Testing thermal and transport properties of effective field theories

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Outline

- Effective field theories
  why we use them?
- Non-local approach
  motivation & formalism: a simple toy model
- Thermal equilibrium
- Shear viscosity to entropy
- Further challenges & plans

Effective field theories

Why?

- ***can't solve the microscopic theory***
- ***strong interaction: PT seems to fail***

effective theory

- scale dependence
- same symmetries
- minimal

relevant DoF (e.g. bound states): **weakly interacting!**

parametrization ⟶ calculating observables → finite temperature / universality / phenomenology

Non-local effective theories

Parametrization: spectral function(s) = energy levels at fixed quantum numbers

- *consistent treatment (all states are involved)*
- *objects with finite lifetime (no mass-shell)*
- *control on symmetries & causal structure*

✔ ***quasi-particles with short lifetime***

✔ ***bound states***

✔ ***liquids (?)***

✔ ***(radiation of strongly interacting sys. (?))***

$$\mathcal{L} = \frac{1}{2}\varphi G^{-1}\varphi + \mathcal{L}_{\mathrm{int}}$$

$$G^{-1} = G_0^{-1} - \Sigma[G]$$

Berges, Cox: Phys. Lett. B 517, 269

easiest: exactly solvable $\mathcal{Z} \sim \int \mathcal{D}\varphi e^{i\int \varphi \mathcal{K} \varphi}$

Formalism

$$S[\varphi] = \int d^4x \int d^4y \varphi(x) K(x-y) \varphi(y) = \int d^4p \mathcal{K}(p) \widetilde{\varphi}(p) \widetilde{\varphi}(-p)$$

represents a complicated spectrum

-from self. cons. approach (SD)
-from experiment

$$\mathcal{K}(p) = \mathrm{Re}\, G_R^{-1}(p)$$

$$G_R(p) = \int \frac{d\omega}{2\pi} \frac{\rho(\omega, \mathbf{p})}{p_0 - \omega + i0^+}$$

Wick's theorem applies

$$\Longrightarrow \langle \varphi\varphi \dots \varphi \rangle$$

Consistency:

- unitarity
- causality $\quad \rho(x) = 0, \; x^2 < 0$
- energy- and momentum conservation
- Lorentz-invariance

Remarks:
quantum theory?
infinities?

A. Jakovác: PRD 86, 085007 (2012)

Energy-momentum tensor

Noether-current: $$T_{\mu\nu}(x) = \frac{1}{2}\varphi(x)[\mathcal{D}_{\mu\nu}\mathcal{K}(i\partial_x)]_{\mathrm{sym}}\varphi(x)$$

$$\langle T_{\mu\nu}\rangle = \frac{1}{2}\int\frac{\mathrm{d}^4p}{(2\pi)^4}[\mathcal{D}_{\mu\nu}\mathcal{K}]_{\mathrm{sym}}(p,-p)\,iG^{12}(p)$$

KMS-condition

$$= \mathcal{D}_{\mu\nu}\mathcal{K}(p) = p_\mu\frac{\partial\mathcal{K}}{\partial p_\nu} - g_{\mu\nu}\mathcal{K}(p)$$

$$= n(p_0)\rho(p_0,\mathbf{p})$$

Assumptions:
bosonic quantum channel
thermal equilibrium – Gibbs-ensemble of composite objects

A. Jakovác: PRD 86, 085007 (2012)

Thermodynamic quantities

$$\langle \dots \rangle = \mathrm{Tr}[e^{-\beta T_{00}}(\dots)]$$

$$\mathcal{DK}(p) = p_0 \frac{\partial \mathcal{K}}{\partial p_0} - \mathcal{K}(p)$$

~ Hamiltonian-density,
for a delta-peak:

$$= p_0^2 + \mathbf{p}^2 + m^2$$

$$\varepsilon = \langle T_{00} \rangle = \int_p^+ \mathcal{DK}(p) n(p_0) \rho(p) = \frac{\partial \beta f}{\partial \beta}$$

$$p = -f = -\frac{1}{\beta} \int_p^+ \mathcal{DK}(p) \frac{\ln(1 - e^{-\beta p_0})}{p_0} \rho(p)$$

$$s = \beta^2 \frac{\partial f}{\partial \beta} = \int_p^+ \mathcal{DK}(p) \left(\beta n(p_0) - \frac{\ln(1 - e^{-\beta p_0})}{p_0} \right) \rho(p)$$

Euclidean generating functional

$$\mathcal{Z} = e^{-\beta V f}$$

Assumption: temperature independent spectral function!

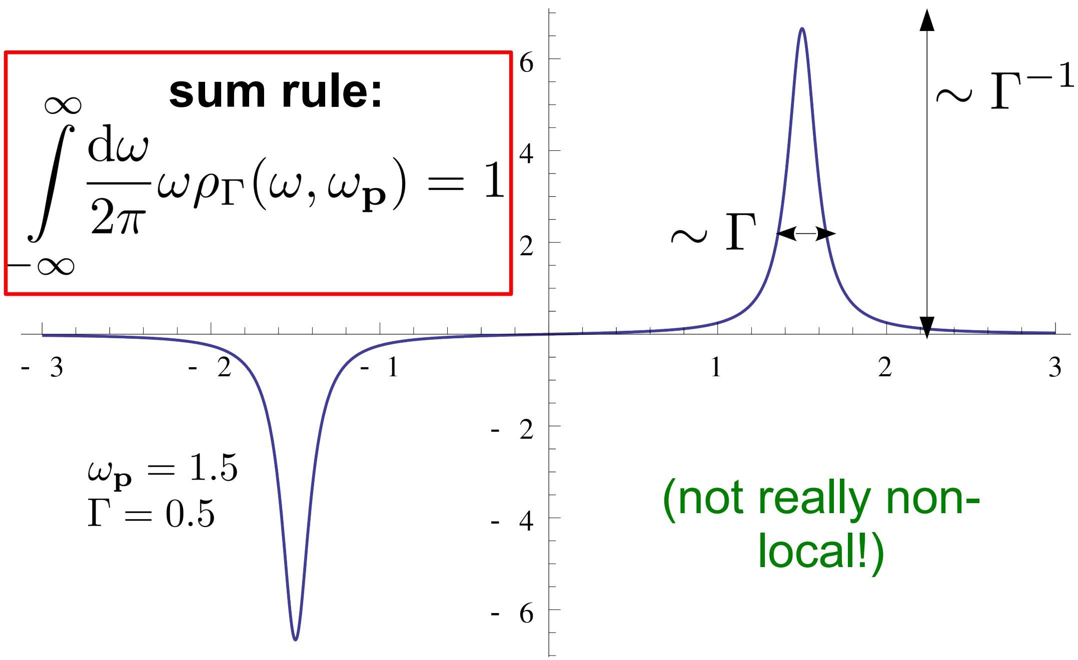
A. Jakovác: PRD 86, 085007 (2012)

A counterexample

$$\boxed{\rho_\Gamma(\omega,\omega_{\mathbf{p}}) = \frac{1}{\omega_{\mathbf{p}}}\left(\frac{\Gamma}{(\omega-\omega_{\mathbf{p}})^2+\Gamma^2} - \frac{\Gamma}{(\omega+\omega_{\mathbf{p}})^2+\Gamma^2}\right) \xrightarrow{\Gamma\to 0} 2\pi\,\mathrm{sgn}(\omega)\delta(\omega^2-\omega_{\mathbf{p}}^2)}$$

sum rule:

$$\int_{-\infty}^{\infty}\frac{\mathrm{d}\omega}{2\pi}\omega\rho_\Gamma(\omega,\omega_{\mathbf{p}}) = 1$$



(not really non-local!)

$$G_R(\omega,\omega_{\mathbf{p}}) = \frac{1}{(\omega+i\Gamma)^2-\omega_{\mathbf{p}}^2}$$

$$\mathcal{K}(\omega,\omega_{\mathbf{p}}) = \omega^2-\omega_{\mathbf{p}}^2-\Gamma^2+\cancel{2i\omega\Gamma}$$

$$\mathcal{D}_{00}\mathcal{K} = \omega^2+\omega_{\mathbf{p}}^2+\Gamma^2$$

$$\rho_\Gamma(x) \sim e^{-\Gamma t}\rho_{\Gamma=0}(x)$$

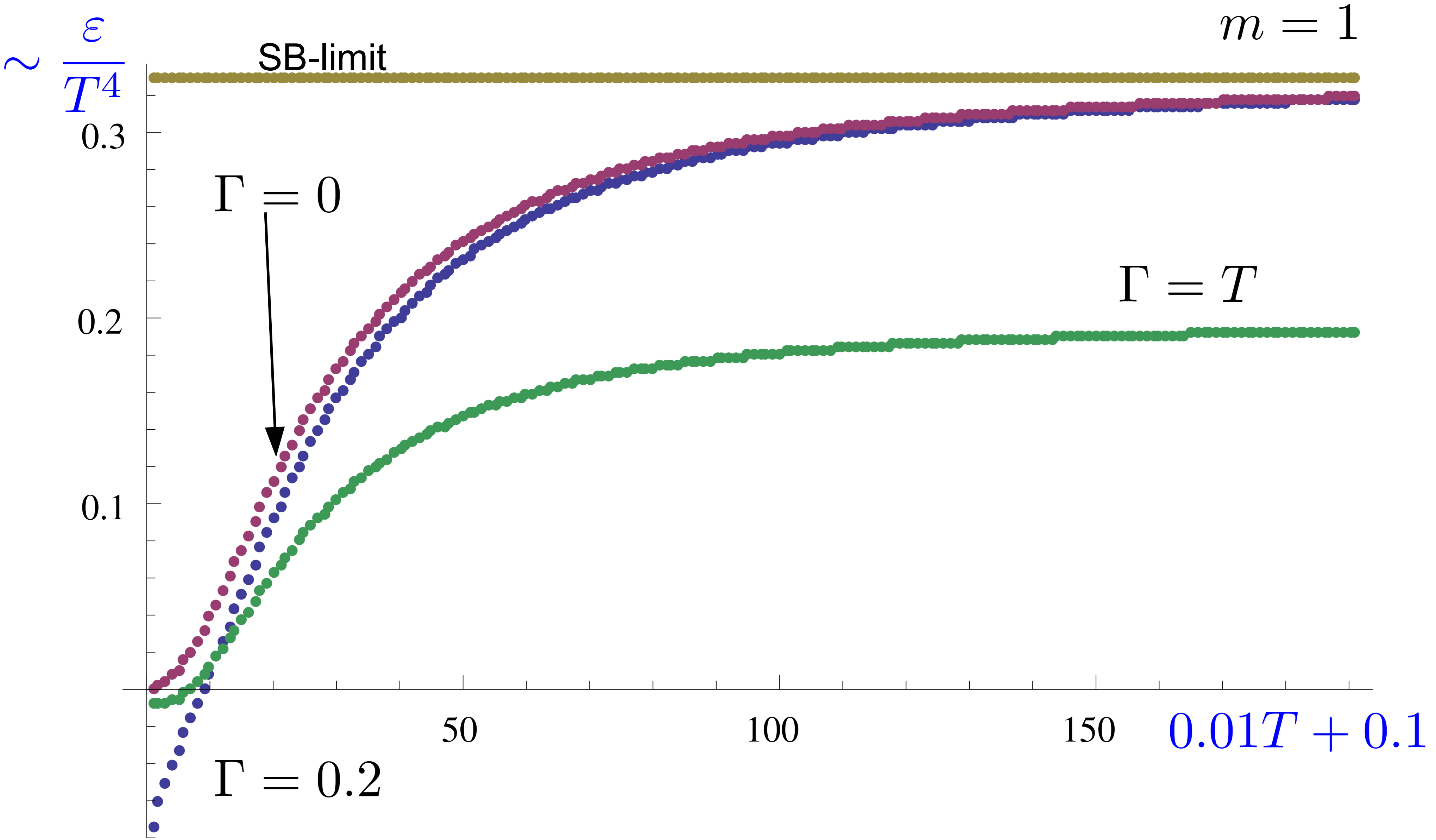
wave packet
finite lifetime

$(\sim \Gamma^{-1})$

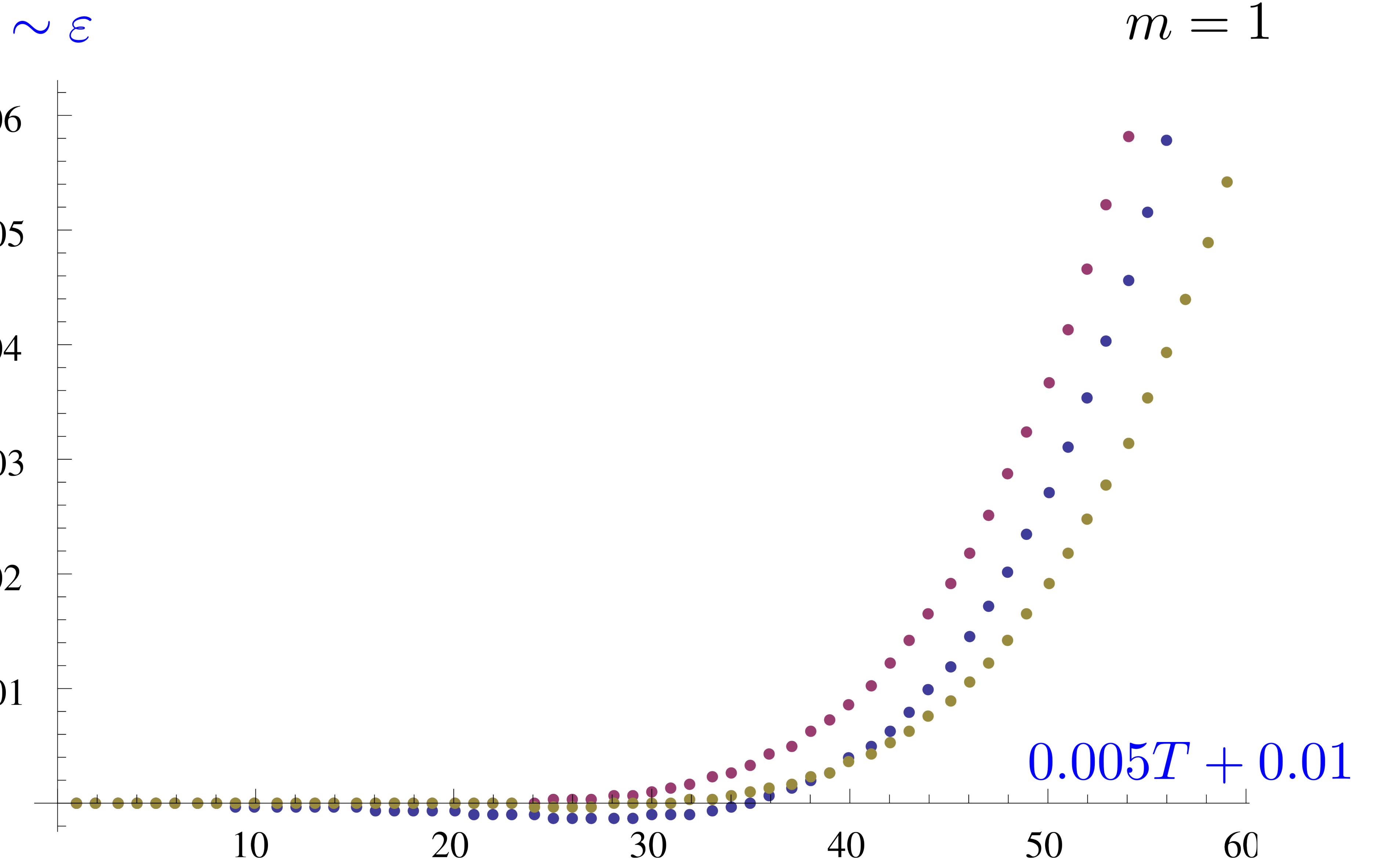
$$\Gamma = 0$$

plane wave
infinite lifetime

Energy density



Energy density



Another counterexample

zero mass, finite width

$$f = -\frac{\pi^2}{90}T^4 + c\Gamma^2 T^2$$

$$s = -\frac{\partial f}{\partial T} = \frac{2\pi^2}{45}T^3 - 2c\Gamma^2 T$$

$$\varepsilon = -T^2 \frac{\partial f/T}{\partial T} = \frac{\pi^2}{30}T^4 - c\Gamma^2 T^2$$

Another counterexample

zero mass, finite T-dependent width

$$f = -\frac{\pi^2}{90}T^4 + c\Gamma^2 T^2$$

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$$\varepsilon = -T^2\frac{\partial f/T}{\partial T} = \frac{\pi^2}{30}T^4 - c\Gamma^2 T^2 - cT^3\frac{\partial \Gamma^2}{\partial T}$$

$\Gamma \sim \frac{1}{T}$ $\longrightarrow$ **no contribution to therm**

$\Gamma \sim T$ $\longrightarrow$ **simple renormalization**

Another counterexample

zero mass, finite T-dependent width

$$\mathcal{Z}_\Gamma = \mathcal{Z}_0 e^{-\beta V \int_0^\Gamma \mathrm{d}\gamma \int \frac{\mathrm{d}^4 p}{(2\pi)^4} n(p_0)\gamma \rho_\gamma(p_0,\mathbf{p})} = \mathcal{Z}_0 e^{-\beta V f_{\Gamma>0}}$$

$$s = -\frac{\partial f}{\partial T} = \frac{2\pi^2}{45}T^3 - 2c\Gamma^2 T - cT^2\frac{\partial \Gamma^2}{\partial T}$$

$$\varepsilon = -T^2\frac{\partial f/T}{\partial T} = \frac{\pi^2}{30}T^4 - c\Gamma^2T^2 - cT^3\frac{\partial \Gamma^2}{\partial T}$$

$\Gamma \sim \frac{1}{T}$ $\longrightarrow$ **no contribution to therm**

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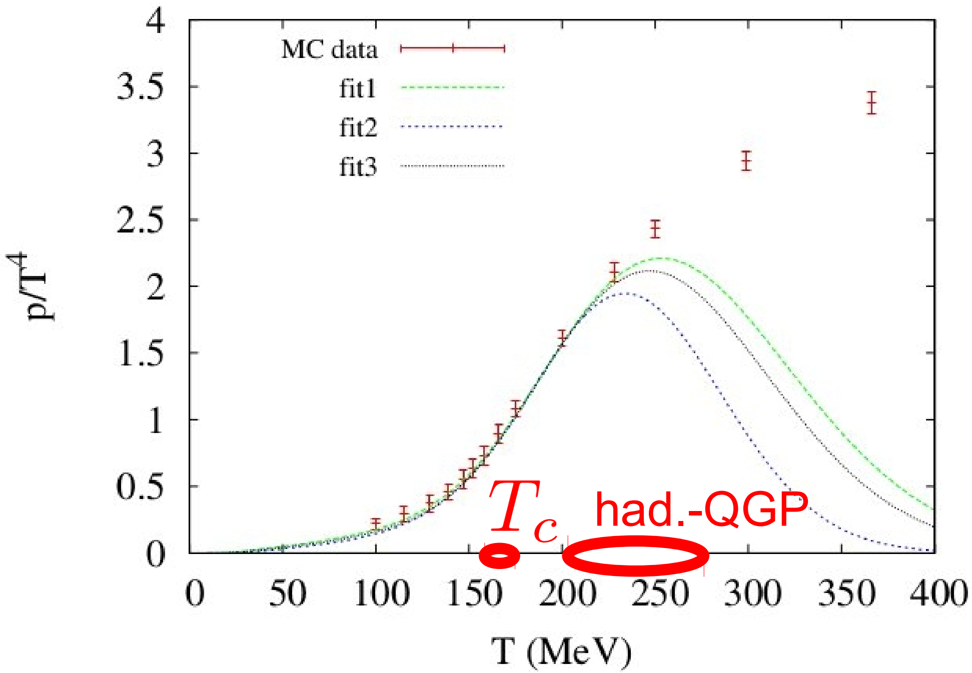
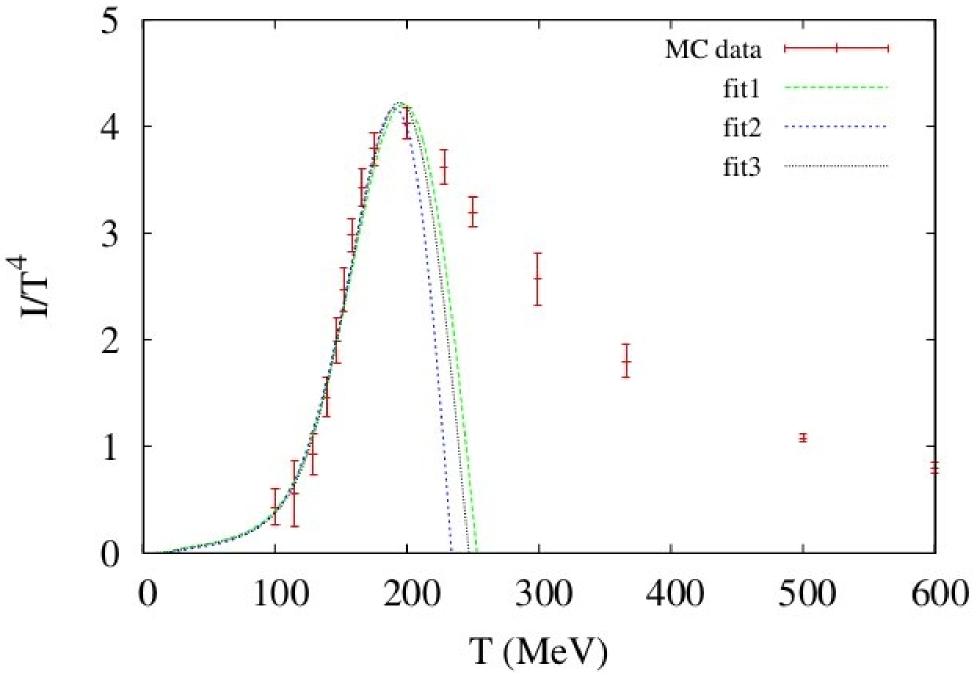
QCD thermodynamics

$$\rho(p) = \rho_\gamma(p) + \frac{p}{p^2+m_{th}^2}\sqrt{\frac{\sqrt{(p^2-m_{th}^2)^2+S^2}+p^2-m_{th}^2}{2}}$$





$$\gamma^2 = \gamma_0^2 + \Theta(T - T_0)\frac{T^2}{T_n^2}$$

3 parameters to fit
+ 2 given by Hagedorn

robust result

A. Jakovác: PRD 88, 065012 (2013)

Shear viscosity – work in progress!

~momentum-diffusion coefficient $\rho\frac{\delta v}{\tau} \sim \eta\frac{\delta v}{l^2}$ $\xrightarrow{\text{QP-appr.}}$ $\frac{\eta}{\rho} \sim \frac{\eta}{s} \sim \langle v\rangle\, l$

Kubo-formula: $$\eta = \frac{1}{3}\sum_{i\neq j}\lim_{\omega\to 0}\frac{\partial}{\partial\omega}\left\langle [T_{ij}(\omega,\mathbf{p}=0);T_{ij}(0)]\right\rangle$$

$$\eta = \frac{1}{6}\int\frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3}\int_0^\infty\frac{\mathrm{d}\omega}{2\pi}(-n'(\omega))(\Delta(\omega,\mathbf{p})\rho_\Gamma(\omega,\mathbf{p}))^2 \sim$$

$$\sim \int_0^\infty \mathrm{d}p\,p^6\int_0^\infty \mathrm{d}\omega\,\frac{\beta}{\cosh(\beta\omega)-1}\left(\frac{\mathrm{d}\mathcal{K}}{\mathrm{d}\mathbf{p}^2}\rho_\Gamma(\omega,\mathbf{p})\right)^2$$

$\Delta(\omega,\mathbf{p}) = \sum_{i\neq j}\mathcal{D}_{ij}\mathcal{K}(\omega,\mathbf{p})$ our case: $\frac{\mathrm{d}\mathcal{K}}{\mathrm{d}\mathbf{p}^2} = 1$

- decreasing with T for gas
- increasing with T for fluid
- lower bound (maybe universal) $\boxed{\eta/s}$

exact:

$$\eta_\Gamma^{m=0} \sim c_1\Gamma T^2 + c_2\frac{T^4}{\Gamma}$$

$\Rightarrow$ $\frac{\eta_\Gamma^{m=0}}{s}\sim\text{const.},\ \ \Gamma\sim T$

$\Rightarrow$ $\frac{\eta_\Gamma^{m=0}}{s}\sim c_1 T^{-2} + c_2 T^2,\ \ \Gamma\sim\frac{1}{T}$

Shear viscosity – work in progress!

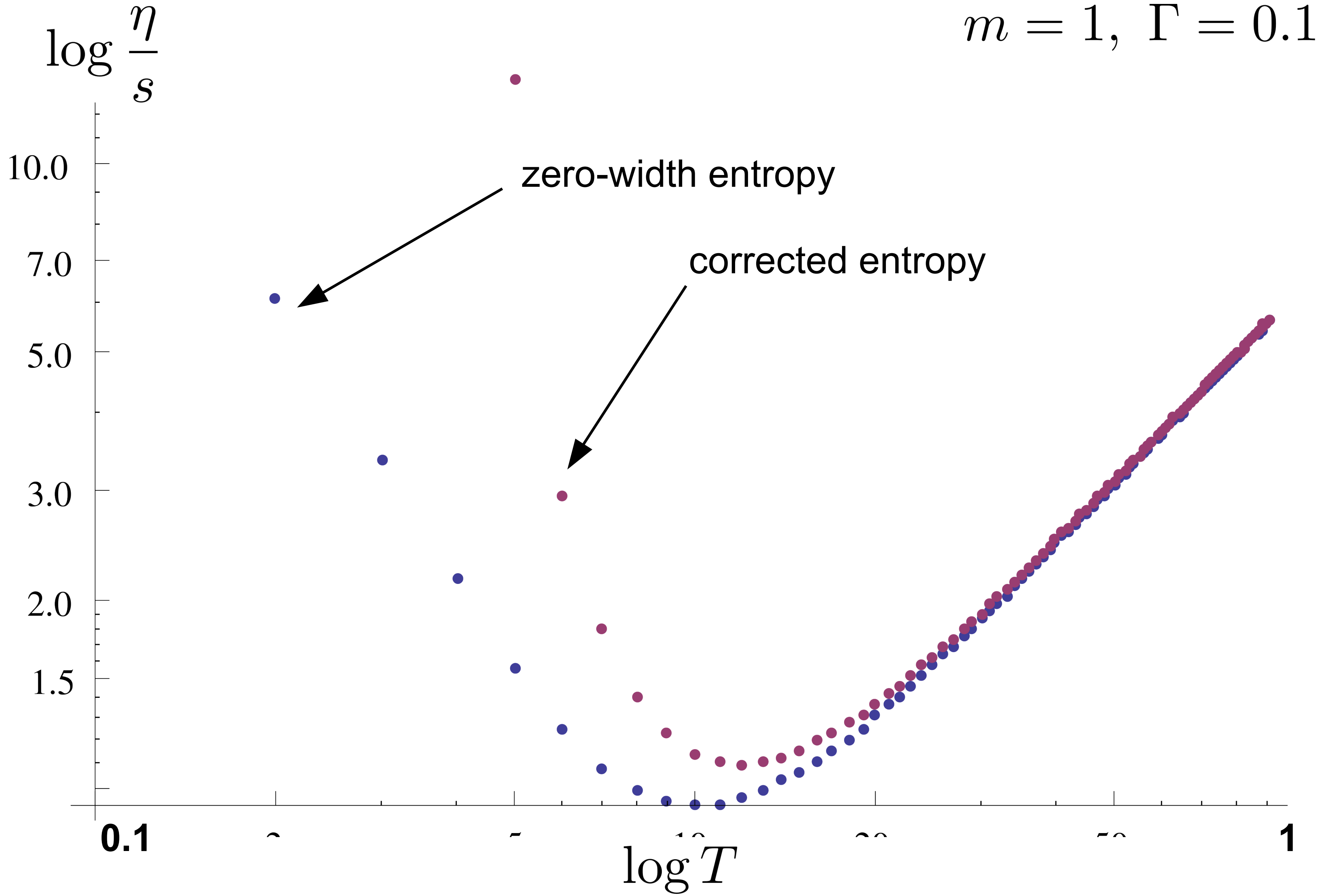
the massless case: $$\eta_\Gamma^{m=0} \sim c_1 \Gamma T^2 + c_2 \frac{T^4}{\Gamma}$$

⇒ $$\frac{\eta_\Gamma^{m=0}}{s} \sim \text{const.}, \;\; \Gamma \sim T \qquad\qquad (s \sim T^3)$$

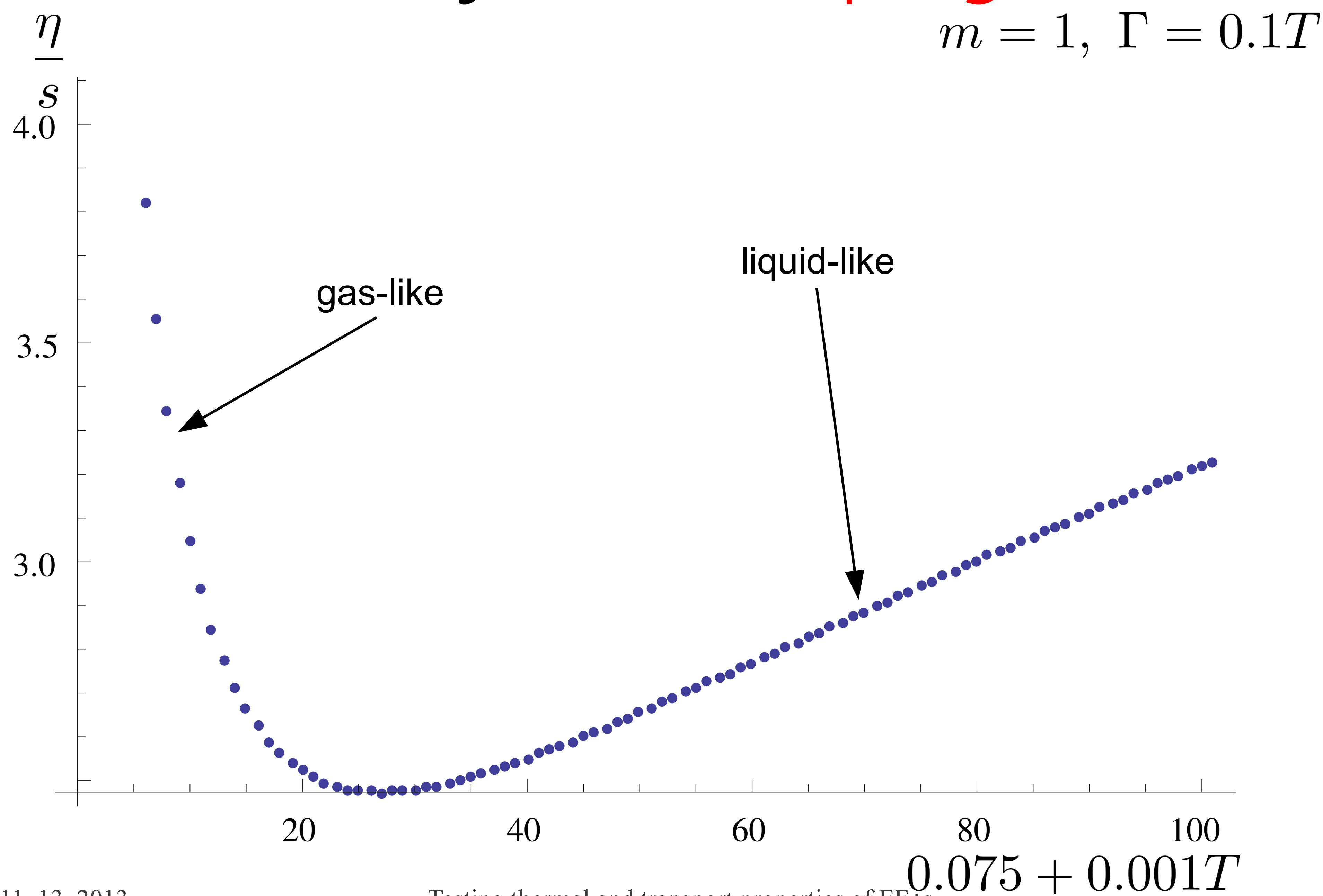
⇒ $$\frac{\eta_\Gamma^{m=0}}{s} \sim c_1 T^{-2} + c_2 T^2, \;\; \Gamma = \frac{g}{T}$$

⇒ **lower bound:** $$\left.\frac{\eta_\Gamma^{m=0}}{s}\right|_{\text{min.}} \sim \frac{1}{\# + g} \xrightarrow{g \to \infty} 0$$

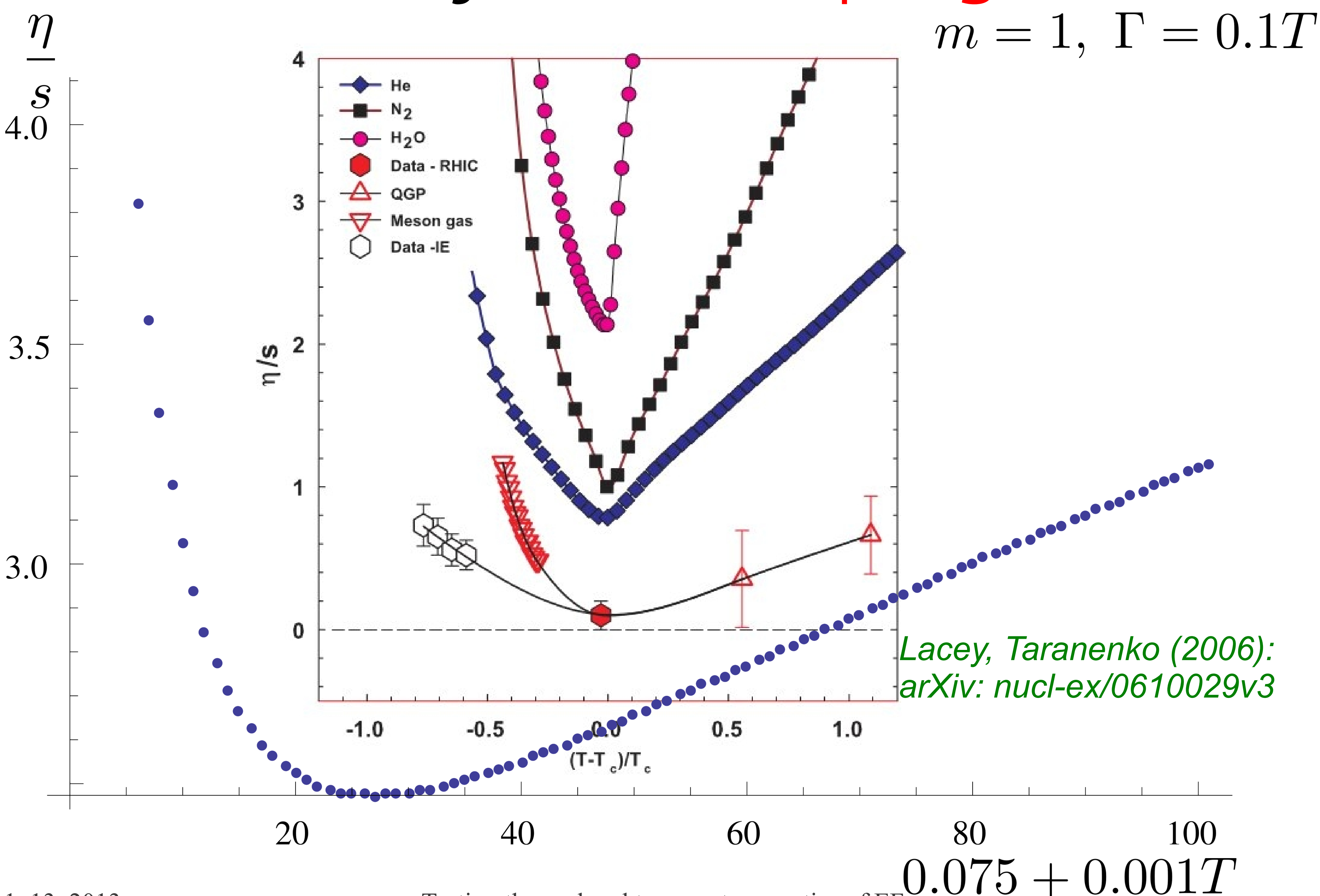
Shear viscosity – work in progress!



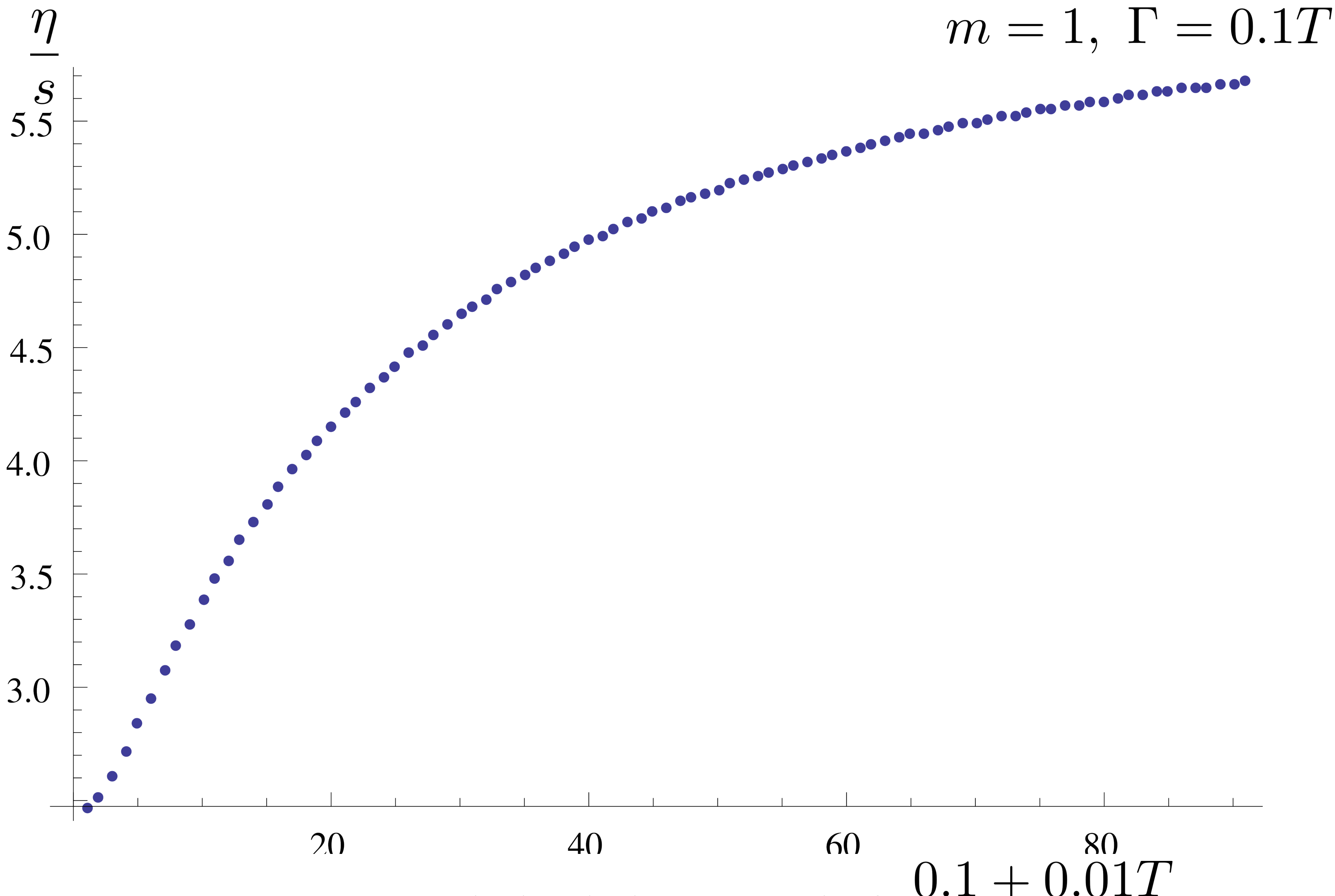
Shear viscosity – work in progress!



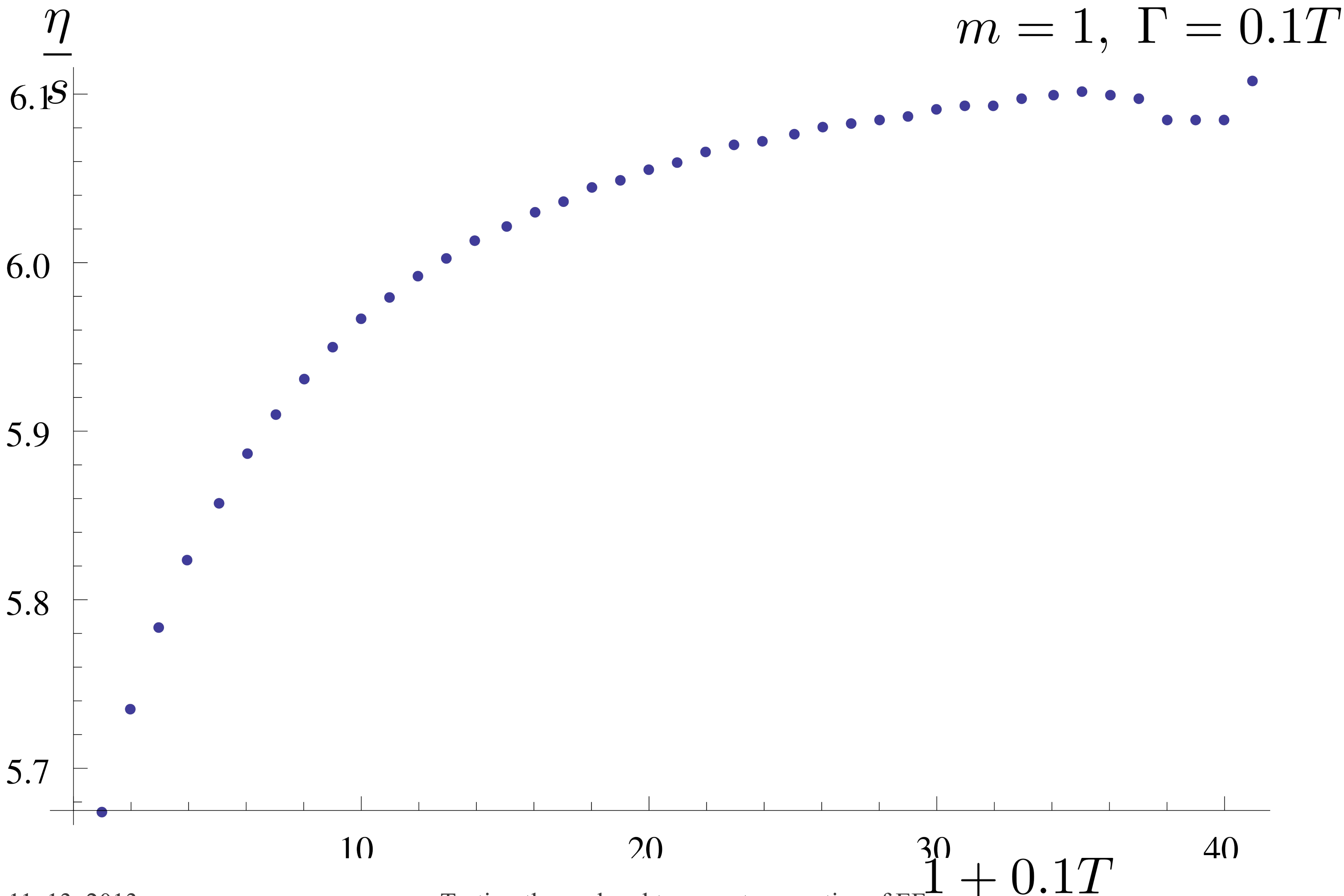
Shear viscosity – work in progress!



Shear viscosity – work in progress!



Shear viscosity – work in progress!



Problems, plans

interacting theory
- –possible construction, restrictions for the spectral function
- –renormalization?

phenomenological QCD thermodynamics

thermodynamic consistency

finite chemical potential
- –complex scalar field, fermions
- –conserved charge of finite lifetime objects?

mimic asymptotic safety – thermodynamics?

presence of continuum (work in progr.) – modified lifetime?

What have we learned?

- how to (get started to) test the thermal and transport properties of a quantum channel with known spectral density in an ~~lazy~~ easy way

- qualitative changes in shear viscosity to entropy ratio for wide peak

- funny, exactly calculable models to liquid-gas crossover

Thank you for the attention!

Questions?

Backup slides

Energy-momentum tensor

backups

$$T_{\mu\nu}(x) = \frac{1}{2}\varphi(x)[\mathcal{D}_{\mu\nu}\mathcal{K}(i\partial_x)]_{\mathrm{sym}}\varphi(x)$$

$$T_{\mu\nu}(x) = \frac{1}{2}\int\frac{\mathrm{d}^4 p}{(2\pi)^4}\int\frac{\mathrm{d}^4 q}{(2\pi)^4}[\mathcal{D}_{\mu\nu}\mathcal{K}]_{\mathrm{sym}}(p,q)\widetilde{\varphi}(p)\widetilde{\varphi}(q)e^{-ix(p+q)}$$

$$[\mathcal{F}]_{\mathrm{sym}}(p,q) = \sum_j\sum_{a=0}^{j}\frac{c_j}{j+1}p^a q^{j-a} \qquad \mathcal{F}(p) = \sum_j c_j p^j$$

$$\langle T_{\mu\nu}\rangle = \frac{1}{2}\int\frac{\mathrm{d}^4 p}{(2\pi)^4}[\mathcal{D}_{\mu\nu}\mathcal{K}]_{\mathrm{sym}}(p,-p)\,iG^{12}(p)$$

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