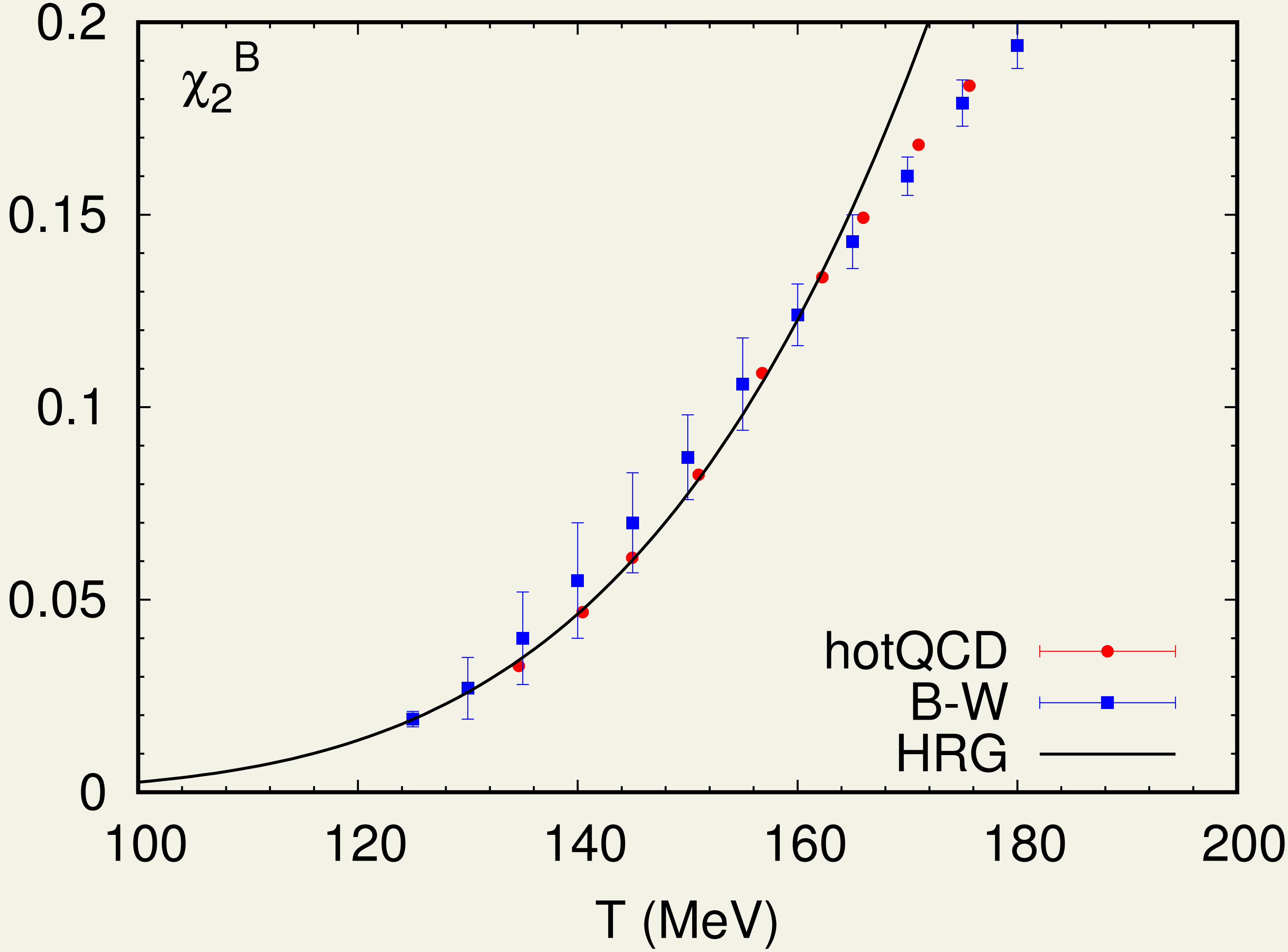
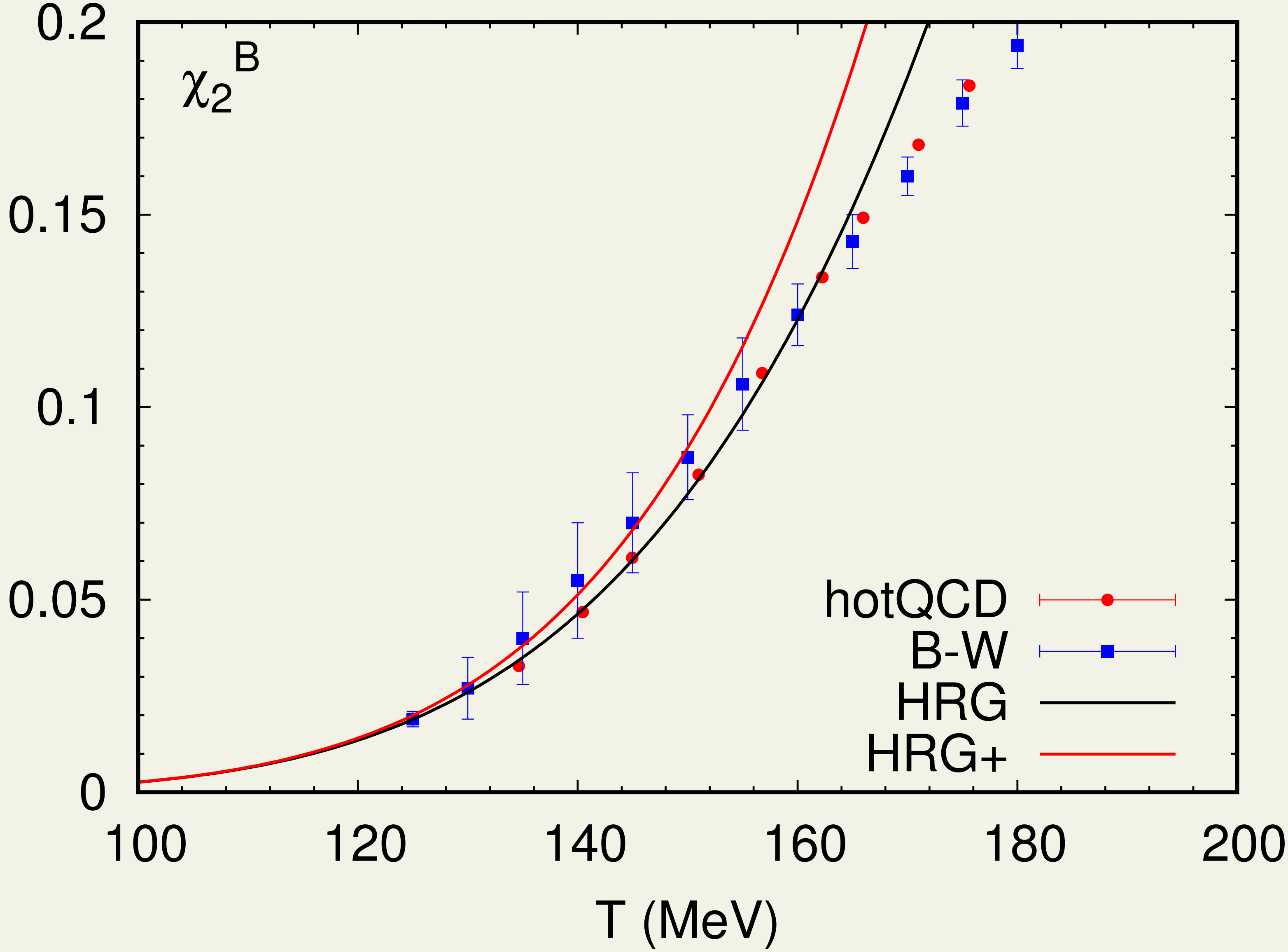


χ_B^2



χ_B^2



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T (MeV)
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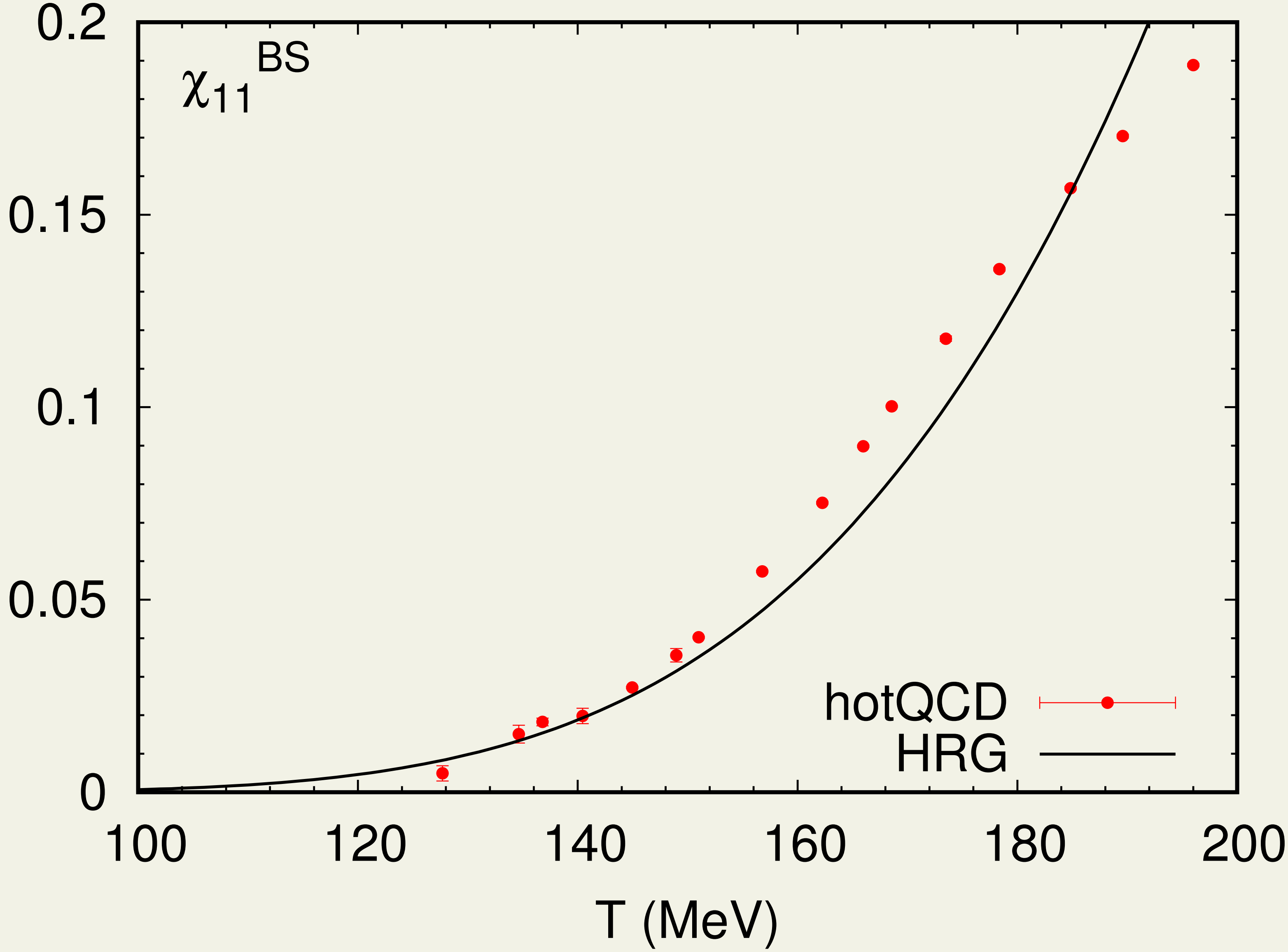
hotQCD
B-W
HRG
HRG+

χ_{BS}^{11}



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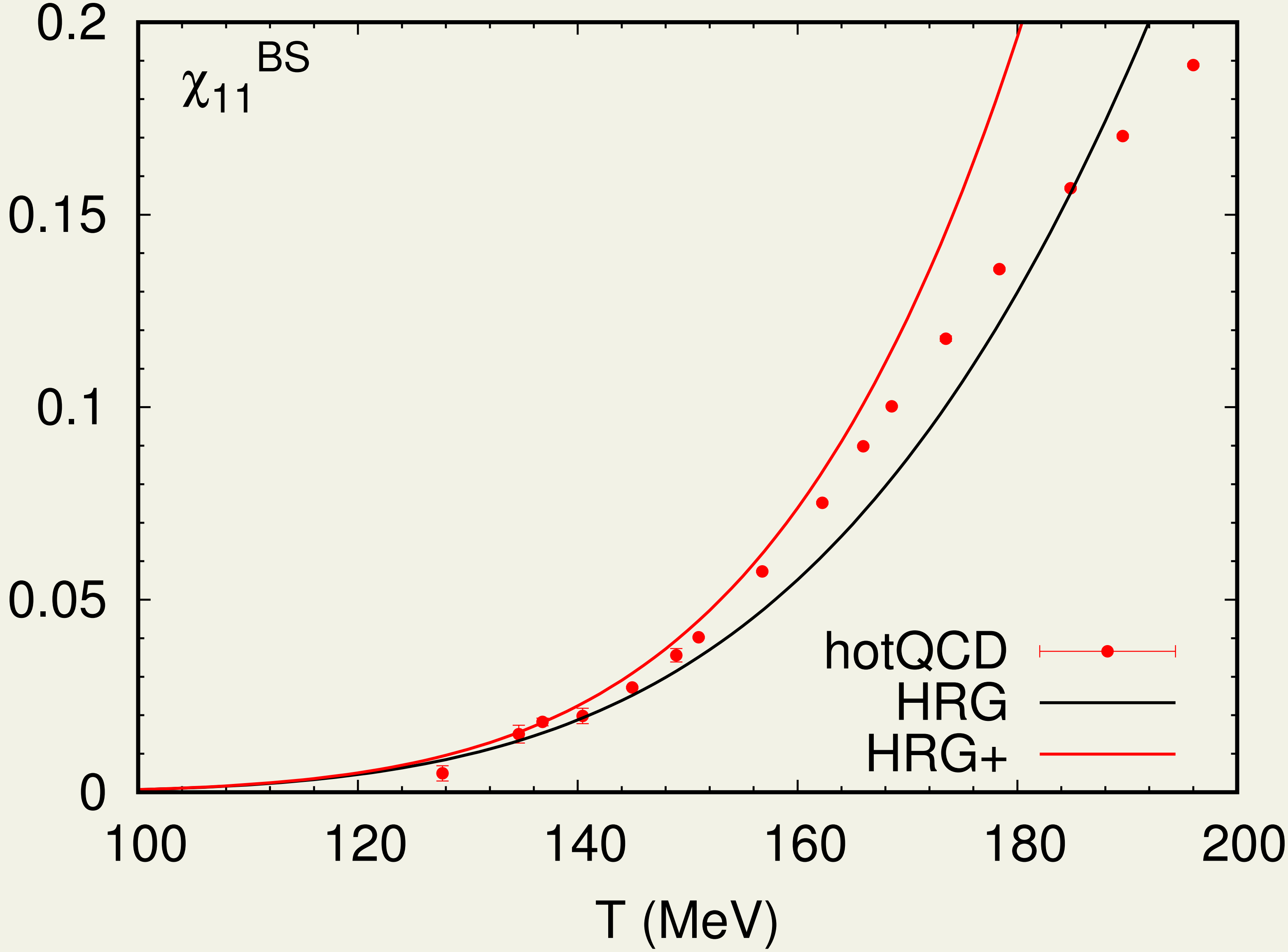

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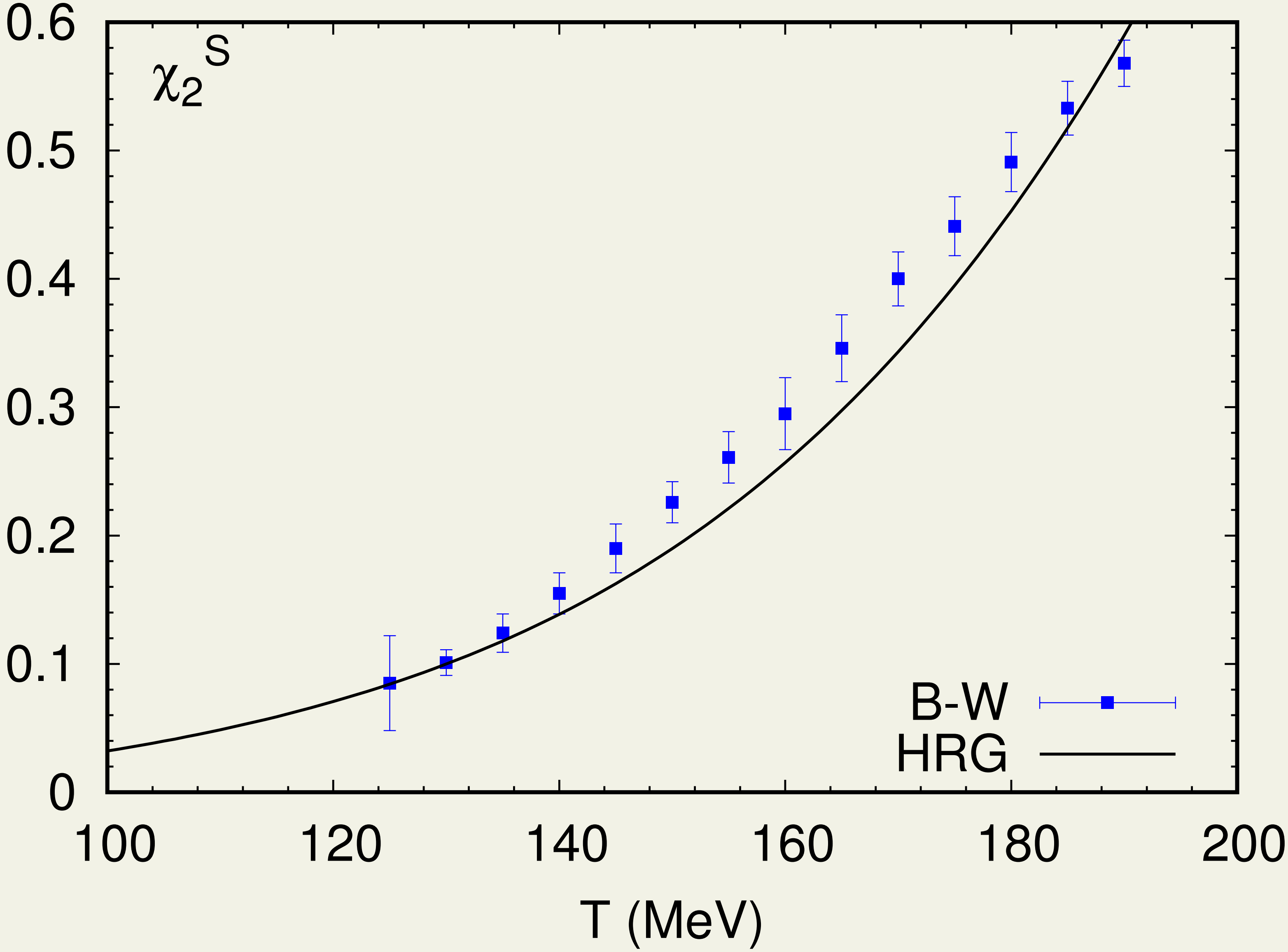
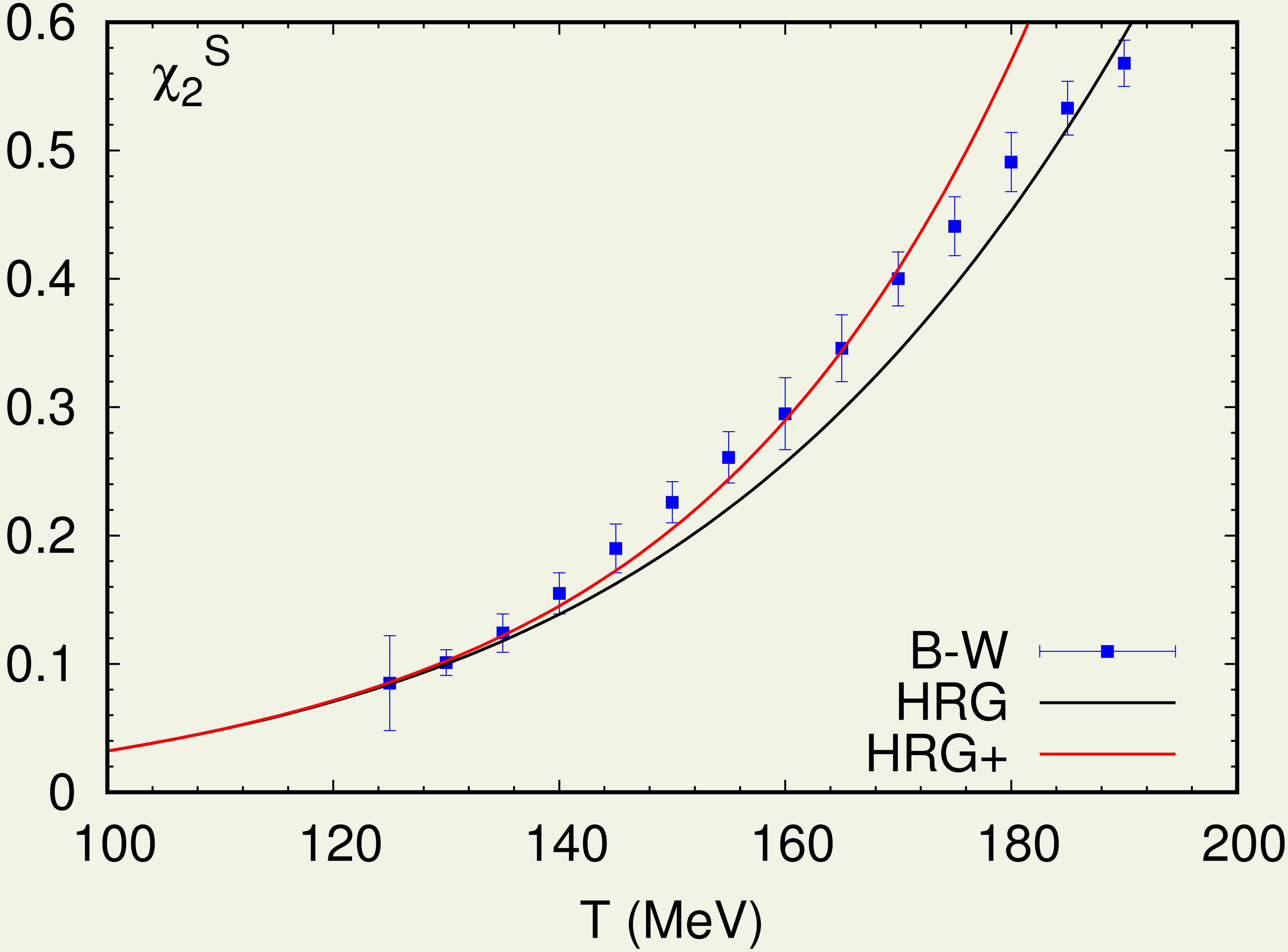
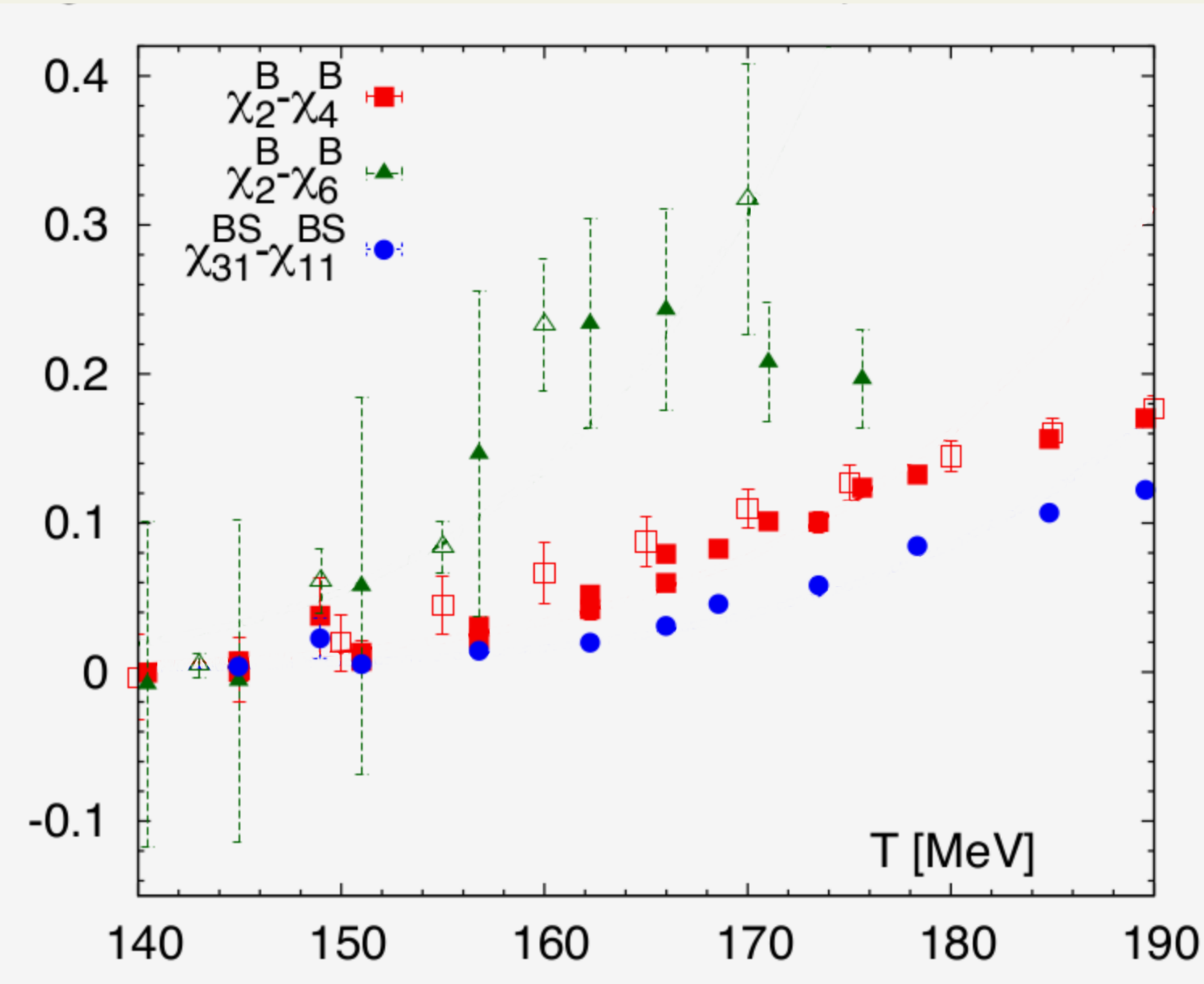
χ^2_S



χ_S^2



Differences of fluctuations



Filled symbols: HISQ

Bazavov et al.,

PRL111, 082301 (2013)

PRD95, 054504 (2017)

Open symbols: stout

4th order

Bellwied et al.,

PRD92, 114505 (2015)

6h order

D'Elia et al,.

PRD95, 094503 (2017)

- **These zero in Boltzmann approximation**

Virial expansion

$$P = P^{\mathrm{ideal}} + T \sum_{ij} b_2^{ij}(T) e^{\beta\mu_i} e^{\beta\mu_j}$$

b_2^{ij} can be related to the S-matrix of scattering of particles i and j

- **$\pi\pi$, πN, etc. scatterings dominated by resonance formation**
- **no resonances in NN scatterings**

Virial expansion in nucleon gas

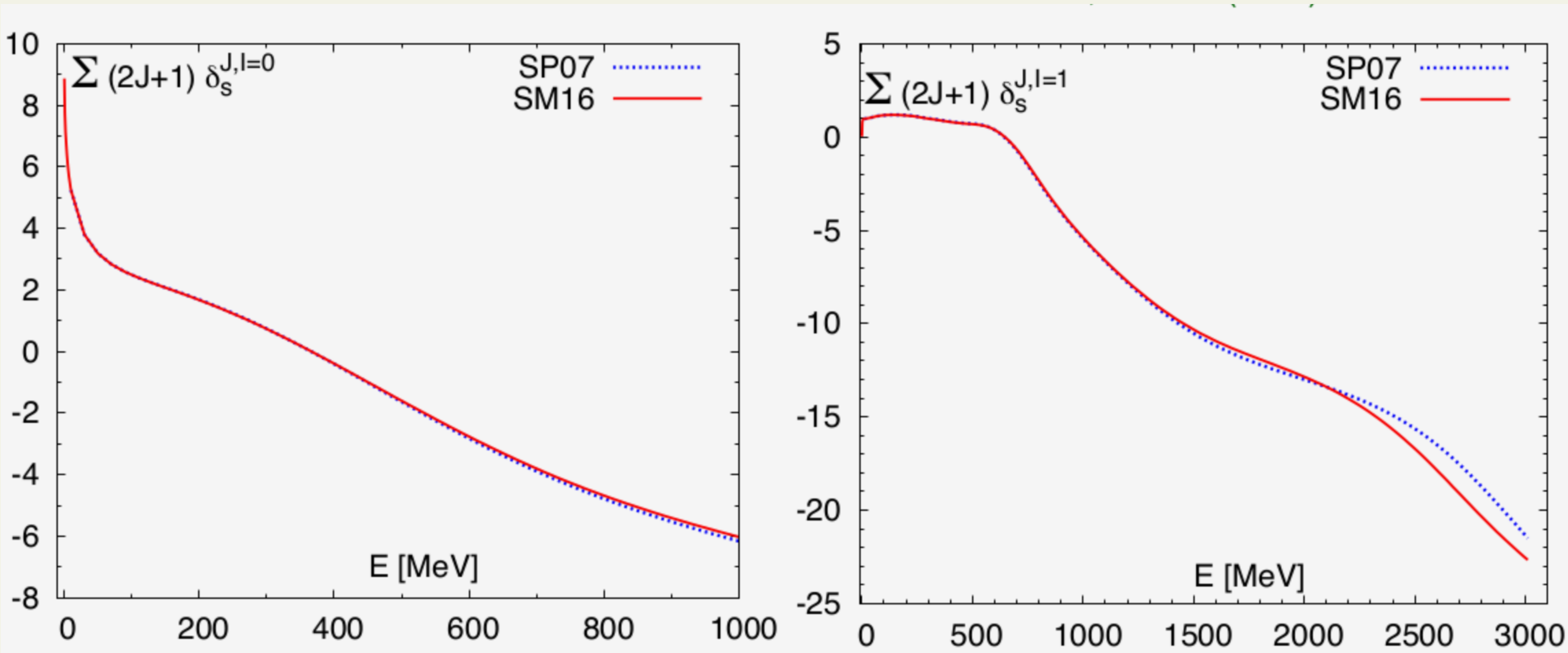
$$\begin{aligned} P(T,\mu) &= P_0(T)\cosh(\beta\mu) + 2b_2(T)\,T\cosh(2\beta\mu) \\ P_0(T) &= \frac{4m^2T^2}{\pi^2}K_2(\beta m) \end{aligned}$$

$$b_2(T) = \frac{2T}{\pi^3}\int_0^\infty \mathrm{d}E\left(\frac{mE}{2}+m^2\right)K_2\left(2\beta\sqrt{\frac{mE}{2}+m^2}\right)\frac{1}{4i}\mathrm{Tr}[S^\dagger\frac{\mathrm{d}S}{\mathrm{d}E}-\frac{\mathrm{d}S^\dagger}{\mathrm{d}E}S]$$

Virial expansion in nucleon gas

Elastic part of the S-matrix from scattering phase shift:

$$\frac{1}{4i}\mathrm{Tr}[S^\dagger\frac{\mathrm{d}S}{\mathrm{d}E}-\frac{\mathrm{d}S^\dagger}{\mathrm{d}E}S]\to\sum_s\sum_J(2J+1)\left(\frac{\mathrm{d}\delta_s^{J,I=0}}{\mathrm{d}E}+3\frac{\mathrm{d}\delta_s^{J,I=1}}{\mathrm{d}E}\right)$$



Workman et al., PRC94, 065203 (2016); Arndt et al., PRC76, 025209 (2007)

Repulsive mean field

Assume: interactions reduce single partice energy by $U = Kn_b$ where n_b is single nucleon density (Olive, NPB190, 483 (1981))

$$n_b = \int \frac{\mathrm{d}^3p}{(2\pi)^3} e^{-\beta(E_p - \mu + U)}$$

Small $\mu \Rightarrow \beta K n_b \ll 1$ and

$$\begin{aligned} n_b &\approx n_b^0(1 - \beta K n_b^0) \Rightarrow \\ P(T,\mu) &= T(n_b + n_{\bar{b}}) - \frac{K}{2}\left((n_b^2)^2 + (n_{\bar{b}}^0)^2\right) \end{aligned}$$

or

$$P(T,\mu) = P_0(T)\left(\cosh(\beta\mu) - \frac{Km}{\pi^2} K_2(\beta m)\cosh(2\beta\mu)\right)$$

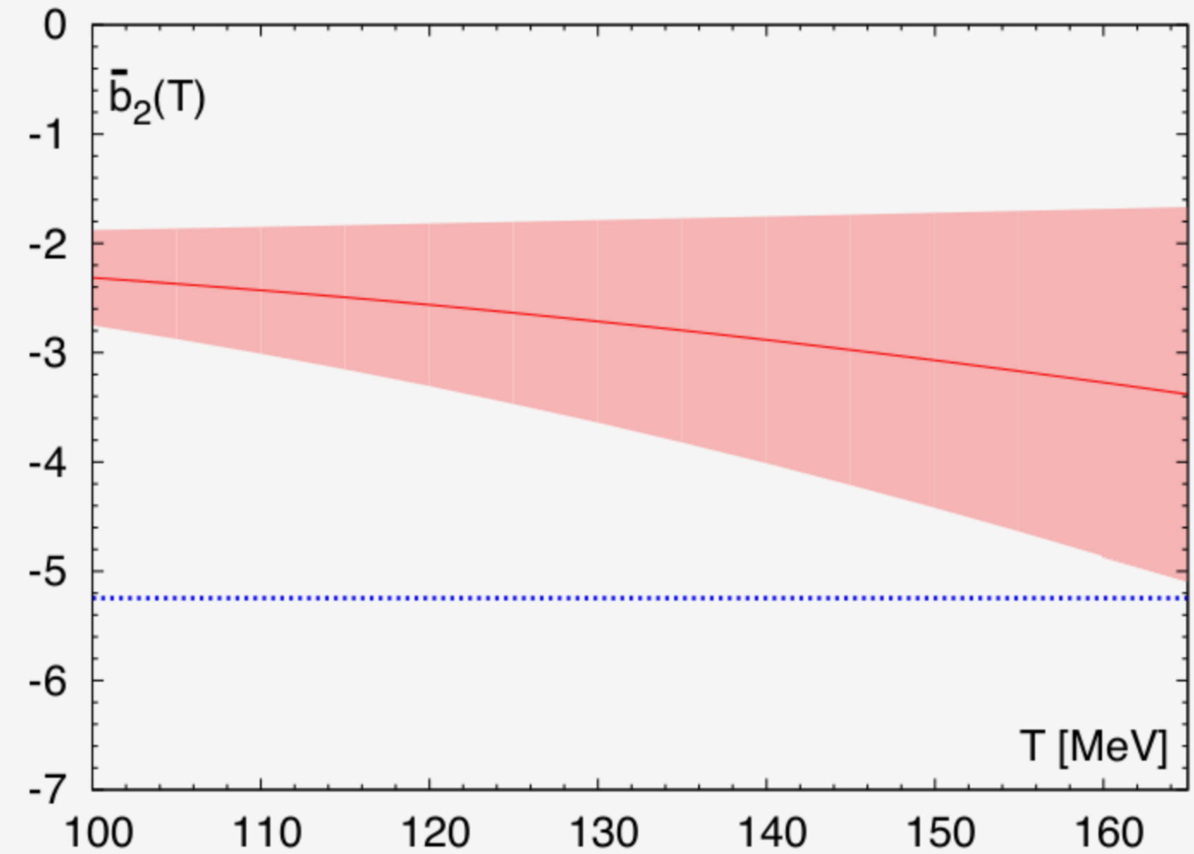
Virial expansion vs. mean field

Repulsive mean field

$P(T,\mu) = P_0(T) \times$
$\left(\cosh(\beta\mu) - \frac{Km}{\pi^2} K_2(\beta m)\cosh(2\beta\mu)\right)$

Virial expansion

$P(T,\mu) = P_0(T) \times$
$\left(\cosh(\beta\mu) - \bar{b}_2(T) K_2(\beta m)\cosh(2\beta\mu)\right)$
where $\bar{b}_2 = \frac{2T b_2(T)}{P_0(T) K_2(\beta m)}$



In-elastic interactions become important for $E > 400$ MeV
$\Rightarrow$ use σ_{el}/σ_{tot}
to estimate the uncertainties in $b_2(T)$ due to these effects

$-\frac{KM^2}{\pi^2}$

for typical phenomenological value $K = 450$ MeV fm^3

Hadron Resonance Gas with mean field

Assume: only members of baryon octet and decuplet repel each other

$$P(T,\mu) = Tn - \frac{K}{2}\left((n^0_{od})^2 + (n^0_{\bar{o}d})^2\right)$$

where

$$n_{od}(T) = \frac{T}{2\pi^2}\sum_i g_i m_i^2 K_2(\beta m_i)$$

$i = N, \Sigma, \Xi, \Delta, \Sigma^*, \Xi^*, \Omega$

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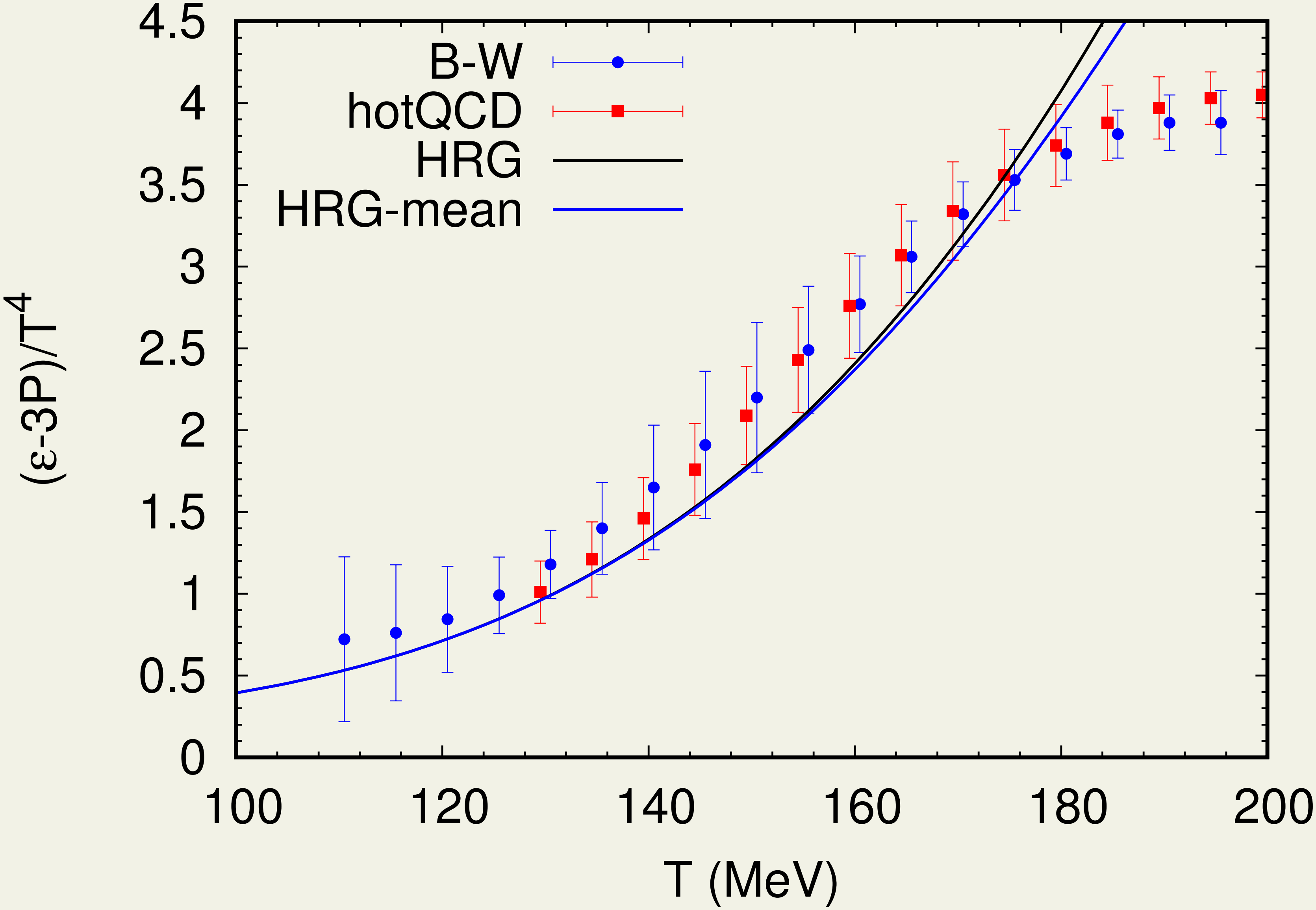
$$\begin{aligned}
\chi^B_n &= \chi^{B(0)}_n - 2^n\beta^4 K (n^0_{od})^2 \\
\chi^{BS}_{n1} &= \chi^{BS(0)}_{n1} + 2^{n+1}\beta^5 K n^0_{od}(P^{S1}_B + 2P^{S2}_B + 3P^{S3}_B)
\end{aligned}$$

Trace anomaly



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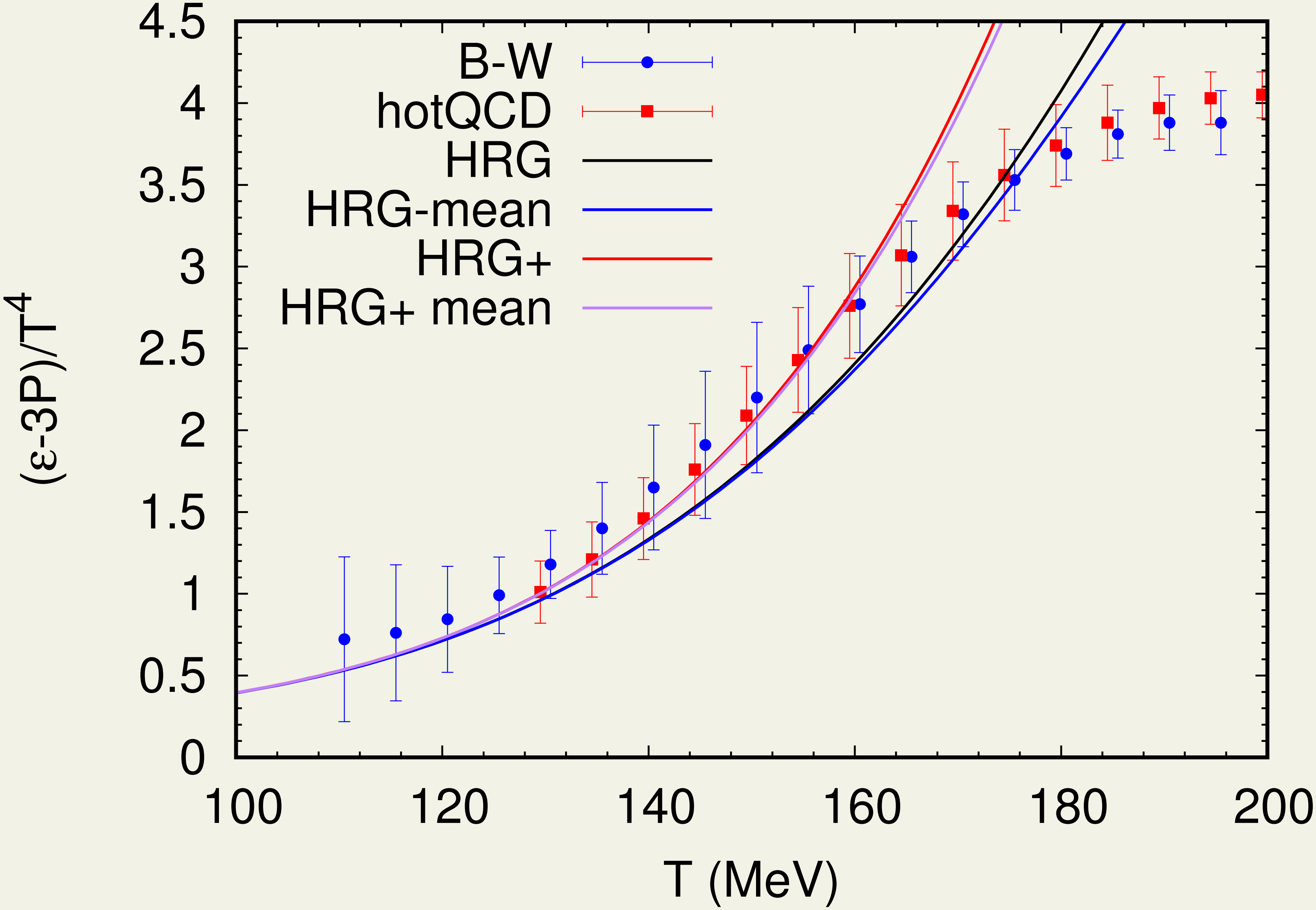
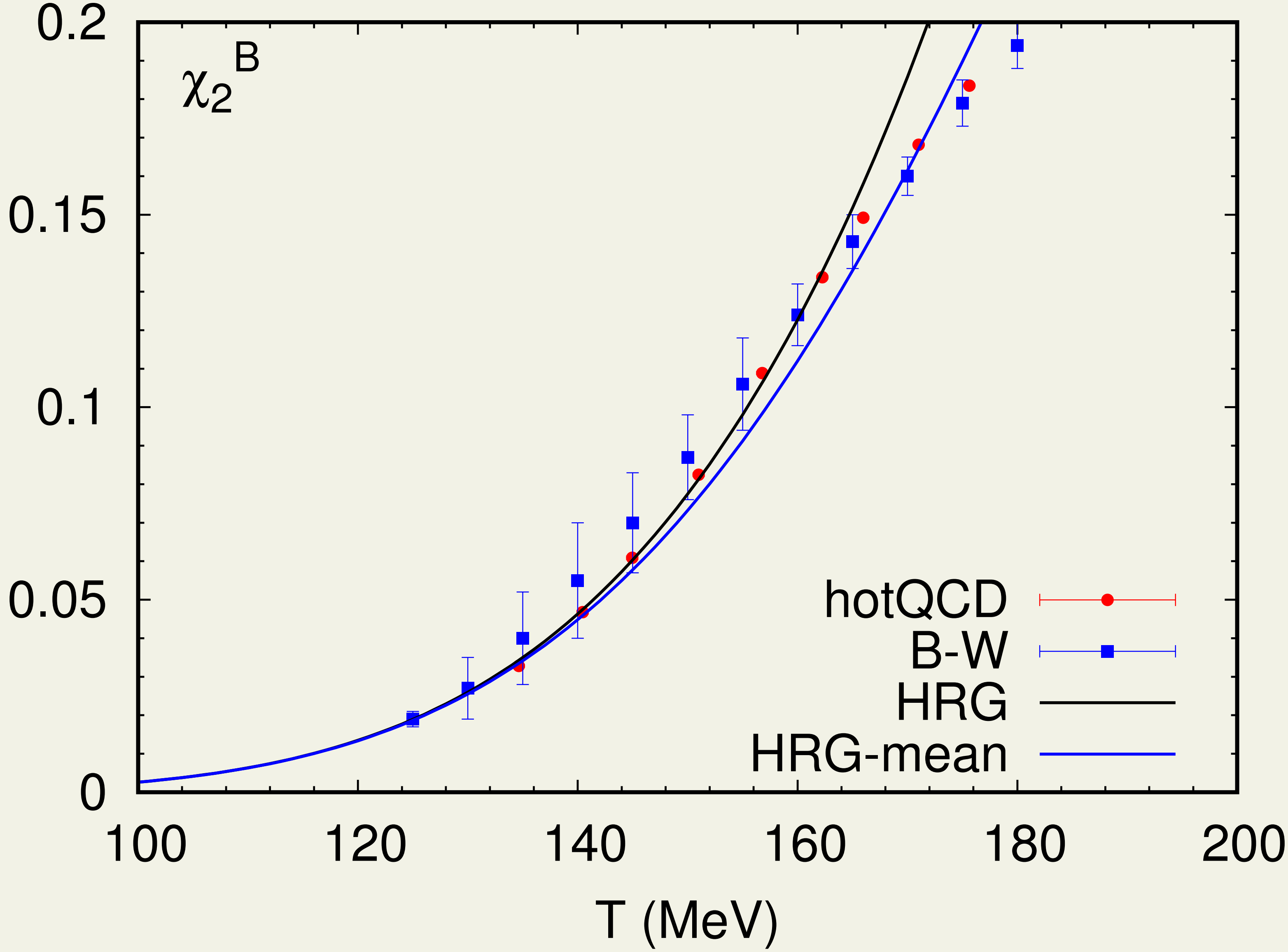


χ^2_B



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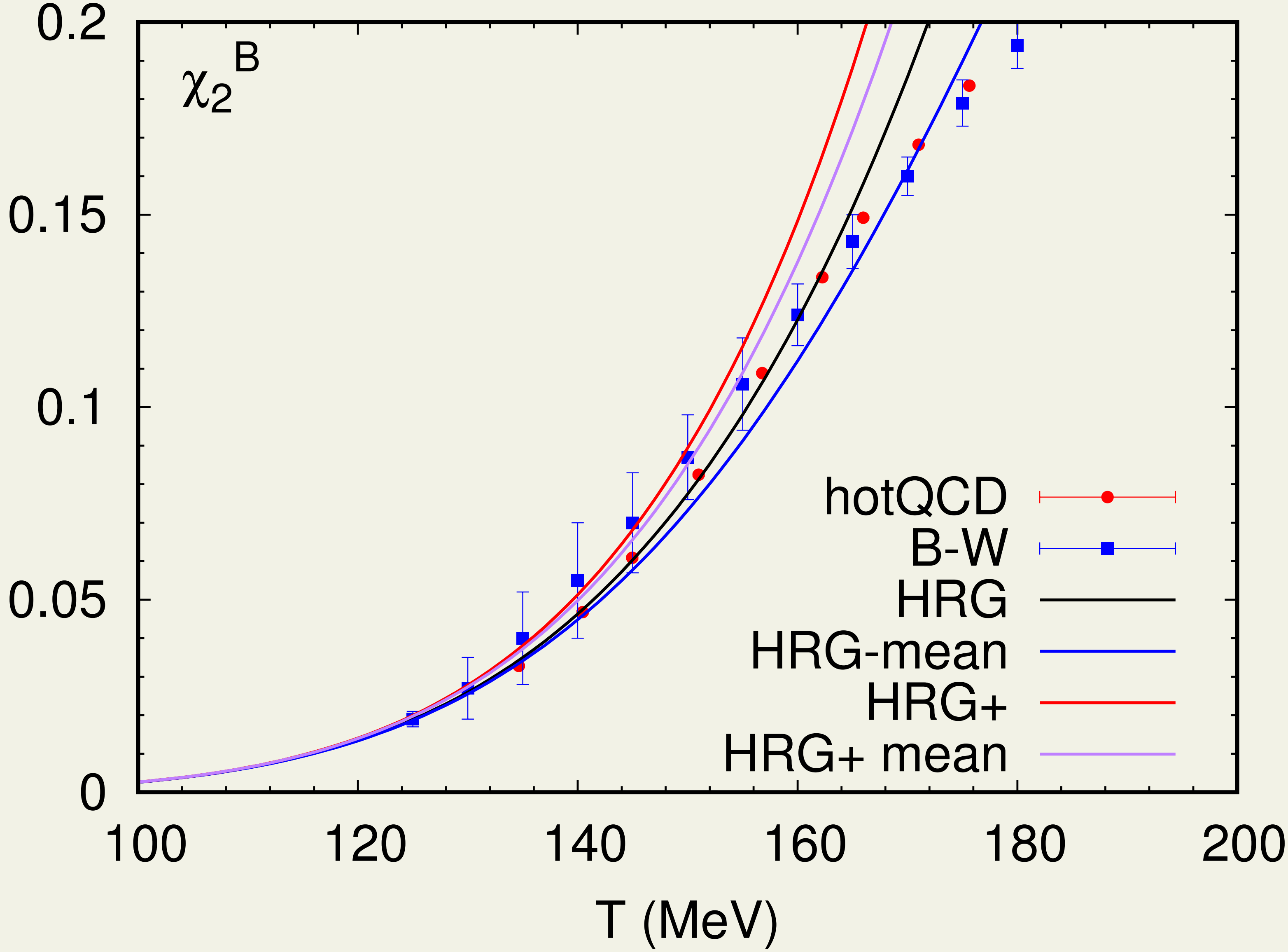

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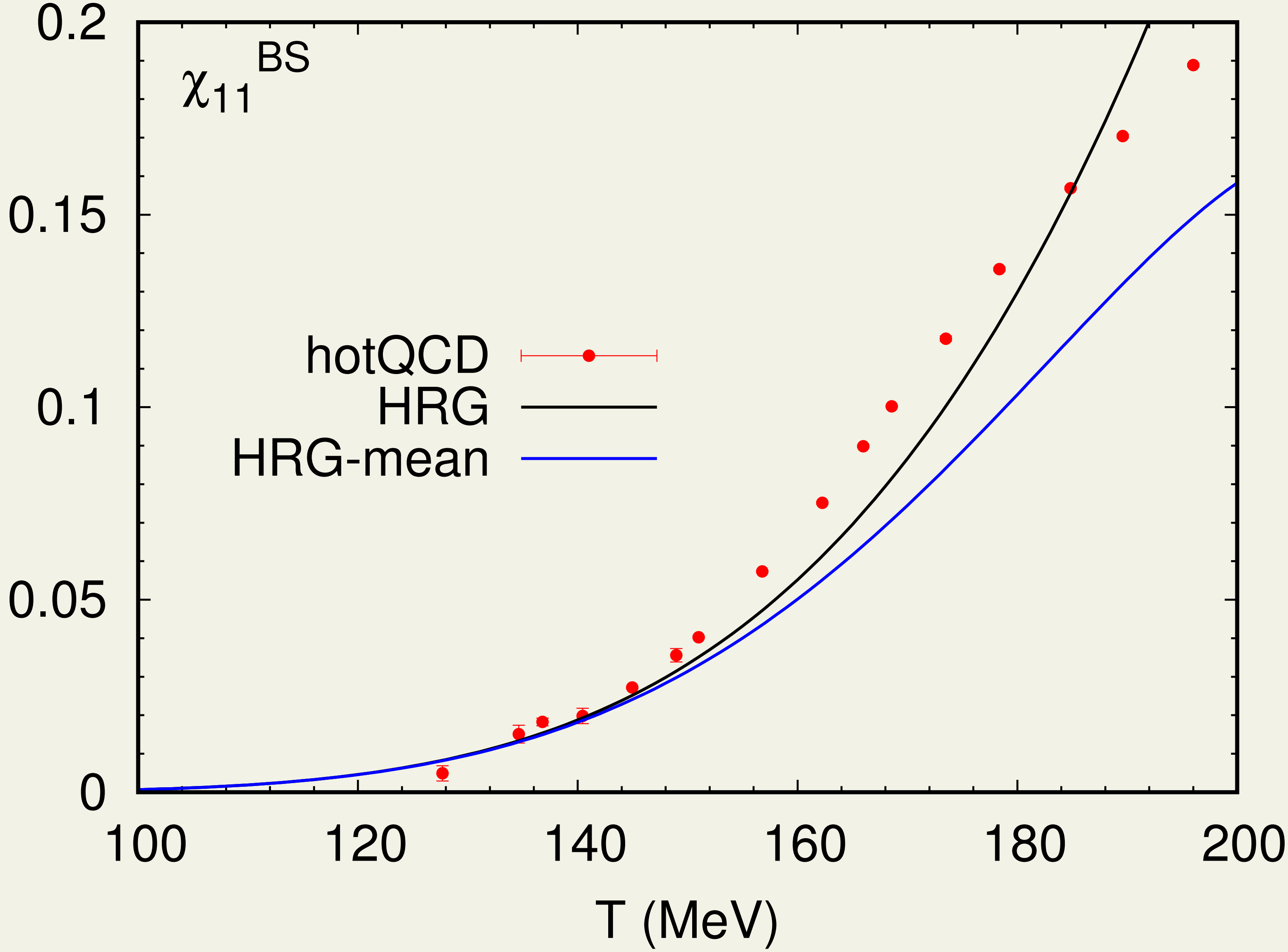
χ^{11}_{BS}



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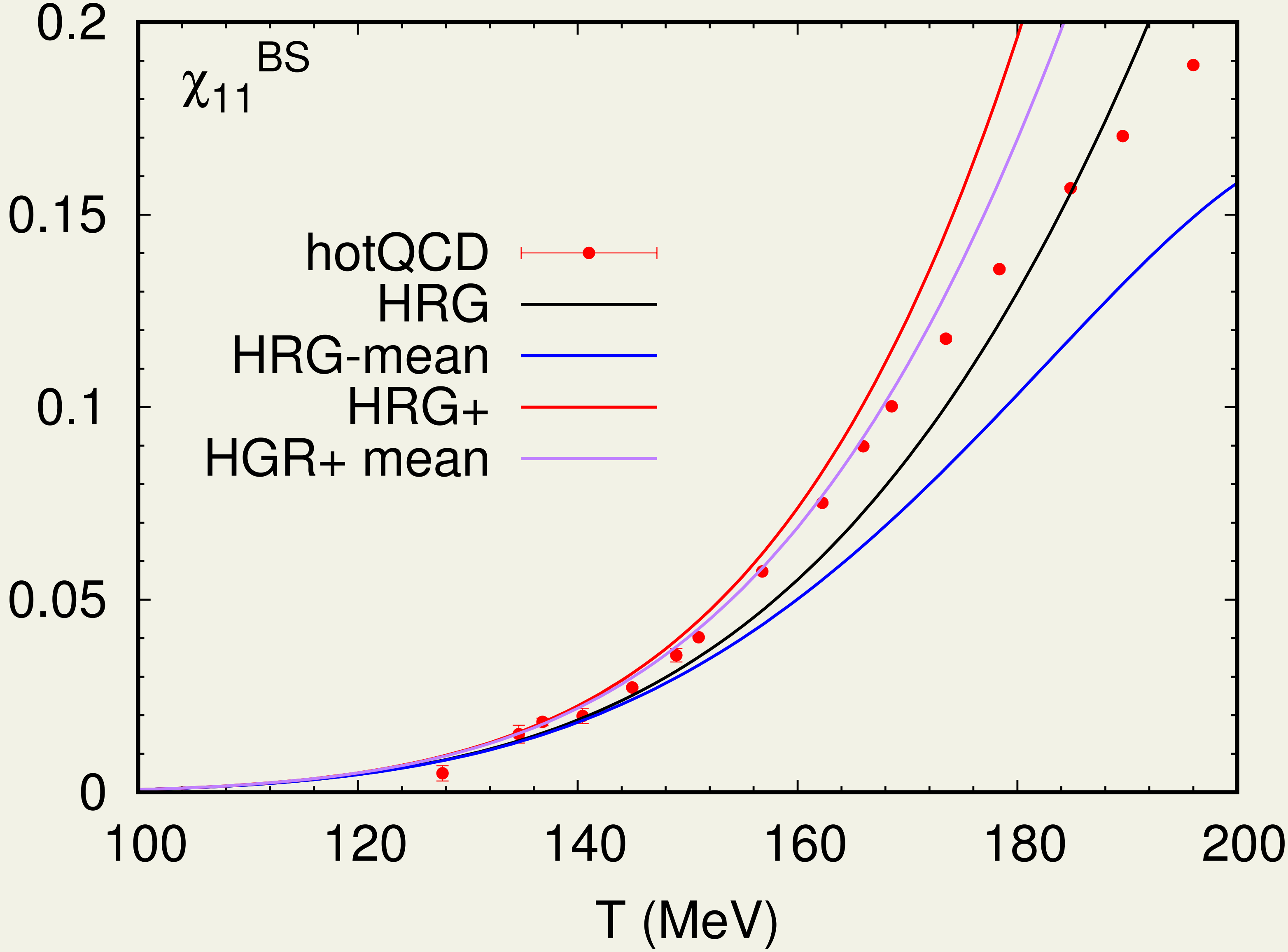
χ_{BS}^{11}



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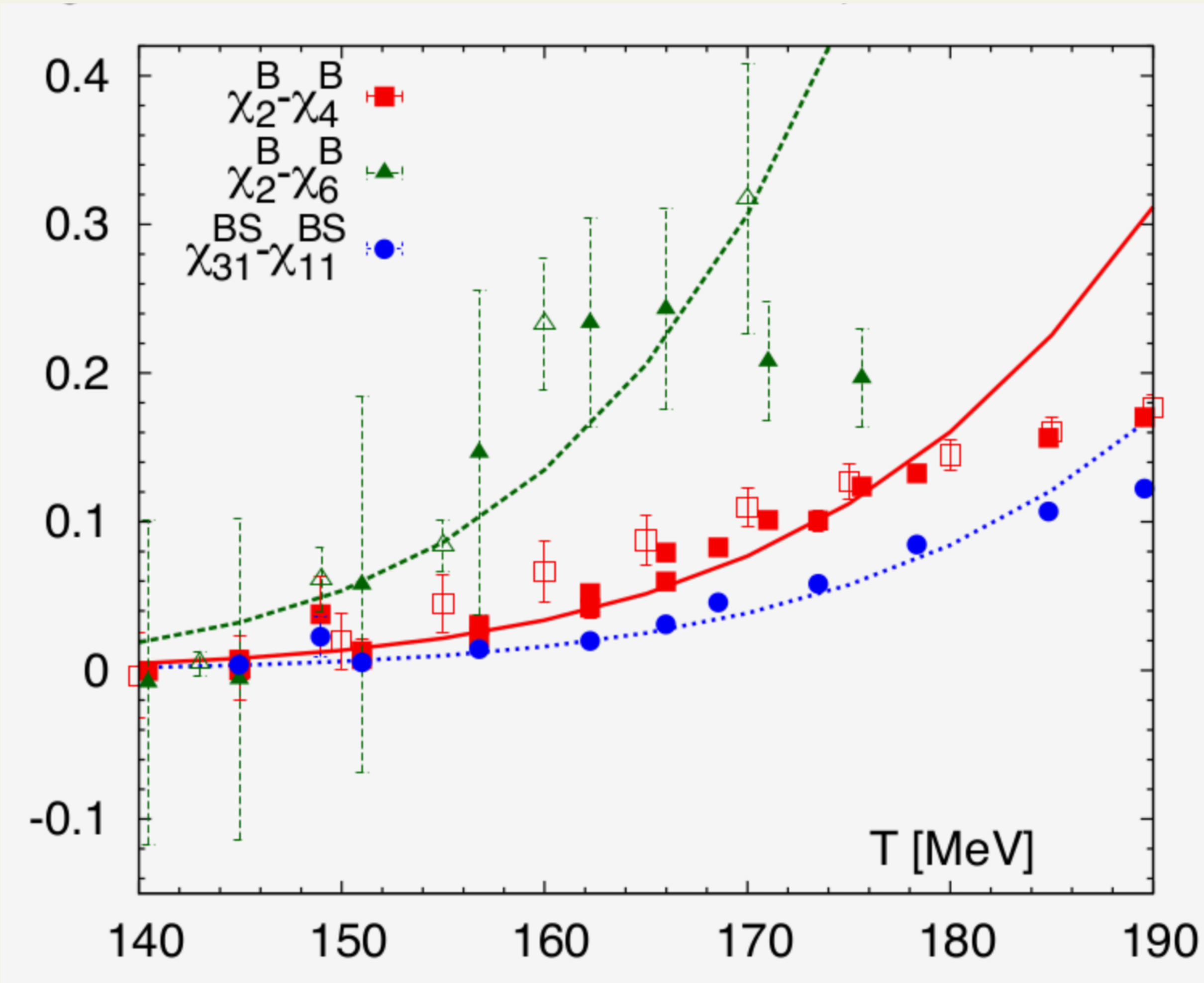
Differences of fluctuations



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- **These zero in Boltzmann approximation**
- **Repulsive interactions create similar differences**

Summary

- **lattice QCD indicates there are more resonances than observed**
  - **inclusion fo quark model states improves the fit to some, and weakens the fit to some observables**

- **repulsive mean field can describe the differences between baryonic fluctuations of different orders**

- **mean field strength can be constrained by phase shifts**