CRC-TR 211
Strong-interaction matter under extreme conditions

ITP Institute for Theoretical Physics

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Diffusion of conserved charges in relativistic heavy ion collisions

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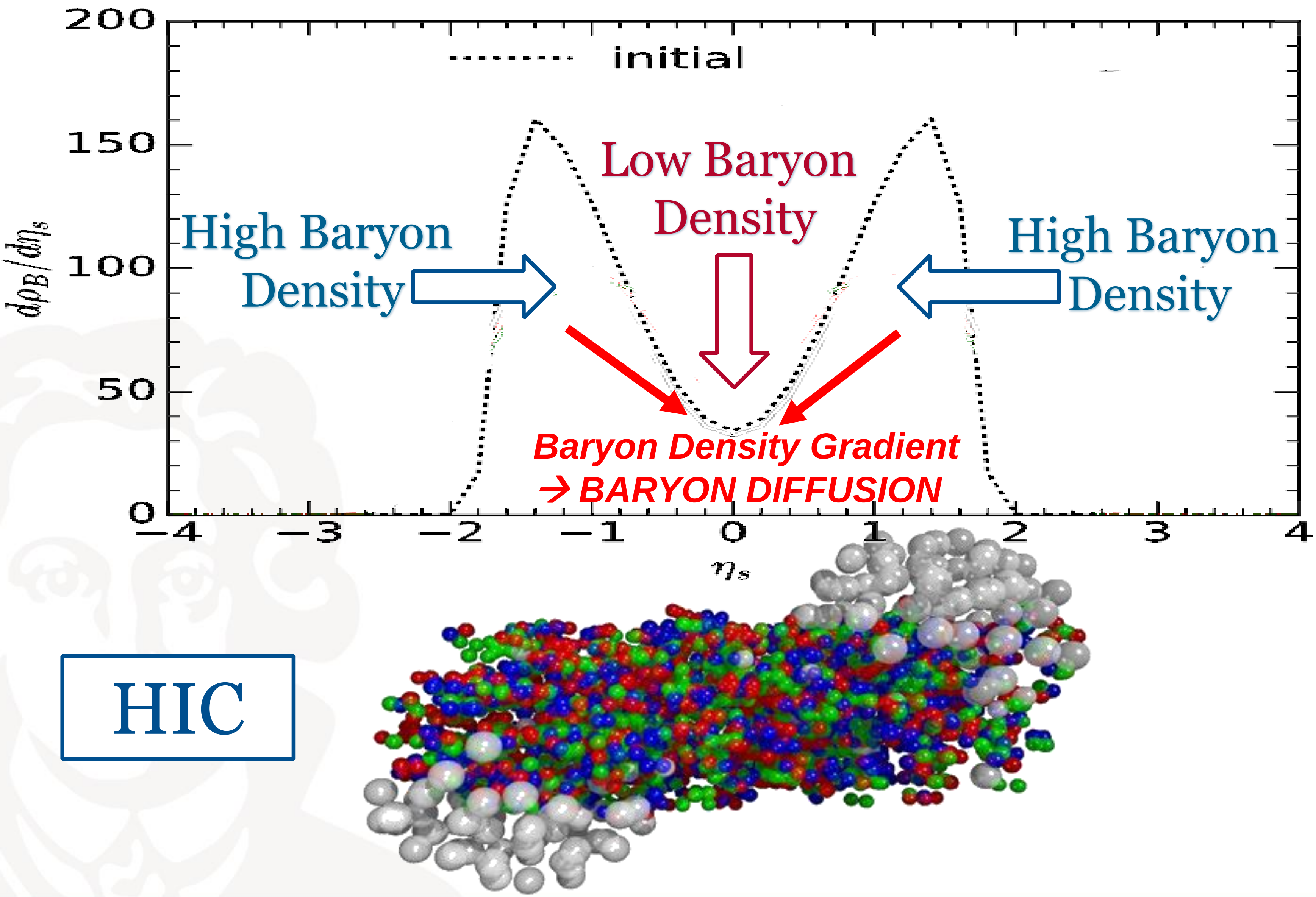
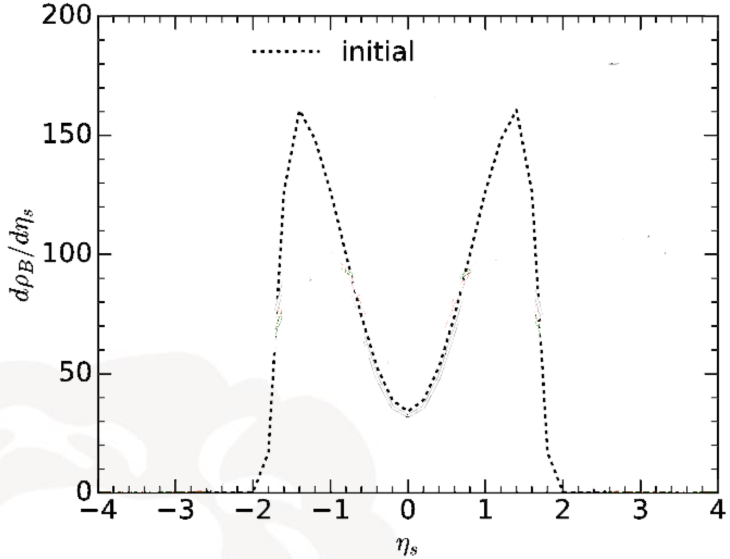
arXiv:1711.08680

Why is Diffusion Important?



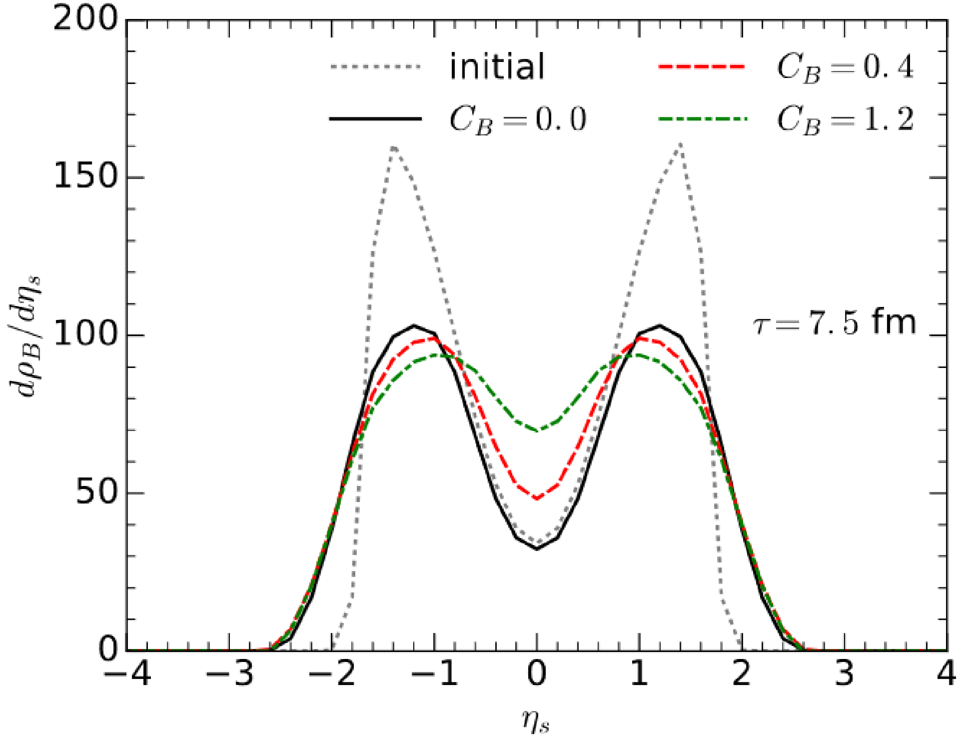
The Evolution in (3+1)-Viscous Hydro

Chun Shen et. al. Nucl. Phys. A 967 (2017) 796-799



0-5% Au-Au collision at 19.7 GeV

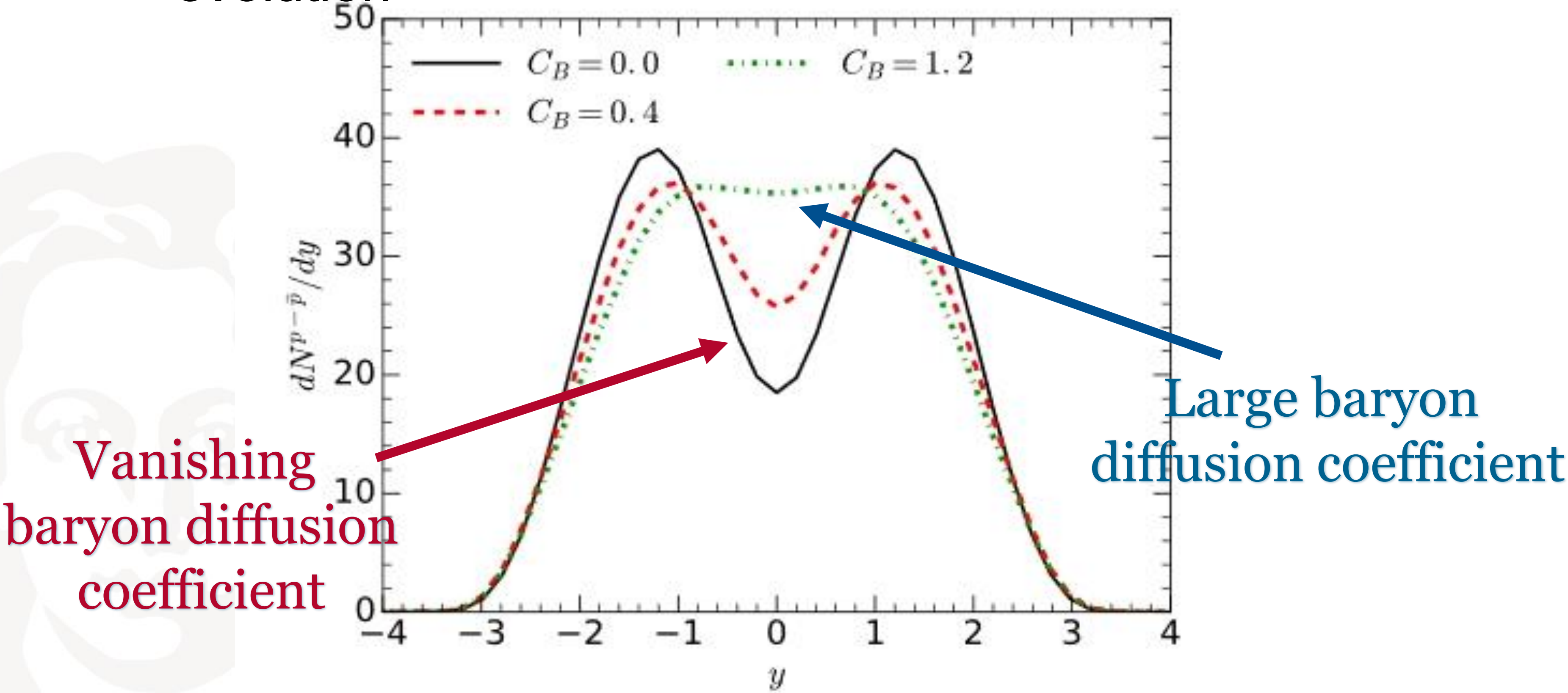
Hydrodynamical evolution after 7.5 fm



Hadronization on next slide

Why is Diffusion Important

- At Low-Energy Heavy Ion Collisions (e.g. RHIC BES): diffusion could have great impact on dynamical evolution



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Description of Diffusion

- Early dynamical evolution of HIC modeled in Relativistic Dissipative Fluid-Dynamics
- For *large evolution times*: Navier-Stokes Theory applicable
- One conserved charge (q):

j_q^μ : **Net-charge diffusion current**

Particle 4-current: $N_q^\mu = n_0 u^\mu + \kappa_q \nabla^\mu \left(\mu_q / T\right)$

Net-charge diffuison coefficient

Gradient in thermal potential ~ Gradient in net-charge density

Description of Diffusion

- In multi-component system with multiple conserved charges: particles can have any combination of charges
(e.g. proton: electric and baryon charge)

- Net-charge diffusion currents effect each other

$$\begin{pmatrix} j_B^\mu \\ j_Q^\mu \\ j_S^\mu \end{pmatrix} = \begin{pmatrix} \kappa_{BB} & \kappa_{BQ} & \kappa_{BS} \\ \kappa_{QB} & \kappa_{QQ} & \kappa_{QS} \\ \kappa_{SB} & \kappa_{SQ} & \kappa_{SS} \end{pmatrix} \cdot \begin{pmatrix} \nabla^\mu \alpha_B \\ \nabla^\mu \alpha_Q \\ \nabla^\mu \alpha_S \end{pmatrix}$$

Off-diagonal coefficients: Gradients of given charge can effect diffusion currents of other charges

Are the off-diagonal coefficients important?

The Chapman-Enskog Expansion

- Assume dilute Boltzmann gas with N_s particle species and conserved baryon, strangeness and electric charge close to local equilibrium → describe with kinetic theory

$$f_k^i = \boxed{f_{0,k}^i} + \epsilon \boxed{f_{1,k}^i} + \mathcal{O}(\epsilon^2)$$

small deviation from equilibrium

local equilibrium term

book-keeping parameter counts gradients

- Neglect non-linear contributions → Navier-Stokes limit

The Chapman-Enskog Expansion

- Relativistic Boltzmann equation determines evolution of system

$$k_i^\mu \partial_\mu f_k^i = -\sum_{j=1}^{N_s} C_{ij}[f_k^i]$$

$\Downarrow$ Chapman-Enskog expansion

$$\epsilon k_i^\mu \partial_\mu \left(f_{0k}^i + \epsilon f_{0k}^i\right) \approx \epsilon k_i^\mu \partial_\mu f_{0k}^i = -\epsilon \sum_{j=1}^{N_s} C_{ij}[f_{1k}^i]$$

With linearized collision term:

$$\sum_{j=1}^{N_s} C_{ij}[f_{1k}^i] = \sum_{j=1}^{N_s} \gamma_{ij} \int \mathrm{d}K_j' \mathrm{d}P_i \mathrm{d}P_j' \boxed{W_{kk'\to pp'}^{ij}} f_{0k}^i f_{0k'}^j \left(\frac{f_{1k}^i}{f_{0k}^i} + \frac{f_{1k'}^j}{f_{0k'}^j} - \frac{f_{1p}^i}{f_{0p}^i} - \frac{f_{1p'}^i}{f_{0p'}^i} \right)$$

Transition rate: contains (isotropic) cross sections = information of microscopic interactions

The Chapman-Enskog Expansion

Evaluating derivatives leads to source equation for deviation f_{1k}^{i}

$$k_i^{\mu}\partial_{\mu}f_{0k}^{i} = -\sum_{j=1}^{N_s} C_{ij}[f_{1k}^{i}]$$

$$\sum_{q\in\{B,S,Q\}} f_{0k}^{i} k_i^{\mu}\left(\frac{E_{ik}n_q}{\epsilon_0 + P_0} - q_i\right)\nabla_{\mu}\left(\frac{\mu_q}{T}\right) = -\sum_{j=1}^{N_s} C_{ij}[f_{1k}^{i}]$$

Gradient in thermal potential

Sum over all conserved charges
→ coupling of diffusion currents

L.H.S. of eq. ~ force term due to gradients in particle density → Navier Stokes currents

The Chapman-Enskog Expansion

Diffusion currents in kinetic theory:

We want to calculate **THIS**

$$j_q^\mu = \sum_{i=1}^{N_s} q_i \int \mathrm{d}K \; k_i^{\langle\mu\rangle} f_{1k}^i \overset{!}{=} \sum_{q'} \kappa_{qq'} \nabla^\mu \left(\frac{\mu_{q'}}{T}\right)$$

Navier-Stokes limit

In order to do so, we need to solve:

$$\sum_{q\in\{B,S,Q\}} f_{0k}^i k_i^\mu \left(\frac{E_{ik} n_q}{\epsilon_0 + P_0} - q_i\right) \nabla_\mu \left(\frac{\mu_q}{T}\right) = -\sum_{j=1}^{N_s} C_{ij}[f_{1k}^i]$$

The Chapman-Enskog Expansion

$$\sum_{q \in \{B,S,Q\}} f^i_{0k} k^\mu_i \left(\frac{E_{ik} n_q}{\epsilon_0 + P_0} - q_i \right) \nabla_\mu \left(\frac{\mu_q}{T} \right) = - \sum_{j=1}^{N_s} C_{ij}[f^i_{1k}]$$

Since collision term is linear in f^i_{1k} the solutions have the general form:

scalar function in energy

$$f^i_{1k} = \sum_q \boxed{a^i_q} k^\mu_i \nabla_\mu \left(\frac{\mu_q}{T} \right)$$

Expand coefficients in power series in energy:

$$a^i_q = \sum_{m=0}^{\infty} a^i_{q,m} E^m_{ik}$$

The Chapman-Enskog Expansion

$$\sum_{q\in\{B,S,Q\}} f_{0k}^{i} k_i^{\mu} \left(\frac{E_{ik} n_q}{\epsilon_0 + P_0} - q_i \right) \nabla_\mu \left(\frac{\mu_q}{T} \right) = - \sum_{j=1}^{N_s} C_{ij}[f_{1k}^{i}]$$

Truncate series at finite integer M and calculate n-th moment of source equation → set of linear equations for expansion Coefficients

Solutions of matrix equation → gives us f_{1k}^{i}

$$\sum_{m=0}^{M} \sum_{j=1}^{N_s} \left(A_{nm}^{i} \delta^{ij} + C_{nm}^{ij} \right) a_{q,m}^{j} = b_{q,n}^{i}$$

moments of collision term → complicated integrals with information about microscopic interactions

Source term for diffusion

The Chapman-Enskog Expansion

$$j_q^\mu = \sum_{i=1}^{N_s} q_i \int \mathrm{d}K \; k_i^{\langle\mu\rangle} f_{1k}^i \stackrel{!}{=} \sum_{q'} \kappa_{qq'} \nabla^\mu \left(\frac{\mu_{q'}}{T} \right)$$

By comparing both sides we find:

$$\kappa_{qq'} = \frac{1}{3} \sum_{i=1}^{N_s} q_i \sum_{m=0}^{M} a_{q',m}^i \int \mathrm{d}K_i E_{ik}^m (m^2 - E_{ik}^2) f_{0k}^i$$

In our most detailed calculation: M = 1 and N_s = 19

The Relaxation Time Approximation

Calculated for $p \; n \; \bar{p} \; \bar{n} \; K \; \pi$ gas
(11 hadron species)

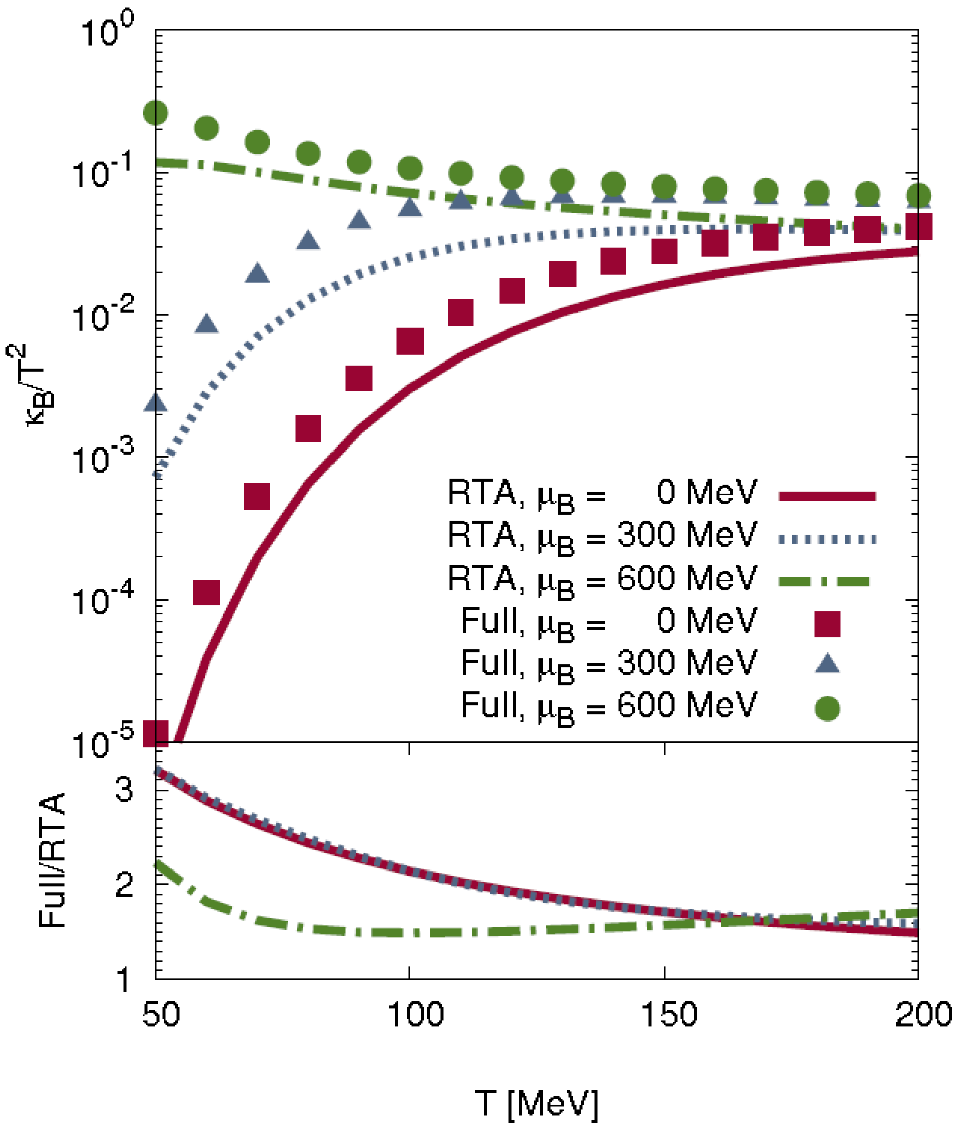
$$\sum_{j=1}^{N_s} C_{ij}[f_{1k}^i] = -\frac{E_{ik}}{\tau} f_{1k}^i$$

Relaxation time:

Total baryon density

$$\tau^{-1} = \frac{2}{3} \boxed{n_{B,\text{tot}}} \boxed{\sigma_0}$$

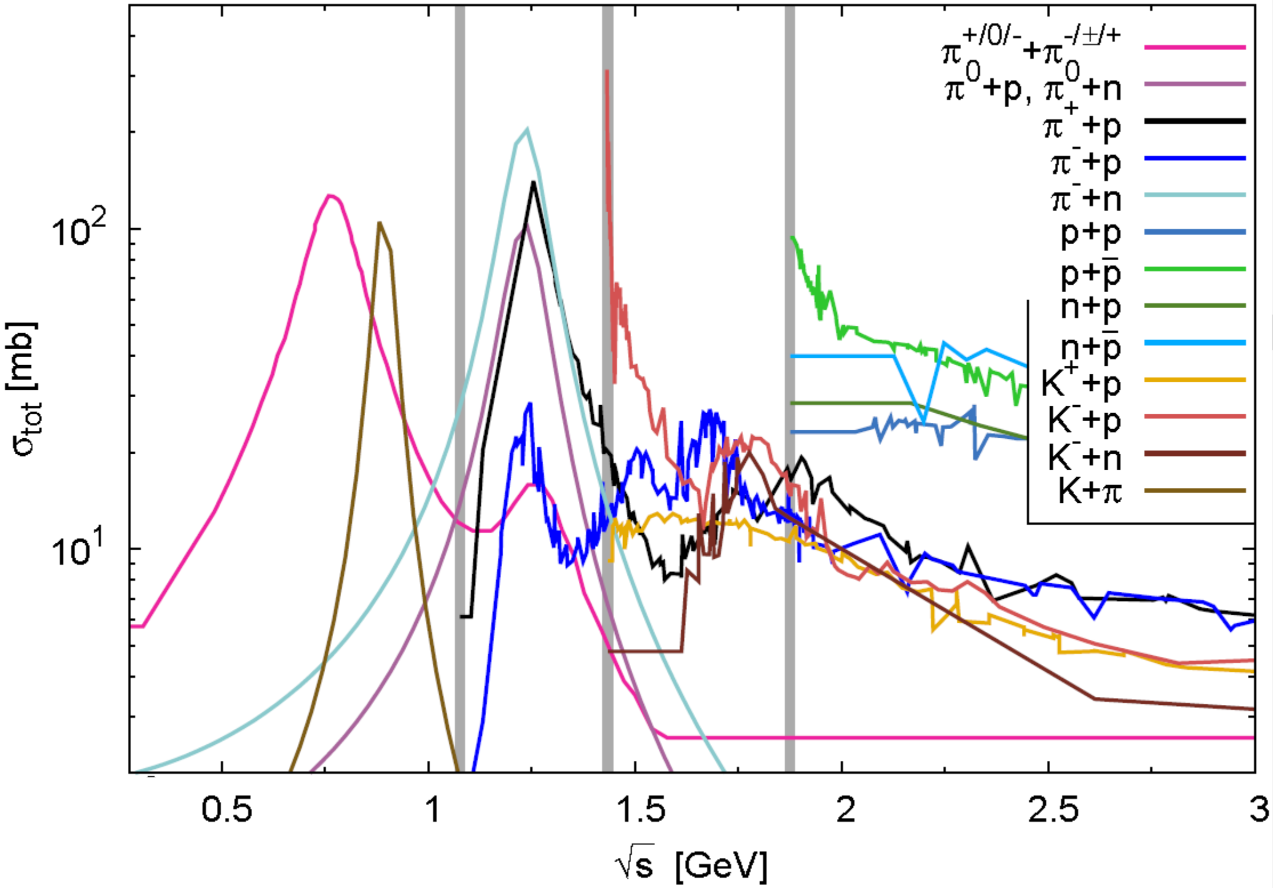
Constant cross section



Hadronic resonance gas...

- Use 19 different, massive species
- Isotropic cross sections

$$\pi^{0,\pm}, K^{\pm,0,\bar{0}}, p, \bar{p}, n, \bar{n}, \Sigma^{0,\pm}, \bar{\Sigma}^{0,\pm}, \Lambda, \bar{\Lambda}$$



- Use PDG data
- Other cross sections: GiBUU, UrQMD or constant

Simplified (conformal) QGP model...

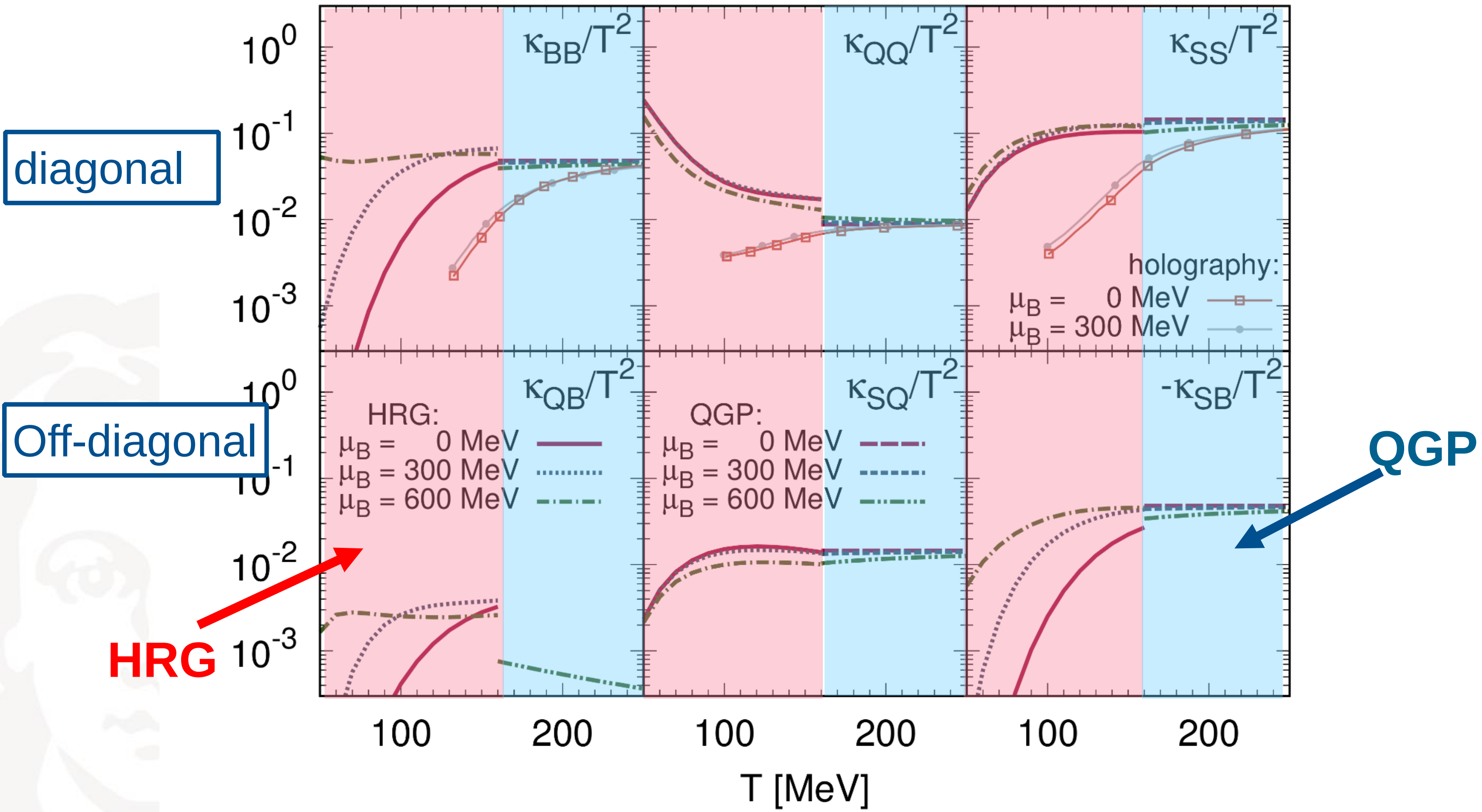
- Use 7 massless species $u, \bar{u}, d, \bar{d}, s, \bar{s}, g$
- Simplified approach: Fix shear viscosity to express isotropic cross section in terms of temperature

$$\frac{\eta}{s} = \frac{1}{4\pi} \quad \Rightarrow \quad \sigma_{tot} = \frac{0.716}{T^2}$$

Calculate diffusion coefficients for the hadron gas for T < 160 MeV and for higher temperatures in the simplified QGP model
→ phase transition area is **NOT** covered by our calculations

$$\begin{pmatrix} j_B^\mu \\ j_Q^\mu \\ j_S^\mu \end{pmatrix} = \begin{pmatrix} \kappa_{BB} & \kappa_{BQ} & \kappa_{BS} \\ \kappa_{QB} & \kappa_{QQ} & \kappa_{QS} \\ \kappa_{SB} & \kappa_{SQ} & \kappa_{SS} \end{pmatrix} \cdot \begin{pmatrix} \nabla^\mu \alpha_B \\ \nabla^\mu \alpha_Q \\ \nabla^\mu \alpha_S \end{pmatrix}$$

The diffusion matrix

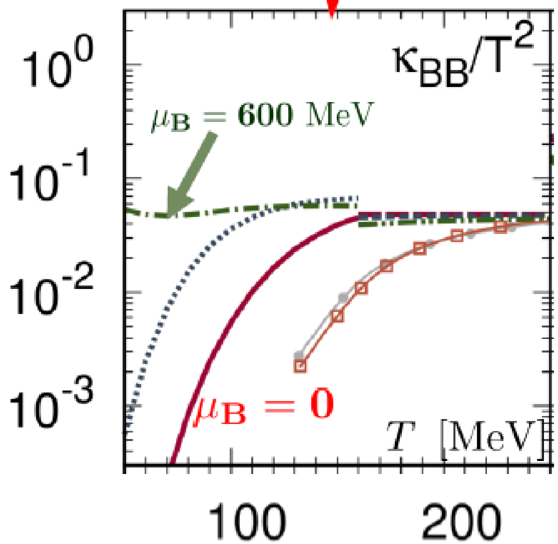


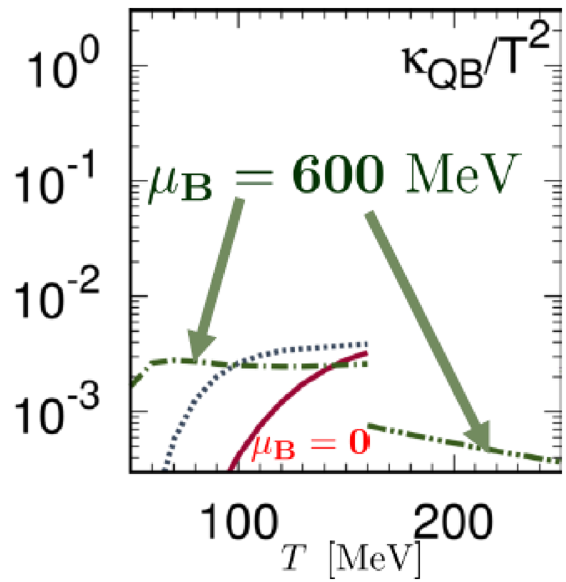
Diffusion matrix is symmetric! → Onsager Theorem holds

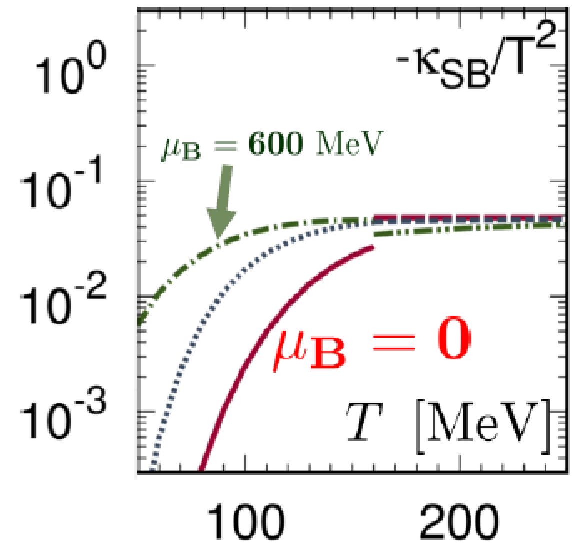
Baryon current

$$\begin{pmatrix} j_B^\mu \\ j_Q^\mu \\ j_S^\mu \end{pmatrix} = \begin{pmatrix} \kappa_{BB} & \kappa_{BQ} & \kappa_{BS} \\ \kappa_{QB} & \kappa_{QQ} & \kappa_{QS} \\ \kappa_{SB} & \kappa_{SQ} & \kappa_{SS} \end{pmatrix} \cdot \begin{pmatrix} \nabla^\mu \alpha_B \\ \nabla^\mu \alpha_Q \\ \nabla^\mu \alpha_S \end{pmatrix}$$

$$j_B^\mu = \kappa_{BB} \nabla^\mu \alpha_B + \kappa_{BQ} \nabla^\mu \alpha_Q + \kappa_{BS} \nabla^\mu \alpha_S$$







- Largest contribution
- Nearly constant at $\mu_B = 600\ MeV$
- So far only used coefficient

- Much smaller than others
- QGP-part vanishes at $\mu_B = 0$
- Strong μ_B dependence

- Negative contribution!
- Similar strength as κ_{BB}
- Could drastically reduce baryon current

Electric current

$$\begin{pmatrix} j_B^\mu \\ j_Q^\mu \\ j_S^\mu \end{pmatrix} = \begin{pmatrix} \kappa_{BB} & \kappa_{BQ} & \kappa_{BS} \\ \kappa_{QB} & \kappa_{QQ} & \kappa_{QS} \\ \kappa_{SB} & \kappa_{SQ} & \kappa_{SS} \end{pmatrix} \cdot \begin{pmatrix} \nabla^\mu \alpha_B \\ \nabla^\mu \alpha_Q \\ \nabla^\mu \alpha_S \end{pmatrix}$$

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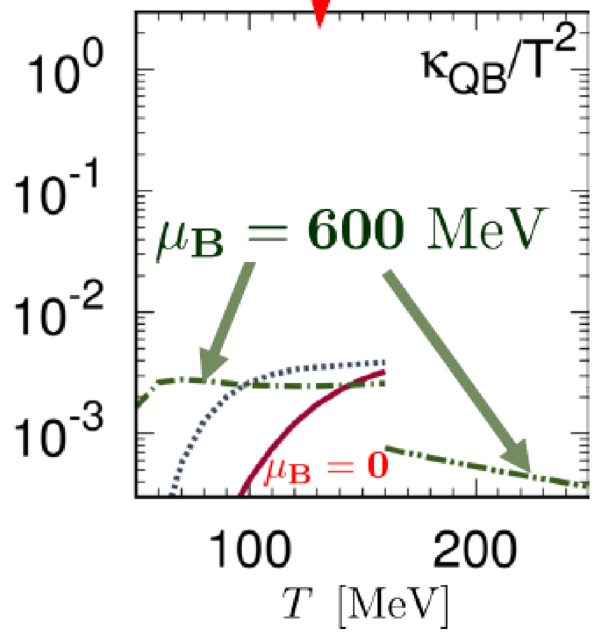
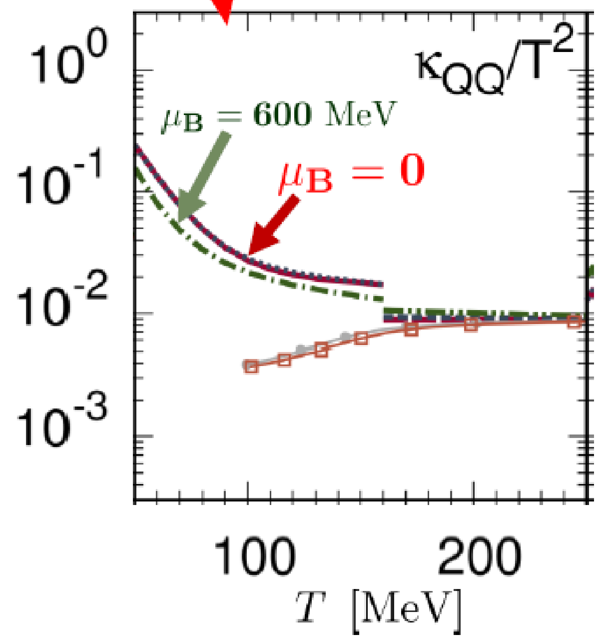
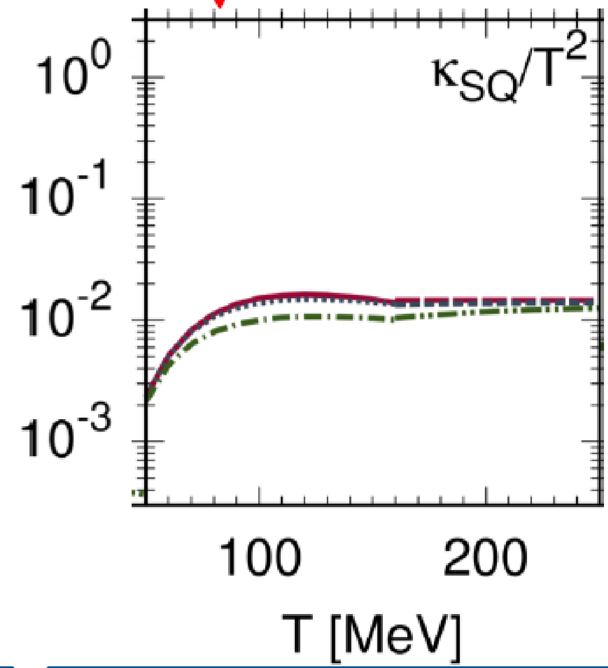
$$j_Q^\mu = \kappa_{QB} \nabla^\mu \alpha_B + \kappa_{QQ} \nabla^\mu \alpha_Q + \kappa_{QS} \nabla^\mu \alpha_S$$







- Smaller than others
- QGP-part vanishes at $\mu_B = 0$
- Strong μ_B dependence

- $\mu_B = 0$ same as electric conductivity
- Only decreasing behavior in T

- QGP: strongest contribution

Strangeness current

$$\begin{pmatrix} j_B^\mu \\ j_Q^\mu \\ j_S^\mu \end{pmatrix} = \begin{pmatrix} \kappa_{BB} & \kappa_{BQ} & \kappa_{BS} \\ \kappa_{QB} & \kappa_{QQ} & \kappa_{QS} \\ \kappa_{SB} & \kappa_{SQ} & \kappa_{SS} \end{pmatrix} \cdot \begin{pmatrix} \nabla^\mu \alpha_B \\ \nabla^\mu \alpha_Q \\ \nabla^\mu \alpha_S \end{pmatrix}$$

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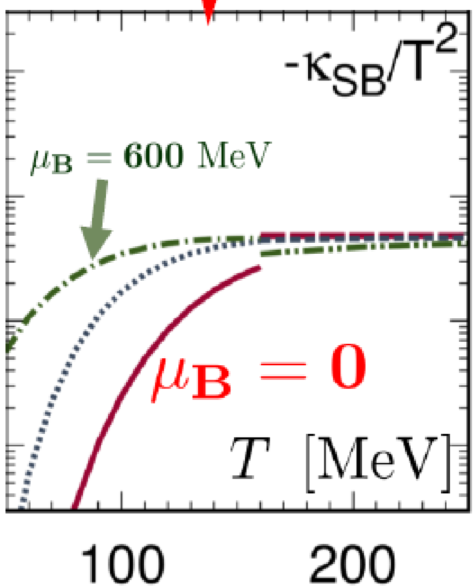
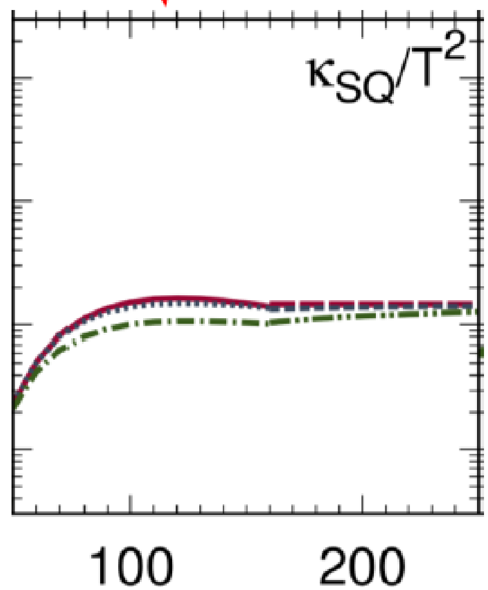
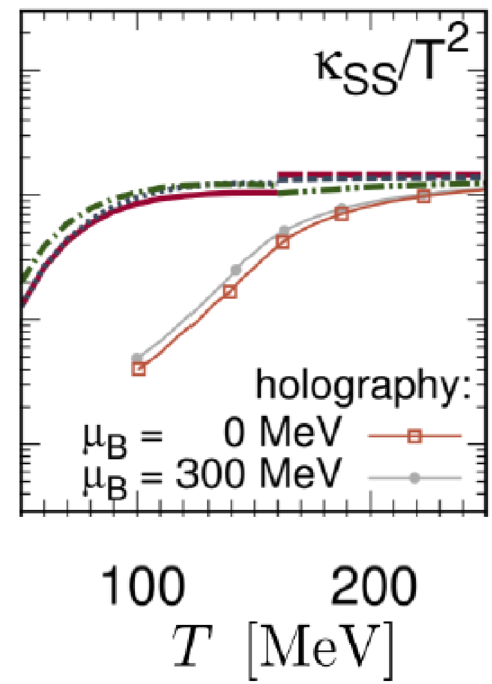
$$j_S^\mu = \kappa_{SB} \nabla^\mu \alpha_B + \kappa_{SQ} \nabla^\mu \alpha_Q + \kappa_{SS} \nabla^\mu \alpha_S$$







- Negative contribution
- Could also drastically reduce strange currents

- 1 Magnitude smaller than κ_{SS}
- Charged Kaons contribute to electric currents (see κ_{QQ})

- By far most important contribution

Conclusion

- First calculation of complete diffusion matrix of baryon, electric and strangeness charges in Navier-Stokes limit with first order Chapman-Enskog expansion
- Classical hadron gas with realistic isotropic cross sections and simple conformal QGP model were used

- HRG: dependence of coefficients on temperature and baryo-chemical potential
- Strong coupling of all gradients to (almost) all currents → large off-diagonal coefficients
- Suggestion: Off-diagonal terms should not be neglected!
- Can be used in (hydro) models

Outlook

- Calculation scheme can be used to calculate other Navier-Stokes coefficients
- Investigate effects in viscous hydro simulations → Observables?
- Compare to other models: SMASH? BAMPS? lQCD?