Transport coefficients of two-flavor superconducting quark matter

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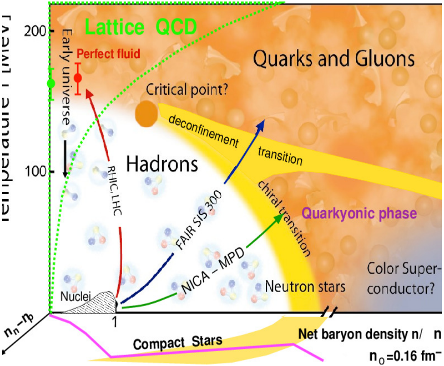
I. Phases of Dense QCD

Neutron stars as probes of dense matter



Neutron stars and supernovae provide complementary information on the state of matter at very high densities to that one hopes to gain from future experiments such as FAIR and NICA.

General form of order parameter

$$\Delta \propto \langle 0|\psi^a_{\alpha\sigma}\psi^b_{\beta\tau}|0\rangle$$

- Antisymmetry in spin σ, τ for the BCS mechanism to work
- Antisymmetry in color a, b for attraction
- Antisymmetry in flavor α, β to avoid Pauli blocking

At low densities 2SC phase (Bailin and Love '84)

$$\boxed{\Delta(2SC) \propto \Delta\epsilon^{ab3}\epsilon_{\alpha\beta}}$$

At high densities we expect 3 flavors of u, d, s massless quarks. The ground state is the color-flavor-locked phase (Alford, Rajagopal, Wilczek '99)

$$\boxed{\Delta(CFL) \propto \langle 0|\psi^a_{\alpha L}\psi^b_{\beta L}|0\rangle = -\langle 0|\psi^a_{\alpha R}\psi^b_{\beta R}|0\rangle = \Delta\epsilon^{abC}\Delta\epsilon_{\alpha\beta C}}$$

Important variations on 2SC phase (crystalline-color-superconductor)

$$\boxed{\Delta(CSC) \propto \Delta\epsilon^{ab3}\epsilon_{\alpha\beta}, \qquad \delta\mu \neq 0, \qquad m_s \neq 0.}$$

Color-superconductivity within the NJL model

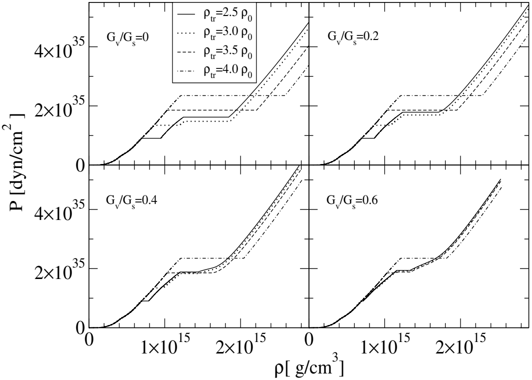
$$
\begin{aligned}
\mathcal{L}_Q &= \bar\psi(i\gamma^\mu\partial_\mu - \hat m)\psi + G_V(\bar\psi i\gamma^0\psi)^2 + G_S\sum_{a=0}^{8}[(\bar\psi\lambda_a\psi)^2 + (\bar\psi i\gamma_5\lambda_a\psi)^2] \\
&+ G_D\sum_{\gamma,c}[\bar\psi^a_\alpha i\gamma_5\epsilon^{\alpha\beta\gamma}\epsilon_{abc}(\psi_C)^b_\beta][(\bar\psi_C)^r_\rho i\gamma_5\epsilon^{\rho\sigma\gamma}\epsilon_{rsc}\psi^8_\sigma] \\
&- K\left\{\mathrm{det}_f[\bar\psi(1+\gamma_5)\psi] + \mathrm{det}_f[\bar\psi(1-\gamma_5)\psi]\right\},
\end{aligned}
$$

quark spinor fields ψ^a_α, color $a = r, g, b$, flavor ($\alpha = u, d, s$) indices, mass matrix $\hat m = \mathrm{diag}_f(m_u, m_d, m_s)$, λ_a $a = 1, ..., 8$ Gell-Mann matrices. Charge conjugated $\psi_C = C\bar\psi^T$ and $\bar\psi_C = \psi^T C$ $C = i\gamma^2\gamma^0$.

Simple but efficient model to treat CSC phases:

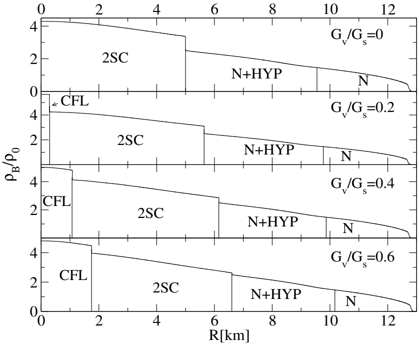
- a sum is over the 8 gluons
- G_S is the scalar coupling fixed from vacuum physics
- G_D is the di-quark coupling, which is related to the G_S via Fierz transformation
- G_V and ρ_{tr} are treated as a free parameter

EOS with equilibrium between nuclear, hypernuclear, 2SC and CFL phases of matter



- Phase equilibrium is constructed via Maxwell prescription
- Sequential phase transition $NM \to 2SC \to CFL$

Internal structure of hybrid star with color superconducting phases



- Small G_V 2SC only. For lagre G_V CFL phase appears.
- Stability is achieved for $G_V > 0.2$ and transition densities few ρ_0

II. Transport coefficients of dense quark matter: Variational calculation

Boltzmann equation for fermions:

$$\left(\frac{\partial}{\partial t} + \mathbf{v}_1 \cdot \nabla_{\mathbf{x}}\right) f_1 = -(2\pi)^4 \sum_j \nu_j \sum_{234} |M_{ij}|^2 \times [f_1 f_2 (1-f_3)(1-f_4) - f_3 f_4 (1-f_1)(1-f_2)]\, \delta^4(p_{\rm in} - p_{\rm out})$$

f - fermion distribution function, M_{ij} scattering matrix element.
ν_j - the degeneracy factors (spin, flavor, color)

What are the degrees of freedom?

Fermions in the basis:

$$\Psi_i = \{\Psi_{bu}, \Psi_{bd}, \Psi_e\} = \{\text{blue up quark } (bu), \text{ blue down quark } (bd), \text{ electron } (e)\}.$$

the indices i and j specify the species of the ungapped fermions in this basis.

Further assumptions:

- Red and green colors are gapped and do not contribute to the transport
- No strangeness (number of s-quarks too small)
- High-density, low-temperature regime $T, m \ll \mu_q$
- Light flavor (isospin) asymmetry typical for neutron stars $\mu_u \ll \mu_d$ (β-equilibrium)

Gauge bosons: write the covariant derivative as

$$\boxed{D_\mu \Psi = \left(\partial_\mu - i \sum_a A^a_\mu Q^a\right)\Psi} \tag{1}$$

Two basis for gauge bosons - standard $(T_8, Q)$and rotated $(X, \tilde{Q})$

$$A_\mu = A^{T_8}_\mu T_8 + A^Q_\mu Q = A^X_\mu X + A^{\tilde{Q}}_\mu \tilde{Q}\,, \tag{2}$$

related by rotations via mixing angle φ

$$A^X_\mu = \cos\varphi\, A^{T_8}_\mu + \sin\varphi\, A^Q_\mu \tag{3}$$

$$A^{\tilde{Q}}_\mu = -\sin\varphi\, A^{T_8}_\mu + \cos\varphi\, A^Q_\mu \qquad \cos\varphi = \frac{\sqrt{3}g}{\sqrt{e^2+3g^2}}. \tag{4}$$

-In the rotated basis the $\tilde{Q}$ charge is massless, i.e., $\tilde{Q}$ color magnetic field penetrates the 2SC phase
-In the rotated basis the X charge is massive, i.e., there is a Meissner effect (more precisely color magnetic flux tubes)

The charges Q^a are defined to be the product of the coupling constant and the charge matrix for the ungapped fermions:

$$\begin{aligned} Q^{T_8} &= g \cdot \mathrm{diag}\left(-\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, 0\right) \\ Q^{Q} &= e \cdot \mathrm{diag}\left(+\frac{2}{3}, -\frac{1}{3}, -1\right) \end{aligned} \quad (5)$$

in the standard (T_8, Q) basis and

$$\begin{aligned} Q^{X} &= g \cos\varphi \cdot \mathrm{diag}\left(-\frac{1-2\tan^2\varphi}{\sqrt{3}}, -\frac{1+\tan^2\varphi}{\sqrt{3}}, -\sqrt{3}\tan^2\varphi\right) \\ Q^{\tilde{Q}} &= e \cos\varphi \cdot \mathrm{diag}\,(1, 0, -1) \end{aligned} \quad (6)$$

in the rotated $(X, \tilde{Q})$ basis.

- The longitudinal part of the screening is evaluated in the standard basis
- The transverse part of the screening is evaluated in the rotated basis

Computing the matrix element for scattering: $p_{1i} + p_{2j} \to p_{3i} + p_{4j}$ (flavor i, j)

Standard Feynman rules give:

$$M_{ij} = J^{\mu}_{a,i} \left(D^{ab}_{\mu\nu} \right) J^{\nu}_{b,j} \quad (7)$$

$$J^{\mu}_{a,i} = Q^a_i \bar{u}\left(\mathbf{p}_3\right) \gamma^{\mu} u\left(\mathbf{p}_1\right) / 2p_1 \qquad J^{\nu}_{b,j} = Q^b_j \bar{u}\left(\mathbf{p}_4\right) \gamma^{\nu} u\left(\mathbf{p}_2\right) / 2p_2 \quad (8)$$

where the most general form of the propagator is given by

$$\left(D^{ab}_{\mu\nu} \right)^{-1} = g_{\mu\nu} \left(\omega^2 - q^2 \right) \delta^{ab} + \Pi^{ab}_{\mu\nu} \quad (9)$$

Screening in a plasma is taken into account via self-energies $\Pi_{\mu\nu}$

Decomposition all the quantities (matrix elements, gauge propagators) into longitudinal and transverse parts:

$$M_{ij} = \sum_{a=\{T_8,Q\}} \frac{J^0_{a,i} J^0_{a,j}}{q^2 + \Pi^{aa}_l} - \sum_{a=\{X,\tilde{Q}\}} \frac{\mathbf{J}^t_{a,i} \cdot \mathbf{J}^t_{a,j}}{q^2 - \omega^2 + \Pi^{aa}_t} \quad (10)$$

$$\Pi^{aa}_l = \sum_i \left(q^a_{D,i}\right)^2 \chi_l + 4 \left(q^a_{D,C}\right)^2 \chi_l \quad \text{in the } (T_8, Q) \text{ basis}$$

$$\Pi^{aa}_t = \sum_i \left(q^a_{D,i}\right)^2 \chi_t + 4 \left(q^a_{D,C}\right)^2 \chi_t + 4 \left(q^a_{D,C}\right)^2 \chi_{sc} \quad \text{in the } (X, \tilde{Q}) \text{ basis} \quad (11)$$

where $q^a_{D,i}$ and $q^a_{D,C}$ are the Debye masses for a given flavor i and the Cooper pair.

The screening functions, χ_l and χ_t in the static limit (Hard Thermal Loop approximation)

$$\chi_l = 1, \qquad \chi_t = i\frac{\pi}{4}\frac{\omega}{q}, \qquad \chi_{sc} = \frac{1}{3}. \tag{12}$$

(better done by Rischke and co-workers). To leading order in ω/q, we thus have

$$\Pi_l^{T_8T_8} = \sum_i (Q_i^{T_8})^2\frac{\mu_i^2}{\pi^2} + 4(Q_C^{T_8})^2\frac{\mu_C^2}{\pi^2} \tag{13}$$

$$\Pi_l^{QQ} = \sum_i (Q_i^{Q})^2\frac{\mu_i^2}{\pi^2} + 4(Q_C^{Q})^2\frac{\mu_C^2}{\pi^2} \tag{14}$$

$$\Pi_t^{XX} = \frac{4}{3}(Q_C^X)^2\frac{\mu_C^2}{\pi^2} \tag{15}$$

$$\Pi_t^{\tilde{Q}\tilde{Q}} = i\frac{\omega}{q}\Lambda^2 \qquad \text{where} \quad \Lambda^2 \equiv \sum_i (Q_i^{\tilde{Q}})^2\frac{\mu_i^2}{4\pi} \tag{16}$$

The Q's can be found in the paper.

We expect some modifications in the χ functions because of threshold behavior of the response functions (non-zero only for $\omega > 2\Delta$.)

The squared matrix element summed over the final spins and averaged over the initial spins is

$$
\begin{aligned}
|M_{ij}|^2 \; = \; & L_l \left| \sum_{a=\{T_8,Q\}} \frac{Q_i^a\, Q_j^a}{q^2+\Pi_l^{aa}} \right|^2 + L_t \left| \sum_{a=\{X,\tilde{Q}\}} \frac{Q_i^a\, Q_j^a}{q^2-\omega^2+\Pi_t^{aa}} \right|^2 \\
& -2L_{lt}\, \Re \left[\left(\sum_{a=\{T_8,Q\}} \frac{Q_i^a\, Q_j^a}{q^2+\Pi_l^{aa}} \right) \left(\sum_{a=\{X,\tilde{Q}\}} \frac{Q_i^a\, Q_j^a}{q^2-\omega^2+\Pi_t^{aa}} \right)^* \right] + \delta_{ij}\gamma_{\rm int} \qquad (17)
\end{aligned}
$$

where

$$
\begin{aligned}
L_l &= \left(1-\frac{q^2}{4p_1^2}\right)\left(1-\frac{q^2}{4p_2^2}\right) \\
L_{lt} &= \left(1-\frac{q^2}{4p_1^2}\right)^{1/2}\left(1-\frac{q^2}{4p_2^2}\right)^{1/2}\cos\theta \qquad (18) \\
L_t &= \left(1-\frac{q^2}{4p_1^2}\right)\left(1-\frac{q^2}{4p_2^2}\right)\cos^2\theta + \frac{q^2}{4p_1^2} + \frac{q^2}{4p_2^2}
\end{aligned}
$$

The interference γ_{int} term is small and is neglected.

Transport coefficients - definitions of electrical and thermal conductivities and shear viscosity

$$j_\alpha = -\sigma \partial_\alpha U \tag{19}$$

$$h_\alpha = -\kappa \partial_\alpha T \tag{20}$$

$$\sigma_{\alpha\beta} = -\eta V_{\alpha\beta} \tag{21}$$

where $V_{\alpha\beta}$ is the traceless part of the spatial derivative of fluid velocity $\mathbf{V}$,

$$V_{\alpha\beta} = \partial_\alpha V_\beta + \partial_\beta V_\alpha - \frac{2}{3}\delta_{\alpha\beta}\nabla\cdot\mathbf{V}. \tag{22}$$

From kinetic theory, we can write the fluxes on the left-hand

$$j_\alpha = \int \frac{d^3p}{(2\pi)^3} e v_\alpha \, \delta f \tag{23}$$

$$h_\alpha = \int \frac{d^3p}{(2\pi)^3} (\epsilon - \mu)\, v_\alpha \, \delta f \tag{24}$$

$$\sigma_{\alpha\beta} = \int \frac{d^3p}{(2\pi)^3} p_\alpha v_\beta \, \delta f \tag{25}$$

Comparing the left-hand-sides we obtain a universal relation

$$\boxed{\xi Y = \sum_i \nu_i \int \frac{d^3p}{(2\pi)^3}\phi_i\,\delta f_i} \tag{26}$$

where ν_i is a spin factor for a particle flavor i, ξ stands σ, κ, or η, - Y stands $-\partial_\alpha U$, $-\partial_\alpha T$, or $-V_{\alpha\beta}$ *and*

$$\phi_i = \begin{cases} e_i v_\alpha & \text{Electrical conductivity} \\ (\epsilon-\mu_i)v_\alpha & \text{Thermal conductivity} \\ p_\alpha v_\beta & \text{Shear viscosity} \end{cases} \tag{27}$$

So far completely general.
Linearization of the Boltzmann equation is given by

$$f_i = f_i^0 + \delta f_i = \frac{1}{e^{(\epsilon-\mu_i)/T}+1} - \frac{\partial f_i^0}{\partial \epsilon}\Phi_i \tag{28}$$

Relaxation time approximation

$$\boxed{\Phi_i = 3\tau_i\psi_i\cdot Y} \tag{29}$$

ψ_i reflects the structure of perturbation and has the standard form

$$\psi_i = \begin{cases} e_i v_\alpha & \text{Electrical conductivity} \\ (\epsilon - \mu_i)\, v_\alpha / T & \text{Thermal conductivity} \\ \left(p_\alpha v_\beta - \frac{1}{3}\delta_{\alpha\beta}\, \mathbf{p}\cdot\mathbf{v}\right)/2 & \text{Shear viscosity} \end{cases} \tag{30}$$

From Eq. (26), we can now define transport coefficient of each component ξ_i as

$$\xi = \sum_i \xi_i = \xi_{bu} + \xi_{bd} + \xi_e \tag{31}$$

with

$$\xi_i Y = -3\tau_i \nu_i \int \frac{d^3p}{(2\pi)^3} \phi_i \left(\psi_i \cdot Y\right) \frac{\partial f_i^0}{\partial \epsilon}. \tag{32}$$

Following the standard procedure, we rewrite Y as

$$Y = \begin{cases} -\partial_\alpha U = -\delta_\alpha^\lambda\, \partial_\lambda U & \text{Electrical conductivity} \\ -\partial_\alpha T = -\delta_\alpha^\lambda\, \partial_\lambda T & \text{Thermal conductivity} \\ -V_{\alpha\beta} = -\frac{1}{2}\left(\delta_\alpha^\lambda\, \delta_\beta^\rho + \delta_\alpha^\rho\, \delta_\beta^\lambda - \frac{2}{3}\delta_{\alpha\beta}\delta^{\lambda\rho}\right) V_{\lambda\rho} & \text{Shear viscosity} \end{cases} \tag{33}$$

-divide the common factor of Y on both sides
-contract the indices α and λ for the electrical and thermal conductivities and the pairs of indices α, λ and β, ρ for the shear viscosity:

$$\boxed{\xi_i = -\frac{3\tau_i\nu_i}{\gamma}\int\frac{d^3p}{(2\pi)^3}\,(\phi_i\cdot\psi_i)\,\frac{\partial f_i^0}{\partial\epsilon}} \tag{34}$$

$\gamma = \delta^\alpha_\alpha = 3$ for the electrical and thermal conductivities
$\gamma = \left(\delta^\alpha_\alpha\delta^\beta_\beta + \delta^\alpha_\alpha - 2\delta^\alpha_\alpha/3\right)/2 = 5$ for the shear viscosity.

Linearization of Boltzmann's collision integral

$$\psi_i\cdot Y\frac{\partial f_1^0}{\partial\epsilon_1} = -\frac{(2\pi)^4}{T}\sum_j\nu_j\sum_{234}|M_{ij}|^2 \\ f_1^0f_2^0(1-f_3^0)(1-f_4^0)\,\delta^4(p_{\rm in}-p_{\rm out})\,(\Phi_1+\Phi_2-\Phi_3-\Phi_4)\,. \tag{35}$$

Acting with $-3\tau_i\nu_i\sum_1\phi_1$ on both sides, we obtain

$$\xi_iY = 3\tau_i\frac{(2\pi)^4}{T}\sum_j\nu_i\nu_j\sum_{1234}|M_{ij}|^2 \\ f_1^0f_2^0(1-f_3^0)(1-f_4^0)\delta^4(p_{\rm in}-p_{\rm out})\phi_1\left[3\tau_i(\psi_1-\psi_3)+3\tau_j(\psi_2-\psi_4)\right]\cdot Y \tag{36}$$

Using the same procedure as for the drift term

$$\xi_i = \frac{9\tau_i}{\gamma}\frac{(2\pi)^4}{T}\sum_j \nu_i\nu_j \sum_{1234}|M_{ij}|^2 f_1^0 f_2^0(1-f_3^0)(1-f_4^0)\delta^4(p_{\rm in}-p_{\rm out})\phi_1\cdot[\tau_i(\psi_1-\psi_3)+\tau_j(\psi_2-\psi_4)]\,. \quad (37)$$

In the limit $\omega, T \ll \mu_q$

$$\xi_i = \frac{\tau_i}{\gamma}\sum_j \nu_i\nu_j \frac{36T\mu_i^2\mu_j^2}{(2\pi)^5}\int_0^\infty d\omega\left(\frac{\omega/2T}{\sinh(\omega/2T)}\right)^2\int_0^{q_M} dq\int_0^{2\pi}\frac{d\theta}{2\pi}|M_{ij}|^2 \phi_1\cdot[\tau_i(\psi_1-\psi_3)+\tau_j(\psi_2-\psi_4)] \quad (38)$$

$q_M = \min[2p_1, 2p_2] = \min[2\mu_i, 2\mu_j]$ is the maximum momentum transfer, and θ is again the angle between $\mathbf{p}_1+\mathbf{p}_3$ and $\mathbf{p}_2+\mathbf{p}_4$.

In the limit $T/\mu_q \ll 1$ $p_1, p_2 \to \mu_i, \mu_j$.

Comparing Eqs. (34) and (38) we obtain relaxation times τ_i for the three gapless fermion species.

III. Numerical and analytical results for 2SC phase

Qualitative understanding

- Transport in the 2SC phase occurs via the ungapped fermions: the blue up quark, the blue down quark, and the electron.

- Transport is dominated by the fermion that feels the least influence from surrounding particles (i.e. long relaxation time or mean-free-path)

Relevant interactions

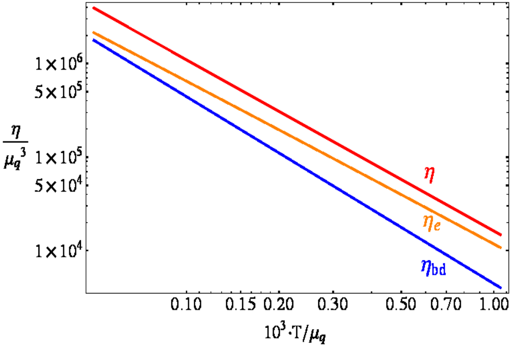
- longitudinal strong interaction (T_8) - Debye screened (short range)
- longitudinal electromagnetic interaction (Q), - Debye screened (short range)
- transverse "rotated" strong interaction (X) - Meissner screening (short ranged)
- transverse "rotated" electromagnetic interaction ($\tilde{Q}$) (not screened, only Landau damped - long ranged at low T)

At low-T the *bu* quark and electron carry $\tilde{Q}$ charge, *bd* does not. Transport is dominated by *bd* quarks (!)

At high T the Landau damping of the $\tilde{Q}$ is more significant. Relaxation times are dominated by the X and T_8 interactions.

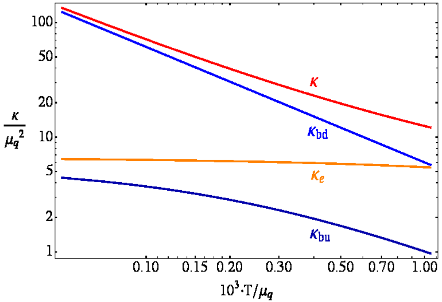
Electron, which has no T_8 charge and only a very small X charge, dominates transport.

A transition from the regime dominated by the *bd* quark to a regime dominated by electrons as the temperature is rised.



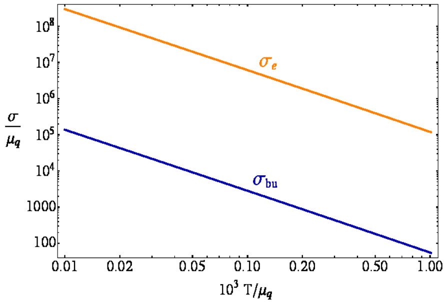
$$\frac{\eta_{bu}}{\mu_q} = \frac{0.150}{(T/\mu_q)^{5/3} + 2490\,(T/\mu_q)^2}, \qquad \frac{\eta_e}{\mu_q} = \frac{0.171}{(T/\mu_q)^{5/3} + 2.78\,(T/\mu_q)^2} \qquad (39)$$

Numerical calculation of shear viscosity as a function of temperature, taking $\alpha_s = 1$. In this temperature range we see the crossover from electron domination at high temperature to blue down quark domination at low temperature.



$$\frac{\kappa_{bu}}{\mu_q} = \frac{5.69}{1 + 3720\,(T/\mu_q)}, \quad \frac{\kappa_e}{\mu_q} = \frac{6.70}{1 + 6.92\,(T/\mu_q)^{2/3}} \tag{40}$$

Numerically calculated thermal conductivity in units of quark chemical potential μ_q in the 2SC phase with $\alpha_s = 1$. I n this temperature range we see the crossover from electron domination at high temperature to blue down quark domination at low temperature.



$$\frac{\sigma_{bu}}{\mu_q} = \frac{0.000672}{(T/\mu_q)^{5/3} + 2.11\,(T/\mu_q)^2}, \quad \frac{\sigma_e}{\mu_q} = \frac{1.46}{(T/\mu_q)^{5/3} + 2.11\,(T/\mu_q)^2} \tag{41}$$

Numerically calculated electrical ($\tilde{Q}$) conductivity as a function of temperature, both expressed in units of the quark chemical potential μ_q, taking strong interaction coupling $\alpha_s = 1$. The electrons dominate because the *bu* relaxation time is shortened by its strong interaction with the *bd* quarks.

IV. Color magnetic flux-tubes (vortices)

Color-magnetic flux tubes in the 2SC phase of compact stars

Ginzburg-Landau functional for 2SC superconductor

$$\mathcal{F} = \mathcal{F}_n + \alpha\psi^*\psi + \frac{1}{2}\beta(\psi^*\psi)^2 + \gamma(\boldsymbol{\nabla}\psi^* - 2ie\boldsymbol{A}\psi^*)(\boldsymbol{\nabla}\psi + 2ie\boldsymbol{A}\psi) + \frac{1}{2\mu_0}(\boldsymbol{B} - \mu_0\boldsymbol{H})^2.$$

The boundary between the type-I and type-II superconductors is set by the GL parameter

$$\kappa_{2SC} \approx 11\frac{\Delta}{\mu}\,. \tag{43}$$

Need to have fields larger than the lower critical field

$$H_{c1} = \frac{\Phi_X}{4\pi\lambda^2}\ln\kappa \simeq 6.5\times 10^{17}\left(\frac{\mu}{400\text{ MeV}}\right)^2\left(\frac{g^2}{4\pi}\right)\left(1-\frac{T}{T_c}\right)\quad \text{G}, \tag{44}$$

The density of color-magnetic flux tubes is of the order of those in the core

$$n_v = \frac{B_X}{\Phi_X} = \frac{2\sqrt{3}e}{g}\frac{B_X}{\Phi_0} \simeq 0.3\,\frac{B_X}{\Phi_0}, \tag{45}$$

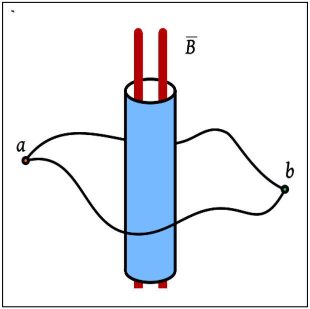
Suppose that we have a network of flux-tubes: how the transport will be modified?

Aharonov-Bohm scattering of fermions of color-magnetic flux tubes

AB effect is a quantum-mechanical interference effect: The cross-section (per unit length)

$$\frac{d\sigma}{d\vartheta} = \frac{\sin^2(\pi\beta)}{2\pi k \sin^2(\vartheta/2)}, \quad \beta = \frac{q_p}{q_c}.$$

q_p is the charge of the scattering particle.



- The cross-section vanishes if β is an integer, but is otherwise non-zero.
- The cross section is *independent of the thickness of the flux tube*: the scattering is not suppressed in the limit where the symmetry breaking energy scale goes to infinity, and the flux tube thickness goes to zero.
- The cross section diverges both at low energy and for forward scattering.

In the complete fermion basis: $\Psi = (ru, gd, rd, gu, bu, bd, e^-)$

$$\beta = \text{diag}\Bigg(\underbrace{\frac{1}{2}+\frac{e^2}{2g^2}, \frac{1}{2}-\frac{e^2}{2g^2}, \frac{1}{2}-\frac{e^2}{2g^2}, \frac{1}{2}+\frac{e^2}{2g^2}}_{\text{gapped quarks}}, -1+\frac{e^2}{g^2}, -1, -\frac{e^2}{g^2}\Bigg). \tag{46}$$

We conclude

- The gapped quarks have β close to $1/2$, which means that they have near-maximal Aharonov-Bohm interactions with an X-flux tube.
- Among the light fermions the $\tilde{Q}$-neutral bd has zero Aharonov-Bohm interaction with the flux tube.
- The bu and electron have a β that differs from an integer by $e^2/g^2 = \alpha/\alpha_s \sim 1/100$, near-maximal Aharonov-Bohm interactions with an X-flux tube.

For a given transport coefficient $\xi = \{\sigma, \kappa, \eta\}$ the relaxation time τ_i^ξ :

$$\frac{1}{\tau_i^\xi} = \frac{1}{\tau_{i,v}^\xi} + \sum_j \frac{1}{\tau_{ij}^\xi} \tag{47}$$

-Thermal conductivity Dominated by the bd quarks, which do not interact with vortices $\rightarrow$ Subdominant effect

-Electrical conductivity and shear viscosity dominated by electrons – mostly $\tilde{Q}$ exchange

For conductivity and shear viscosity the scattering rate is given by

$$\sum_j \frac{1}{\tau^{\xi}_{e,j}} = c_\xi \, \alpha^{5/3} \frac{T^{5/3}}{\mu_q^{2/3}} \tag{48}$$

where $\xi = \{\sigma, \eta\}$ and $\xi, c_\xi \sim 10$ depends on fermion charges. Momentum relaxation rate for the electron-vortex scattering is

$$\frac{1}{\tau_{e,v}} = \frac{\pi^{3/2}\alpha^{5/2}}{3\alpha_s^2} \frac{B}{\mu_e}. \tag{49}$$

Electron-vortex scattering becomes important when

$$\sum_j \frac{1}{\tau^{\xi}_{e,j}} \sim \frac{1}{\tau_{e,v}} \tag{50}$$

For $\alpha_s = 1$, $\mu_q = 400$ MeV and $T = 10^7$ K the equipartition is when

$$\boxed{B \sim 10^{12}\,\mathrm{G} \left(\frac{T}{10^7\,\mathrm{K}}\right)^{5/3} \left(\frac{\mu_q}{400\,\mathrm{MeV}}\right)^{1/3}.} \tag{51}$$

Summary

Thermal conductivity: the crossover from blue-down to electron domination occurs at $T/\mu_q \sim \alpha/7.7 \sim 10^{-3}$, so most of the temperature range of interest for neutron stars is in the blue-down-dominated regime where $\kappa \sim 1/T$.

Shear viscosity: the crossover from blue-down to electron domination occurs at $T/\mu_q \sim 10^{-5}$, so electrons are dominant down to $T \sim 10\,\text{keV}$. The crossover temperature for the shear viscosity is much smaller than for the thermal conductivity because the relevant collisions for shear viscosity are those that transfer higher momentum, so the increase in the range of $\tilde{Q}$ interaction has a smaller impact on the mean free path since the long-range interactions involve low momentum transfer.

Electrical conductivity: this is a special case because the transported quantity is $\tilde{Q}$ charge, so the blue down quarks, which are $\tilde{Q}$ neutral, do not contribute to the electrical conductivity. The electron contribution therefore dominates over the entire temperature range.

Other possible excitations - the color-magnetic flux tubes and gluons in the unbroken gauge sector. Flux tubes are at sufficiently low temperature and high magnetic field, the vortex-fermion scattering via the Aharonov-Bohm effect may suppress the electron contributions to the electrical conductivity and shear viscosity.