Hadronic Multiplicities and Energy Spectra from Hagedorn States

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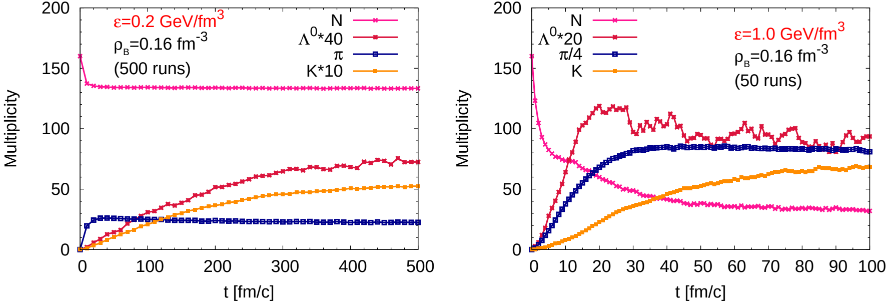
Problem: Chem. equilibration in UrQMD too slow

cf. M. Belkacem et al., PRC 58 (1998) 1727
see also E. Bratkovskaya et al., NPA 675 (2000) 661



- initial particles: 80 n + 80 p (uniformly distributed)
- string production disabled

Application of Hagedorn States (HS):

J. Noronha-Hostler, C. Greiner, I. A. Shovkovy., PRL 100 (2008) 252301

- SPS energies: strong increase of antiprotons/antihyperons through 'clustering' of mesons

$$n_1\pi + n_2K \leftrightarrow \bar{Y} + p$$

- chemical equilibration time of $t_{\rm eq} \approx 1 - 3\,{\rm fm}/c$
- RHIC energies: $t_{\rm eq} \sim 10\,{\rm fm}/c$ for antibaryons
- quick chemical equilibration mechanism through HS

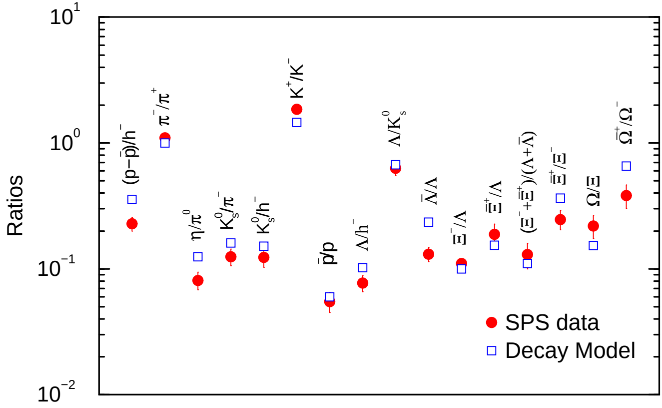
$$\left(n_1\pi + n_2K + n_3\bar{K} \leftrightarrow\right)\; HS \leftrightarrow \bar{B} + B + X$$

- dynamical evolution through set of coupled rate equations
- HS and pions in equilibrium $\rightarrow t_{\rm eq} \approx 5\,{\rm fm}/c$ for $B\bar{B}$-pairs

Application of Hagedorn States (HS) :

S. Pal, P. Danielewicz, Phys.Lett. B627 (2005) 55

- statistical model for decay and formation of HS
- Hagedorn Temperature of $T_H \simeq 170$ MeV
- subsequent decay of one single heavy resonance (HS)



Intention: UrQMD[1] and Hagedorn States (HS)

- UrQMD = Ultrarelativistic Quantum Molecular Dynamics
- microscopic transport model for p+p, p+N and A+A for Bevalac and SIS up to AGS, SPS and RHIC energies
- detailed balance
  - is enforced: meson-baryon, meson-meson, resonance-nucleon, resonance-resonance
  - is violated by string and some hadron decays $(\omega \rightarrow 3\pi)$

- for $\sqrt{s} \geq 2.5 - 10\,\mathrm{GeV}$: HS production replaces strings

Observables:

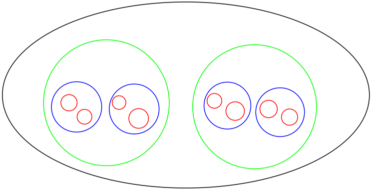
- particle multiplicities
- chemical equilibration times
- estimates for $\frac{\eta}{s}$ (shear viscosity over entropy density)
- ...

[1] S. A. Bass et al., Prog.Part.Nucl.Phys. 41 (1998) 225

Historical Background of Statistical Bootstrap Model

1965: Rolf Hagedorn postulates "Statistical Bootstrap Model"[2]

- highly excited lumps of matter are not essentially different from observed hadronic resonances at lower excitation
- fireballs and their constituents are the same
- nesting fireballs into each other leads to
  self-consistency condition (bootstrap-equation)



- solution: "Hagedorn Spectrum", exponentially rising

[2] R. Hagedorn, Nuovo Cim.Suppl. 3 (1965) 147-186

Covariant Formulation of Hagedorn Spectrum

- number of states of particle in (resting) volume V

$$\mathrm{d}N = \left(V\frac{m}{E}\right)\frac{\mathrm{d}^3p}{(2\pi)^3} = 2V\frac{\mathrm{d}^4p}{(2\pi)^4}\,m\,(2\pi)\delta\left(p^2 - m^2\right)$$

- include mass degeneration $\tilde{\tau}\left(m^2\right)$

$$\mathrm{d}N = 2V\frac{\mathrm{d}^4p}{(2\pi)^4}m\,\mathrm{d}m^2\,\tilde{\tau}\left(m^2\right)(2\pi)\delta\left(p^2 - m^2\right)$$

- convolution of single state densities

$$\tilde{\tau}\left(m^2\right) = \frac{V}{m}\frac{1}{(2\pi)^3}\prod_{i=1}^{2}\int \mathrm{d}m_i^2\tilde{\tau}\left(m_i^2\right)m_i\Phi_2\left(m\right)$$

Generalization for additional quantum numbers

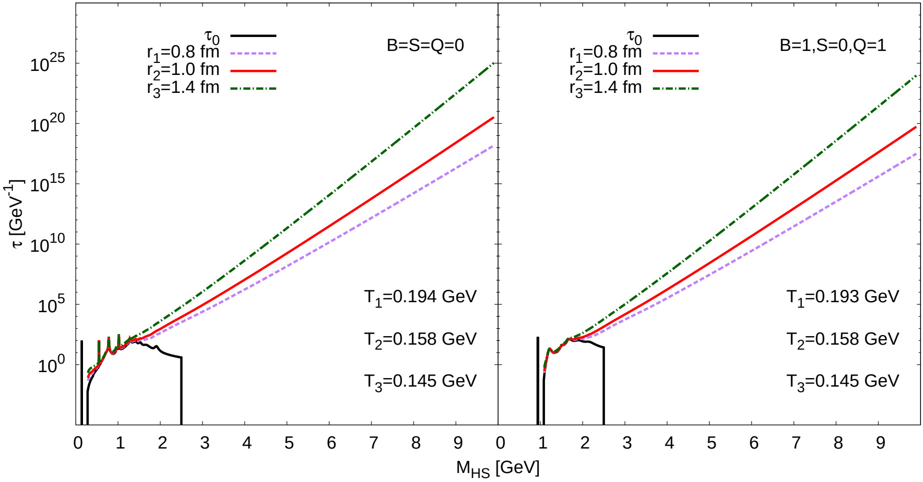
- $\tilde{\tau}\left(m^2\right) = \tau(m)/(2m)$ and $V = 4\pi R^3/3$

$$\tau\left(m\right) = \frac{R^3}{3\pi m} \iint \mathrm{d}m_1 \mathrm{d}m_2 \tau\left(m_1\right) \tau\left(m_2\right) m_1 m_2 p_{cm}$$

- conserve baryon number B, strangeness S and electric charge Q $\left(\vec{C} = (B, S, Q)\right)$

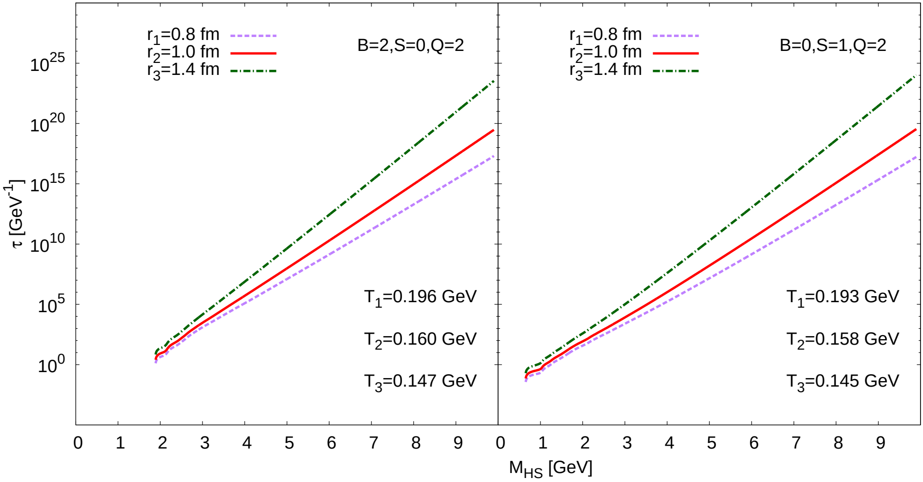
$$\tau_{\vec{C}}\left(m\right) = \frac{R^3}{3\pi m} \sum_{\vec{C}_1, \vec{C}_2} \iint \mathrm{d}m_1 \mathrm{d}m_2$$
$$\times \tau_{\vec{C}_1}\left(m_1\right) \tau_{\vec{C}_2}\left(m_2\right) m_1 m_2 p_{cm} \delta^3\left(\vec{C} - \vec{C}_1 - \vec{C}_2\right)$$

Mesonic (left), baryonic (right) Hagedorn Spectra



fit function: $f(m) = cm^a \exp\left(\frac{m}{T}\right)$

Exotic Hagedorn Spectra



fit function: $f(m) = cm^a \exp\left(\frac{m}{T}\right)$

Cross Section and Decay Width

- cross section for creation of HS in binary collisions

$$\sigma = \frac{\pi \left|\mathcal{M}_c\right|^2}{4m^2 p_{cm}} \tau\left(m\right)$$

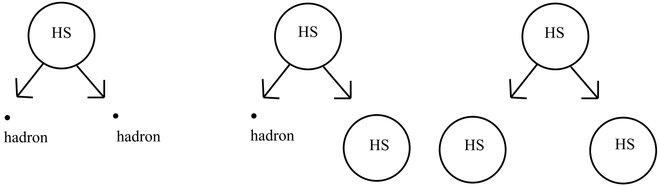
- decay width of HS in two particles only

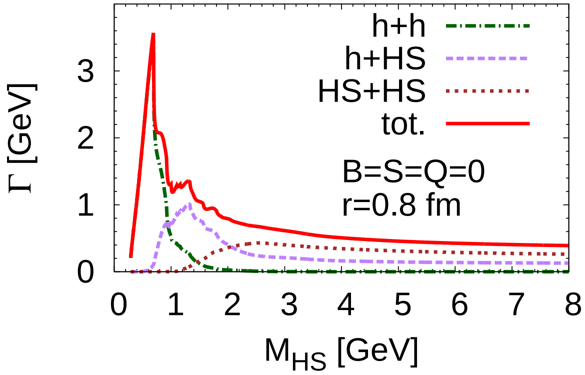
$$\Gamma = \frac{\left|\mathcal{M}_d\right|^2}{8\pi m^2} \iint \mathrm{d}m_1 \mathrm{d}m_2 \tau\left(m_1\right) \tau\left(m_2\right) p_{cm}$$

- detailed balance: $\left|\mathcal{M}_c\right|^2 = \left|\mathcal{M}_d\right|^2$

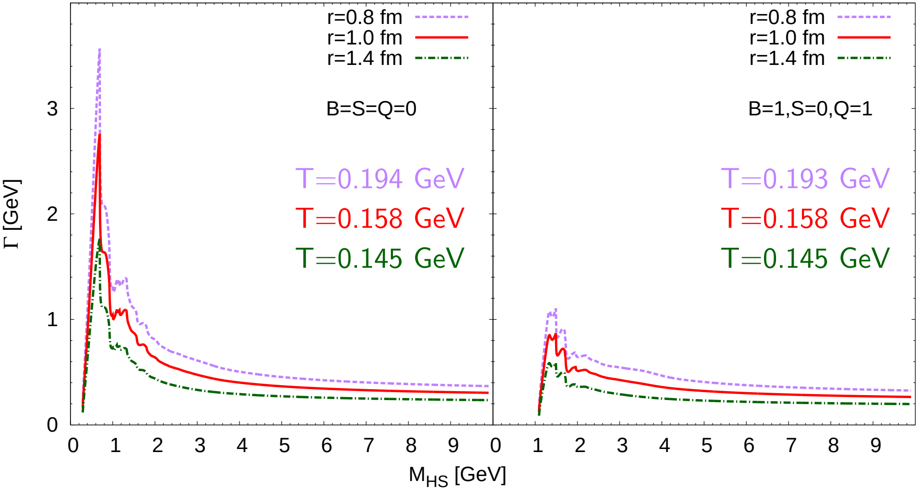
$$\Gamma\left(m\right) = \frac{\sigma}{2\pi^2 \tau\left(m\right)} \iint \mathrm{d}m_1 \mathrm{d}m_2 \tau\left(m_1\right) \tau\left(m_2\right) p_{cm}^2$$

Hagedorn state decay modes

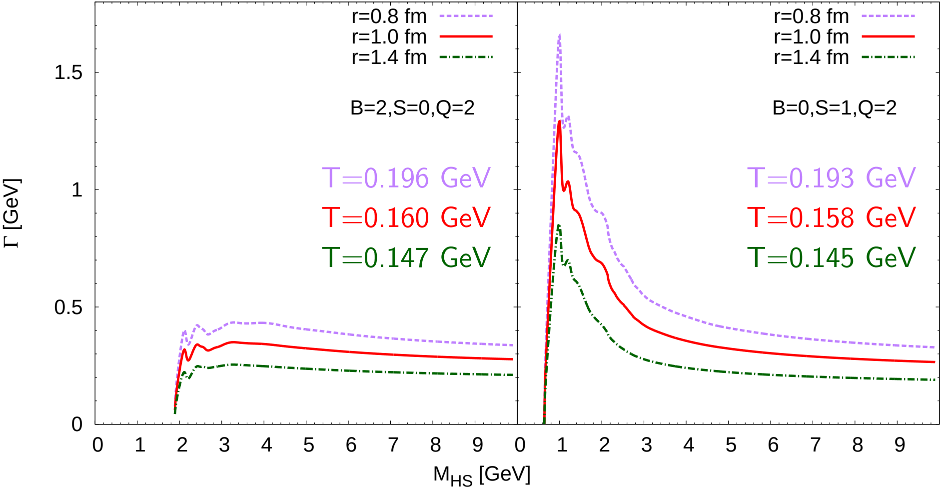




Decay Width for mesonic (left) and baryonic (right) HS

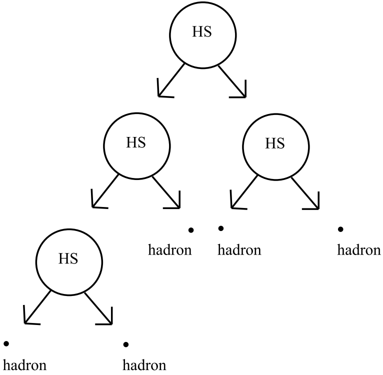


Decay Width for exotic HSs

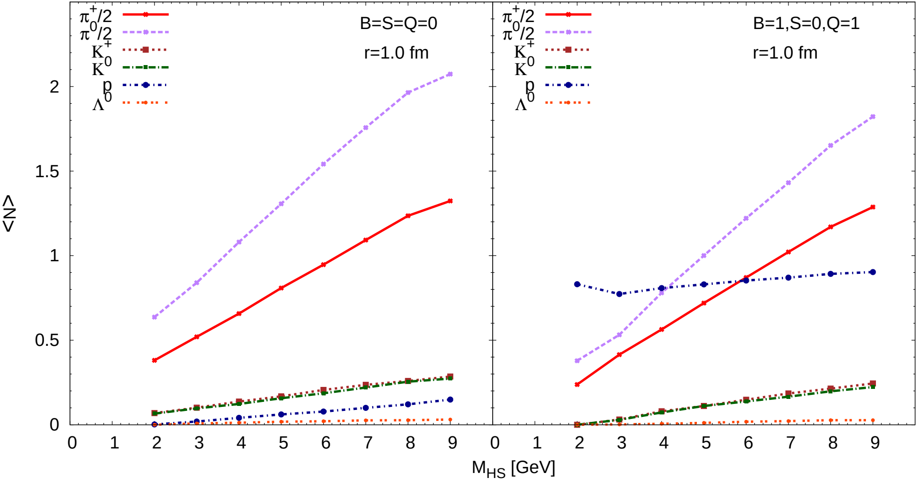


Hagedorn State Cascade Simulation

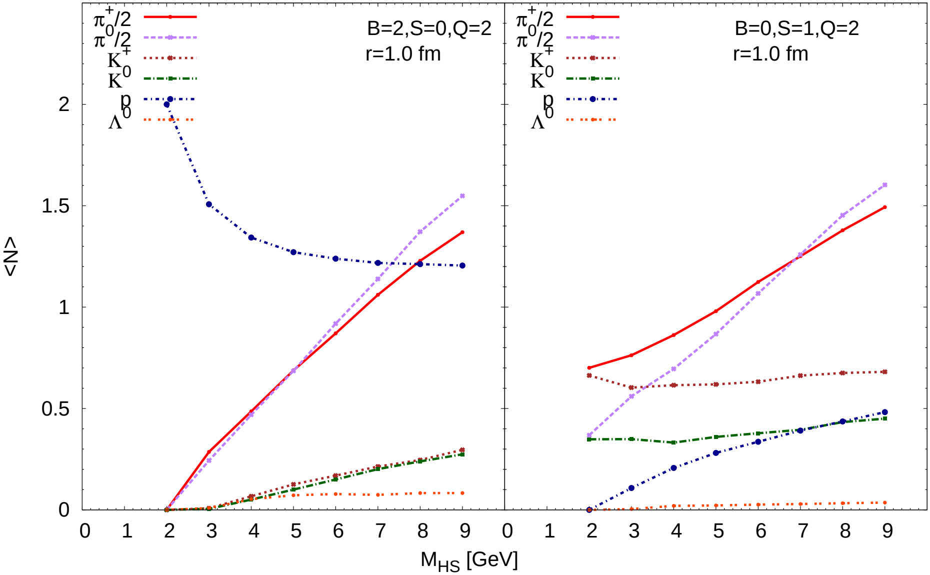
- **One** massive charged HS cascades down to hadrons (resonances)
- resonances further cascade down to stable hadrons (hadronic feeddown)



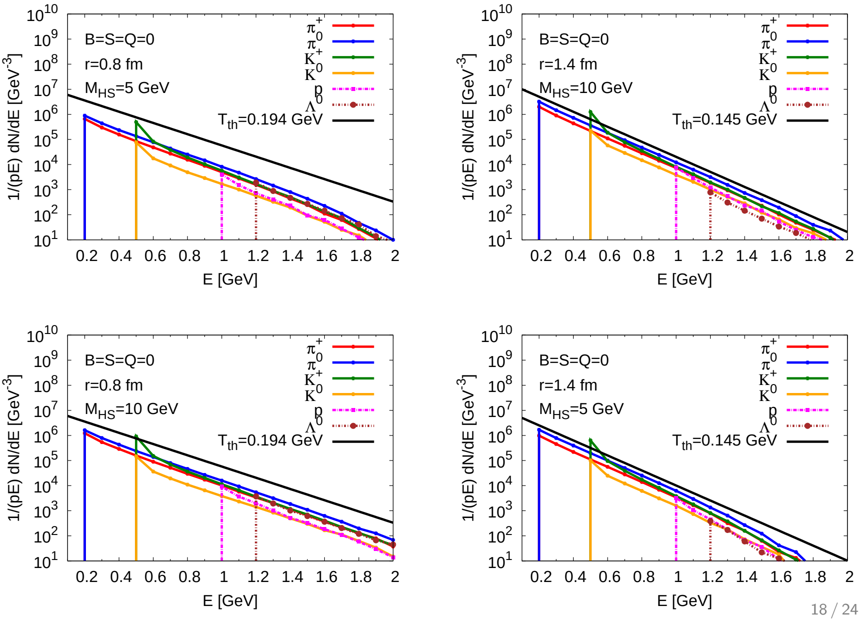
Multiplicities from mesonic (left) and baryonic (right) HS



Multiplicities from exotic HSs



Energy spectra of hadrons from Hagedorn state decay



Summary and Outlook

- chem. equilibration times in UrQMD (box) too long
- derivation of covariant formulated Hagedorn Spectrum $\tau\left(m\right)$
- presentation of full Hagedorn Spectra to show main properties
- presentation of full and detailed HS decay widths
- presentation of hadronic multiplicities from HS cascade simulations
- presentation of hadronic (thermal) energy spectra from HS cascade simulations
- use of HS to lower chem. equilibration times
- impact of HS on η/s in UrQMD

Phase-Space Approach: S. Frautschi, 1971[3]

- no use of temperature introduction needed (microcanonical approach)
- two particle system in rest frame of HS with volume V

$$\rho(m) = \frac{V}{(2\pi)^3} \frac{1}{2!} \prod_{i=1}^{2} \int \mathrm{d}m_i \, \rho(m_i) \times \int \mathrm{d}^3 p_i \, \delta\left(\sum_{i=1}^{2} E_i - m\right) \delta^{(3)}\left(\sum_{i=1}^{2} \vec{p}_i\right) \tag{1}$$

- non-covariant formulation in rest frame of HS
- mass degeneration considered by integrals over m_i

[3] doi=10.1103/PhysRevD.3.2821

Non-Covariant Formulation of Hagedorn Spectrum

- indroduce (inverse) Lorentz factors $\gamma^{-1} = m/E$

$$\tau(m) = \frac{R^3}{3\pi m} \iint \mathrm{d}m_1 \mathrm{d}m_2 \tau(m_1)\,\tau(m_2) \times \gamma_1^{-1}\gamma_2^{-1} E_1 E_2 p_{cm} \tag{2}$$

- neglect relativistic effects $\gamma_i \approx 1$ [4]

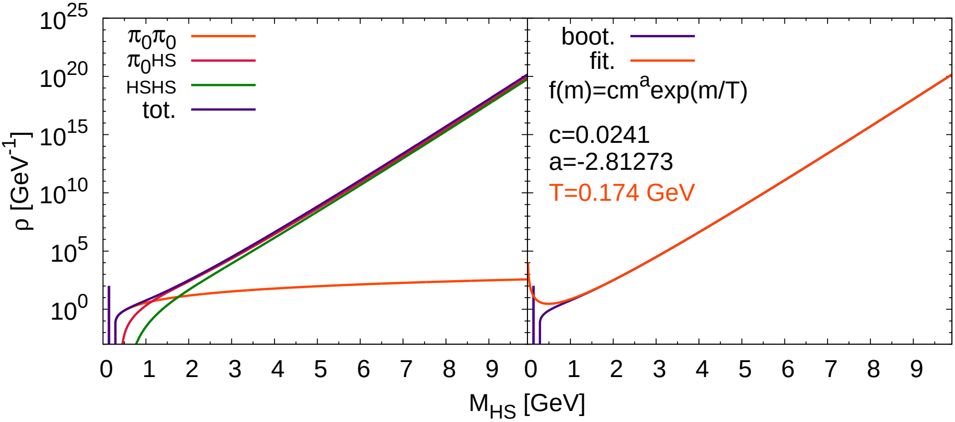
$$\rho(m) = \frac{R^3}{3\pi m} \iint \mathrm{d}m_1 \mathrm{d}m_2 \rho(m_1)\,\rho(m_2)\, E_1 E_2 p_{cm}$$

[4] S. C. Frautschi, Phys.Rev., D3:2821-2834, 1971

Toy Model: Hagedorn Spectrum

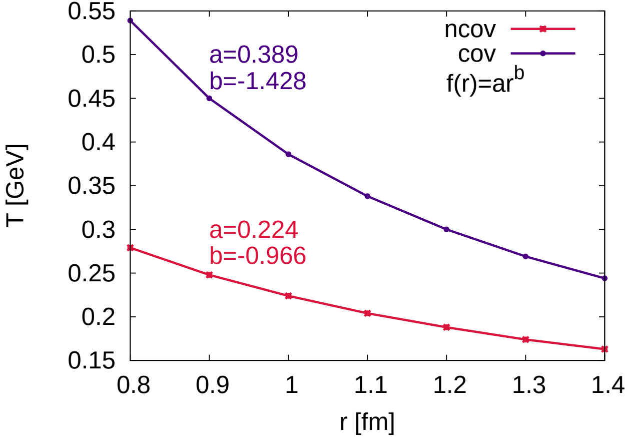
- π_0 as low mass input only

$$\rho\left(m\right) = \delta\left(m - m_{\pi_0}\right)$$



Volume Temperature Dependence

- fit Hagedorn temperatures for different radii



Toy Model Decay Width (Cov.)