Critical fluctuations near the QCD critical point

Lijia Jiang

I. Introduction

II. critical fluctuations along the Freeze-out surface

III. Dynamical critical fluctuations--- Langevin dynamics

IV. Summary

Jiang, Li & Song, PRC, 94, 024918; Jiang, Wu, Song, in preparation

QCD phase transition & CP

Critical Point --- the landmark of the QCD phase diagram.

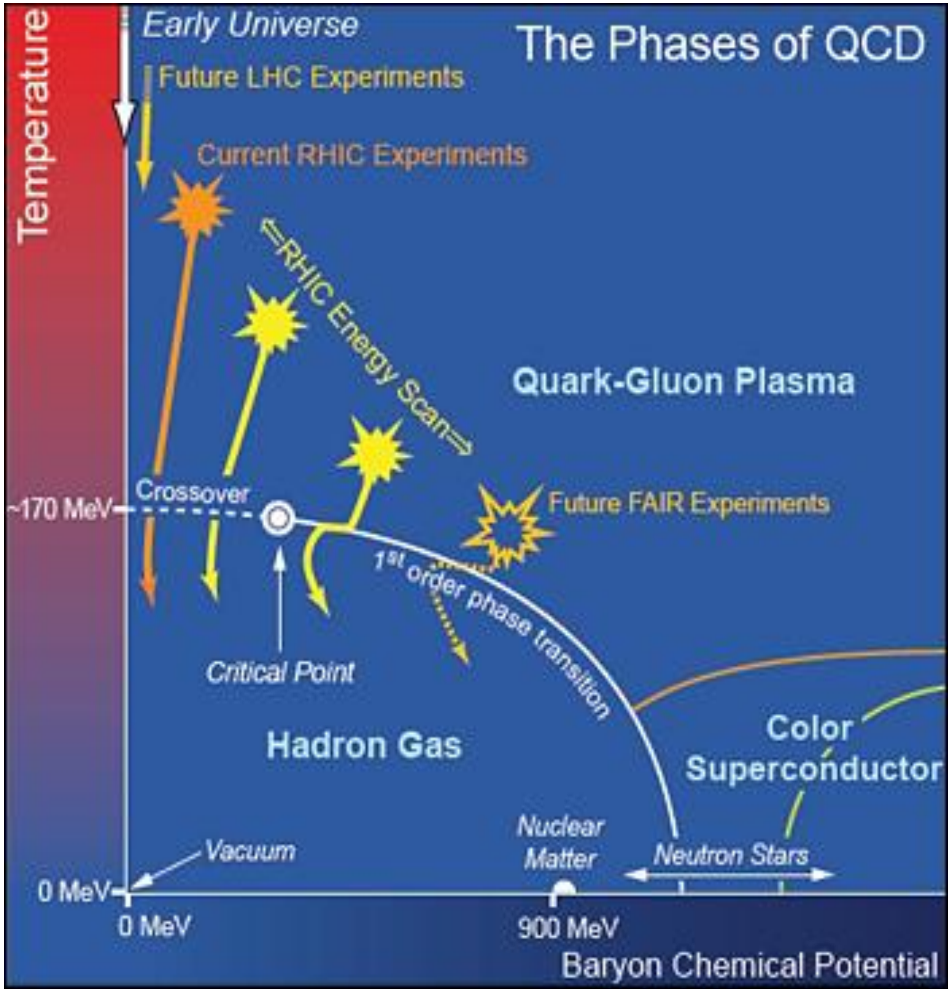
- **Lattice simulation :**

  - μ=0, finite T
  - crossover

- **Effective theories:**

  - (P)NJL, QM, FRG, DSE, RM)
  - finite T and μ
  - first order
  - **CP** is predicted.

➢ **The location of CP? The signals?**



Theoretical predictions

M. Stephanov, PRL 102, 032301(2009)

$$P[\sigma] \sim \exp\{-\Omega[\sigma]/T\}, \qquad \Omega = \int d^3x \left[\frac{1}{2}(\nabla\sigma)^2 + \frac{m_\sigma^2}{2}\sigma^2 + \frac{\lambda_3}{3}\sigma^3 + \frac{\lambda_4}{4}\sigma^4 + \cdots\right].$$

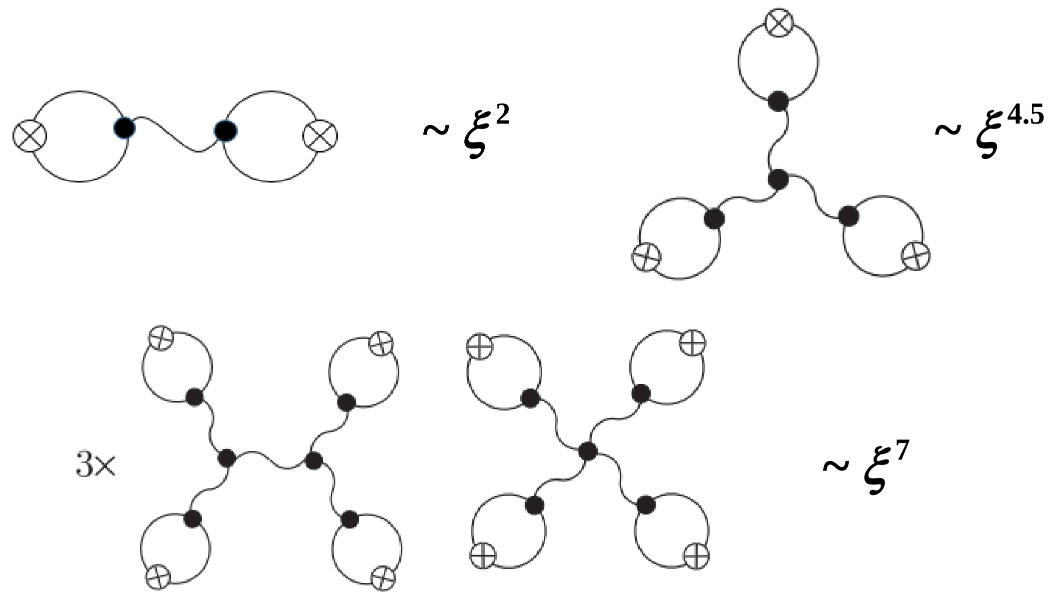
$$\langle\sigma_0^2\rangle = \frac{T}{V}\xi^2 \qquad \langle\sigma_0^3\rangle = \frac{2\lambda_3 T}{V}\xi^6; \qquad \langle\sigma_0^4\rangle_c = \frac{6T}{V}[2(\lambda_3\xi)^2 - \lambda_4]\xi^8.$$

Fluctuations of particles:

$$\left\langle(\delta N)^2\right\rangle \sim \xi^2$$

$$\left\langle(\delta N)^3\right\rangle \sim \xi^{4.5}$$

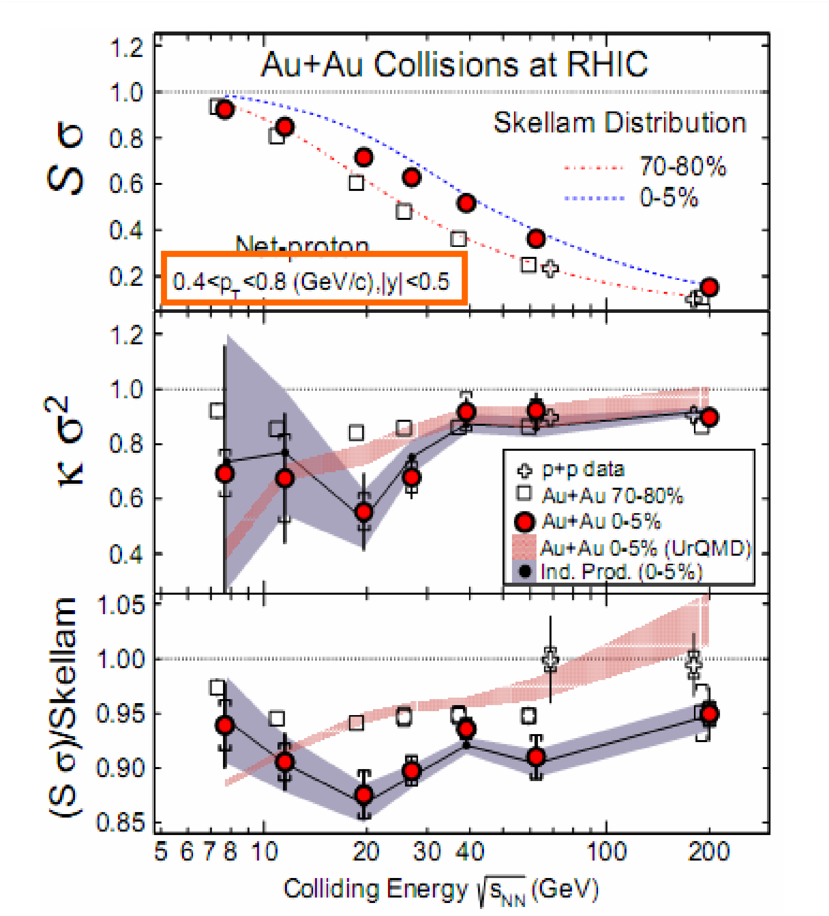
$$\left\langle(\delta N)^4\right\rangle \sim \xi^7$$



- **Static and infinite system, on the critical point :** $\xi \to \infty$
- **Fireball, finite size & finite evolution time:** $\xi \sim O\ (3\ fm)$

B. Berdnikov and K. Rajagopal, Phys. Rev. D 61, 105017 (2000).

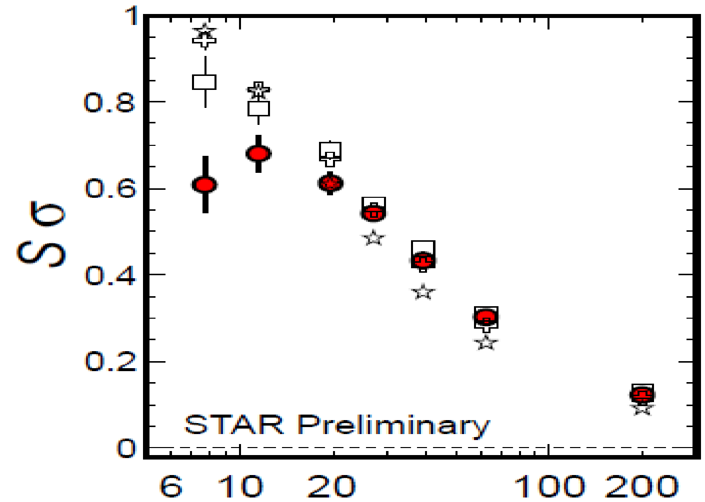
STAR BES: Cumulants ratios

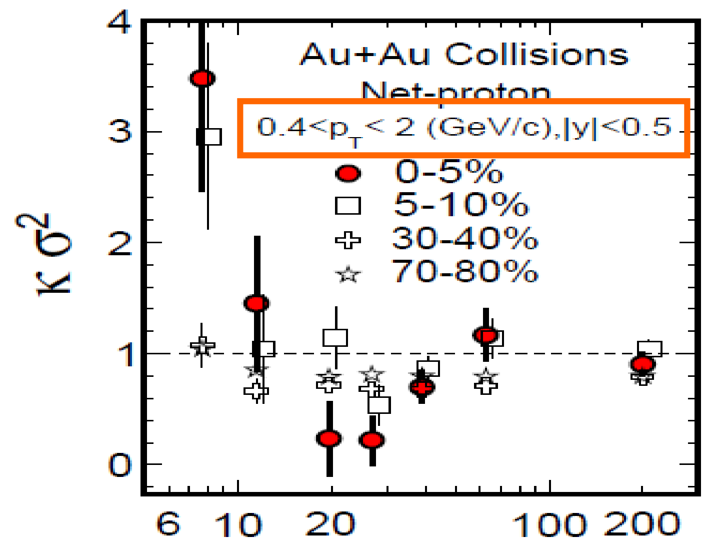


STAR Collaboration, PRL, 112, 032302 (2013)

$$S\,\sigma = \frac{C_3}{C_2} \sim \chi_B^{(3)}/\chi_B^{(2)}$$

$$\kappa\sigma^2 = \frac{C_4}{C_2} \sim \chi_B^{(4)}/\chi_B^{(2)}$$





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Static --> dynamical??

It is important to address the effects from **dynamical** evolutions!

Dynamical Model near the QCD critical point

Essential ingredients for dynamical models near critical point:

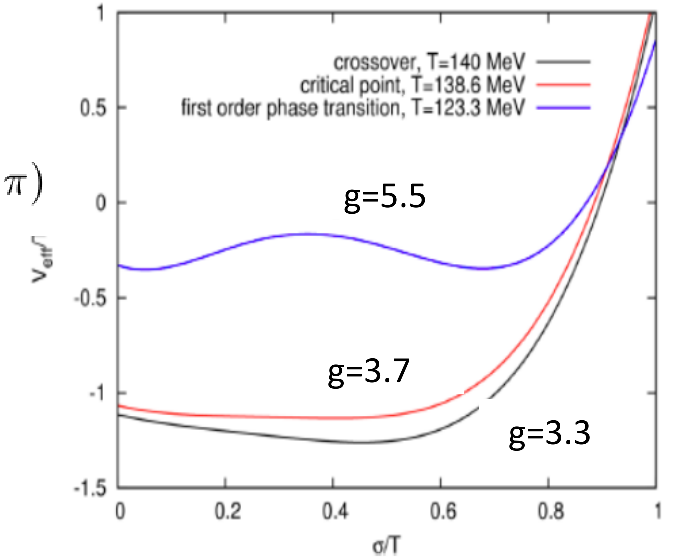
1. **evolution of bulk matter with external field**
2. **EOS with CP**
3. **A proper treatment of freezeout scheme**

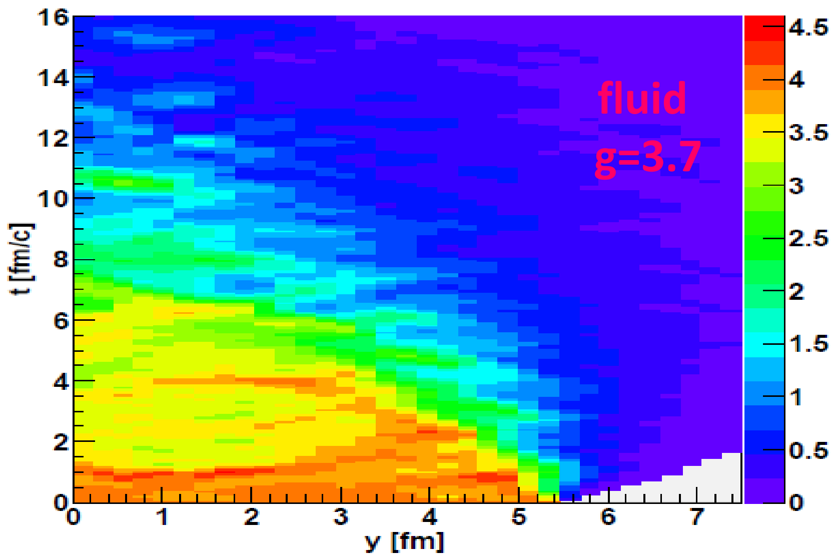
Chiral Hydrodynamics

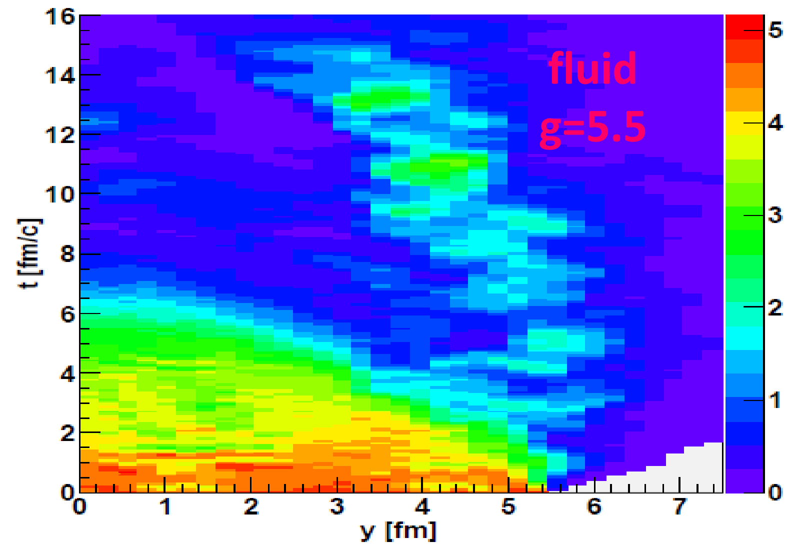
K. Paech, H. Stoecker and A. Dumitru, PRC 68, 044907 (2003)

$$\mathcal{L} = \bar{q}\left[i\gamma - m - g\left(\sigma + i\gamma_5\tau\pi\right)\right]q + \frac{1}{2}\left[\partial_\mu\sigma\partial^\mu\sigma + \partial_\mu\pi\partial^\mu\pi\right] - U\left(\sigma,\pi\right)$$

$$\begin{cases} \partial_\mu\partial^\mu\sigma + \frac{\delta U_{eff}}{\delta\sigma} + g\left\langle\bar{q}q\right\rangle = 0 & \text{(order parameter)} \\ \partial_\mu T^{\mu\nu}_{fluid} = S^\nu,\ S^\nu = -\left(\partial^2\sigma + \frac{\delta U_{eff}}{\delta\sigma}\right)\partial^\nu\sigma & \text{(heat bath)} \end{cases}$$





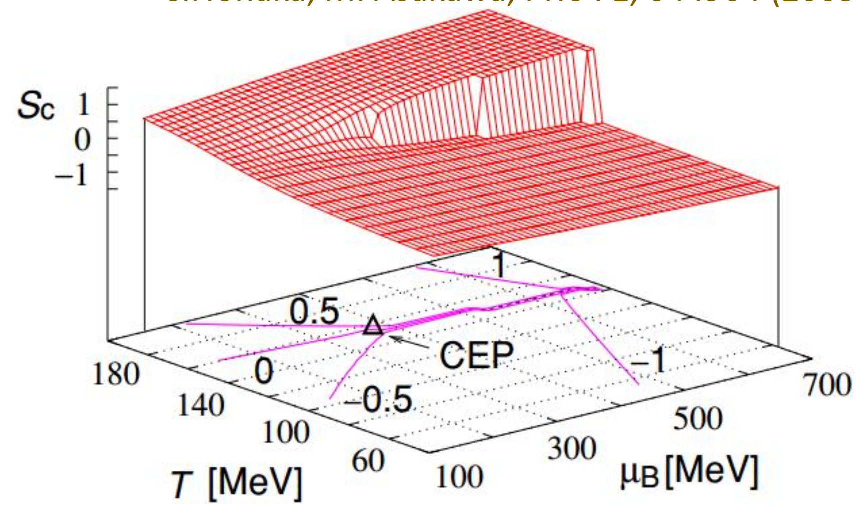


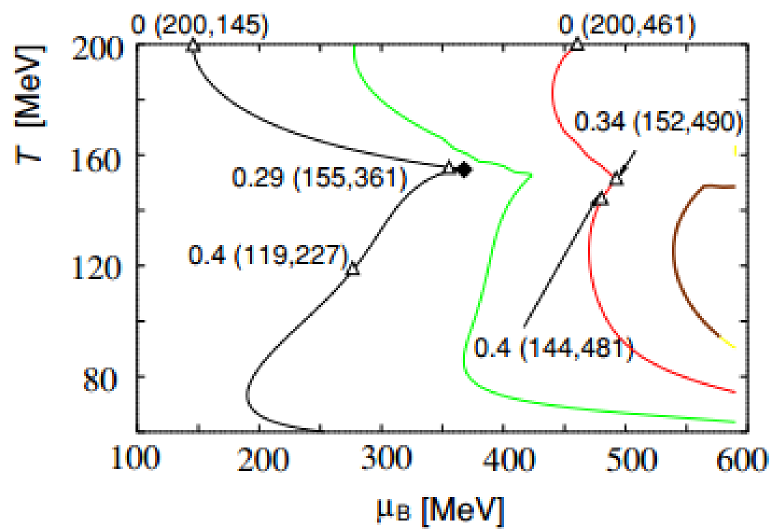
- **Chiral fluid dynamics with dissipation & noise** Nahrgang,et al., PRC 84, 024912 (2011)
- **Chiral fluid dynamics with a Polyakov loop (PNJL)** Herold, et al., PRC 87, 014907 (2013)

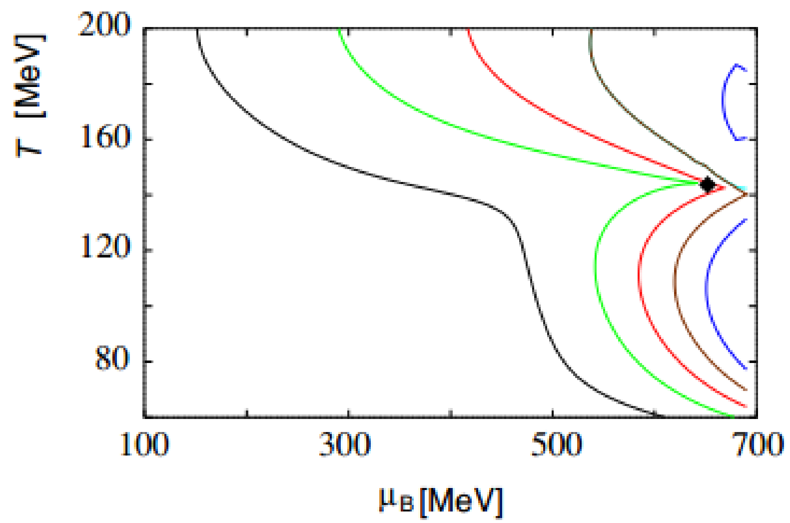
EOS with CP employed in hydrodynamics

C.Nonaka, M. Asakawa, PRC 71, 044904 (2005)

$$s(T,\mu_B) = \frac{1}{2}(1-\tanh[S_c(T,\mu_B)])s_H(T,\mu_B) + \frac{1}{2}(1+\tanh[S_c(T,\mu_B)])s_Q(T,\mu_B),$$

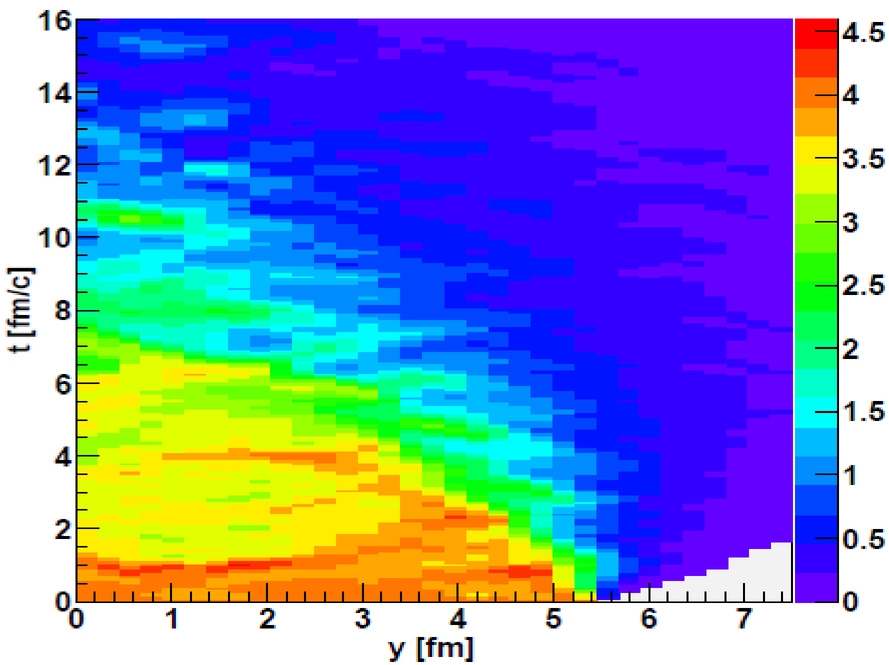


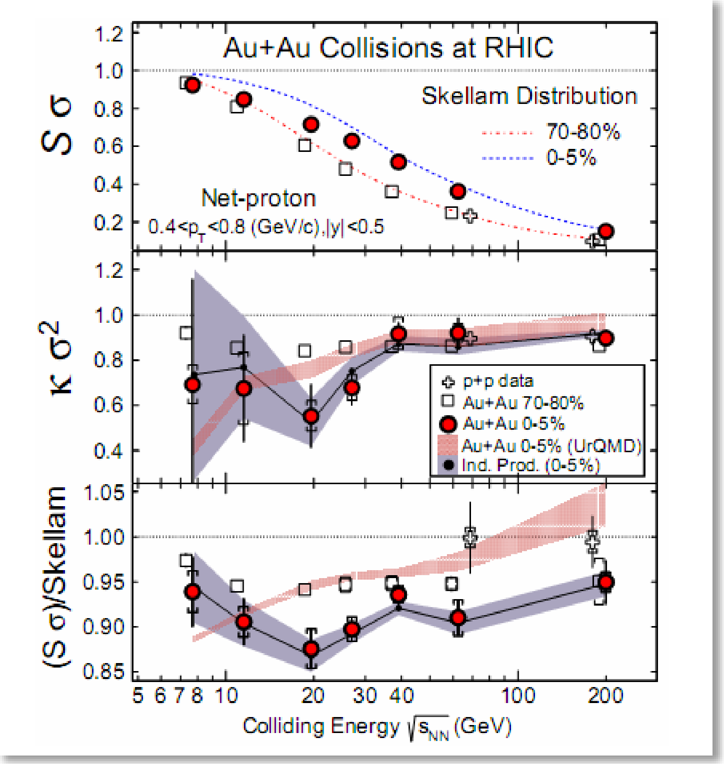




- **Pure hydrodynamics, needs to be extended to chiral hydrodynamics.**

dynamical models to experimental data





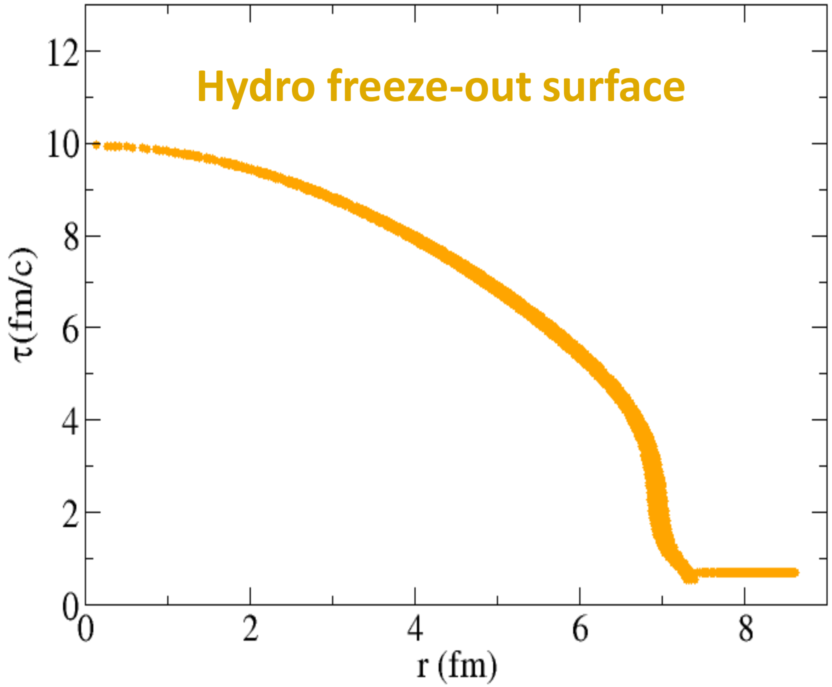
Essential ingredients for dynamical models:

1. Evolution of bulk matter with external field √
2. EOS with CP √
3. A proper treatment of freezeout scheme ?

Jiang, Li & Song, PRC, 94, 024918

Freeze-out scheme near the critical point

Particles emission near Tc with external field



Jiang, Li & Song , PRC, 94, 024918

Particle emissions in HIC, Cooper-Frye formula:

$$E\frac{dN}{d^3p} = \int_\Sigma \frac{p_\mu d\sigma^\mu}{2\pi^3} f(x,p)$$

Particle emissions with fluctuated external field:

$$M \rightarrow g\left(\bar{\sigma} + \sigma(x)\right)$$

$$\begin{aligned} f(x,p) &= f_0(x,p)\left[1 - g\sigma(x)/(\gamma T)\right] \\ &= f_0 + \delta f \end{aligned}$$

$$\begin{aligned} \langle \delta f_1 \delta f_2 \rangle_\sigma &= \frac{f_{01} f_{02}}{\gamma_1 \gamma_2} \frac{g^2}{T^2} \langle \sigma_1 \sigma_2 \rangle_c \,, \\ \langle \delta f_1 \delta f_2 \delta f_3 \rangle_\sigma &= -\frac{f_{01} f_{02} f_{03}}{\gamma_1 \gamma_2 \gamma_3} \frac{g^3}{T^3} \langle \sigma_1 \sigma_2 \sigma_3 \rangle_c \,, \\ \langle \delta f_1 \delta f_2 \delta f_3 \delta f_4 \rangle_\sigma &= \frac{f_{01} f_{02} f_{03} f_{04}}{\gamma_1 \gamma_2 \gamma_3 \gamma_4} \frac{g^4}{T^4} \langle \sigma_1 \sigma_2 \sigma_3 \sigma_4 \rangle_c \,. \end{aligned}$$

infinite volume

$$\langle \delta f_1 \delta f_2 \rangle_\sigma = \frac{f_{01} f_{02}}{\gamma_1 \gamma_2} \frac{g^2}{T^2} \langle \sigma_1 \sigma_2 \rangle_c ,$$

$$\langle \delta f_1 \delta f_2 \delta f_3 \rangle_\sigma = -\frac{f_{01} f_{02} f_{03}}{\gamma_1 \gamma_2 \gamma_3} \frac{g^3}{T^3} \langle \sigma_1 \sigma_2 \sigma_3 \rangle_c ,$$

$$\langle \delta f_1 \delta f_2 \delta f_3 \delta f_4 \rangle_\sigma = \frac{f_{01} f_{02} f_{03} f_{04}}{\gamma_1 \gamma_2 \gamma_3 \gamma_4} \frac{g^4}{T^4} \langle \sigma_1 \sigma_2 \sigma_3 \sigma_4 \rangle_c .$$

$$\Downarrow \quad \frac{\left(\int d^3x\right)^i \left\langle (\delta f)^i \right\rangle}{\left(\int d^3x\right)^i 1}$$

[M. Stephanov, PRD (1999) & PRL (2009).]

$$\langle \delta n_p \delta n_q \rangle_\sigma = \frac{G^2}{VT} \frac{n_p n_q}{\omega_p \omega_q} \frac{1}{m^2}$$

$$\langle \delta n_p \delta n_q \delta n_k \rangle_\sigma = \frac{2\lambda_3}{V^2 T} \frac{n_p n_q n_k}{\omega_p \omega_q \omega_k} \left(\frac{G}{m^2} \right)^3$$

$$\langle \delta n_{p_1} \delta n_{p_2} \delta n_{p_3} \delta n_{p_4} \rangle_\sigma = \frac{6}{V^3 T} \frac{n_{p_1} n_{p_2} n_{p_3} n_{p_4}}{\omega_{p_1} \omega_{p_2} \omega_{p_3} \omega_{p_4}} \left(\frac{G}{m^2} \right)^4 \left[2 \left(\frac{\lambda_3}{m} \right)^2 - \lambda_4 \right]$$

- **For stationary & infinite medium, integrate over coordinate space, the results in Stephanov PRL09 are reproduced.**

Freeze-out scheme near the CP

Jiang, Li & Song , PRC, 94, 024918

$$\left\langle (\delta N)^2 \right\rangle_c = \left(\frac{g_i}{(2\pi)^3}\right)^2 \left(\prod_{i=1,2}\left(\frac{1}{E_i}\int d^3p_i \int_{\Sigma_i} p_{i\mu} d\sigma_i^\mu d\eta_i\right)\right) \frac{f_{01}f_{02}}{\gamma_1\gamma_2}\frac{g^2}{T^2}\langle\sigma_1\sigma_2\rangle_c\,,$$

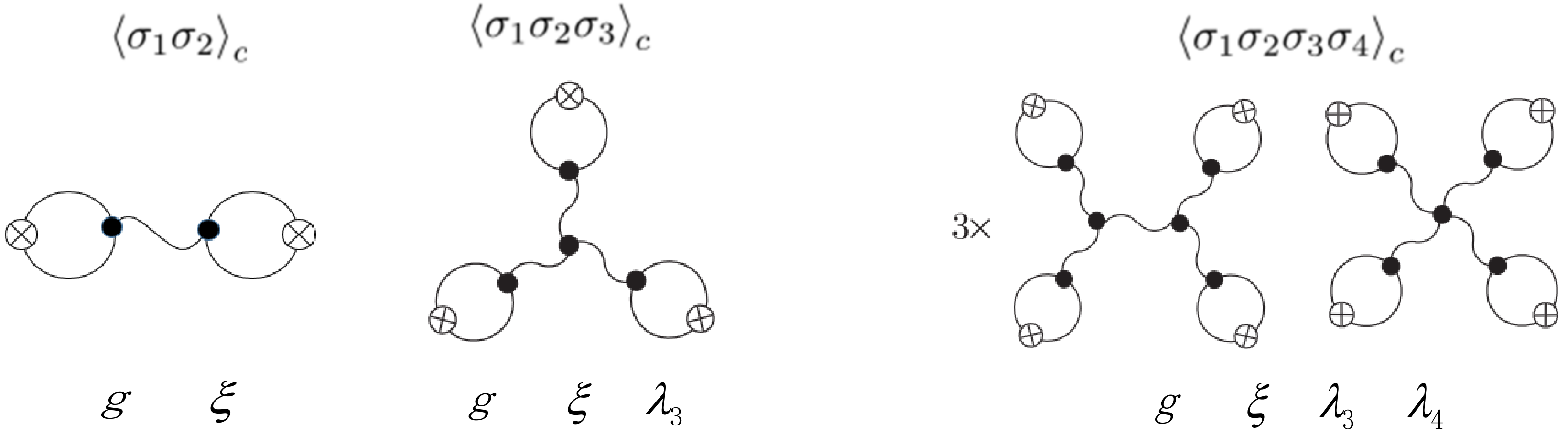
$$\left\langle (\delta N)^3 \right\rangle_c = \left(\frac{g_i}{(2\pi)^3}\right)^3 \left(\prod_{i=1,2,3}\left(\frac{1}{E_i}\int d^3p_i \int_{\Sigma_i} p_{i\mu} d\sigma_i^\mu d\eta_i\right)\right) \frac{f_{01}f_{02}f_{03}}{\gamma_1\gamma_2\gamma_3}\left(-\frac{g^3}{T^3}\langle\sigma_1\sigma_2\sigma_3\rangle_c\right),$$

$$\left\langle (\delta N)^4 \right\rangle_c = \left(\frac{g_i}{(2\pi)^3}\right)^4 \left(\prod_{i=1,2,3,4}\left(\frac{1}{E_i}\int d^3p_i \int_{\Sigma_i} p_{i\mu} d\sigma_i^\mu d\eta_i\right)\right) \frac{f_{01}f_{02}f_{03}f_{04}}{\gamma_1\gamma_2\gamma_3\gamma_4}\frac{g^4}{T^4}\langle\sigma_1\sigma_2\sigma_3\sigma_4\rangle_c$$

$$\begin{aligned}
\langle\sigma_1\sigma_2\rangle_c &= TD(x_1-x_2)\,,\\
\langle\sigma_1\sigma_2\sigma_3\rangle_c &= -2T^2\lambda_3\int d^3z D(x_1-z)D(x_2-z)D(x_3-z)\,,\\
\langle\sigma_1\sigma_2\sigma_3\sigma_4\rangle_c &= -6T^3\lambda_4\int d^3z D(x_1-z)D(x_2-z)D(x_3-z)D(x_4-z)\\
&\quad +12T^3\lambda_3^2\int d^3u\int d^3v D(x_1-u)D(x_2-u)D(x_3-v)D(x_4-v)D(u-v)\,.
\end{aligned}$$

- **no evolution effects.**
- **The evolution of bulk matter is not affected.**

The input parameters



- $g \sim (0, 10)$

  phenomenological model
  in vacuum: $m_p \sim$ 900 MeV -> g $\sim$ 10; large T: non-interacting, g $\sim$ 0

- $\xi \sim (0.5, 4)\text{fm}$

  volume effects, critical slowing down
  ξ increases when the CEP is approaching. (maximum ξ at 27 GeV)

- $\lambda_3 \sim (0, 8)$, $\lambda_4 \sim (4, 20)$

  lattice simulation of the effective potential around critical point.
  increase from the crossover side to the 1st order phase transition side

A. Andronic, et al. NPA (2006); M. A. Stephanov, Phys. Rev. Lett. 102, 032301 (2009); S. P. Klevansky, Rev. Mod. Phys, Vol, 64, No.3 (1992); W. Fu, Y-x, Liu, Phys. Rev. D 79, 074011(2009); M. M. Tsypin, Phys. Rev. Lett. 73, 2015 (1994); M. M. Tsypin, Phys. Rev. B 55, 8911 (1997).; B. Berdnikov and K. Rajagopal, Phys. Rev. D 61, 105017 (2000).

The input parameters

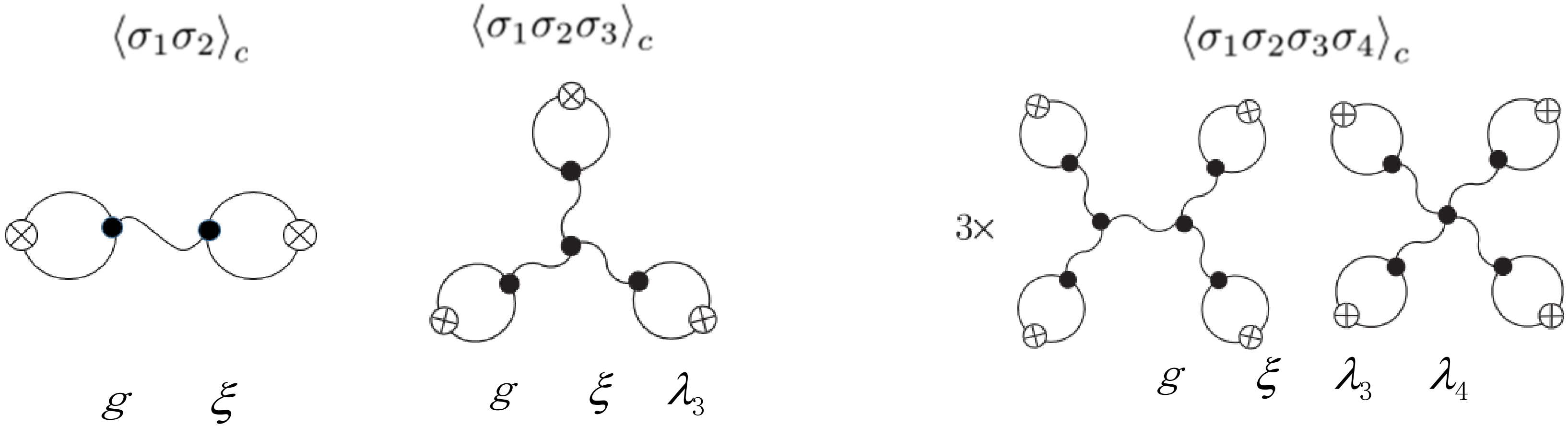


TABLE I. Parameter sets for the critical fluctuations.

$\sqrt{s_{NN}}$ (GeV)		7.7	11.5	19.6	27	39	62.4	200
Para-I	g	3.2	2.5	2.3	2.2	2	1.8	1
	$\tilde{\lambda}_3$	6	4	3	2	0	0	0
	$\tilde{\lambda}_4$	14	13	12	11	10	9	8
	ξ	1	2	3	3	2	1	0.5
Para-II	g	3.2	2.5	2.3	2.2	2	1.8	1
	$\tilde{\lambda}_3$	6	4	3	2	2	1.5	1
	$\tilde{\lambda}_4$	14	13	12	11	10	9	8
	ξ	1.1	2.5	4	4	3	2	1
Para-III	g	2.8	1.8	1.7	1.6	1	0.5	0.1
	$\tilde{\lambda}_3$	6	4	3	2	2	1.5	1
	$\tilde{\lambda}_4$	14	13	12	11	10	9	8
	ξ	1	2	3	3	2	1	0.5

- ~ 0
- /)
- n side

A. Andronic, et al. NI ... ansky, Rev. Mod. Phys, Vol, 64, No.3 (1992); ... 73, 2015 (1994); M. M. Tsypin, Phys. Rev. B 5 ...)0).

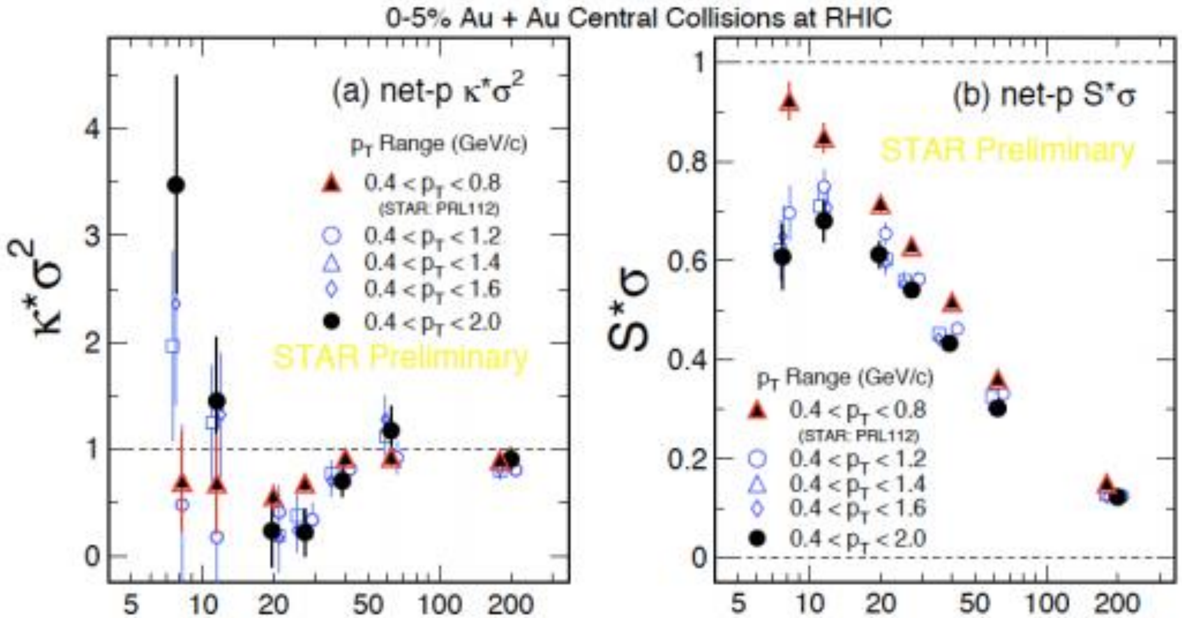
critical fluctuations along the freeze-out surface

---- comparison with the experimental data

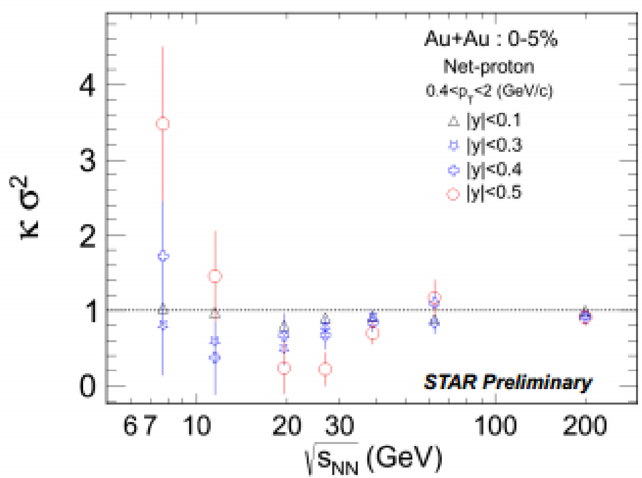
STAR BES: Acceptance dependence

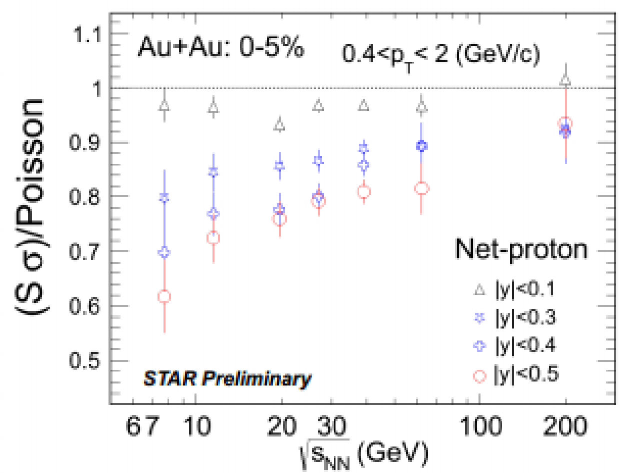
Xiaofeng Luo(for the STAR Collaboration), PoS(CPOD2014)019

Pt dependence:



y dependence:





- **The signals are significantly enhanced when the pt and y acceptance are increased.**

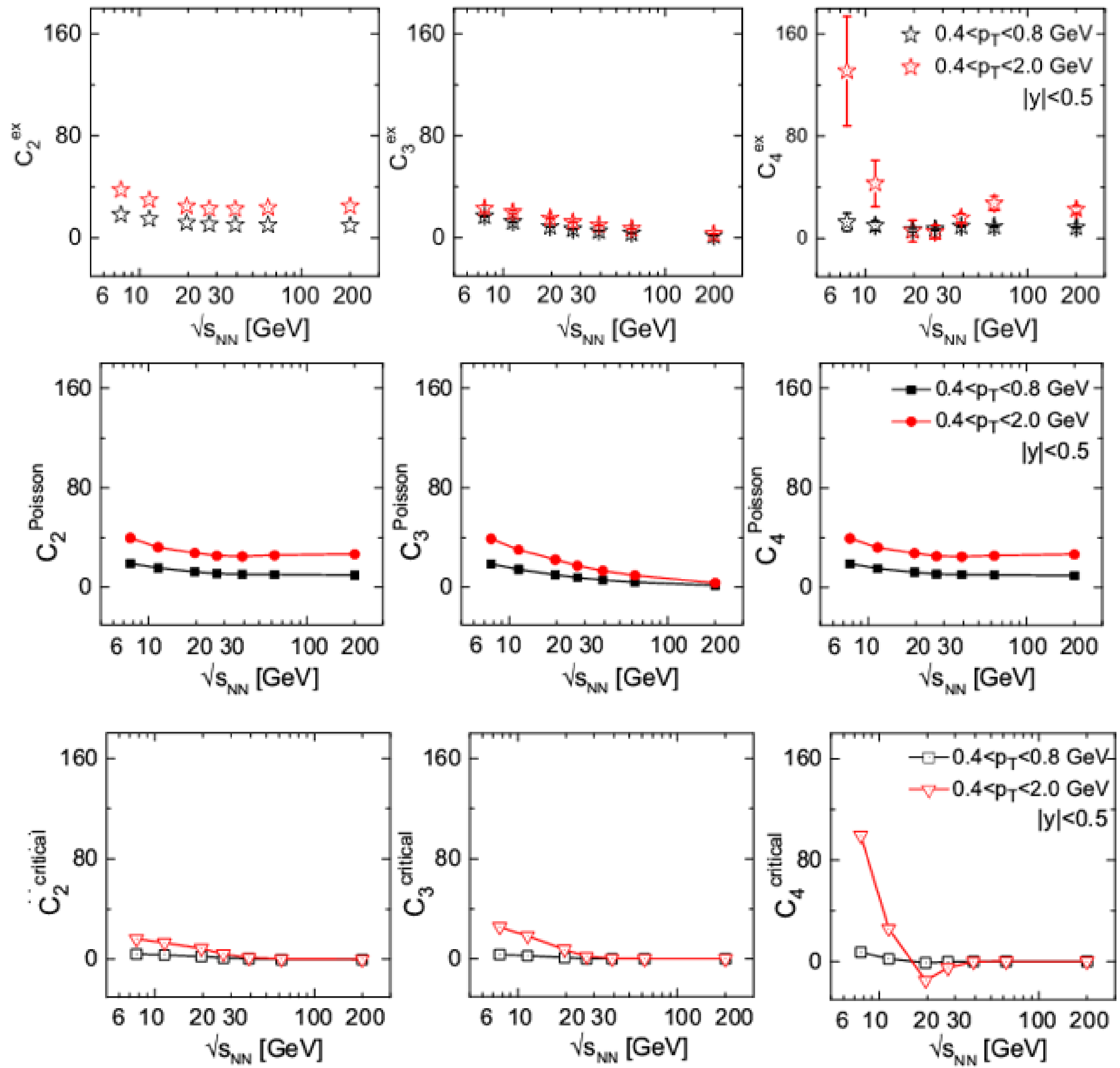
STAR data -- statistical baselines -- critical fluctuations

Note from Jiang

Experimental data:

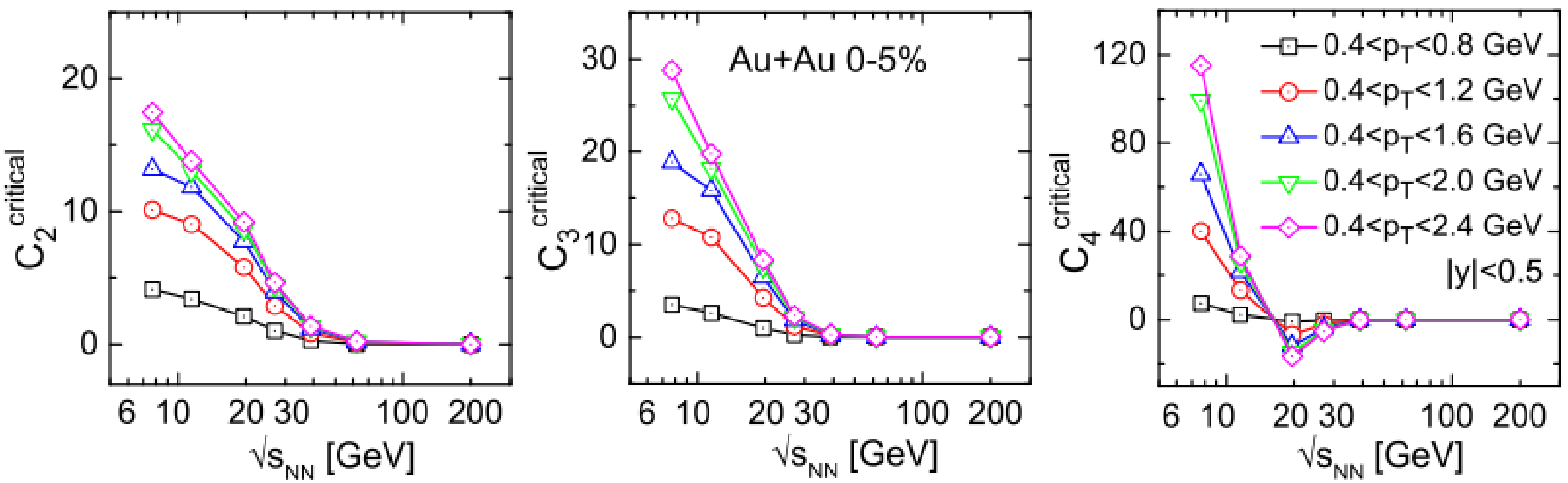
Poisson expectations:

Critical fluctuations:



Pt acceptance dependent critical fluctuations

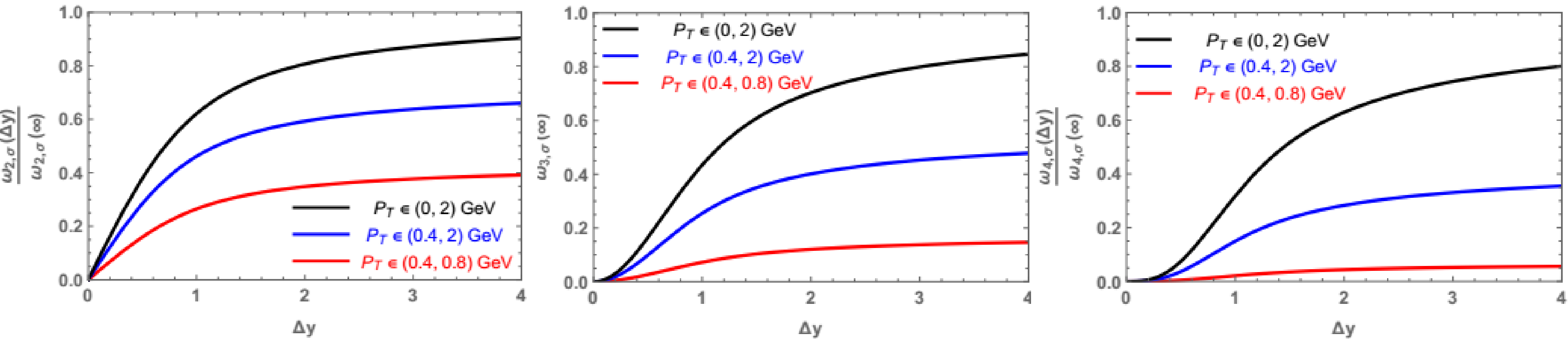
Jiang, Li & Song , PRC, 94, 024918



- **The critical fluctuations are largely enhanced as the increasing of pt acceptance at small collision energies, even xi is very small.**
- **The critical fluctuations saturate at larger pt acceptance.**
- **The critical fluctuations are determined by both N_p and xi.**

y acceptance dependence

Ling & Stephanov PRC2016



Simplified correlators for sigma field:

$$\langle\sigma(\boldsymbol{x})\sigma(\boldsymbol{y})\rangle \to T\xi^2\delta^3(\boldsymbol{x}-\boldsymbol{y})$$

$$\langle\sigma(\boldsymbol{x})\sigma(\boldsymbol{y})\sigma(\boldsymbol{z})\rangle \to -2\tilde{\lambda}_3 T^{3/2}\xi^{9/2}\delta^6(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})$$

$$\langle\sigma(\boldsymbol{x})\sigma(\boldsymbol{y})\sigma(\boldsymbol{z})\sigma(\boldsymbol{w})\rangle_c \to 6(2\tilde{\lambda}_3^2-\tilde{\lambda}_4)T^2\xi^7\delta^9(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\boldsymbol{w})$$

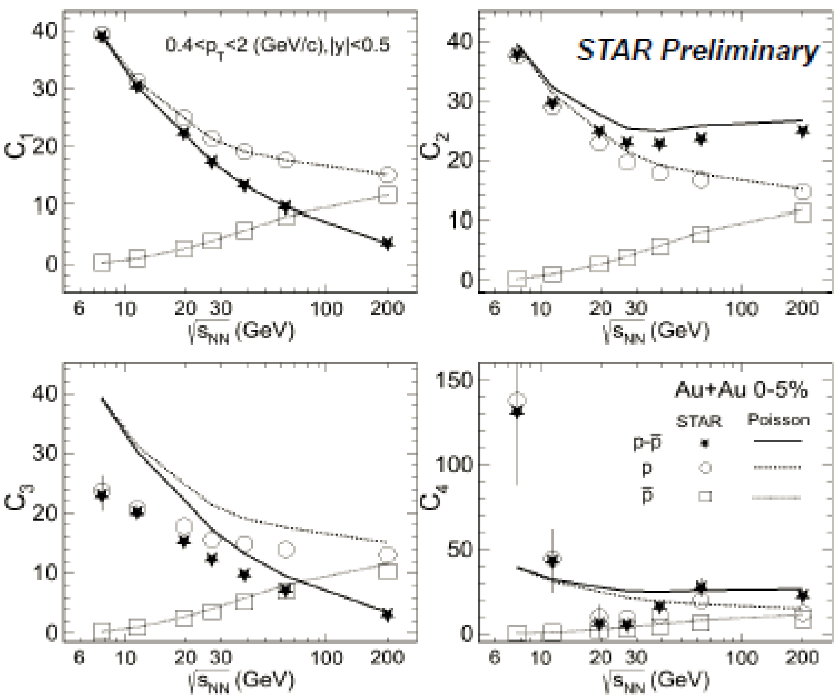
Freeze-out surface: **Blast wave model**.

- **larger acceptance leads to significantly larger critical point signal**
- **saturation at large acceptance.**

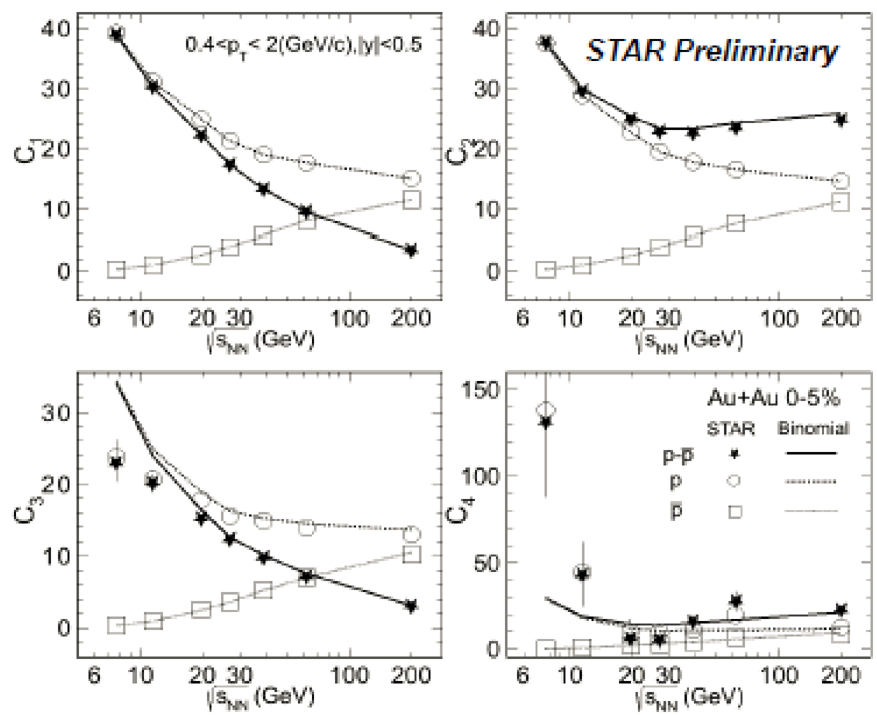
STAR data VS statistical baselines

Xiaofeng Luo(for the STAR Collaboration), PoS(CPOD2014)019

Cumulants vs. Poisson



Cumulants vs. Binomial

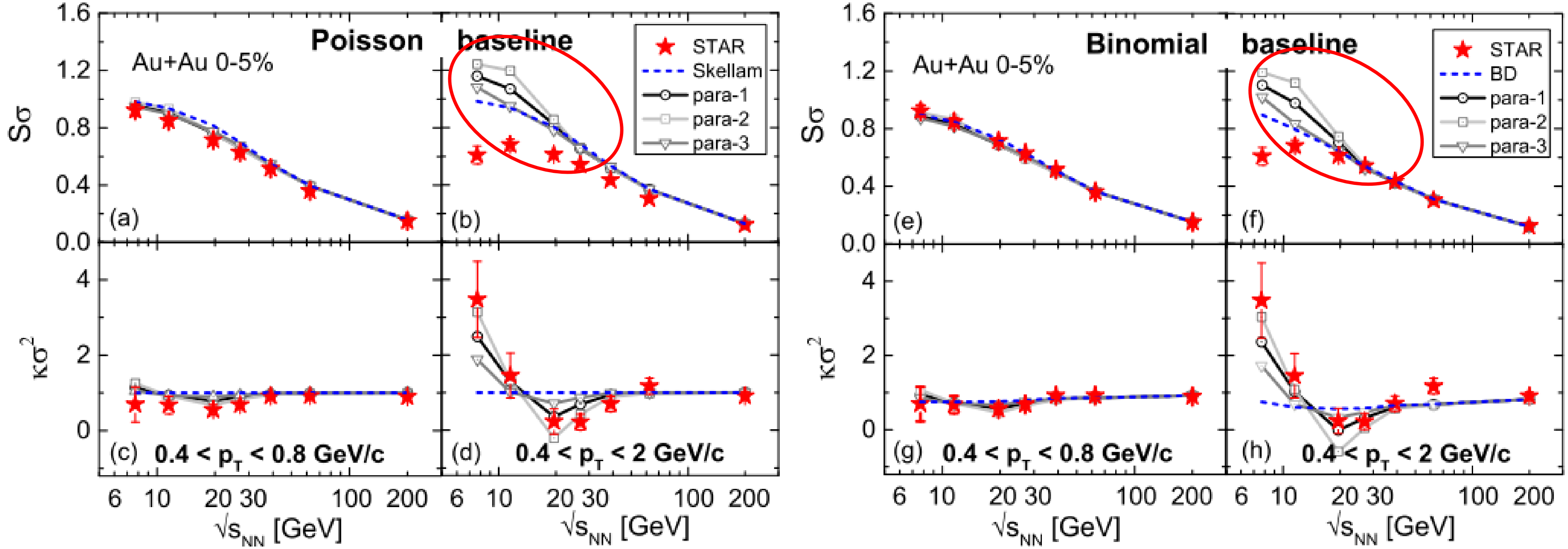


- **Fluctuations measured in Experiment: critical fluc. + statistical fluc. + …**

Cumulants ratios

Net Protons 0-5%

Jiang, Li & Song , PRC, 94, 024918

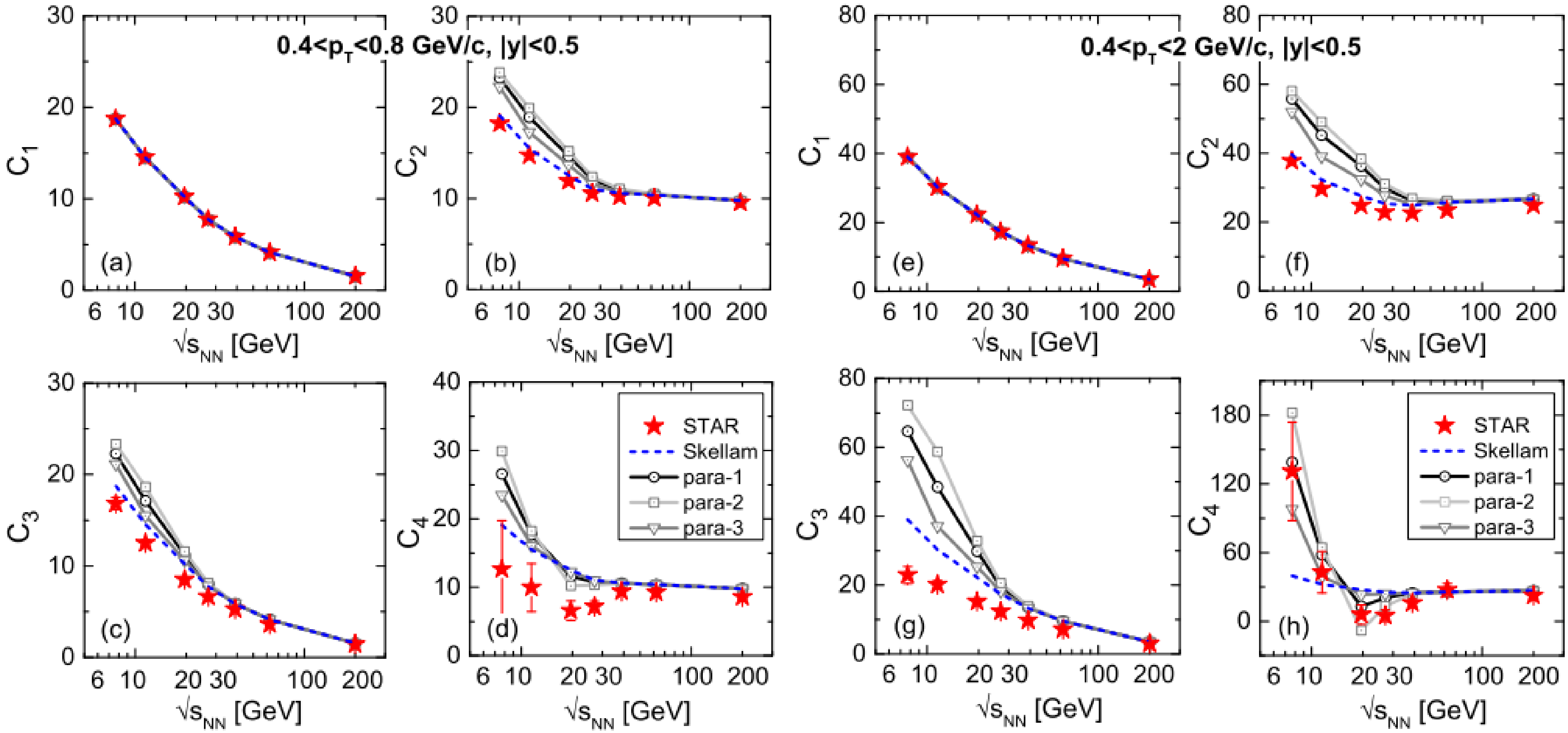


- **The cumulants ratios are better described within both pt ranges with different statistical baselines, except $S\sigma$ at low collision energies.**

Cumulants (Model + Poisson baselines)

Net Protons 0-5%

Jiang, Li & Song , PRC, 94, 024918

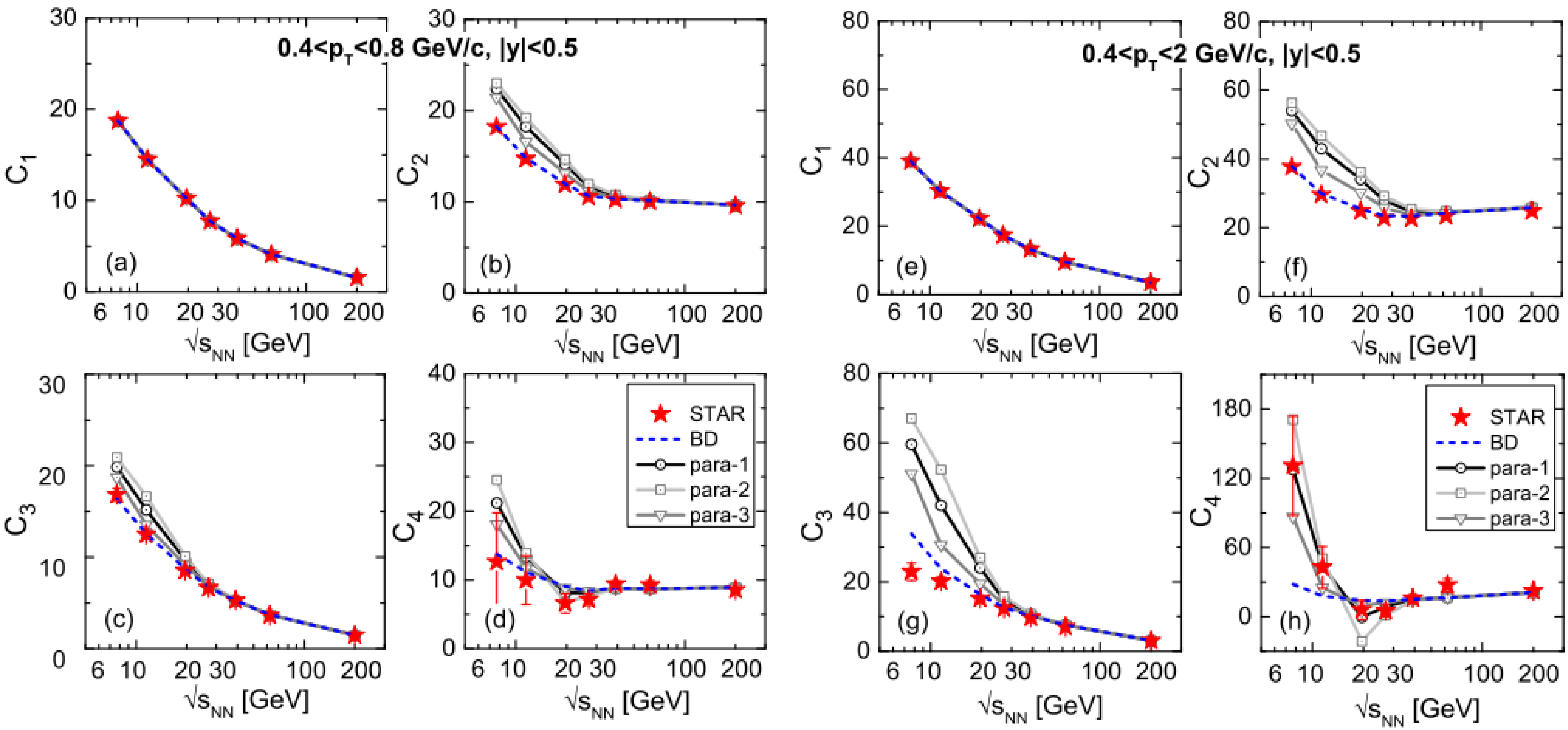


- **C4 can be roughly described**
- **critical fluctuations of C2 and C3 are positive, above the statistical baselines**

Cumulants (Model + Binomial baseline)

Net Protons 0-5%

Jiang, Li & Song , PRC, 94, 024918

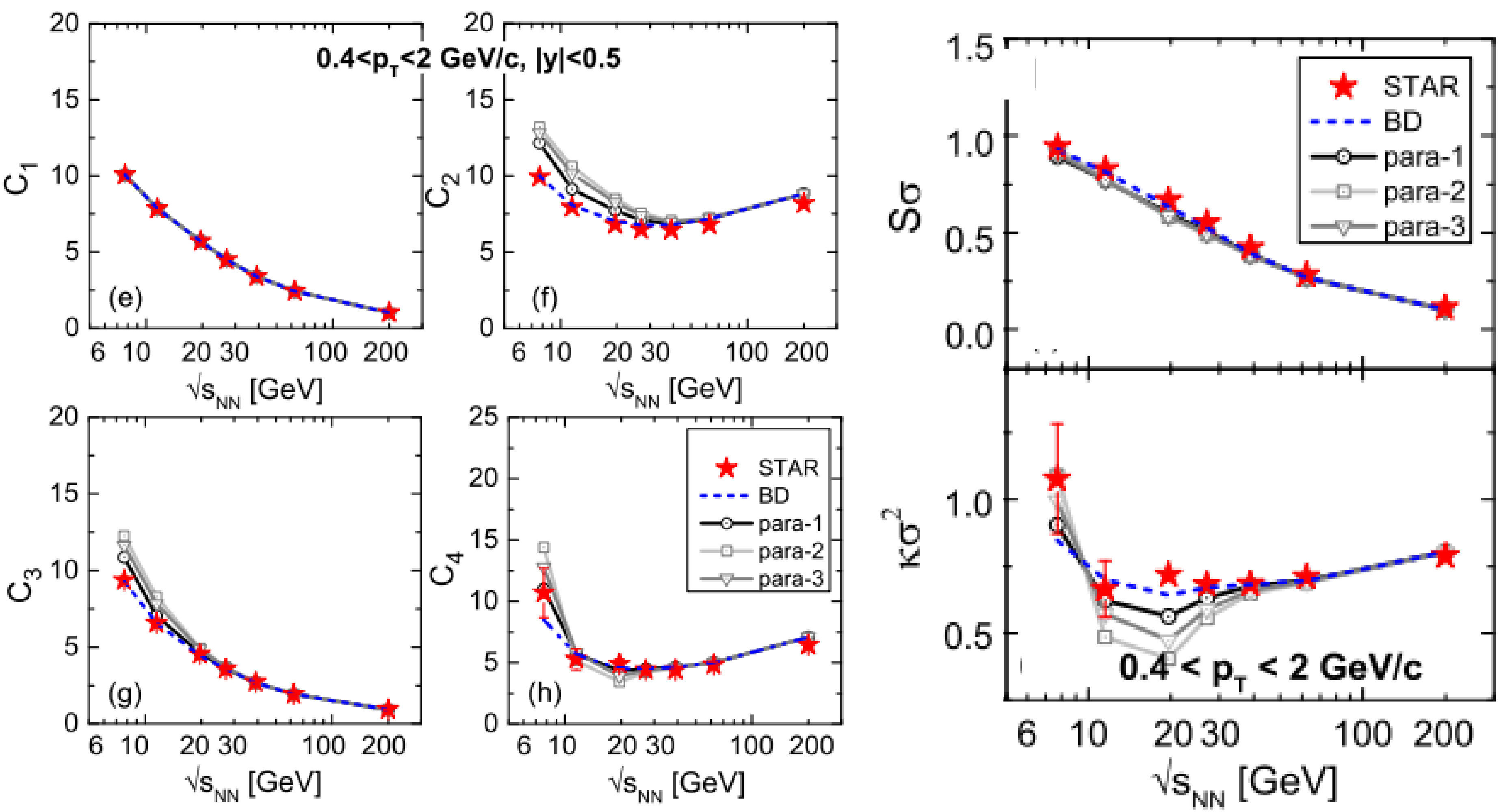


- **For static fluctuations, C4 can be described**
- **critical fluctuations of C2 and C3 are positive, above the statistical baselines, cannot explain the experimental data.**

Results -- non-central collisions

Net Protons 30-40%

Jiang, Li & Song , PRC, 94, 024918



- With the same set of parameters, C4 and $\kappa\sigma^2$ at non-central collisions can be described, but C2 and C3 above the baselines, cannot describe the experimental data.

Summary-critical fluctuations on freezeout surface

- larger acceptance leads to significantly larger critical fluctuations, which are qualitatively in accord with the experimental measurements

- C4 and $\kappa\sigma^2$ can be reproduced through tuning the parameters of the model, at both central and non-central collisions

- C2 , C3 are well above the statistical baselines, which can NOT explain/describe the experimental data

Summary-critical fluctuations on freezeout surface

- larger acceptance leads to significantly larger critical fluctuations, which are qualitatively in accord with the experimental measurements

- C4 and $\kappa\sigma^2$ can be reproduced through tuning the parameters of the model, at both central and non-central collisions

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What can we obtain from dynamical evolution ?

Real time evolution of Sigma's cumulants

Mukherjee, Venugopalan & Yin PRC 2015

The relaxation of critical mode are described by Fokker-Plank equation

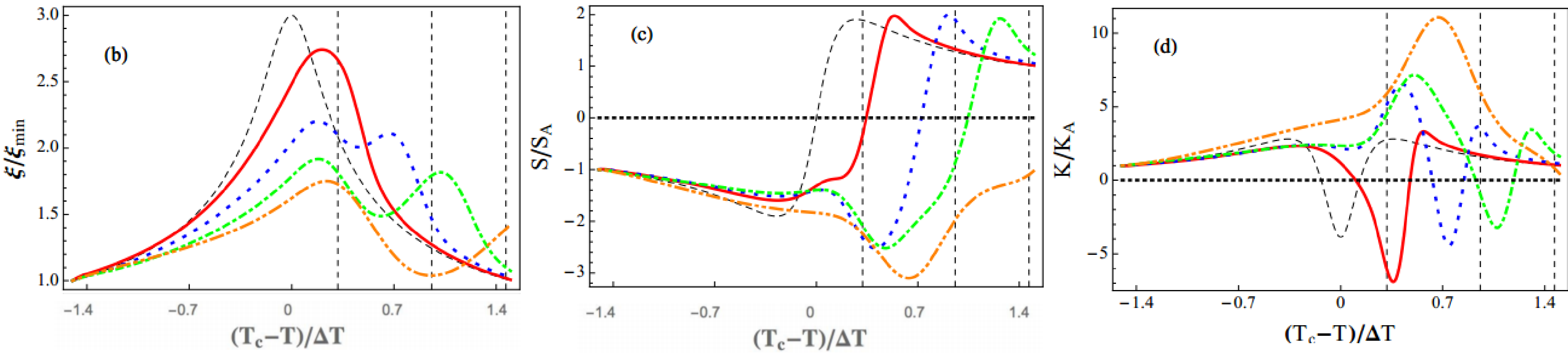
$$\partial_\tau P(\sigma;\tau) = \frac{1}{(m_\sigma^2 \tau_{\rm eff})} \left\{ \partial_\sigma \left[\partial_\sigma \Omega_0(\sigma) + V_4^{-1} \partial_\sigma \right] P(\sigma;\tau) \right\},$$

The higher order cumulants in ϵ expansion can be written as

$$\partial_\tau \kappa_2(\tau) = -2\,\tau_{\rm eff}^{-1}\,(b^2)\left[\left(\frac{\kappa_2}{b^2}\right)F_2(M) - 1\right]\left[1+\mathcal{O}(\epsilon^2)\right],$$

$$\partial_\tau \kappa_3(\tau) = -3\,\tau_{\rm eff}^{-1}\,(\epsilon\, b^3)\left[\left(\frac{\kappa_3}{\epsilon\, b^3}\right)F_2(M) + \left(\frac{\kappa_2}{b^2}\right)^2 F_3(M)\right] \times \left[1+\mathcal{O}(\epsilon^2)\right],$$

$$\partial_\tau \kappa_4(\tau) = -4\,\tau_{\rm eff}^{-1}\,(\epsilon^2\, b^4)\left\{\left(\frac{\kappa_4}{\epsilon^2\, b^4}\right)F_2(M) + 3\left(\frac{\kappa_2}{b^2}\right)\left(\frac{\kappa_3}{\epsilon\, b^3}\right)F_3(M) + \left(\frac{\kappa_2}{b^2}\right)^3 F_4\right\} \times \left[1+\mathcal{O}(\epsilon^2)\right].$$



- **Zero mode only, could not combined with the freeze-out scheme.**

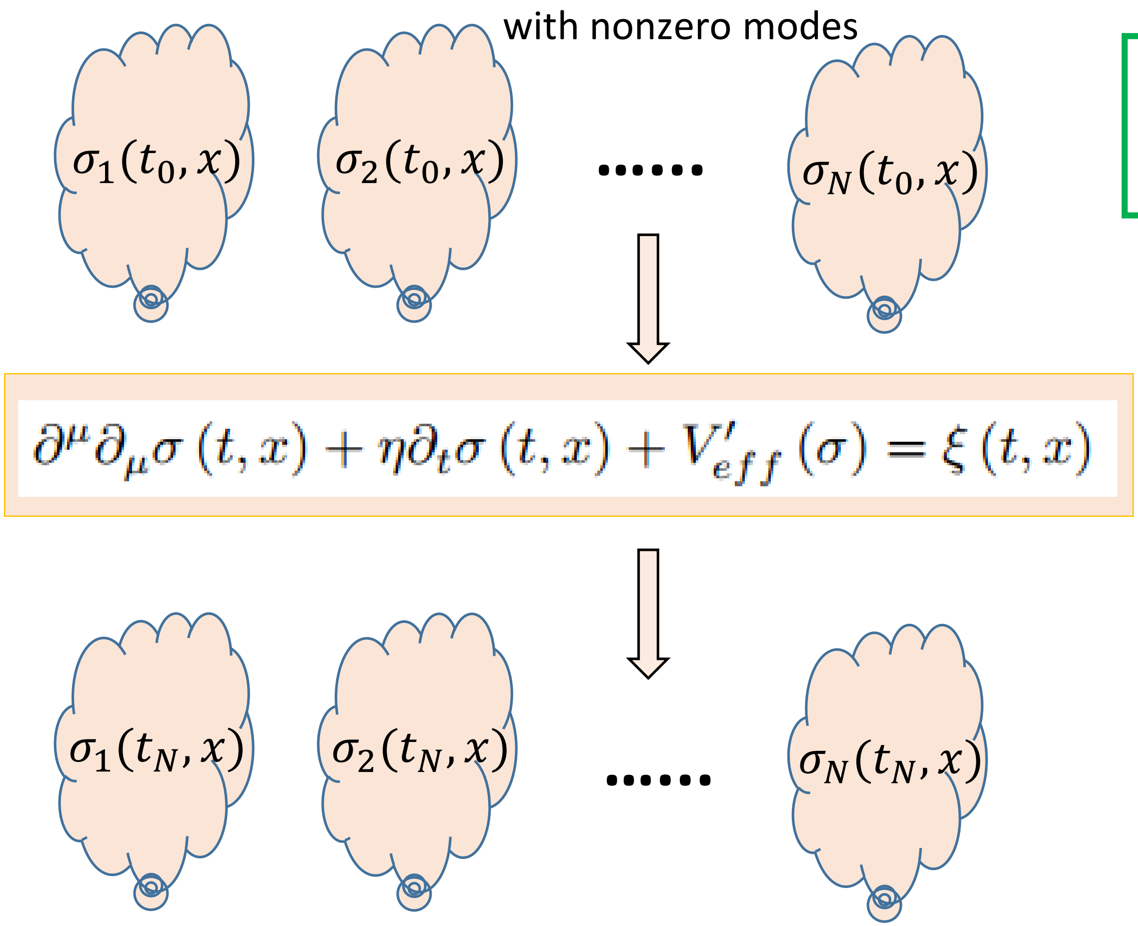
Jiang, Wu, **Song, in preparation**

Dynamical evolution of Sigma’s cumulants

-- Langevin dynamics

e-b-e Langevin dynamics

Jiang, Wu, Song, in preparation



e-b-e initial conditions
(constructed based on partition function)

↓

Langevin equation
(with damping and noise)

↓

Evolving cumulants
(record cumulants at each time)

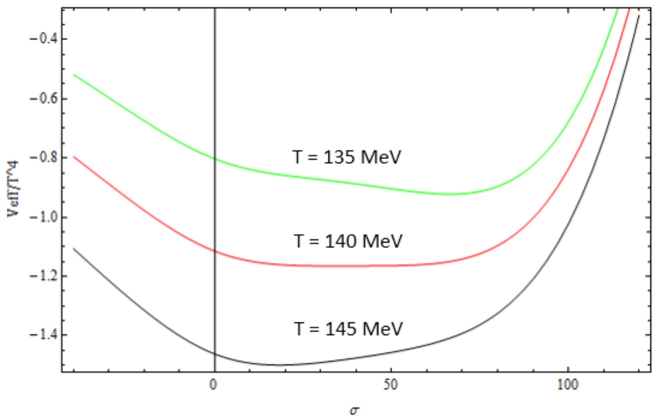
Cumulants and cumulants ratios:

$$S\sigma = \frac{C_3}{C_2}, \quad \kappa\sigma^2 = \frac{C_4}{C_2}$$

$$\begin{aligned}
C_1 &= M_1, \\
C_2 &= M_2 - M_1^2, \\
C_3 &= M_3 - 3M_2M_1 + 2M_1^3, \\
C_4 &= M_4 - 4M_3M_1 - 3M_2^2 + 12M_2M_1^2 - 6M_1^4.
\end{aligned}$$

Sigma’s cumulants from Langevin dynamics

- **10^5 events**
- **uniform and finite volume**
- **effective potential at different T**

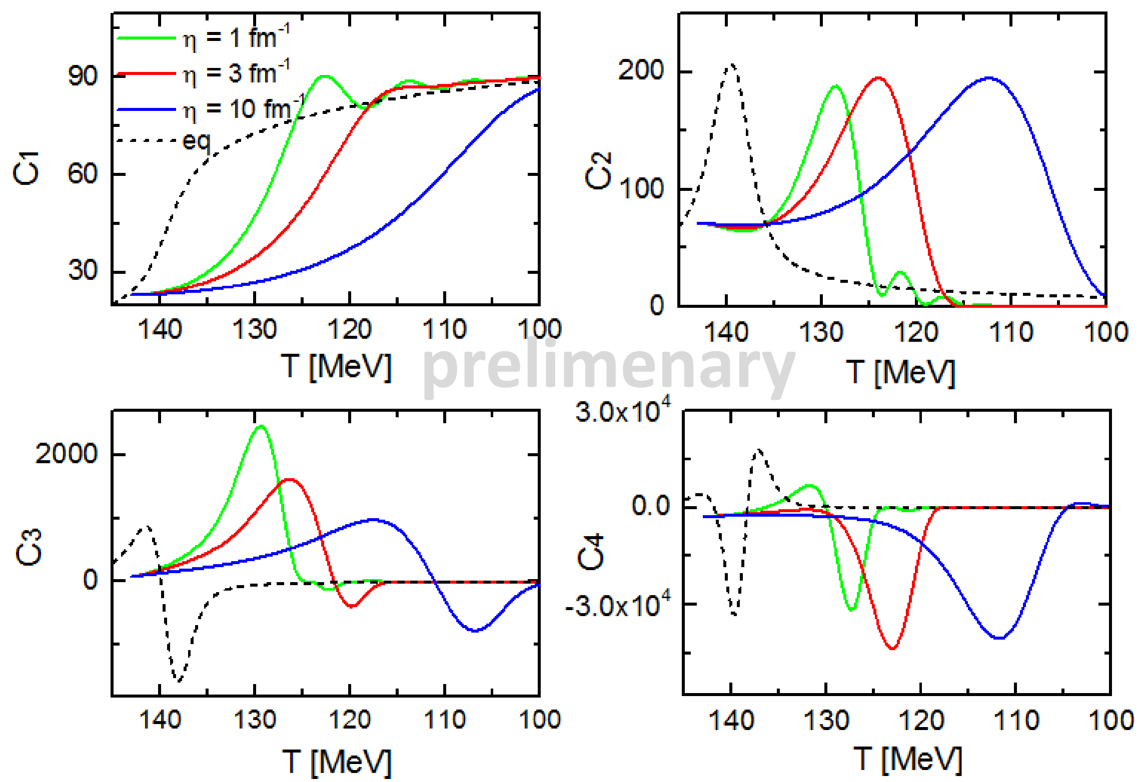


- **decreasing Temperature**

$$\frac{T(t)}{T_0} = \left(\frac{t}{t_0}\right)^{-0.45}$$

(Hubble like)

Jiang, Wu, **Song**, **in preparation**



- **The correlation of sigma field automatically increase.**
- **Memory effects**
- **The sign and value of C3, C4 different from the equilibrium ones**

Summary and Outlook

- **STAR BES provides exciting new measurements on cumulants for net protons.**

- **Critical fluctuations on the freeze-out surface**

  - The acceptance dependence can be qualitatively explained

  - C_4 and $\kappa\sigma^2$ can be roughly reproduced through tuning the parameters of the model

  - C_2 , C_3 are well above the statistical baselines, which CANNOT explain/describe the experimental data

- **critical fluctuations from dynamical evolution**

  - the correlation length automatically increase as the system evolves near the critical point.

  - both Fokker-Plank and e-b-e Langevin dynamics present memory effects, thus the value and the sign can be different from the equilibrium ones for S and K.

- **Future works:**
  - construction of e-b-e critical fluctuations in a non-uniform system
  - micro/macroscopic evolution with external chiral field
  - statistical baselines
  - …

Thank you!