

Dynamical equilibration and transport coefficients of strongly-interacting 'infinite' parton matter

Vitalii Ozvenchuk

Transport Meeting

13 December 2012

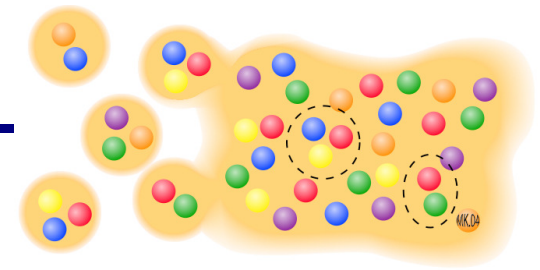


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From hadrons to partons

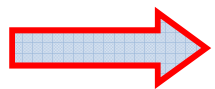


In order to study of the **phase transition** from hadronic to partonic matter – **Quark-Gluon-Plasma** –

we need a **consistent non-equilibrium (transport) model with**

- explicit **parton-parton interactions** (i.e. between quarks and gluons) beyond **strings!**
- explicit **phase transition** from **hadronic** to **partonic** degrees of freedom
- **IQCD EoS** for **partonic** phase

Transport theory: off-shell **Kadanoff-Baym** equations for the **Green-functions** $S_h^<(x,p)$ in phase-space representation for the **partonic** and **hadronic** phase



Parton-Hadron-String-Dynamics (PHSD)

QGP phase described by

Dynamical QuasiParticle Model (DQPM)

W. Cassing, E. Bratkovskaya, PRC 78 (2008) 034919;
NPA831 (2009) 215;
W. Cassing, EPJ ST 168 (2009) 3

A. Peshier, W. Cassing, PRL 94 (2005) 172301;
Cassing, NPA 791 (2007) 365; NPA 793 (2007)



DQPM spectral function

Basic idea: effective strongly-interacting quasiparticles

- massive quarks, antiquarks and gluons (q, q_{bar}, g) with **broad spectral functions**

DQPM: Peshier, Cassing, PRL 94 (2005) 172301;
Cassing, NPA 791 (2007) 365; NPA 793 (2007)

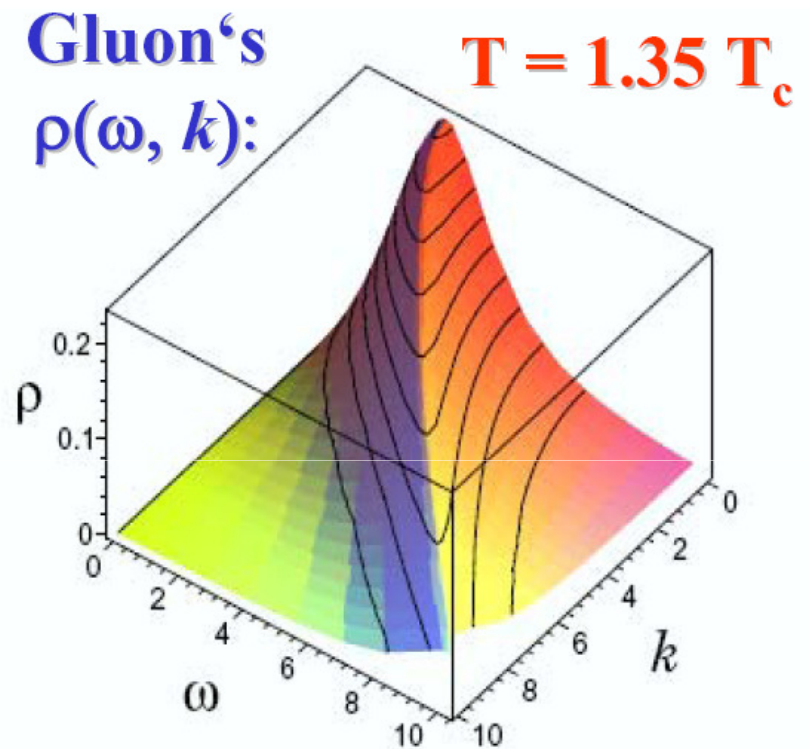
Breit-Wigner spectral function:

$$\rho(\omega, \mathbf{p}) = \frac{4\omega\Gamma}{(\omega^2 - \mathbf{p}^2 - M^2)^2 + 4\Gamma^2\omega^2} \equiv$$
$$\equiv \frac{\Gamma}{E} \left[\frac{1}{(\omega - E)^2 + \Gamma^2} - \frac{1}{(\omega + E)^2 + \Gamma^2} \right]$$

notation: $E^2 = \mathbf{p}^2 + M^2 - \Gamma^2$

□ **mass and width:**
⇒ quasiparticle properties

□ **finite width:**
⇒ two-particle correlations





DQPM running coupling

□ running coupling \Rightarrow fit to the lattice QCD results

IQCD: Kaczmarek *et al.*, PRD 70 (2004) 074505

$$\alpha_s(T) = \frac{g^2(T)}{4\pi} = \frac{12\pi}{(11N_c - 2N_f) \ln[\lambda^2(T/T_c - T_s/T_c)^2]}$$

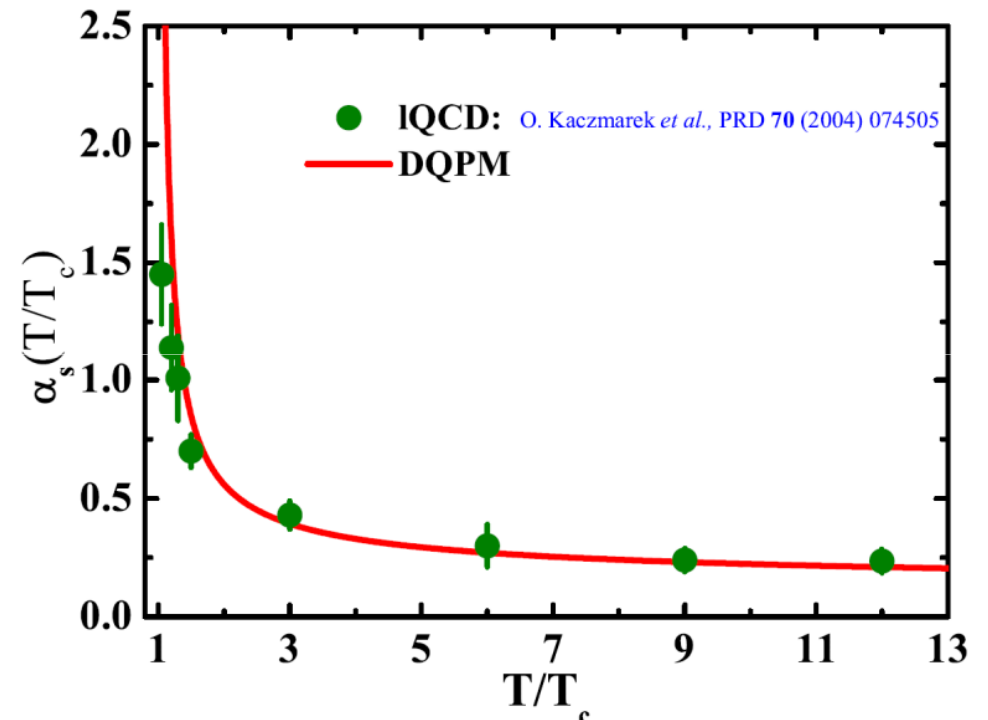
➤ number of **colors** $\Rightarrow N_c = 3$

➤ number of **flavors** $\Rightarrow N_f = 3$

➤ **3 fitting parameters**

$$\Rightarrow \lambda = 2.42 \quad T_s/T_c = 0.56$$

$$c = 14.4$$





DQPM mass and width

□ **spectral function:** $\rho(\omega, \mathbf{p}) = \frac{4\omega\Gamma}{(\omega^2 - \mathbf{p}^2 - M^2)^2 + 4\Gamma^2\omega^2}$

quark (antiquark):

$$M_{q(\bar{q})}^2(T) = \frac{N_c^2 - 1}{8N_c} g^2 \left(T^2 + \frac{\mu_q^2}{\pi^2} \right)$$

$$\Gamma_{q(\bar{q})}(T) = \frac{1}{3} \frac{N_c^2 - 1}{2N_c} \frac{g^2 T}{8\pi} \ln \left(\frac{2c}{g^2} + 1 \right)$$

gluon:

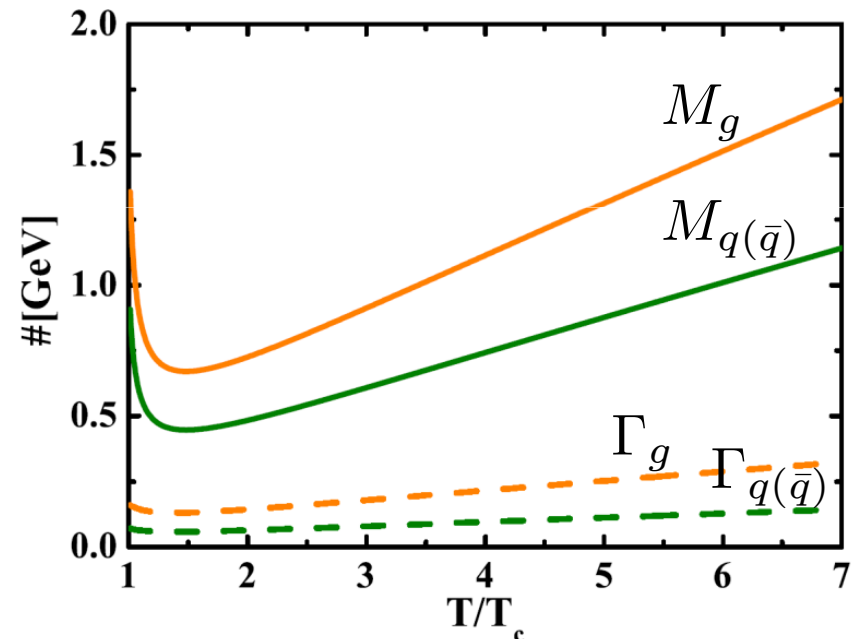
$$M_g^2(T) = \frac{g^2}{6} \left(\left(N_c + \frac{N_f}{2} \right) T^2 + \frac{N_c}{2} \sum_q \frac{\mu_q^2}{\pi^2} \right)$$

$$\Gamma_g(T) = \frac{1}{3} N_c \frac{g^2 T}{8\pi} \ln \left(\frac{2c}{g^2} + 1 \right)$$

Peshier, PRD 70 (2004) 034016

□ **high temperature regime**
⇒ **one-loop perturbative QCD results**

□ **mass and width define quasiparticle properties**





DQPM thermodynamics ($N_f=3$)

□ entropy: $s = \frac{\partial P}{\partial T} \Rightarrow$ pressure

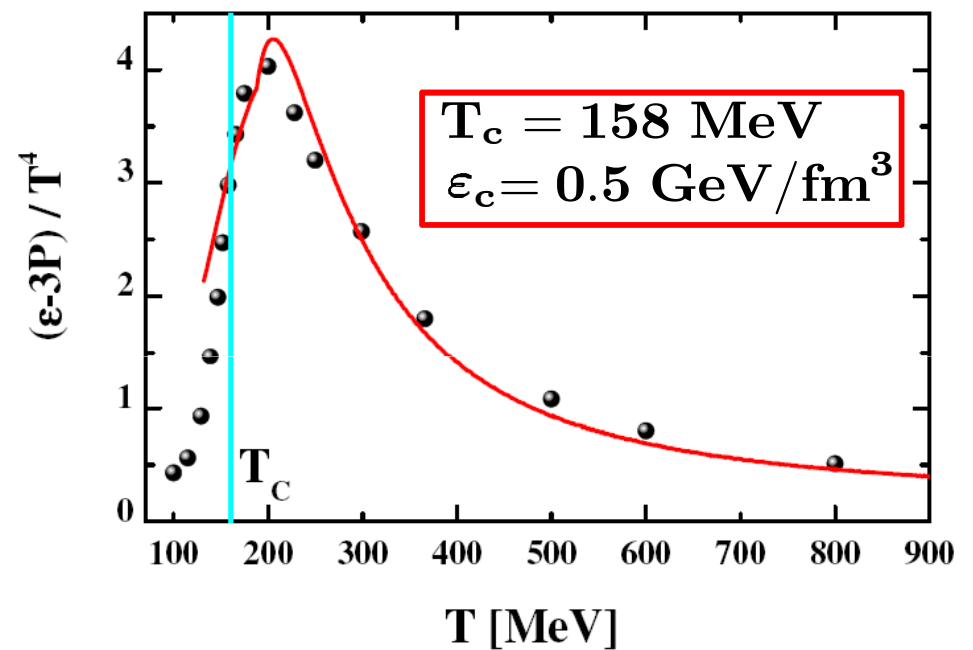
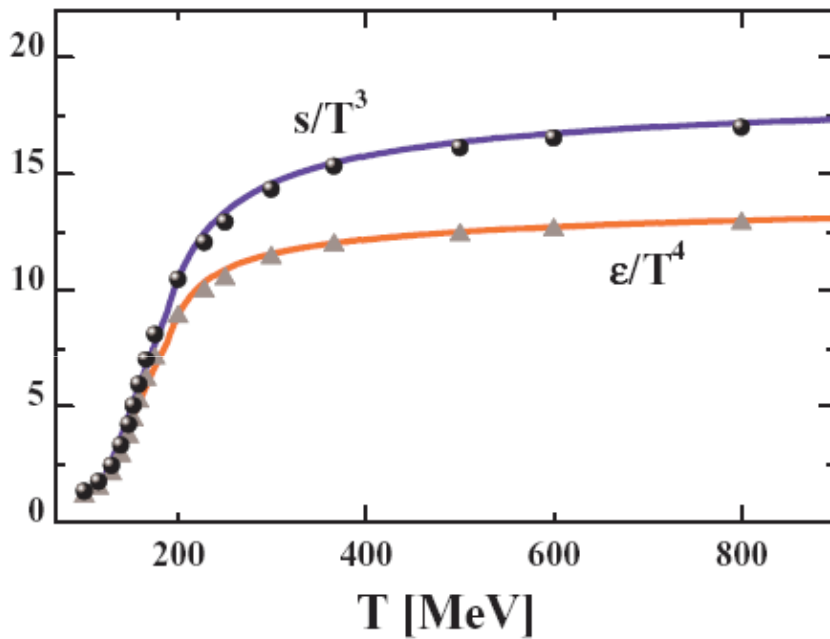
□ energy density: $\varepsilon = Ts - P$

□ interaction measure:

$$W = \varepsilon - 3P = Ts - 4P$$

IQCD: Wuppertal-Budapest group

Y.Aoki et al., JHEP 0906 (2009) 088



DQPM gives a good description of IQCD results !



DQPM overview

□ fit to **lattice QCD** results:

⇒ thermodynamics quantities (pressure, entropy density, energy density) in equilibrium

⇒ running coupling:

$$\alpha_s(T) = \frac{g^2(T)}{4\pi} = \frac{12\pi}{(11N_c - 2N_f) \ln[\lambda^2(T/T_c - T_s/T_c)^2]}$$

□ **DQPM** provides:

⇒ spectral function (mass and width) ⇒ **off-shell** quasiparticle properties

$$\rho(\omega, \mathbf{p}) = \frac{4\omega\Gamma}{(\omega^2 - \mathbf{p}^2 - M^2)^2 + 4\Gamma^2\omega^2}$$

⇒ mean fields (1 PI) for quarks (antiquarks) and gluons as well as effective 2-body interactions (2 PI)



Our goals

- study of the partonic system **out of equilibrium** (beyond the **DQPM**)
 - **dynamical equilibration** of QGP within the non-equilibrium off-shell **PHSD** transport approach
 - influence of the partonic **elastic** and **inelastic** cross sections;

- study of the **thermal** properties of **equilibrated** partonic system in **PHSD**
 - transport coefficients (**shear and bulk viscosities**) of **strongly-interacting** partonic matter;
 - particle number fluctuations (**scaled variance, skewness, kurtosis**).



Parton interactions in PHSD

degrees of freedom in PHSD:

colored quarks (u, d, s), antiquarks (ubar, dbar, sbar) and gluons

interaction processes:

(quasi)-elastic

$$q(m_1) + q(m_2) \rightarrow q(m_3) + q(m_4)$$

$$q + \bar{q} \rightarrow q + \bar{q}$$

$$\bar{q} + \bar{q} \rightarrow \bar{q} + \bar{q}$$

$$g + q \rightarrow g + q$$

$$g + \bar{q} \rightarrow g + \bar{q}$$

$$g + g \rightarrow g + g$$

inelastic

$$q + \bar{q} \rightarrow g$$

$$g \rightarrow q + \bar{q}$$

basic processes
for the **chemical**
equilibration \Leftrightarrow
flavor exchange

$$\text{e.g.} \Rightarrow u + \bar{u} \leftrightarrow g \dots g \leftrightarrow s + \bar{s}$$

~~$$q + \bar{q} \leftrightarrow gg + gg$$

$$gg \leftrightarrow gg + gg$$~~

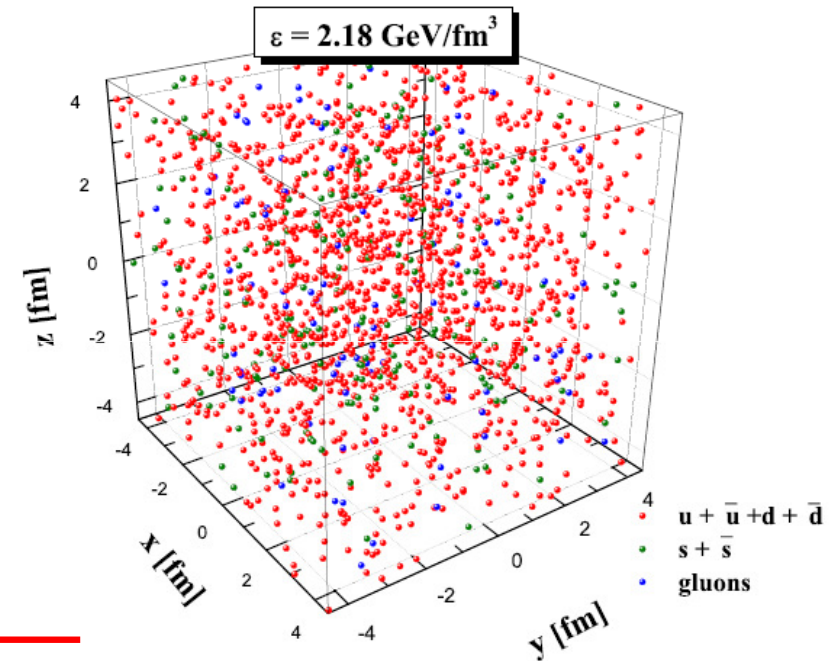
suppressed (<1%)
due to the **large**
mass of gluon



Initialization

□ cubic box:

- periodic boundary conditions;
- size is fixed to 9^3 fm^3 ;
- light and strange quarks, antiquarks and gluons;
- various values for the energy density and quark chemical potential.



□ initialization is:

- ⇒ **close** to the thermal equilibrium with thermal distribution for the momenta;
- ⇒ **far out** of the chemical equilibrium due to the strangeness suppression:

$$N_u \div N_d \div N_s = 3 \div 3 \div 1$$



Partial widths

- **DQPM** provides the **total width Γ** of the dynamical quasiparticles

$$\Gamma_{total} = \Gamma_{elastic} + \Gamma_{inelastic}$$

- **partial widths - (quasi)-elastic and inelastic - cannot** be defined from the **DQPM**

for gluons: $\Gamma_g^{DQPM}(\varepsilon) = \Gamma_{g \rightarrow q + \bar{q}}^{inelastic}(\varepsilon) + \Gamma_{gg}^{elastic}(\varepsilon) + \Gamma_{gq}^{elastic}(\varepsilon) + \Gamma_{g\bar{q}}^{elastic}(\varepsilon)$

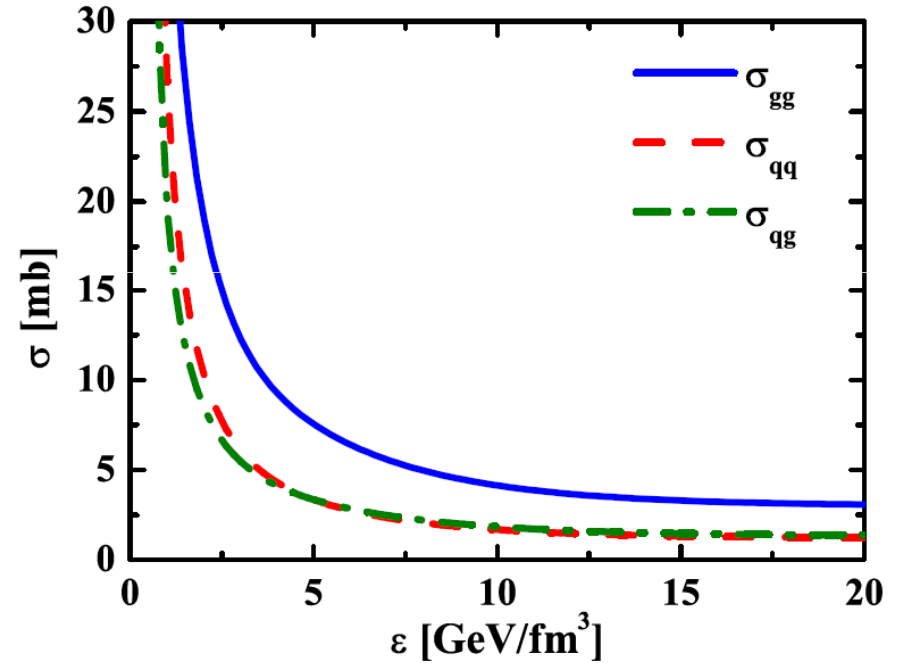
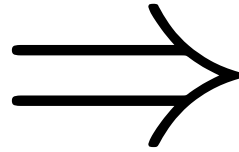
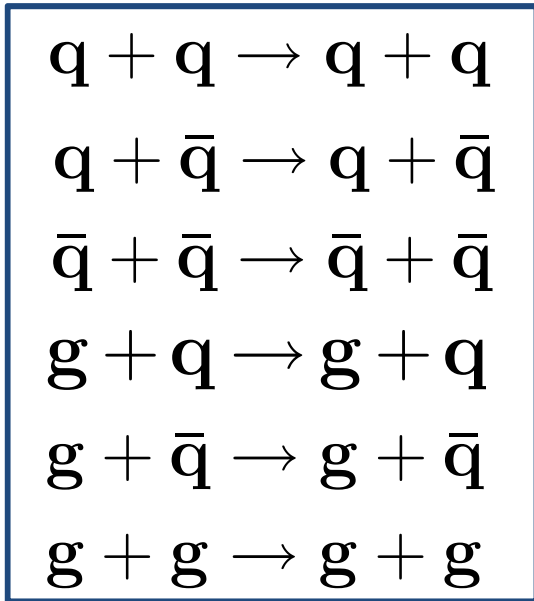
for quarks: $\Gamma_j^{DQPM}(\varepsilon) = \Gamma_{\bar{q}q \rightarrow g}^{inelastic}(\varepsilon) + \Gamma_{jg}^{elastic}(\varepsilon) + \Gamma_{jq}^{elastic}(\varepsilon) + \Gamma_{j\bar{q}}^{elastic}(\varepsilon)$
 $j = q, \bar{q}$

- obtain the **partial widths** \Leftrightarrow **cross sections** for different channels from the **PHSD** simulations in the box
- **final check** \Rightarrow reproduce the **IQCD EoS** within **PHSD** in the box



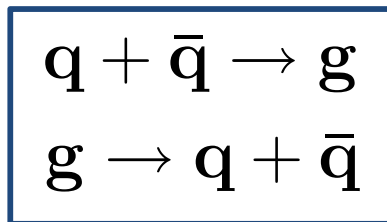
Parton cross sections in PHSD

(Quasi)-elastic cross sections



Inelastic channels

Breit-Wigner cross section



$$\Rightarrow \sigma_{q\bar{q} \rightarrow g}(\epsilon) = \frac{2}{4} \frac{4\pi s \Gamma_{g \rightarrow q+\bar{q}}^2}{(s - M_g^2(\epsilon))^2 + s \Gamma_g^2} / P_{rel}^2$$

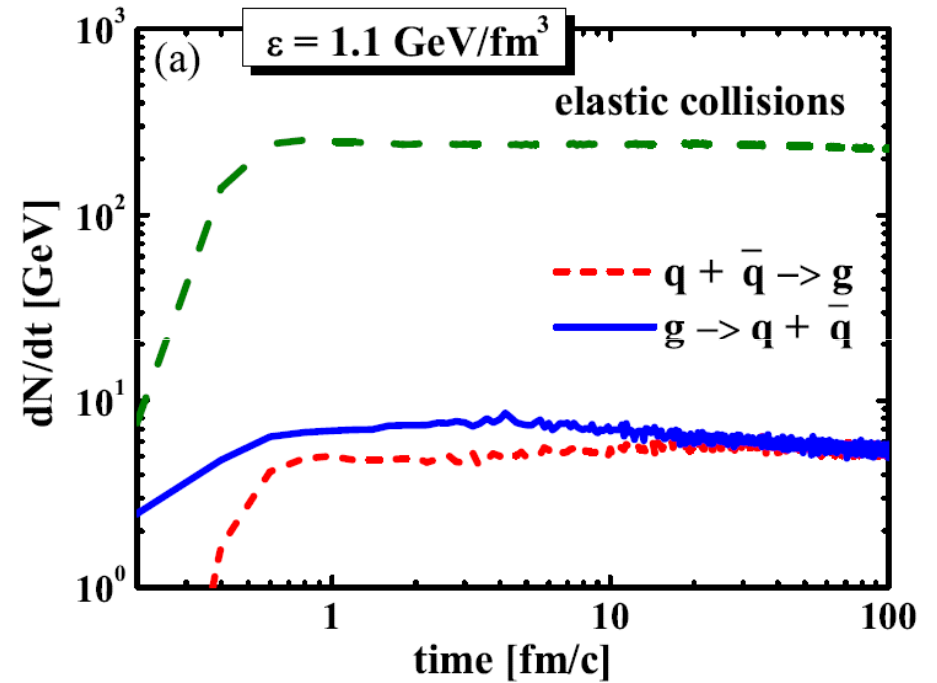
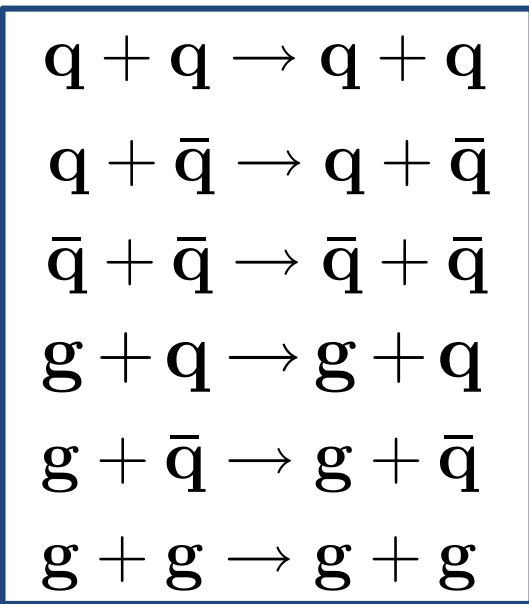


Detailed balance

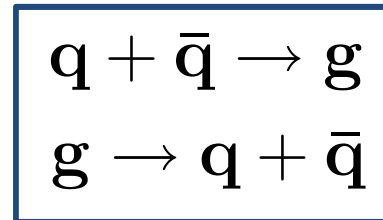
□ reactions rates are practically **constant** and obey detailed balance for

- gluon splitting
- quark + antiquark fusion

□ (quasi)-elastic collisions lead to the **thermalization** of all particle species



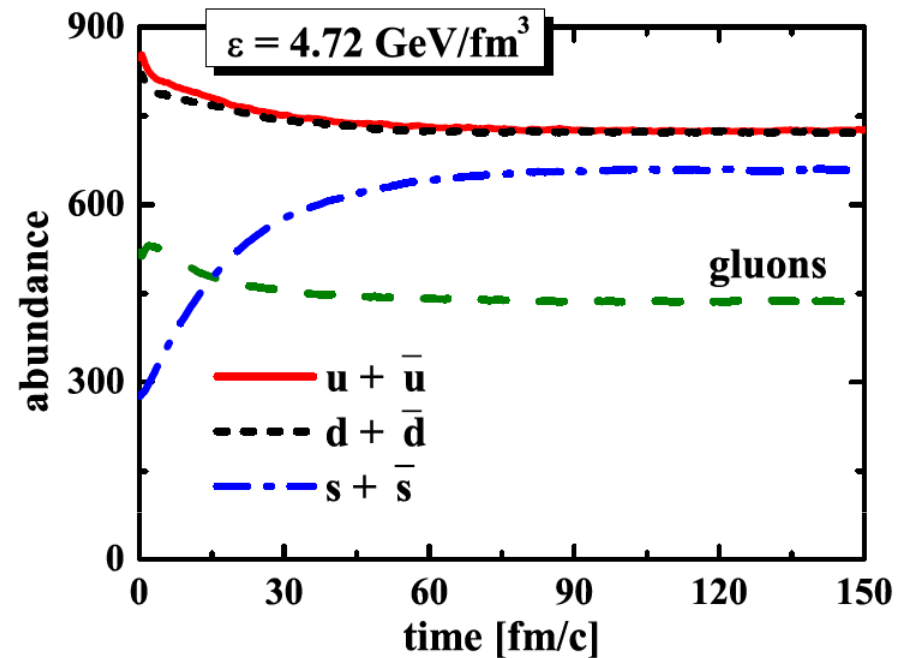
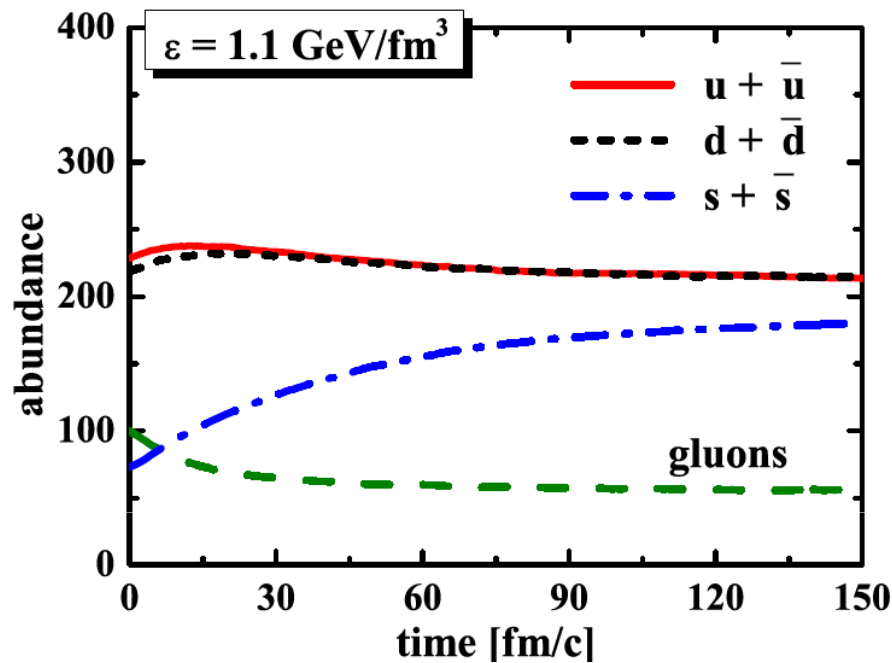
□ the numbers of partons dynamically reach their **equilibrium values** through the **inelastic collisions**





Chemical equilibrium

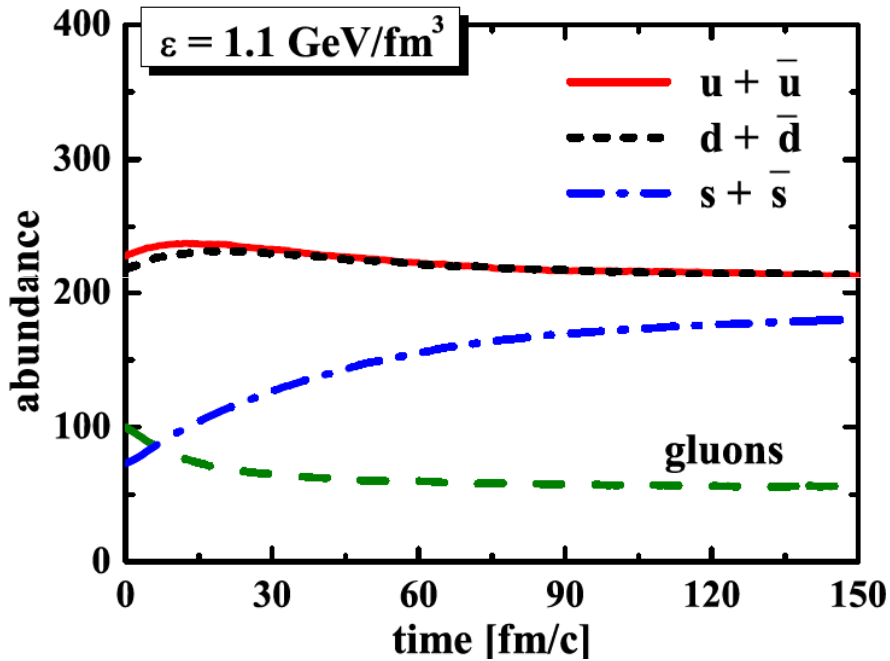
- a sign of **chemical equilibrium** is the **stabilization** of the numbers of partons of the different species in time



- **final abundancies** vary with energy density



Chemical equilibration of strange partons

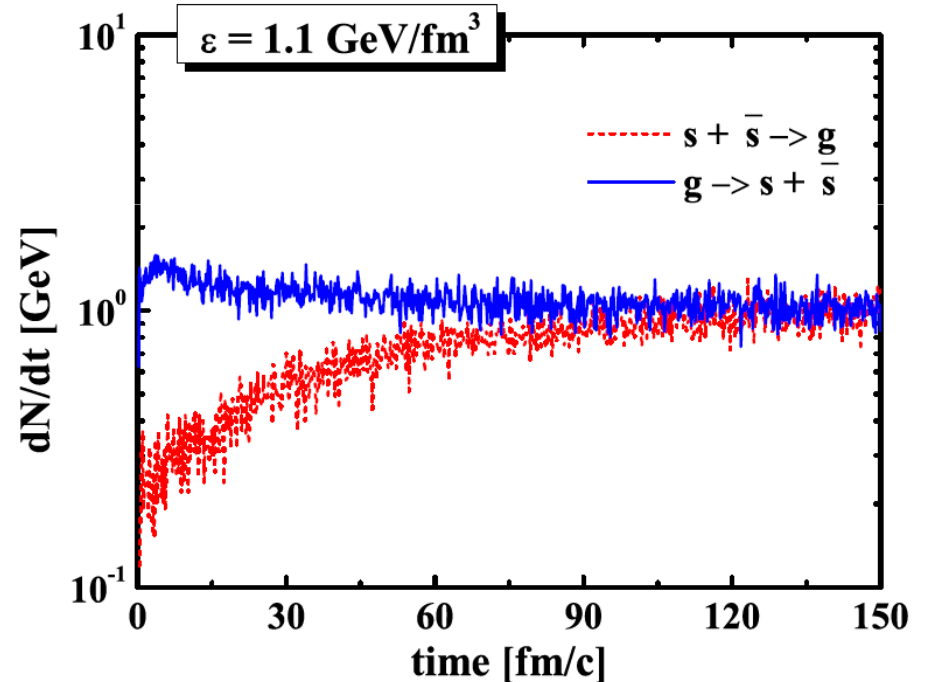


$$N_u \div N_d \div N_s = 3 \div 3 \div 1$$

□ initial rate for $s + \bar{s} \rightarrow g$ is suppressed by a factor of 9

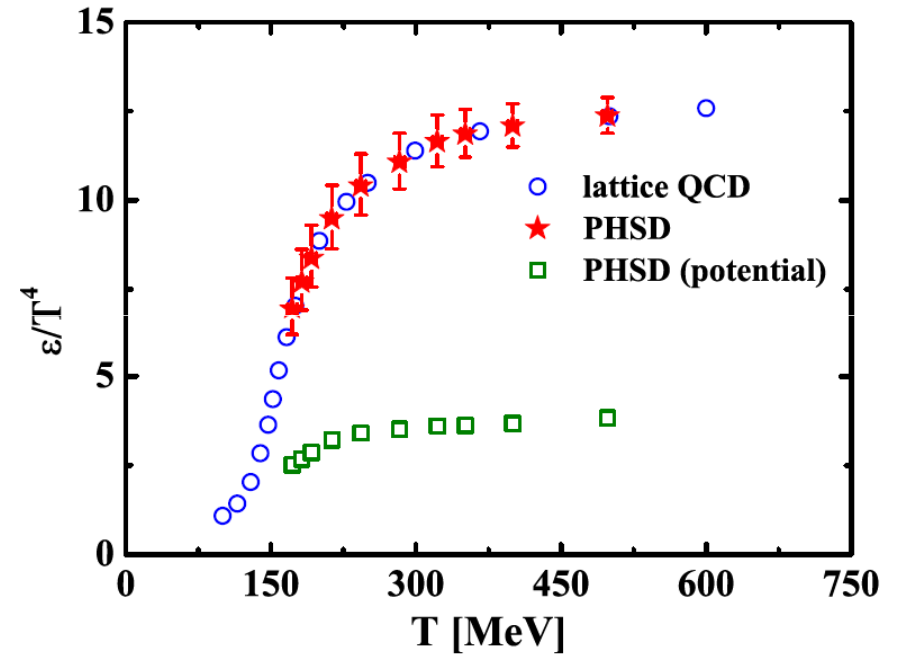
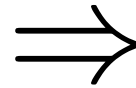
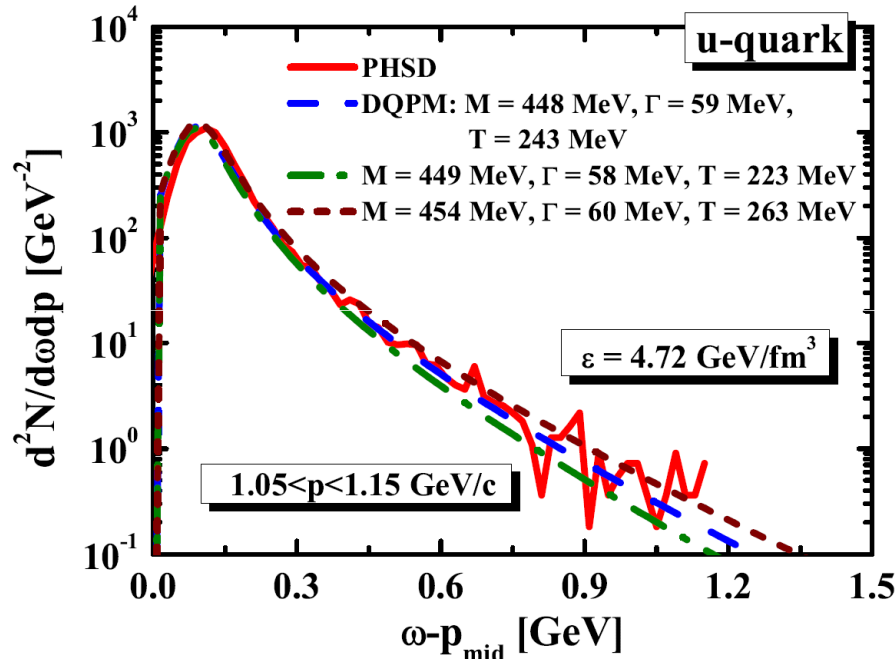
□ slow increase of the total number of strange quarks and antiquarks

⇒ long equilibration times through inelastic processes involving strange partons





Equation of state



□ equation of state implemented in PHSD

- is well in agreement with the DQPM and the IQCD results;
- includes the potential energy density from the DQPM.

IQCD data: Borsanyi et al., JHEP 1009, 073 (2010); JHEP 1011, 077 (2010)



Shear viscosity (Kubo formalism)

□ Kubo formula for the shear viscosity:

$$\eta = \frac{1}{T} \int d^3r \int_0^\infty dt \langle \pi^{xy}(\mathbf{0}, 0) \pi^{xy}(\mathbf{r}, t) \rangle$$

Kubo, J. Phys. Soc. Japan 12, 570 (1957);
Rep. Prog. Phys. 29, 255 (1966).

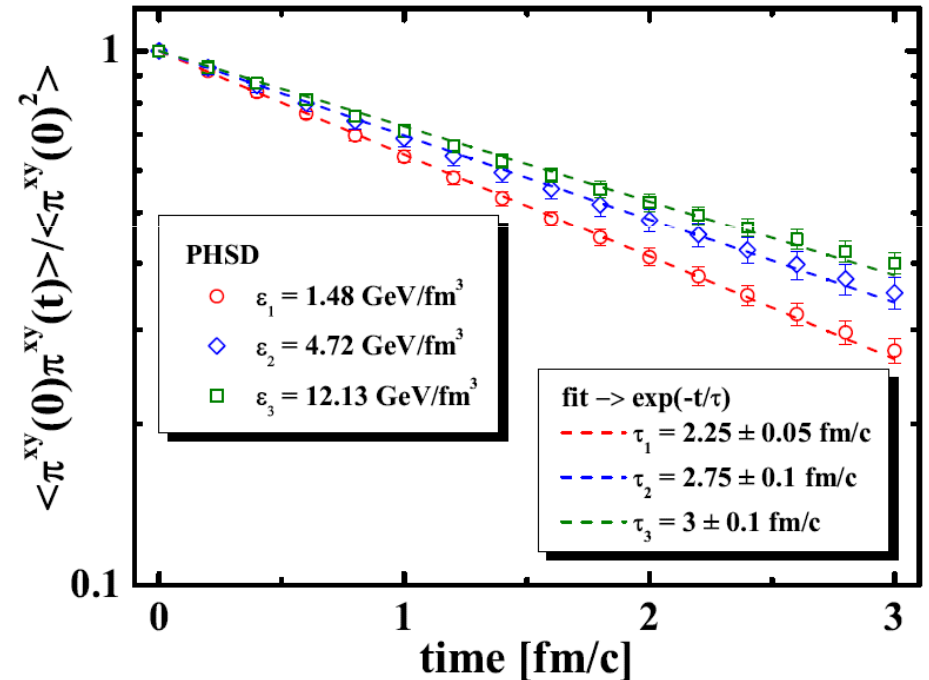
□ shear component (traceless part):

$$\pi^{xy}(\mathbf{r}, t) = \int \frac{d^3p}{(2\pi)^3} \frac{p^x p^y}{E} f(\mathbf{r}, \mathbf{p}, t)$$

□ test-particles ansatz $\Rightarrow \pi^{xy} = \frac{1}{V} \sum_{j=1}^N \frac{p_j^x p_j^y}{E_j}$

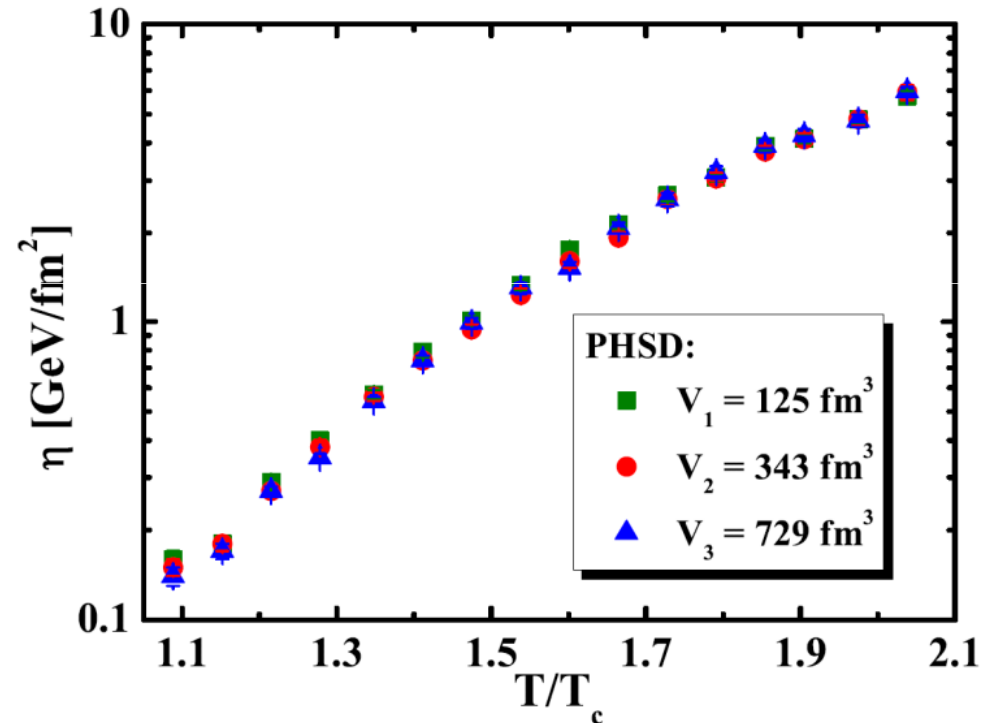
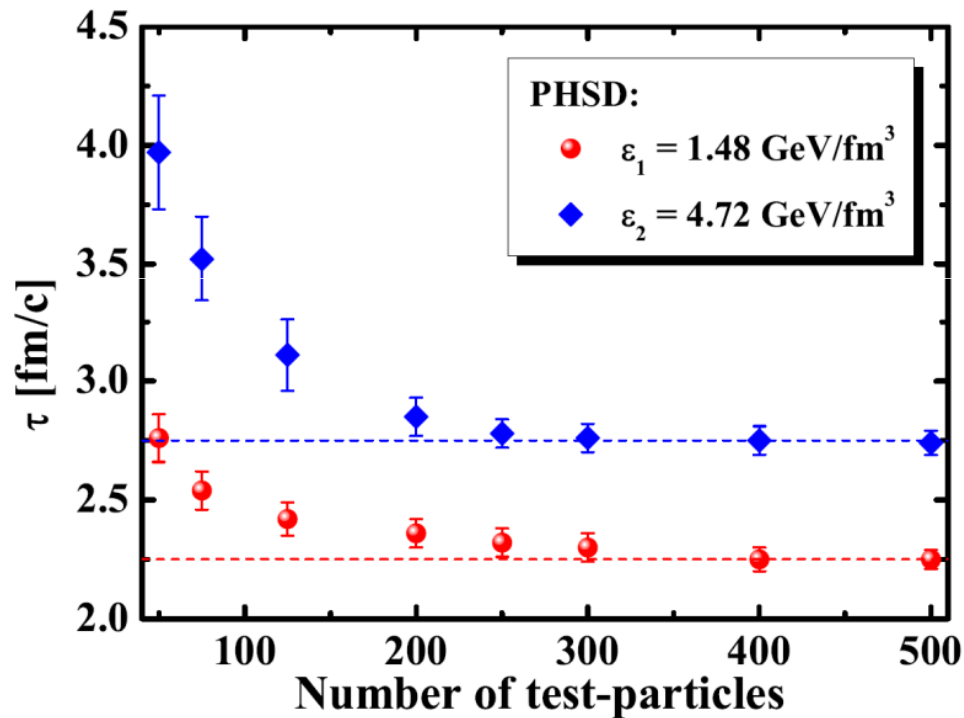
□ correlation functions are empirically found to decay exponentially in time:

$$\langle \pi^{xy}(0) \pi^{xy}(t) \rangle = \langle \pi^{xy}(0) \pi^{xy}(0) \rangle \exp\left(-\frac{t}{\tau}\right) \Rightarrow \eta = \frac{V}{T} \langle \pi^{xy}(0)^2 \rangle \tau$$





Volume and number of TP dependencies



- relaxation time depends on the number of test-particles
⇒ reaches the **constant** value for **large** number of TP
- shear viscosity **does not** depend on the volume of the system



Relaxation time approximation

□ starting hypothesis:

$$C[f] = -\frac{f - f^{eq}}{\tau}$$

τ - relaxation time

$$\tau = \Gamma^{-1}$$

□ shear and bulk viscosities assume the following expressions:

$$\eta = \frac{1}{15T} \sum_a \int \frac{d^3p}{(2\pi)^3} \frac{|\mathbf{p}|^4}{E_a^2} \tau_a(E_a) f_a^{eq}(E_a/T)$$

$$\zeta = \frac{1}{9T} \sum_a \int \frac{d^3p}{(2\pi)^3} \frac{\tau_a(E_a)}{E_a^2} [(1-3v_s^2)E_a^2 - m_a^2] f_a^{eq}(E_a/T)$$

Hosoya, Kajantie, Nucl. Phys. B 250, 666 (1985); Gavin, Nucl. Phys. A 435, 826 (1985);
Chakraborty, Kapusta, Phys. Rev. C 83, 014906 (2011).

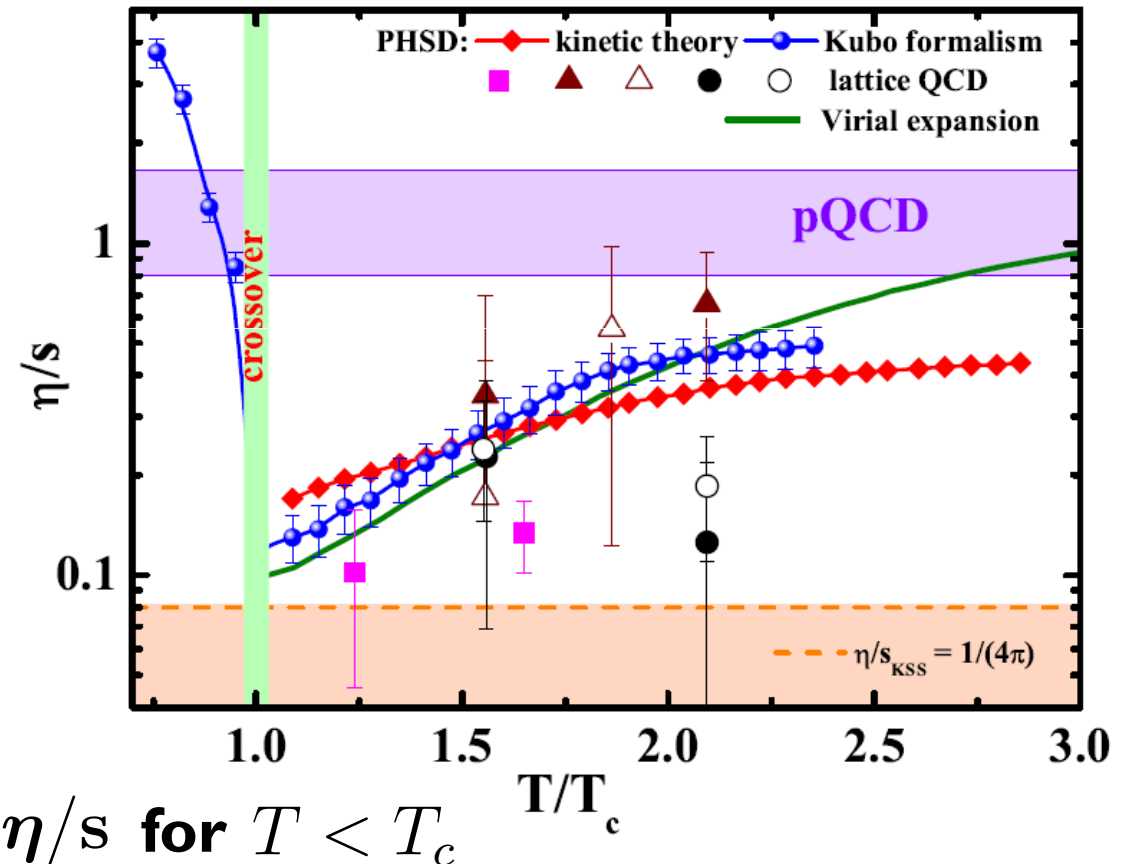
□ in numerical simulations \Rightarrow test-particle ansatz:

$$\eta = \frac{1}{15TV} \sum_{i=1}^N \frac{|\mathbf{p}_i|^4}{E_i^2} \Gamma_i^{-1}$$

$$\zeta = \frac{1}{9TV} \sum_{i=1}^N \frac{\Gamma_i^{-1}}{E_i^2} [(1-3v_s^2)E_i^2 - m_i^2]^2$$

Kubo \approx RTA

- **minimum** close to the **critical temperature**
- **pQCD limit** at higher temperatures



- **fast increase** of the ratio η/s for $T < T_c$
 - \Rightarrow **lower** interaction rate of the **hadronic system**;
 - \Rightarrow **smaller** number of degrees of freedom (or **entropy density**).

QGP in PHSD \Rightarrow strongly-interacting liquid



Bulk viscosity (mean-field effects)

bulk viscosity with mean-field effects:

Chakraborty, Kapusta, Phys. Rev. C 83, 014906 (2011).

$$\zeta = \frac{1}{TV} \sum_{i=1}^N \frac{\Gamma_i^{-1}}{E_i^2} \left[\left(\frac{1}{3} - v_s^2 \right) |\mathbf{p}|^2 - v_s^2 \left(m_i^2 - T^2 \frac{dm_i^2}{dT^2} \right) \right]^2$$

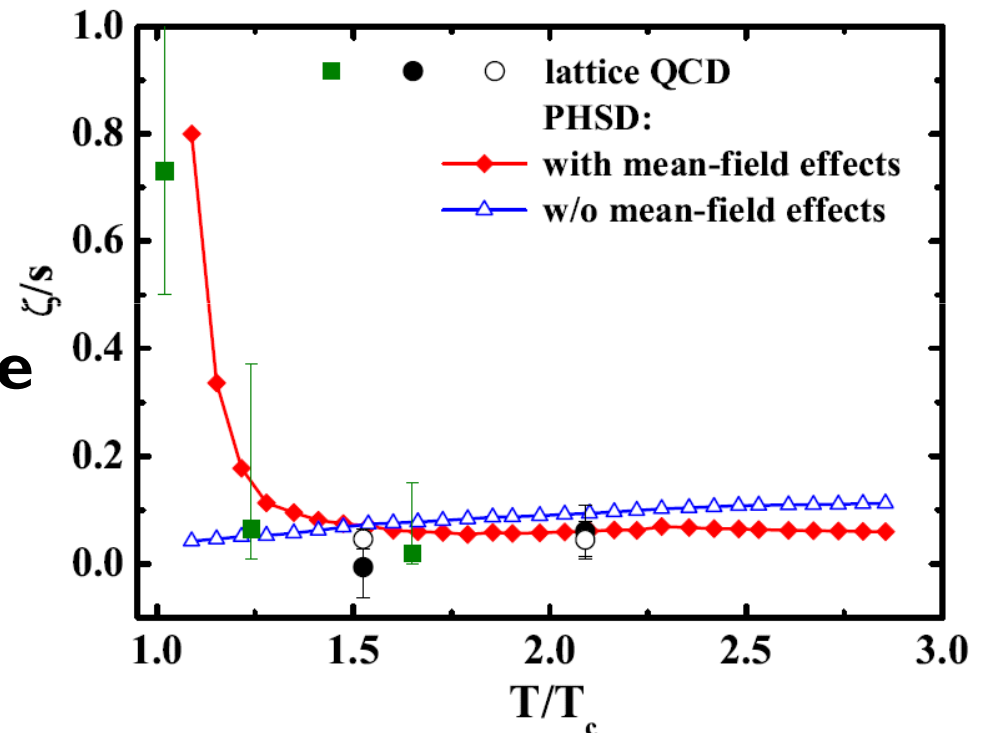
DQPM expressions for masses:

$$m_q^2 = \frac{1}{3} g^2 T^2, \quad m_g^2 = \frac{3}{4} g^2 T^2$$

significant rise in the vicinity of critical temperature

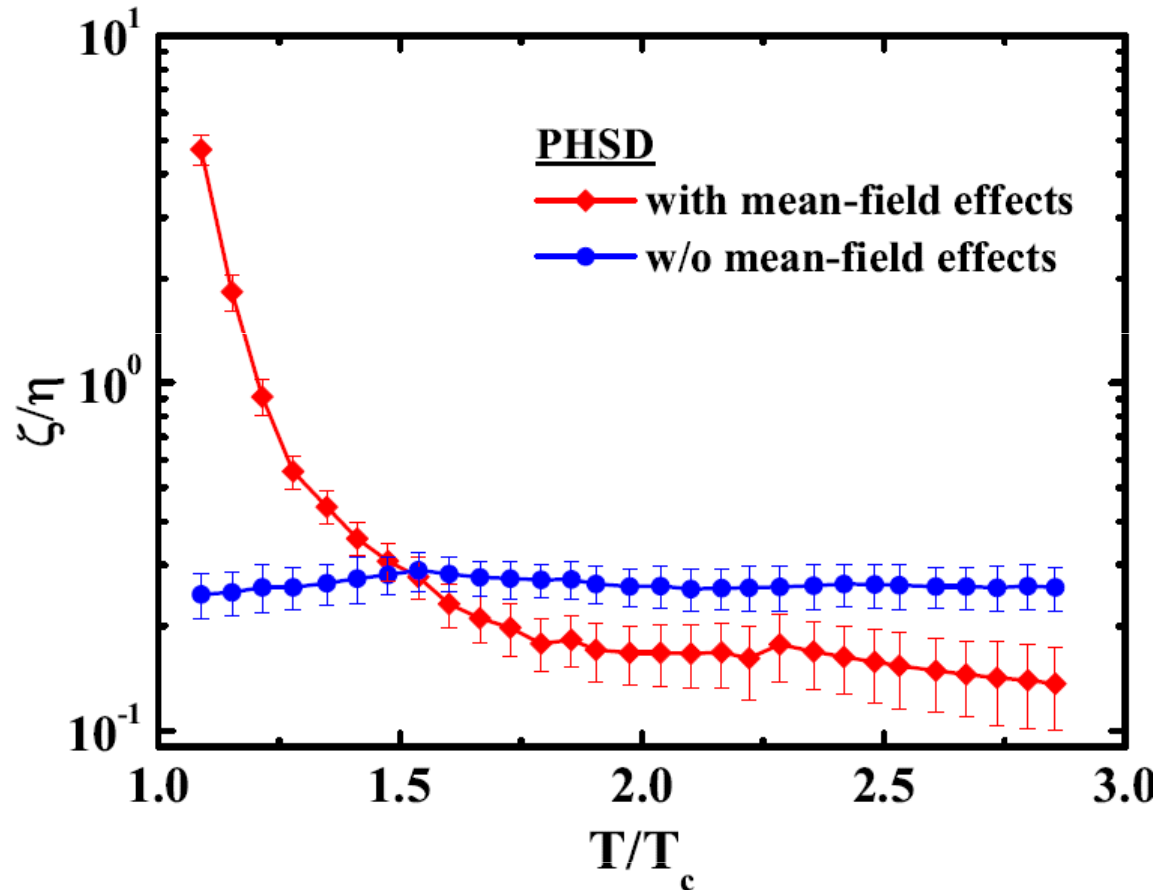
in line with the ratio from the IQCD calculations

Meyer, Phys. Rev. Lett. 100, 162001 (2008);
Sakai, Nakamura, Pos LAT2007, 221 (2007).





Bulk to shear viscosity ratio



□ **without** mean-field effects:

⇒ almost temperature **independent** behavior

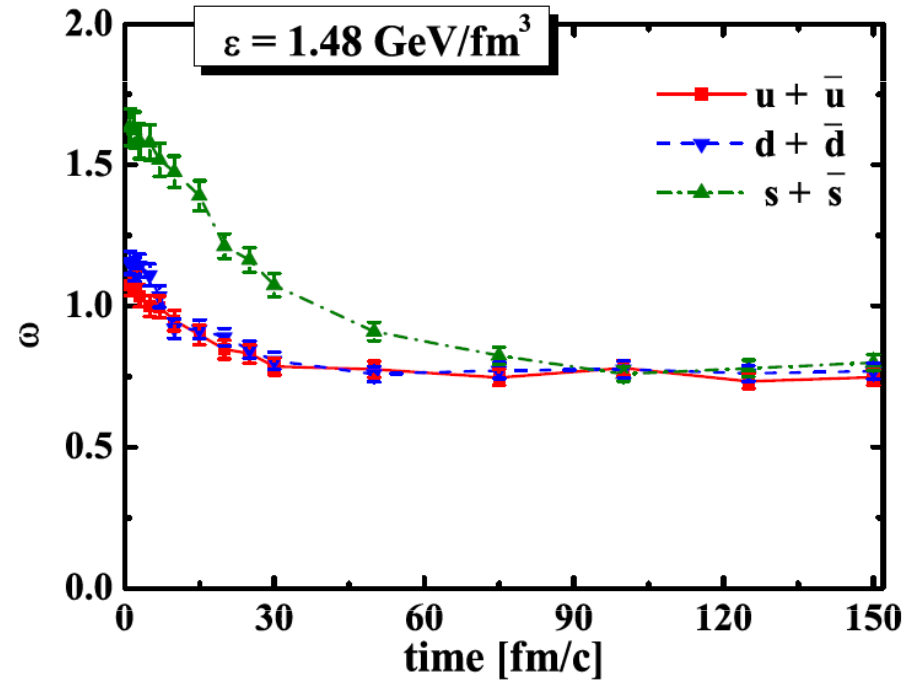
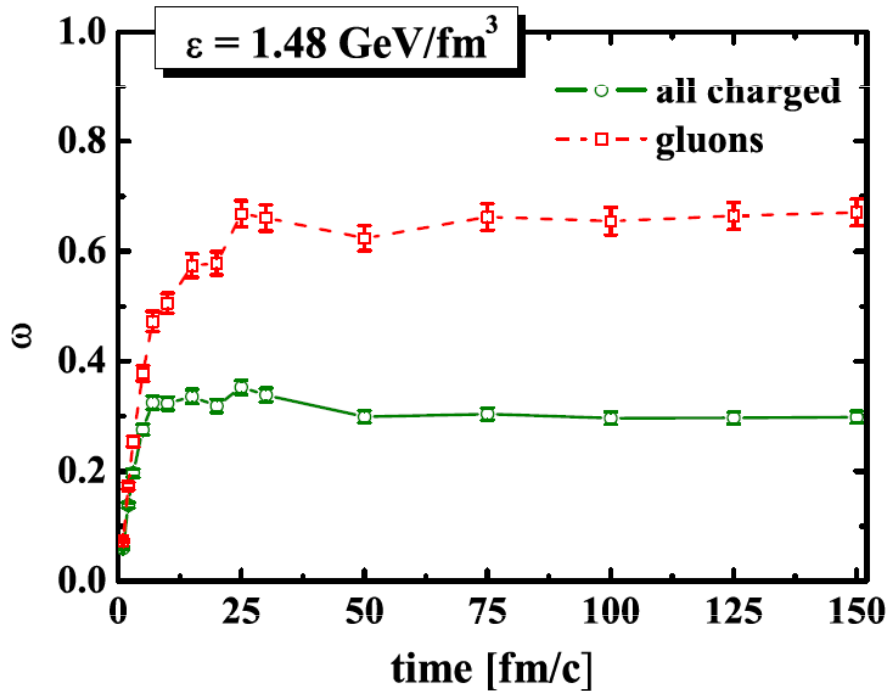
□ **with** mean-field effects:

⇒ **strong increase** close to the **critical** temperature



Scaled variance

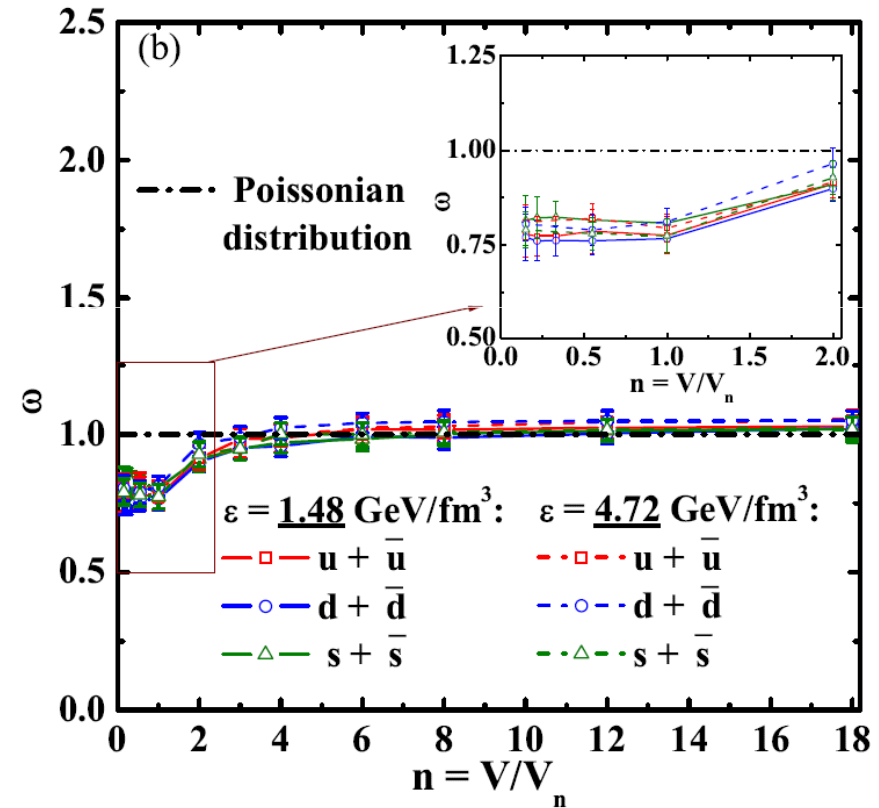
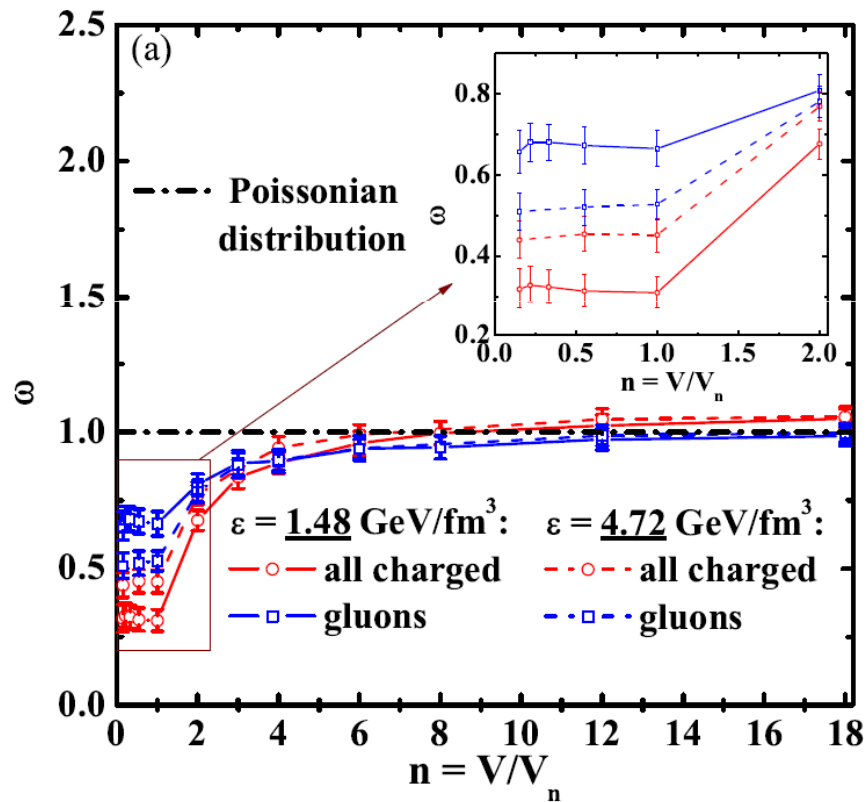
□ scaled variance: $\omega = \frac{\sigma^2}{\mu}$, $\sigma^2 = \frac{1}{N-1} \sum_{i=1}^N (x_i - \mu)^2$



- scaled variances reach a **plateau** in time for all observables
- equilibrium values are **less than 1** for all observables \Rightarrow **MCE**
- particle number fluctuations are **flavor blind**



Cell dependence of scaled variance



□ **impact** of total energy conservation in the **sub-volume V_n** is **less** than in the **total volume V**

⇒ $\omega \cong 1$ for all scaled variances for **large number** of cells ⇒ **GCE**

□ for **larger** box sizes by up to about a **factor of 8** ($n \approx 0.15$)

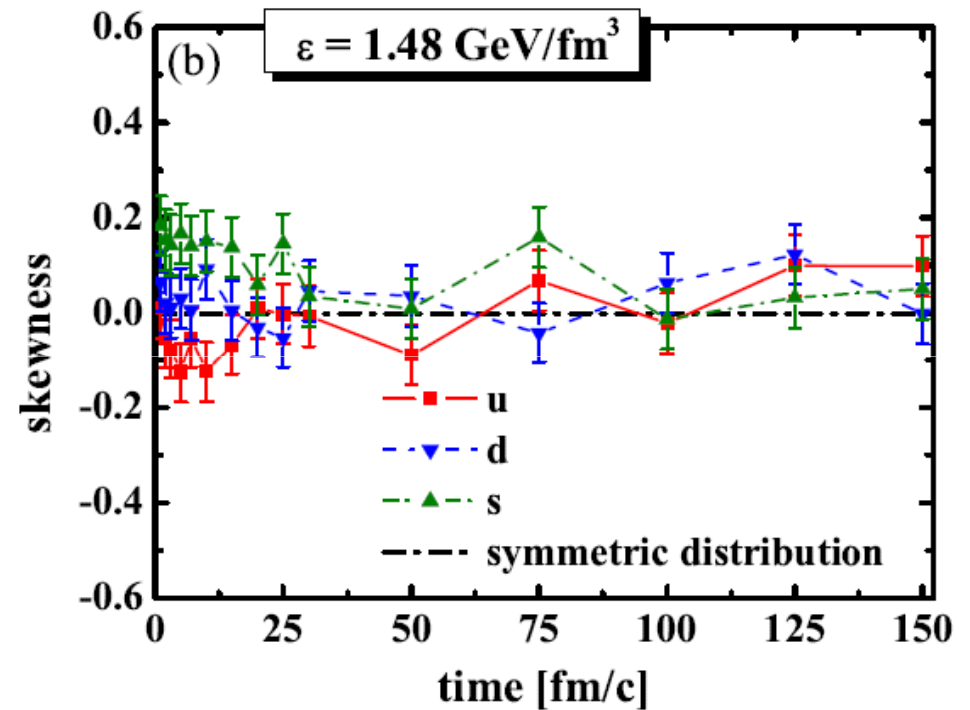
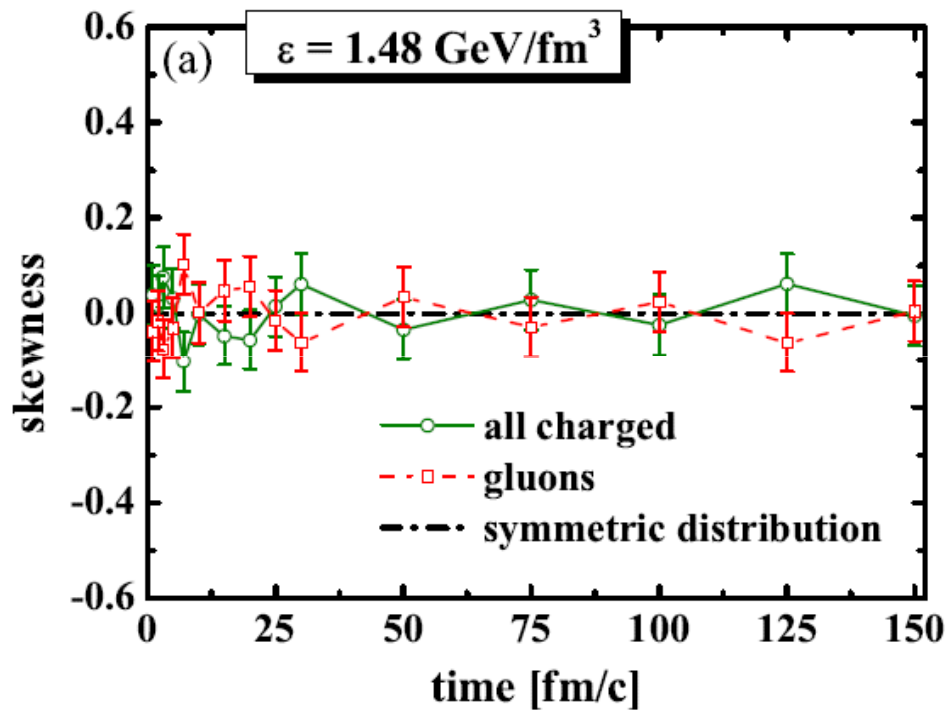
⇒ scaled variances reach the **continuum limit**



Skewness

□ skewness: $g_1 = \frac{m_3}{m_2^{3/2}} = \frac{m_3}{\sigma^3}$, $m_3 = \frac{1}{N} \sum_{i=1}^N (x_i - \mu)^3$

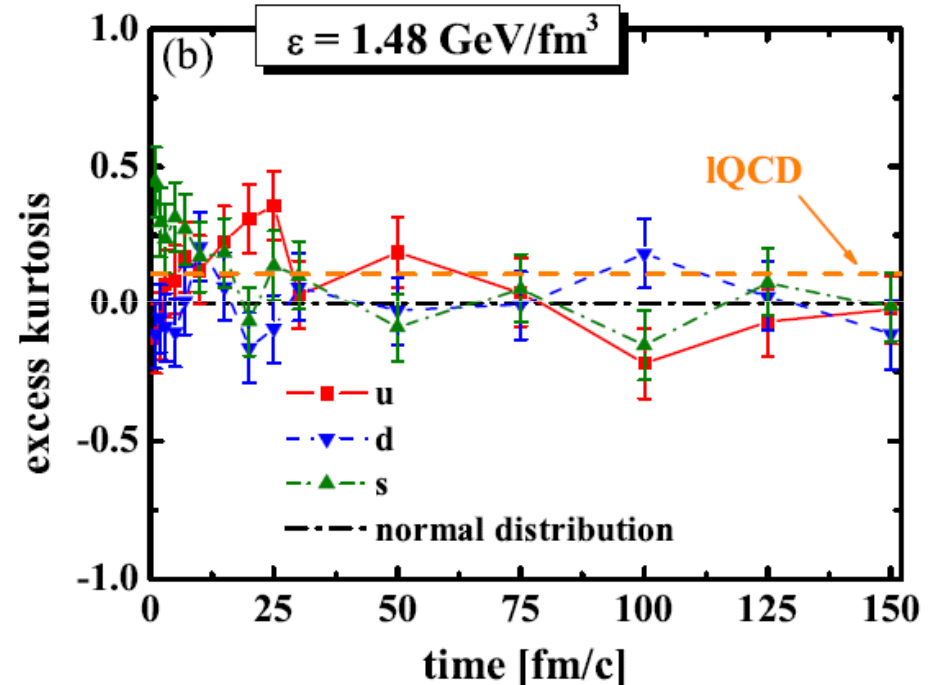
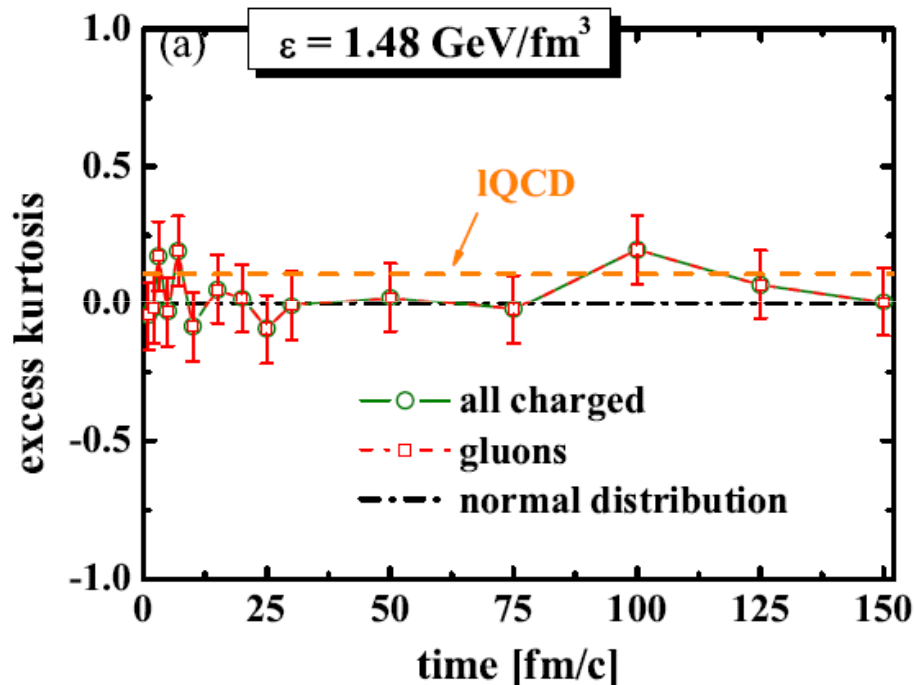
□ skewness characterizes the **asymmetry** of the distribution function with respect to its **average value**



□ **kurtosis:** $\beta_2 = \frac{m_4}{m_2^2} = \frac{m_4}{\sigma^4}$, $m_4 = \frac{1}{N} \sum_{i=1}^N (x_i - \mu)^4$

□ **kurtosis is equal to 3 for normal distribution**

⇒ **excess kurtosis:** $g_2 = \beta_2 - 3$





Summary

- ❑ partonic systems in PHSD achieve **kinetic** and **chemical equilibrium** in time
- ❑ Kubo formalism and the **relaxation time approximation** show the **same results** for the shear viscosity to entropy density ratio
- ❑ QGP in PHSD behaves as a **strongly-interacting liquid**
- ❑ **significant rise** of the bulk viscosity to entropy density ratio in the **vicinity** of the critical temperature when including the **scalar mean-field** from PHSD
- ❑ **scaled variances** for the different particle number fluctuations in the box reach **equilibrium values** in time and behave as a **micro-canonical ensemble**
- ❑ **scaled variances** for all observables approach the Poissonian limit (**GCE**) when the cell volume is much **smaller** than that of the total box
- ❑ **skewness** for all observables are compatible with **zero**
- ❑ **excess kurtosis** is compatible with **IQCD results** for gluons and charged particles

Back up



Initial momentum distributions and abundancies

□ initial number of partons is given by:
$$N_{g(q,\bar{q})} = \int_0^\infty \frac{d\omega}{2\pi} \int \frac{d^3p}{(2\pi)^3} f_{g(q,\bar{q})}(\omega, \mathbf{p})$$

□ with a 'thermal' distribution:
$$f(\omega, \mathbf{p}) = C_i p^2 \omega \rho_i(\omega, \mathbf{p}) n_{F(B)}(\omega/T_{in})$$

□ spectral function:
$$\rho_i(\omega, \mathbf{p}) = \frac{\gamma_i}{E_i} \left(\frac{1}{(\omega - E_i)^2 + \gamma_i^2} - \frac{1}{(\omega + E_i)^2 + \gamma_i^2} \right)$$
$$= \frac{4\omega\gamma_i}{(\omega^2 - \mathbf{p}^2 - M_i^2)^2 + 4\gamma_i^2\omega^2}$$

□ Fermi and Bose distributions:
$$n_{F(B)} = \frac{1}{e^{(\omega-\mu)/T_{in}} \pm 1}$$

□ initial parameters:
$$T_{in}, \mu, C_i$$

□ four-momenta are distributed according to the $f(\omega, \mathbf{p})$ by Monte Carlo

Determination of mean-field parton potentials

Partonic potential energy density:

$$V := \mathbf{T}_{00,g}^- + \mathbf{T}_{00,q}^- + \mathbf{T}_{00,\bar{q}}^- = \tilde{V}_{gg} + \tilde{V}_{qq} + \tilde{V}_{q\bar{q}}$$

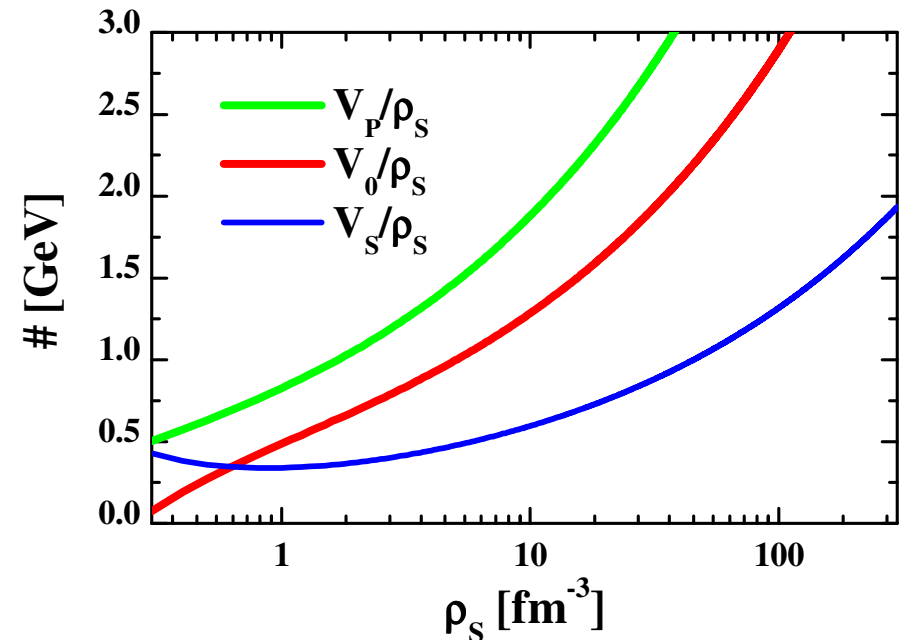
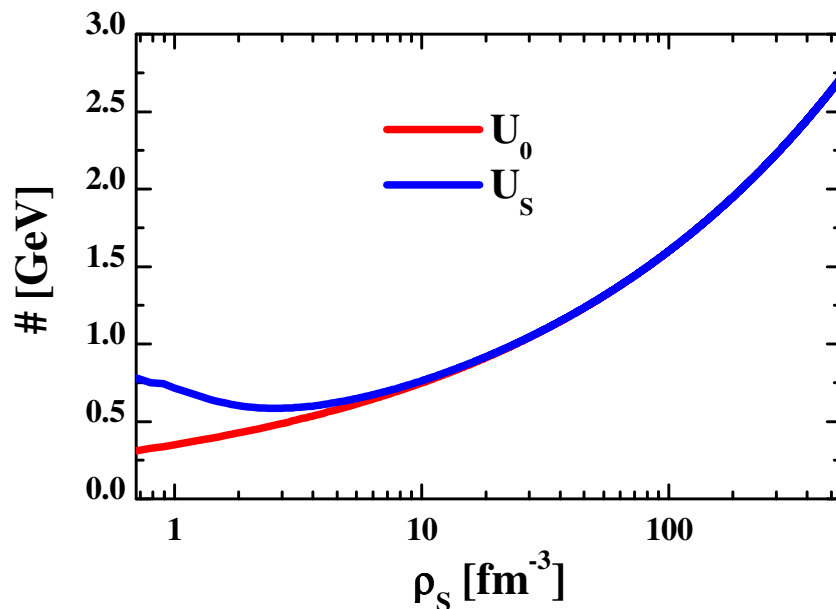
+ Constrain:

$$P = \langle P_{xx} \rangle - V_s + V_0$$

$$\varepsilon = \langle p_0 \rangle + V_s + V_0$$

Mean-field potential:

$$U_s = dV_s/d\rho_s \quad U_0 = dV_0/d\rho_0$$



Parton density:

$$\rho_p = N_g^+ + N_q^+ + N_{\bar{q}}^+, \quad N_x^+ = \tilde{T}r_x^+ 1$$

Scalar parton density:

$$\rho_x^s \equiv N_x^s(T) = \tilde{T}r_x^+ \left(\frac{\sqrt{P^2}}{\omega} \right), \quad x : g, q, \bar{q}$$

→ PHSD

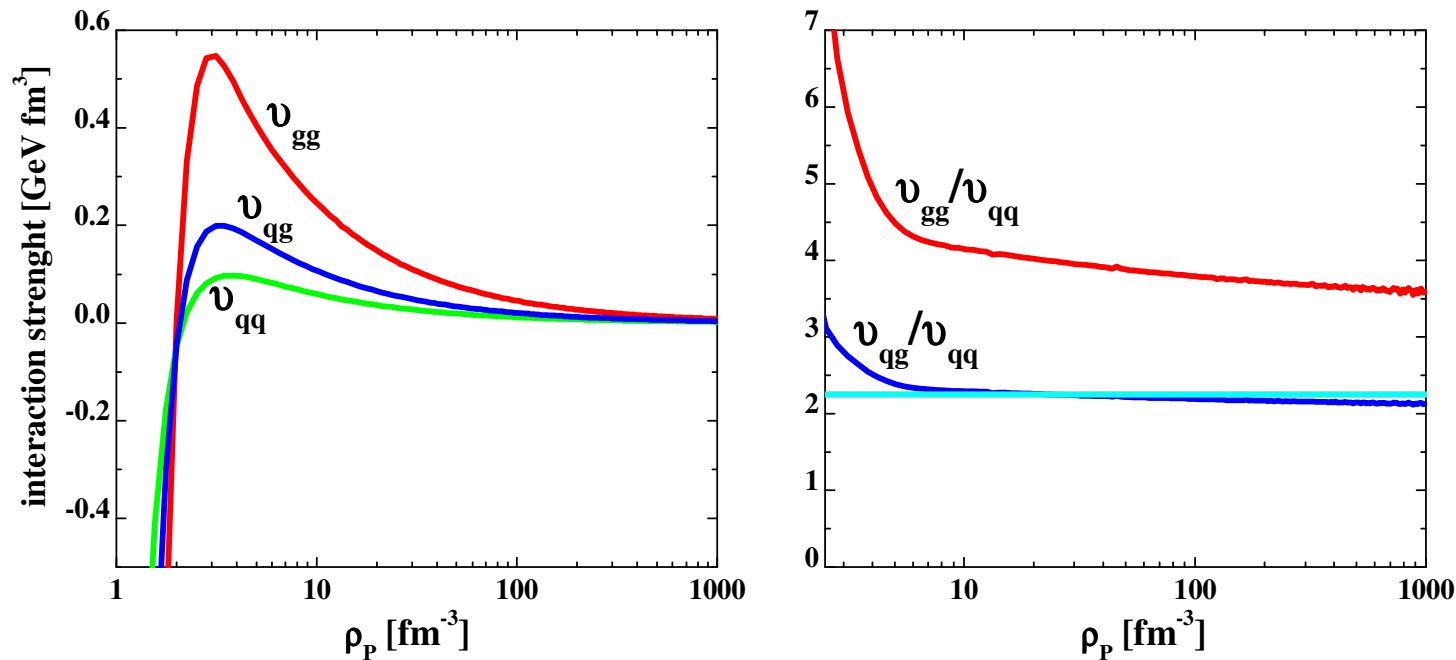
Effective 2-body interactions of time-like partons

2nd derivatives of interaction densities

$$v_{gg}(\rho_p) := \frac{\partial^2 \tilde{V}_{gg}}{\partial N_g^{+2}} \approx \frac{1}{2} \frac{\partial^2 (1 - \beta - \kappa) V}{\partial \rho_p^2} \left(\frac{\partial \rho_p}{\partial N_g^+} \right)^2$$

$$v_{qq}(\rho_p) := \frac{\partial^2 \tilde{V}_{qq}}{\partial (N_q^+ + N_{\bar{q}}^+)^2} \approx \frac{1}{2} \frac{\partial^2 (1 - \beta + \kappa) V}{\partial \rho_p^2} \left(\frac{\partial \rho_p}{\partial (N_q^+ + N_{\bar{q}}^+)} \right)^2$$

$$v_{qg}(\rho_p) := \frac{\partial^2 \tilde{V}_{qg}}{\partial (N_q^+ + N_{\bar{q}}^+) \partial N_g^+} \approx \frac{\partial^2 (\beta V)}{\partial \rho_p^2} \left(\frac{\partial \rho_p}{\partial (N_q^+ + N_{\bar{q}}^+)} \right) \left(\frac{\partial \rho_p}{\partial N_g^+} \right)$$



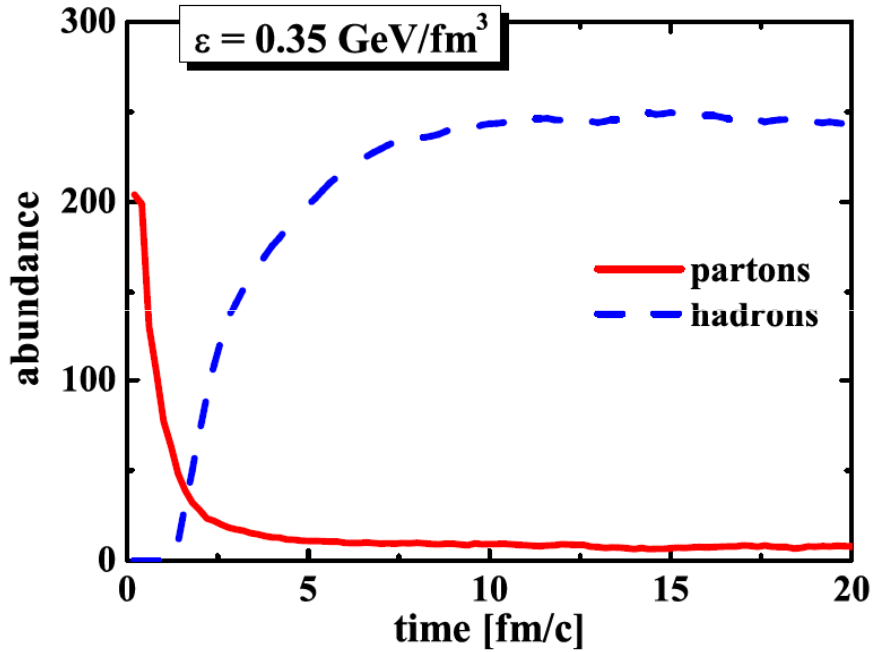
9/4

effective interactions turn strongly attractive below 2.2 fm⁻³ !

→ PHSD



Dynamical phase transition & different initializations

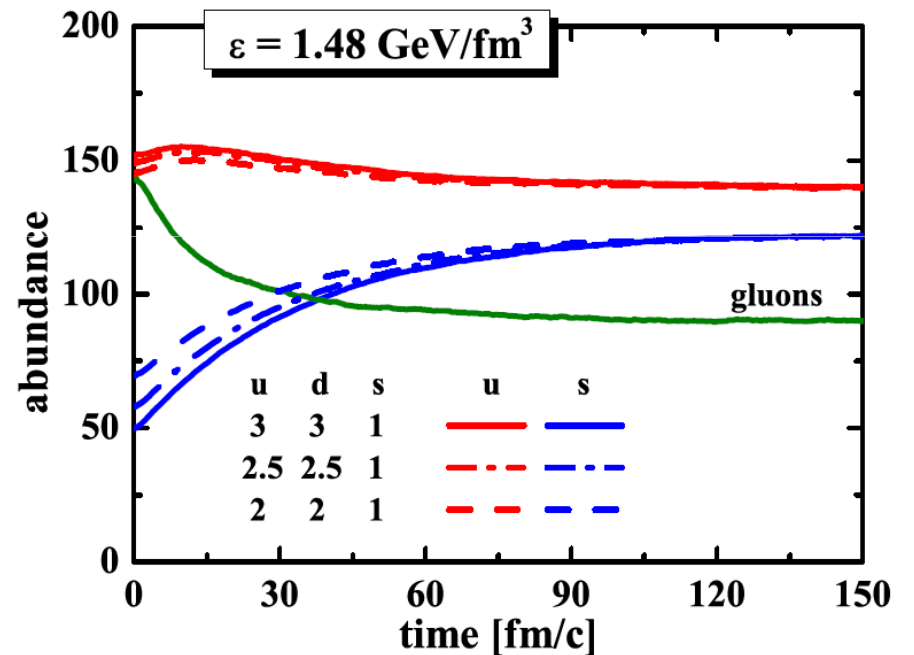


the transition from partonic to hadronic degrees-of-freedom is complete after about 9 fm/c

a small non-vanishing fraction of partons – local fluctuations of energy density from cell to cell

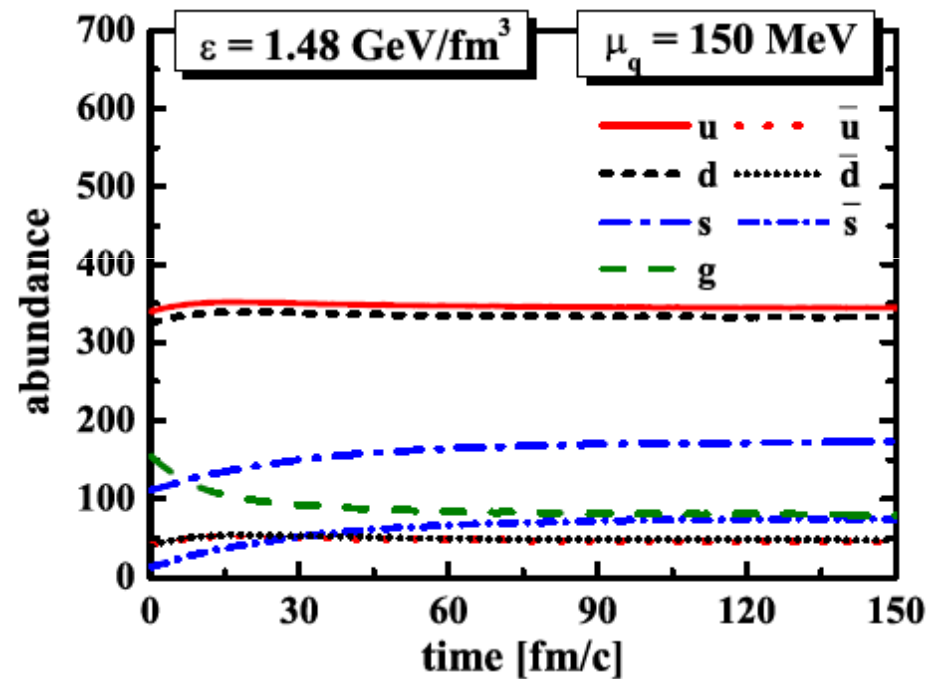
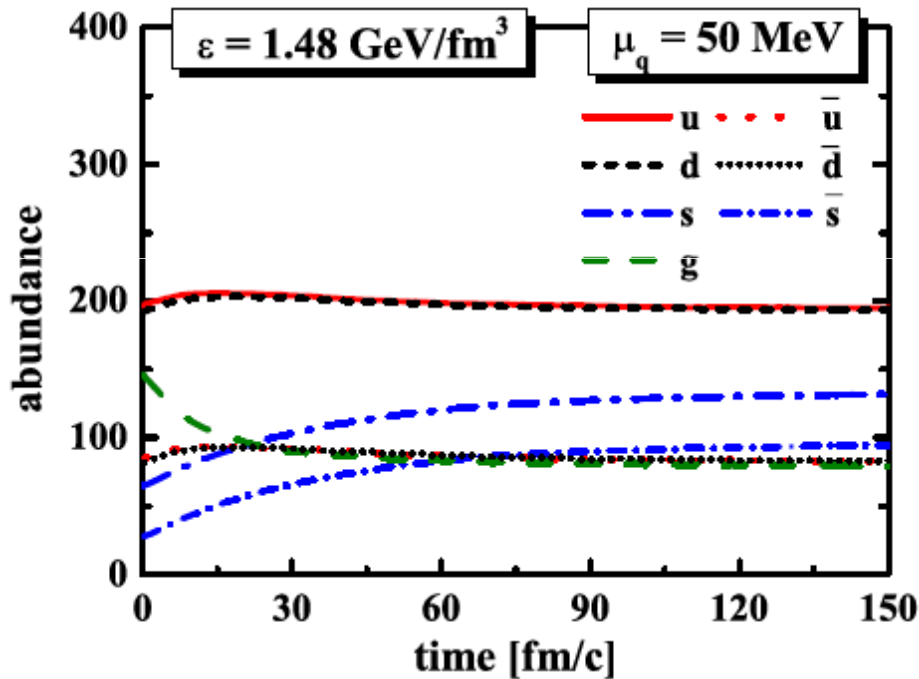
the equilibrium values of the parton numbers do not depend on the initial flavor ratios

our calculations are stable with respect to the different initializations





Finite quark chemical potentials

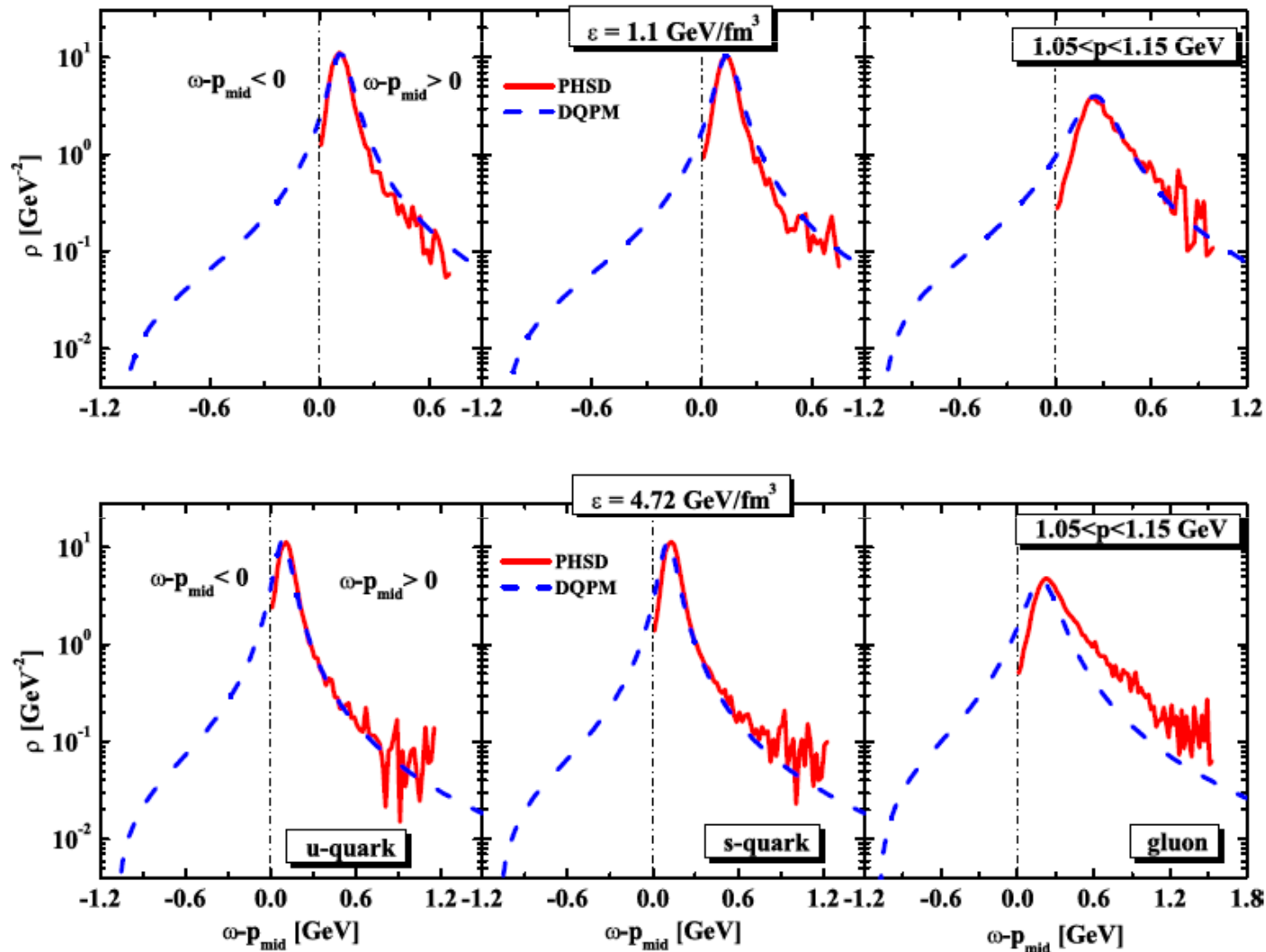


- the phase transition happens at the same critical energy ϵ_c for all μ_q
- in the present version the DQPM and PHSD treat the quark-hadron transition as a smooth crossover at all μ_q

Spectral function

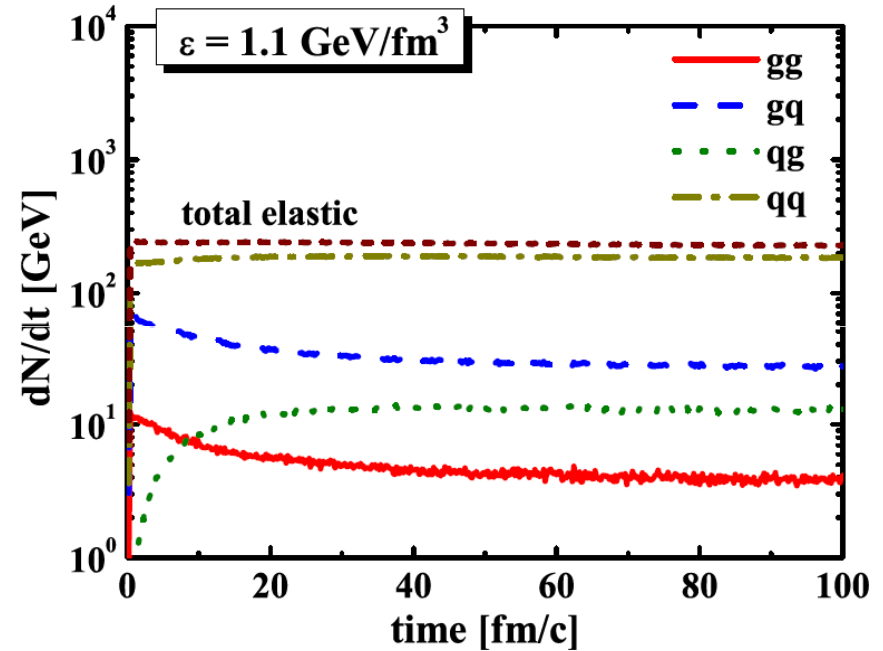
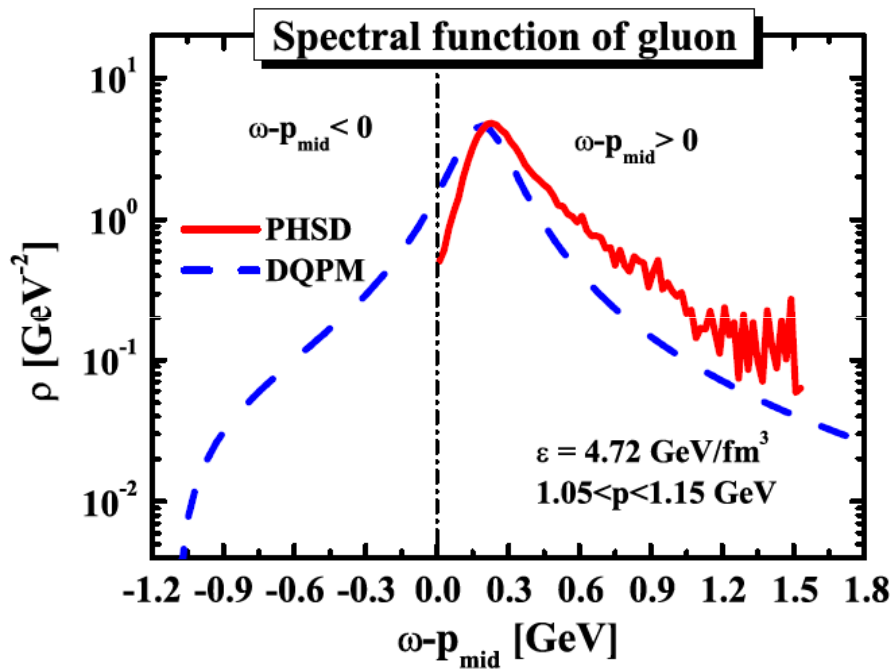
the dynamical spectral function is well described by the DQPM form in the fermionic sector for time-like partons

$$\rho_j(\omega, \mathbf{p}) = \frac{\Gamma_j}{E_j} \left(\frac{1}{(\omega - E_j)^2 + \Gamma_j^2} - \frac{1}{(\omega + E_j)^2 + \Gamma_j^2} \right)$$





Deviation in the gluonic sector



- ❑ the **inelastic** collisions are **more** important at higher parton energies
- ❑ the **elastic** scattering rate of gluons is **lower** than that of quarks
- ❑ the inelastic interaction of partons generates a **mass-dependent width** for the gluon spectral function in contrast to the DQPM assumption of the **constant width**

PHSD: Hadronization details

Local covariant off-shell transition rate for $q+q\bar{q}$ fusion
 \Rightarrow meson formation

$$\frac{dN_m(x, p)}{d^4x d^4p} = Tr_q Tr_{\bar{q}} \delta^4(p - p_q - p_{\bar{q}}) \delta^4\left(\frac{x_q + x_{\bar{q}}}{2} - x\right) \\ \times \omega_q \rho_q(p_q) \omega_{\bar{q}} \rho_{\bar{q}}(p_{\bar{q}}) |v_{q\bar{q}}|^2 W_m(x_q - x_{\bar{q}}, p_q - p_{\bar{q}}) \\ \times N_q(x_q, p_q) N_{\bar{q}}(x_{\bar{q}}, p_{\bar{q}}) \delta(\text{flavor, color}).$$



using $Tr_j = \sum_j \int d^4x_j d^4p_j / (2\pi)^4$

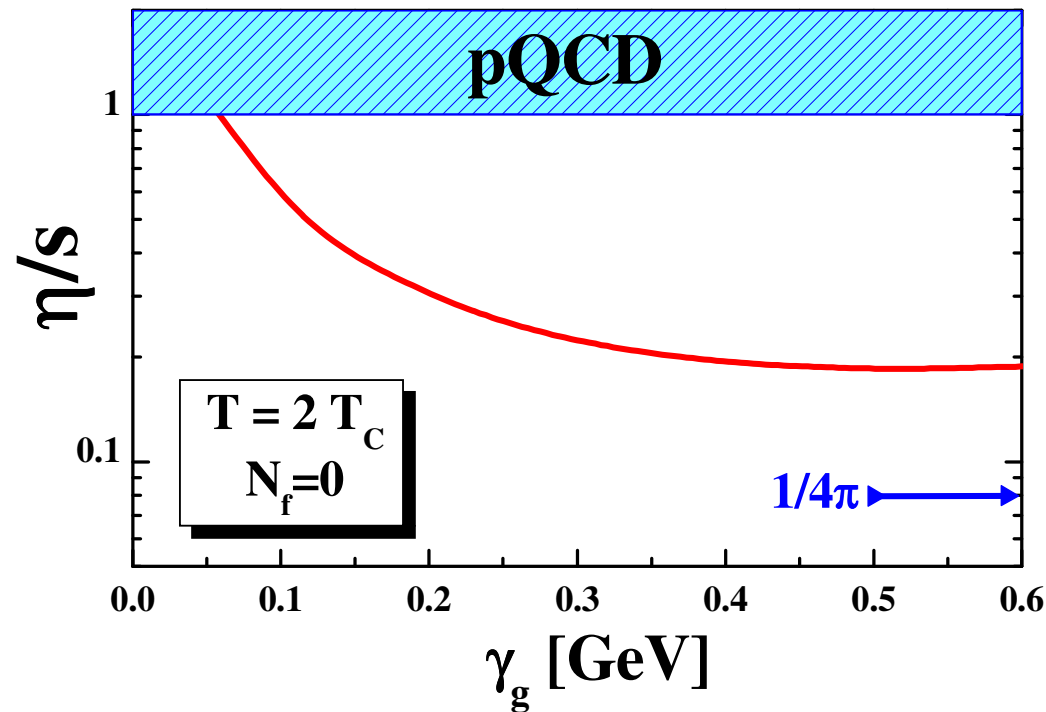
- $N_j(x, p)$ is the phase-space density of parton j at space-time position x and 4-momentum p
- W_m is the phase-space distribution of the formed 'pre-hadrons':
(Gaussian in phase space) $\sqrt{\langle r^2 \rangle} = 0.66$ fm
- $v_{q\bar{q}}$ is the effective quark-antiquark interaction from the DQPM

Transport properties of hot glue

Why do we need broad quasiparticles?

shear viscosity ratio to entropy density:

$$\eta^{\text{DQP}} = -\frac{d_g}{60} \int \frac{d\omega}{2\pi} \frac{d^3 p}{(2\pi)^3} \frac{\partial n}{\partial \omega} \rho^2(\omega) [7\omega^4 - 10\omega^2 p^2 + 7p^4].$$



→ otherwise η/s will be too high!