New formalism for spin alignment

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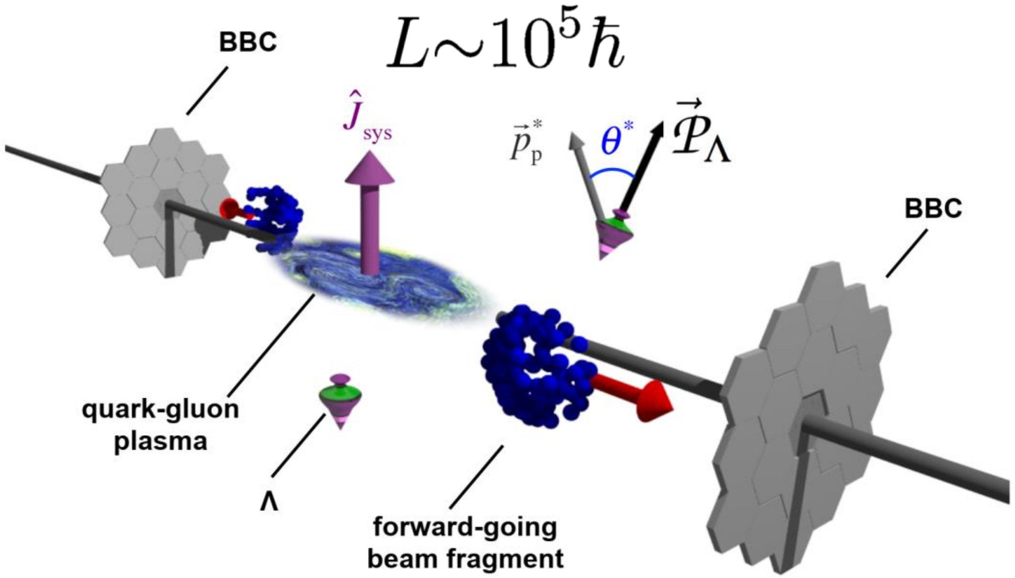
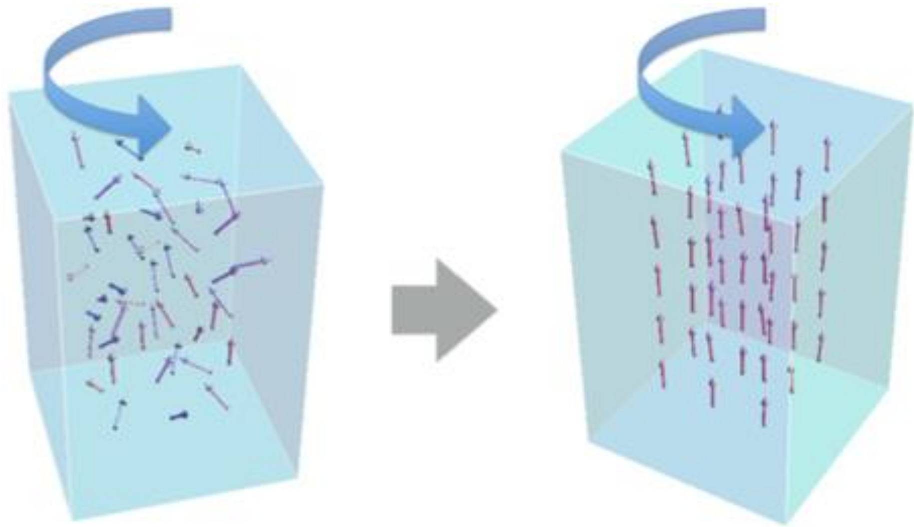
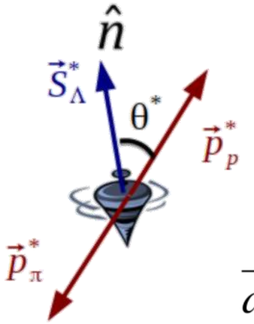
In collaboration with Yi-Liang Yin, Qun Wang et.al
Based on: 2506.XXXXX

Spin polarization final particle





Orbit angular momentum → Spin polarization

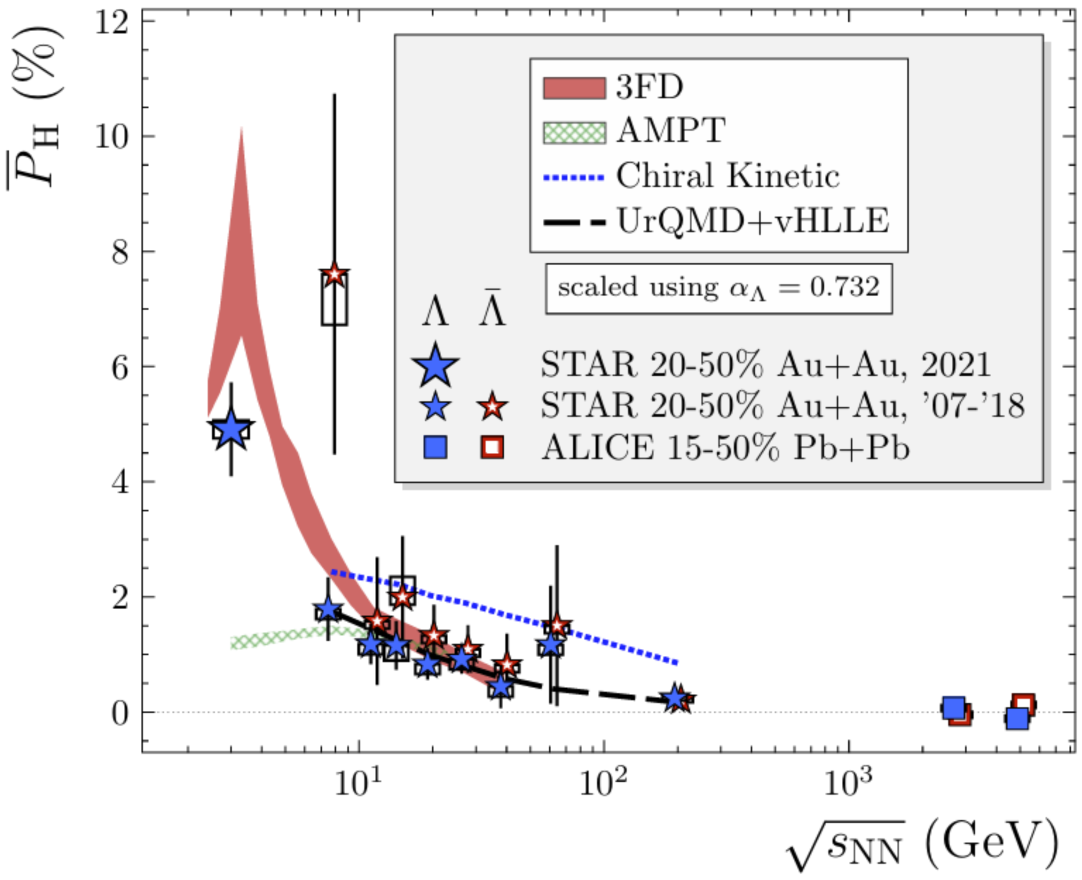


$$\Lambda \to p + \pi^-$$

$$\frac{dN}{d\cos\theta^*} = \frac{1}{2}\left(1 + \alpha_H |\vec{\mathcal{P}}_H| \cos\theta^*\right).$$

Z.-T. Liang and X.-N. Wang, Phys. Rev. Lett. **94**, 102301 (2005)
Z.-T. Liang and X.-N. Wang, Phys. Lett. B **629**, 20 (2005)
J.-H. Gao *et al.*, Phys. Rev. C **77**, 044902 (2008)

global polarization



Spin polarization vector:

$$S^\mu_\varpi(p) = -\frac{1}{8m}\epsilon^{\mu\rho\sigma\tau}p_\tau \frac{\int_\Sigma \mathrm{d}\Sigma \cdot p\, n_F(1-n_F)\varpi_{\rho\sigma}}{\int_\Sigma \mathrm{d}\Sigma \cdot p\, n_F},$$

- *Phys.Rev.Lett.* 94 (2005) 102301
- *Phys.Rev.C* 95 (2017) 5, 054902
- *STAR Nature* 548 (2017) 62-65
- *STAR Phys.Rev.C 104 (2021) 6, L061901*

Conclusions:

- Spin carried by s quark
- Global spin polarization is induced by thermal vorticity

Local polarization

Thermal vorticity:

$$S^{\mu}_{\varpi}(p) = -\frac{1}{8m}\epsilon^{\mu\rho\sigma\tau}p_{\tau}\frac{\int_{\Sigma} d\Sigma \cdot p\, n_F(1-n_F)\varpi_{\rho\sigma}}{\int_{\Sigma} d\Sigma \cdot p\, n_F},$$

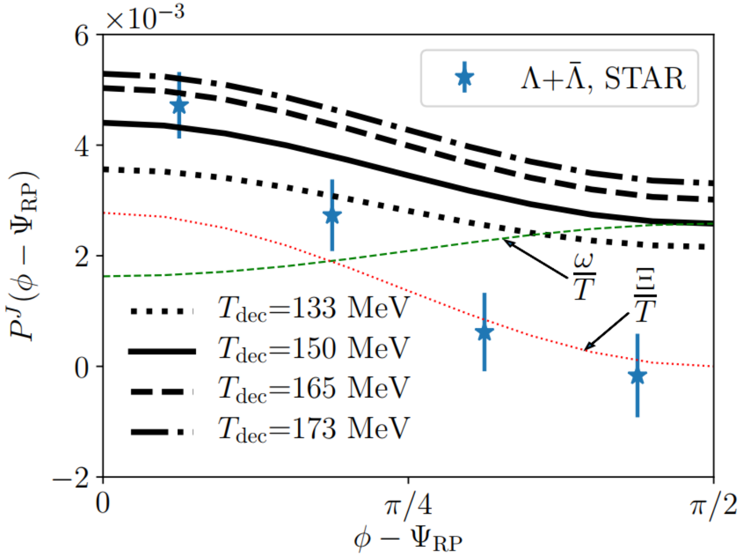
$$\varpi_{\mu\nu} = -\frac{1}{2}\left(\partial_{\mu}\beta_{\nu} - \partial_{\nu}\beta_{\mu}\right).$$

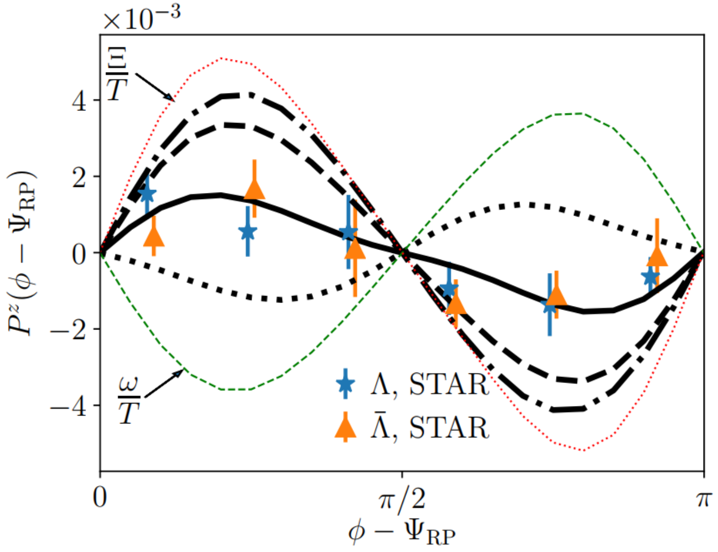
Thermal shear:

$$S^{\mu}_{\xi}(p) = -\frac{1}{4m}\epsilon^{\mu\rho\sigma\tau}\frac{p_{\tau}p^{\lambda}}{\varepsilon}\frac{\int_{\Sigma} d\Sigma \cdot p\, n_F(1-n_F)\hat{t}_{\rho}\xi_{\sigma\lambda}}{\int_{\Sigma} d\Sigma \cdot p\, n_F}$$

$$\xi_{\mu\nu} = \frac{1}{2}\left(\partial_{\mu}\beta_{\nu} + \partial_{\nu}\beta_{\mu}\right).$$

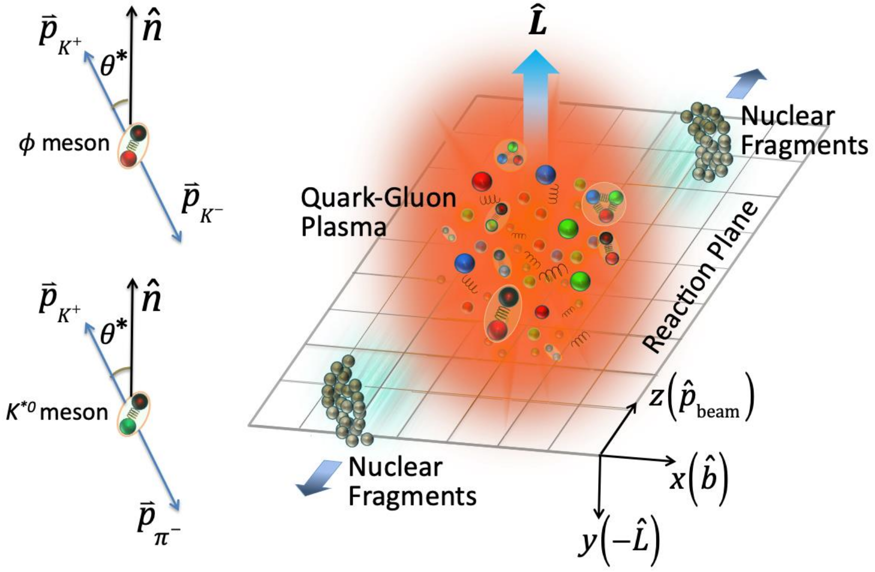
Non-equilibrium effect



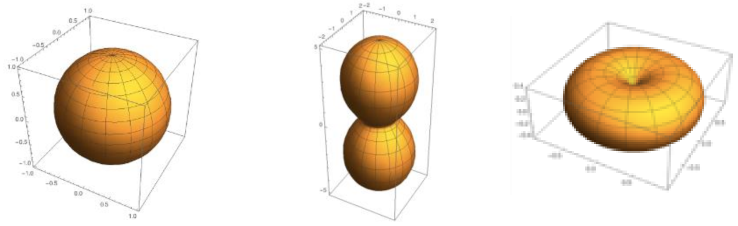


- *Phys.Rev.Lett.* 127 (2021) 14, 142301
- *Phys.Rev.Lett.* 127 (2021) 27, 272302
- *Phys.Rev.C* 104 (2021) 6, 064901

Spin alignment for S=1 particle

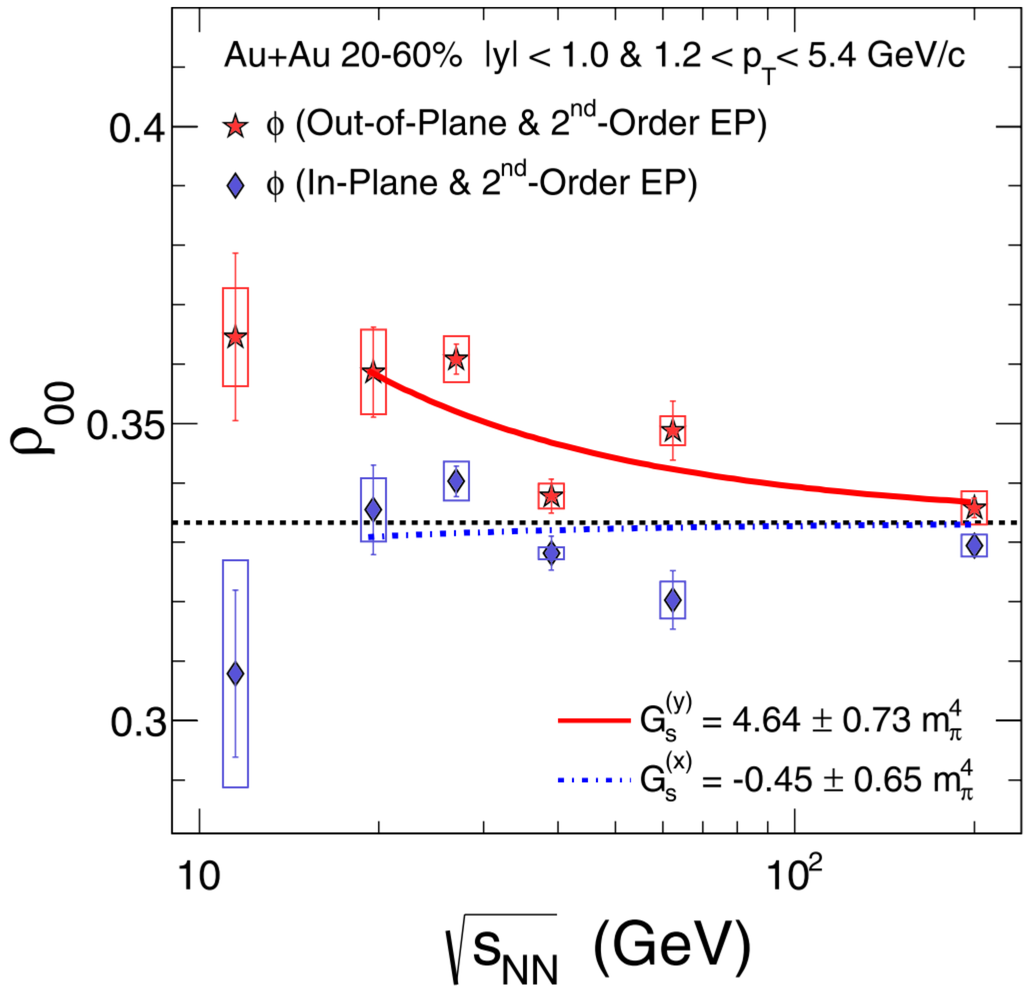


$$\frac{dN}{d(\cos\theta^*)} = N_0 \times \left[(1-\rho_{00}) + (3\rho_{00}-1)\cos^2\theta^*\right]$$



$\rho_{00} = \frac{1}{3}$ $\rho_{00} > \frac{1}{3}$ $\rho_{00} < \frac{1}{3}$

Taken from report of A.H. Tang, Spin2023



STAR Nature 614 (2023) 7947, 244-248

Potential sources for Φ meson

Physics Mechanisms	(ρ_{00})
c_Λ: Quark coalescence vorticity & magnetic field[1]	< 1/3 (Negative ~ 10^{-5})
c_ε: Vorticity tensor[1]	< 1/3 (Negative ~ 10^{-4})
c_E: Electric field[2]	> 1/3 (Positive ~ 10^{-5})
Fragmentation[3]	> or, < 1/3 (~ 10^{-5})
Local spin alignment and helicity[4]	< 1/3
Turbulent color field[5]	< 1/3
c_ϕ: Vector meson strong force field[6]	> 1/3 (Can accomodate large positive signal)

$$\rho_{00} - \frac{1}{3} \approx c_\Lambda + c_\epsilon + c_E + c_\phi + \cdots$$

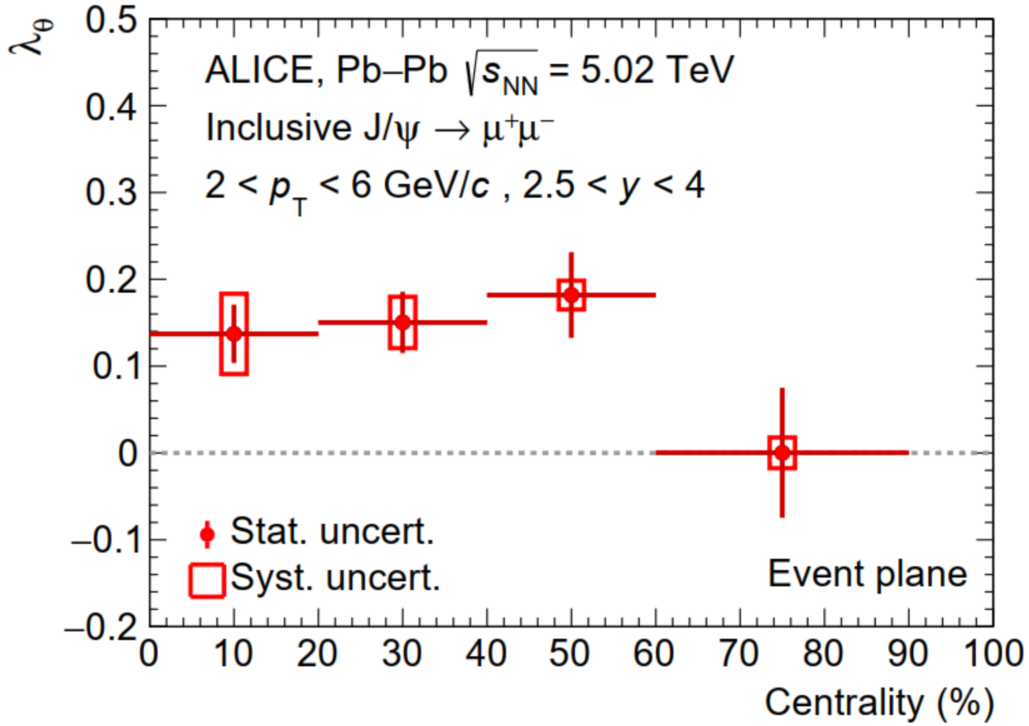
[1]. Liang et., al., Phys. Lett. B 629, (2005);
Yang et., al., Phys. Rev. C 97, 034917 (2018);
Xia et., al., Phys. Lett. B 817, 136325 (2021);
Beccattini et., al., Phys. Rev. C 88, 034905 (2013)
[2]. Sheng et., al., Phys. Rev. D 101, 096005 (2020);
Yang et., al., Phys. Rev. C 97, 034917 (2018)
[3]. Liang et., al., Phys. Lett. B 629, (2005)
[4]. Xia et., al., Phys. Lett. B 817, 136325 (2021);
Guo, Phys. Rev. D 104, 076016 (2021)
[5]. Muller et., al., Phys. Rev. D 105, L011901 (2022)
[6]. Sheng et., al., Phys. Rev. D 101, 096005 (2020);
Sheng et., al., Phys. Rev. D 102, 056013 (2020)

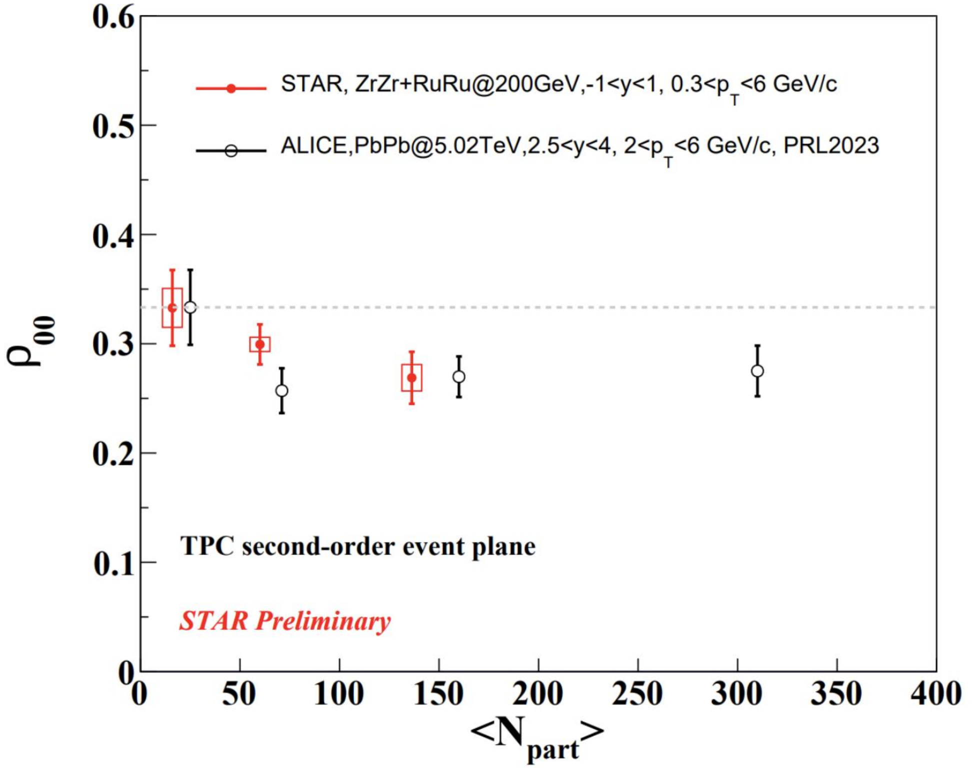
Taken from report of Subhash Singha, QM 2022

Spin alignment for J/ψ

Phys.Rev.Lett. 131 (2023) 4, 042303

D. Shen for STAR, SPIN 2023

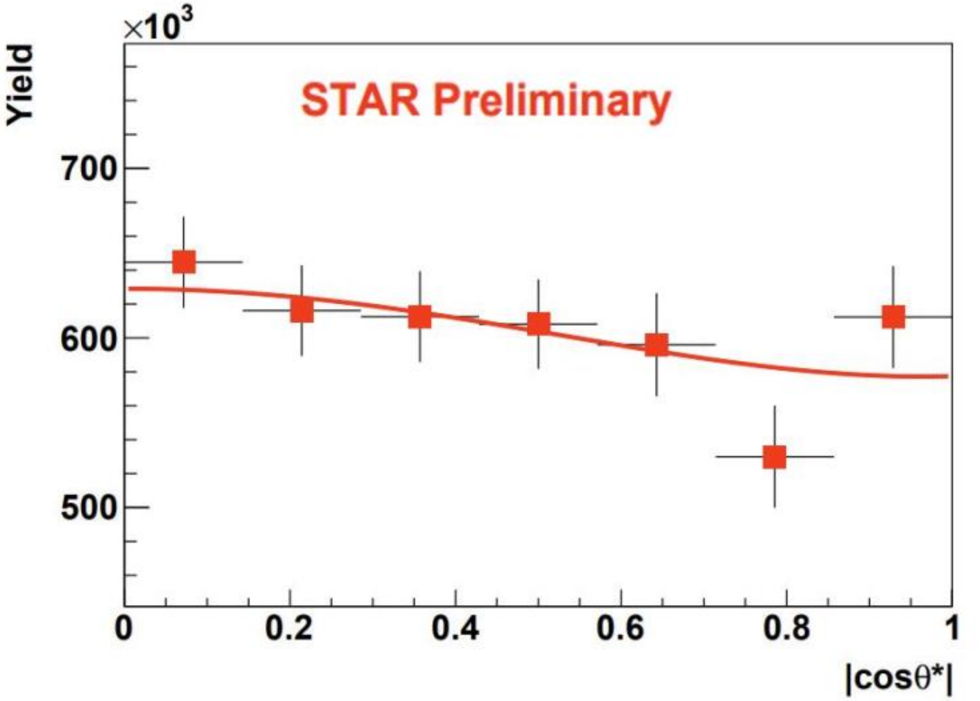




$$\rho_{00}^{J/\Psi} < \frac{1}{3} \quad ?$$

Spin alignment for ρ

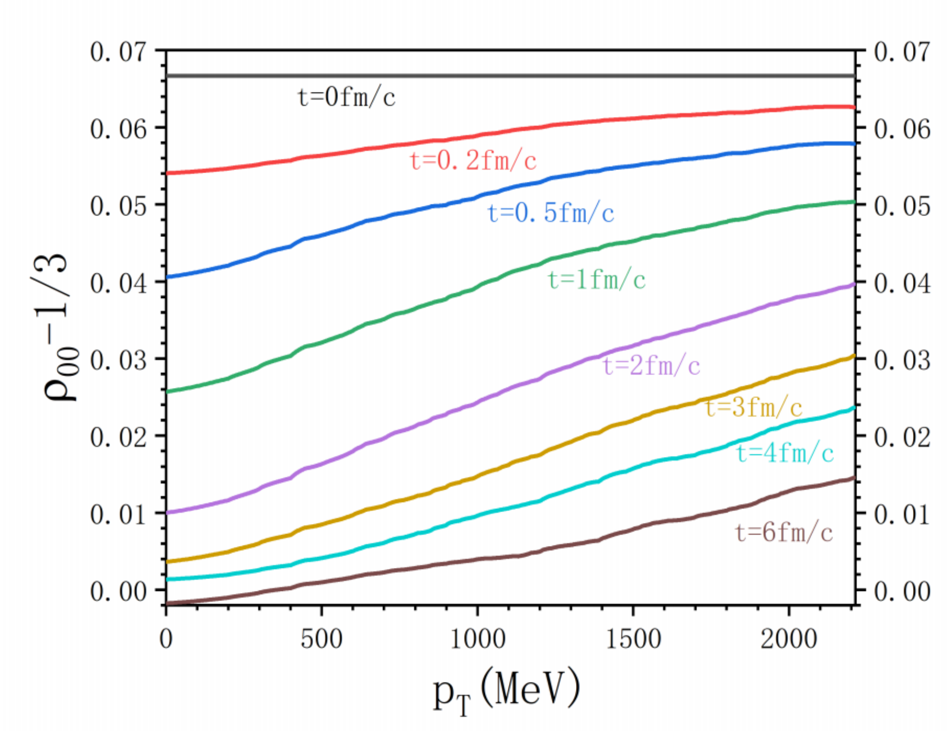
Experimental result:



AuAu for run 2011 at 200 GeV, Centrality: 60-80%, p_T: 1.8-2.4 GeV/c Taken from Baoshan Xi QM23

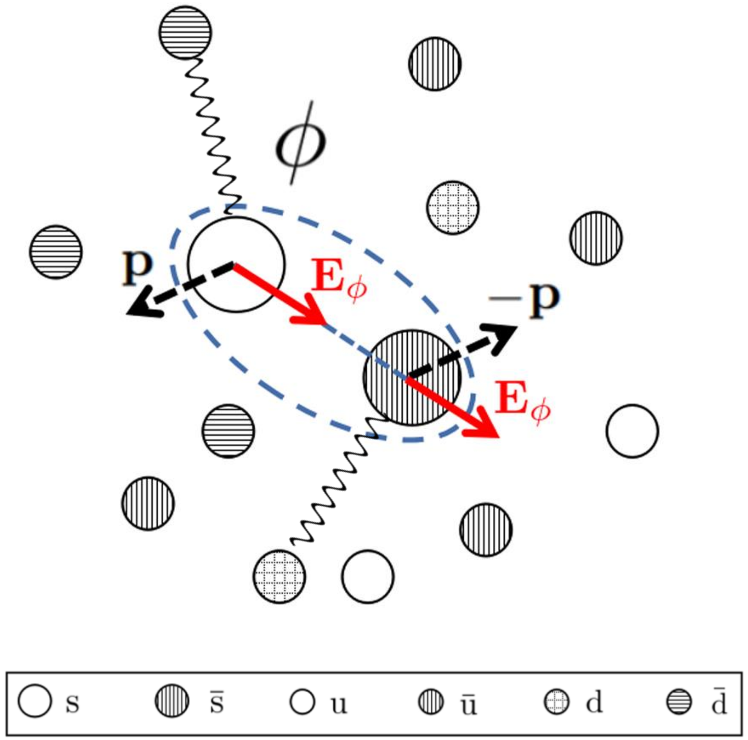
$$\delta\rho_{00}^{\rho} < \frac{1}{3}$$

Prediction from spin Boltzmann equation with local collision term:



Taken from *Phys.Rev.C* 110 (2024) 2, 024905

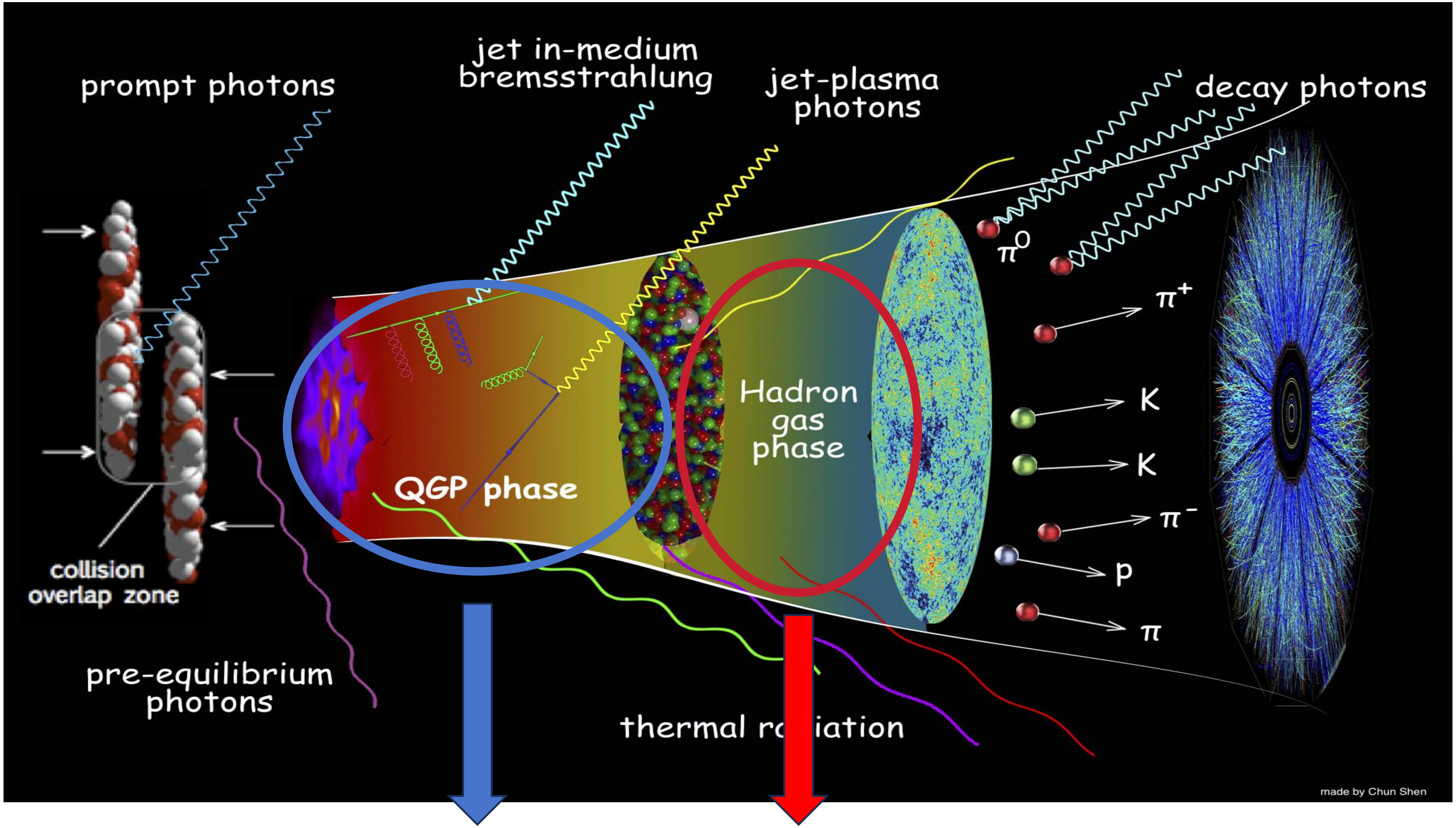
spin alignment in thermal media



mesons interact with media → Different spectra (T and L mode) → Different coupling with shear, vorticity… → Global spin alignment

Take ***ρ* meson** as example

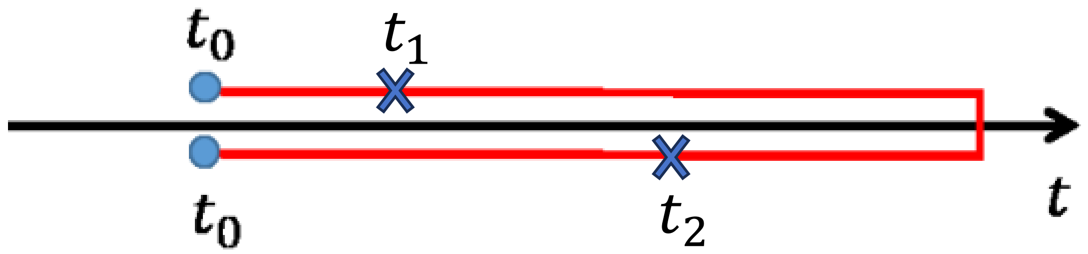
Related paper:
- Wen-Bo Dong et al. Phys.Rev.D 109 (2024) 5, 056025
- Feng Li et al. arXiv:2206.11890
- Zhong-Yuan Sun et al. arXiv:2503.13408
- Xin-Nan Zhu et al. arXiv: 2503.23919



ϕ **meson formed** ρ **meson interacted**

Schwinger-Keldysh contour

Two-point function:



$$Z[J] = Tr\left\{T_p \exp\left[i\int_p J(x)\phi(x)dx\right]\rho\right\}$$

$$G(x_1,x_2) = \frac{(-i)^2\delta^2 \ln(Z)}{\delta J(x_1)\delta J(x_2)}$$

$$G^{++}(x_1,x_2) = G^{F}(x_1,x_2) =< T\phi(x_1)\phi(x_2) >$$
$$G^{+-}(x_1,x_2) = G^{<}(x_1,x_2) =< \phi(x_1)\phi(x_2) >$$
$$G^{-+}(x_1,x_2) = G^{>}(x_1,x_2) =< \phi(x_2)\phi(x_1) >$$
$$G^{--}(x_1,x_2) = G^{\bar{F}}(x_1,x_2) =< \bar{T}\phi(x_1)\phi(x_2) >$$

Physical representation:

3 components independent

$$G^{A}(x_1,x_2) = G^{F}(x_1,x_2) - G^{>}(x_1,x_2)$$
$$G^{R}(x_1,x_2) = G^{F}(x_1,x_2) - G^{<}(x_1,x_2)$$
$$G^{C}(x_1,x_2) = G^{F}(x_1,x_2) + G^{\bar{F}}(x_1,x_2)$$

Choose $\boldsymbol{G^A, G^R, G^<}$ as variables

Dyson equation vs. KB equation

KB equation:

$$(A\star B)^{\mu\nu}(x_1,x_2)=\int dy A_{\rho}^{\mu}(x_1,y)B^{\rho\nu}(y,x_2)$$

$$\begin{aligned}&\left[\left(\partial_{x_1}^2+m^2\right)g^{\mu\rho}-\partial_{x_1}^{\mu}\partial_{x_1}^{\rho}\right]G_{\rho}^{<,\nu}(x_1,x_2)\\&=i\hbar(\Sigma^F\star G^<)^{\mu\nu}(x_1,x_2)-i\hbar\left(\Sigma^<\star G^{\bar{F}}\right)^{\mu\nu}(x_1,x_2)\end{aligned}$$

Under Wigner transformation, KB equation → Boltzmann equation (equation of motion)
Widely used in spin transport theory(1902.06513, 2103.10636, 2206.05868…)

Dyson equation:

$$\begin{aligned}G_{A/R}^{\mu\nu}(x_1,x_2)&=G_{A/R}^{\mu\nu,0}(x_1,x_2)+\left(G_{A/R}^0\star\Sigma_{A/R}\star G_{A/R}\right)^{\mu\nu}(x_1,x_2)\\G_{<}^{\mu\nu}(x_1,x_2)&=G_{<}^{\mu\nu,0}(x_1,x_2)+\left(G_R^0\star\Sigma_R\star G_<\right)^{\mu\nu}(x_1,x_2)\\&\quad+\left(G_R^0\star\Sigma_<\star G_A\right)^{\mu\nu}(x_1,x_2)+\left(G_<^0\star\Sigma_A\star G_A\right)^{\mu\nu}(x_1,x_2)\end{aligned}$$

1.Equation of state, solvable
2.R/A-component equations are independent with “<“ component.

Wigner transformation

Wigner transformation:

$$O(X,p)=\int dy e^{ipy} O(X,y)=\int dy e^{ipy} O(\frac{x_1+x_2}{2}, x_1-x_2)$$

Wigner function:

$$G^{\mu\nu}_{R/A,0}(X,p)=\left(g^{\mu\nu}-\frac{p^\mu p^\nu}{m^2}\right)\frac{-i}{p^2-m^2\pm ip^0\epsilon}$$

$$G^{\mu\nu}_{<,0}(X,p)=2\pi\delta(p^2-m^2)\theta(-p^0)\epsilon^*_\mu(\lambda_1,-p)\epsilon_\nu(\lambda_2,-p)$$
$$+\theta(p^0)\epsilon_\mu(\lambda_1,p)\epsilon^*_\nu(\lambda_2,p)f^{V,(0)}_{\lambda_1\lambda_2}(X,p)$$
$$+\theta(-p^0)\epsilon^*_\mu(\lambda_1,-p)\epsilon_\nu(\lambda_2,-p)f^{V,(0)}_{\lambda_2\lambda_1}(X,-p)$$

$$\epsilon^\mu(\lambda,p)=\left(\frac{\mathbf{p}\cdot\vec{\epsilon}_\lambda}{\sqrt{p^2}}, \vec{\epsilon}_\lambda+\frac{\mathbf{p}\cdot\vec{\epsilon}_\lambda}{\sqrt{p^2}\left(p^0+\sqrt{p^2}\right)}\mathbf{p}\right)$$

Matrix-valued spin dependent distribution(MVSD)

$$G^{\mu\nu}_{R/A,0}(X,p)\sim O(g^0\partial^0),\quad G^{\mu\nu}_{<,0}(X,p)\sim O(g^0)=O(g^0\partial^0)+O(g^0\partial^1)+\cdots$$

Leading order DSE

$O(\partial^0)$: Spatial gradient expansion = $\hbar$ expansion

$$G^{\mu\nu}_{R,L}(X,p) = G^{\mu\nu}_{R,L,0}(X,p) + G^{\mu\rho}_{R,L,0}(X,p)\Sigma^{R,L}_{\rho\sigma}(X,p)G^{\sigma\nu}_{R,L}(X,p)$$
$$= A^{\mu\nu}_{R,L}\left[G^{\mu\nu}_{R,L,0}\right] + B^{\mu}_{L,\sigma}(X,p)G^{\sigma\nu}_{R,L}(X,p) \qquad (1)$$

$$G^{\mu\nu}_{<,L}(X,p) = G^{\mu\nu}_{<,L,0}(X,p) + G^{\mu\rho}_{R,L,0}(X,p)\Sigma^{<,L}_{\rho\sigma}(X,p)G^{\sigma\nu}_{A,L}(X,p)$$
$$+G^{\mu\rho}_{<,L,0}(X,p)\Sigma^{A,L}_{\rho\sigma}(X,p)G^{\sigma\nu}_{A,L}(X,p) + G^{\mu\rho}_{R,L,0}(X,p)\Sigma^{R,L}_{\rho\sigma}(X,p)G^{\sigma\nu}_{<,L}(X,p)$$
$$= A^{\mu\nu}_{<,L}\left[G^{\mu\nu}_{L,0}, \Sigma^{\mu\nu}_{L}, G^{\mu\nu}_{R/A,L}\right] + B^{\mu}_{L,\sigma}(X,p)G^{\sigma\nu}_{<,L}(X,p) \qquad (2)$$

$A^{\mu\nu}$: independent with variable.
$B^{\mu\nu}$: same for two identities.

Solution: $G \sim \frac{A}{1-B}$

Next-to-Leading order DSE

$O(\partial^1)$:

$$
\begin{aligned}
G^{R,\mathrm{NL}}_{\mu\nu}(X,p) &= G^{R,\mathrm{L}}_{0,\mu\rho}(X,p)\,\Sigma^{R,\rho\sigma}_{\mathrm{NL}}(X,p)\,G^{R,\mathrm{L}}_{\sigma\nu}(X,p) \\
&\quad + \frac{i}{2}\left[G^{R,\mathrm{L}}_{0,\mu\rho}(X,p)\,,\Sigma^{R,\rho\sigma}_{\mathrm{L}}(X,p)\,G^{R,\mathrm{L}}_{\sigma\nu}(X,p)\right]_{P.B.} \\
&\quad + \frac{i}{2}G^{R,\mathrm{L}}_{0,\mu\rho}(X,p)\left[\Sigma^{R,\rho\sigma}_{\mathrm{L}}(X,p)\,,G^{R,\mathrm{L}}_{\sigma\nu}(X,p)\right]_{P.B.}, \\
&\quad + G^{R,\mathrm{L}}_{0,\mu\rho}(X,p)\,\Sigma^{R,\rho\sigma}_{\mathrm{L}}(X,p)G^{R,\mathrm{NL}}_{\sigma\nu}(X,p) \\
&= A^{R,\mathrm{NL}}_{\mu\nu}\left[G^{0}_{R,\mathrm{L}}, G_{R,\mathrm{L}}, \Sigma^{R}_{\mathrm{L}}, \Sigma^{R}_{\mathrm{NL}}\right] + B^{\mathrm{L},\sigma}_{\mu}(X,p)G^{R,\mathrm{NL}}_{\sigma\nu}(X,p)
\end{aligned}
$$

Self energy correction

Poisson bracket

A^{NL}_{R}: Poisson bracket + self energy correction

$[A,B]_{P.B.} = \partial_X A \partial_p B - \partial_p A \partial_X B$

Wigner transformation

Non-trivial distribution

Next-to-Leading order DSE

$O(\partial^1)$:

$$
\begin{aligned}
G^{<,\mathrm{NL}}_{\mu\nu}(X,p) &= B^{\mathrm{L},\sigma}_{\mu}(X,p)G^{<,\mathrm{NL}}_{\sigma\nu}(X,p) + A^{<}_{\mathrm{NL},\mu\nu}[G^{R/A}_{\mathrm{L},0}, G^{R/A}_{\mathrm{NL}}\cdots] \\
&= G^{R,\mathrm{L}}_{0,\mu\rho}(X,p)\,\Sigma^{R,\rho\sigma}_{\mathrm{L}}(X,p)G^{<,\mathrm{NL}}_{\sigma\nu}(X,p) \\
&\quad + G^{R,\mathrm{L}}_{0,\mu\rho}(X,p)\,\Sigma^{R,\rho\sigma}_{\mathrm{NL}}(X,p)\,G^{<,\mathrm{L}}_{\sigma\nu}(X,p) \\
&\quad + G^{<,\mathrm{L}}_{0,\mu\rho}(X,p)\,\Sigma^{A,\rho\sigma}_{\mathrm{NL}}(X,p)\,G^{A,\mathrm{L}}_{\sigma\nu}(X,p) \\
&\quad + G^{R,\mathrm{L}}_{0,\mu\rho}(X,p)\,\Sigma^{<,\rho\sigma}_{\mathrm{NL}}(X,p)\,G^{A,\mathrm{L}}_{\sigma\nu}(X,p) \\
&\quad + G^{<,\mathrm{NL}}_{0,\mu\rho}(X,p)\,\Sigma^{A,\rho\sigma}_{\mathrm{L}}(X,p)\,G^{A,\mathrm{L}}_{\sigma\nu}(X,p) + G^{<,\mathrm{NL}}_{0,\mu\rho}(X,p)\,\Sigma^{A,\rho\sigma}_{\mathrm{L}}(X,p)\,G^{A,\mathrm{L}}_{\sigma\nu}(X,p) \\
&\quad + G^{<,\mathrm{L}}_{0,\mu\rho}(X,p)\,\Sigma^{A,\rho\sigma}_{\mathrm{L}}(X,p)\,G^{A,\mathrm{NL}}_{\sigma\nu}(X,p) + G^{R,\mathrm{L}}_{0,\mu\rho}(X,p)\,\Sigma^{<,\rho\sigma}_{\mathrm{L}}(X,p)\,G^{A,\mathrm{NL}}_{\sigma\nu}(X,p) \\
&\quad + \frac{i}{2}\left[G^{R,\mathrm{L}}_{0,\mu\rho}(X,p)\,,\Sigma^{R,\rho\sigma}_{\mathrm{L}}(X,p)\,G^{<,\mathrm{L}}_{\sigma\nu}(X,p)\right]_{P.B.} \\
&\quad + \frac{i}{2}\left[G^{<,\mathrm{L}}_{0,\mu\rho}(X,p)\,,\Sigma^{A,\rho\sigma}_{\mathrm{L}}(X,p)\,G^{A,\mathrm{L}}_{\sigma\nu}(X,p)\right]_{P.B.} \\
&\quad + \frac{i}{2}\left[G^{R,\mathrm{L}}_{0,\mu\rho}(X,p)\,,\Sigma^{<,\rho\sigma}_{\mathrm{L}}(X,p)\,G^{A,\mathrm{L}}_{\sigma\nu}(X,p)\right]_{P.B.} \\
&\quad + \frac{i}{2}G^{R,\mathrm{L}}_{0,\mu\rho}(X,p)\left[\Sigma^{R,\rho\sigma}_{\mathrm{L}}(X,p)\,,G^{<,\mathrm{L}}_{\sigma\nu}(X,p)\right]_{P.B.} \\
&\quad + \frac{i}{2}G^{<,\mathrm{L}}_{0,\mu\rho}(X,p)\left[\Sigma^{A,\rho\sigma}_{\mathrm{L}}(X,p)\,,G^{A,\mathrm{L}}_{\sigma\nu}(X,p)\right]_{P.B.} \\
&\quad + \frac{i}{2}G^{R,\mathrm{L}}_{0,\mu\rho}(X,p)\left[\Sigma^{<,\rho\sigma}_{\mathrm{L}}(X,p)\,,G^{A,\mathrm{L}}_{\sigma\nu}(X,p)\right]_{P.B.}.
\end{aligned}
$$

Self energy correction

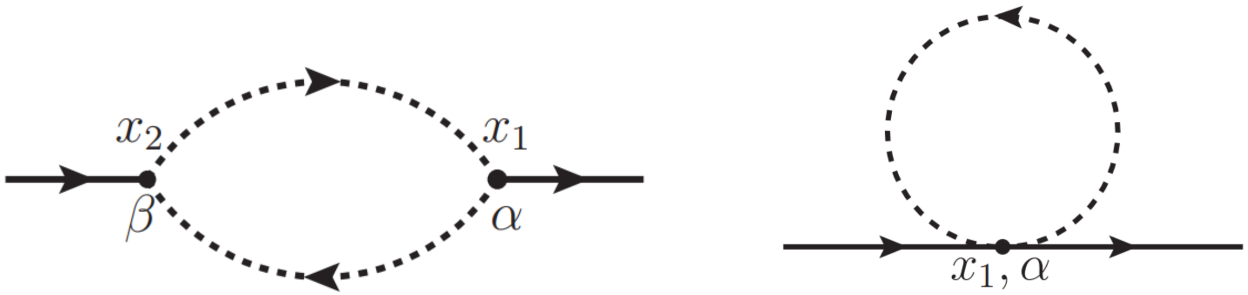
Non-trivial distribution

Poisson bracket

Self energy

$\rho-\pi$ interaction(example):

$$\mathcal{L}=\mathcal{L}_{kin}+ig_{\rho\pi}\left(\partial_\mu\pi^+\pi^- -\pi^+\partial_\mu\pi^-\right)\rho_\mu+g_{\rho\pi}^2\pi^+\pi^-\rho_\mu\rho^\mu,$$



$$\begin{aligned}\Sigma^{\mathrm{CTP}}_{\alpha\beta}\left(x_1,x_2\right) &= g_{\rho\pi}^2\left[\partial_\alpha^{x_1}S\left(x_1,x_2\right)\partial_\beta^{x_2}S\left(x_2,x_1\right)+\partial_\alpha^{x_1}S\left(x_2,x_1\right)\partial_\beta^{x_2}S\left(x_1,x_2\right)\right]\\ &\quad -g_{\rho\pi}^2\left\{\left[\partial_\alpha^{x_1}\partial_\beta^{x_2}S\left(x_1,x_2\right)\right]S\left(x_2,x_1\right)+\left[\partial_\alpha^{x_1}\partial_\beta^{x_2}S\left(x_2,x_1\right)\right]S\left(x_1,x_2\right)\right\}\\ &\quad +2ig_{\rho\pi}^2 g_{\alpha\beta}S\left(x_1,x_2\right)\delta\left(x_1-x_2\right)\end{aligned}$$

Self energy

$O(\partial^0)$:

$$\begin{aligned}\Sigma^{\mu\nu}_{R,\mathrm{L}}(X,p) &= -\Delta^{\mu\nu}_L\Sigma_L\left(p^2,u\cdot p,\beta^2\right)-\Delta^{\mu\nu}_T\Sigma_T\left(p^2,u\cdot p,\beta^2\right)\\ \Sigma^{\mu\nu}_{<,\mathrm{L}}(X,p) &= -\Delta^{\mu\nu}_L\Sigma^<_L\left(p^2,u\cdot p,\beta^2\right)-\Delta^{\mu\nu}_T\Sigma^<_T\left(p^2,u\cdot p,\beta^2\right)\end{aligned}$$

$$\begin{aligned}\Delta^{\mu\nu} &= g^{\mu\nu}-\frac{p^\mu p^\nu}{p^2}\\ \Delta^{\mu\nu}_L &= \frac{\Delta^{\mu\rho}\Delta^{\nu\sigma}u_\rho u_\sigma}{\Delta^{\mu\nu}u_\mu u_\nu}\\ \Delta^{\mu\nu}_T &= \Delta^{\mu\nu}-\Delta^{\mu\nu}_L\end{aligned}$$

$O(\partial^1)$:

$$\begin{aligned}\Sigma^{<,\mathrm{NL}}_{\alpha\beta} =\;& \partial^X_\alpha\beta_\beta E_1\left(p^2,u\cdot p,\beta^2\right)+\partial^X_\alpha\beta_\rho u_\beta p^\rho E_2\left(p^2,u\cdot p,\beta^2\right)\\ &+\partial^X_\alpha\beta_\rho u_\beta u^\rho E_3\left(p^2,u\cdot p,\beta^2\right)-(\alpha\leftrightarrow\beta)\\ &+p_{\alpha,\beta}\text{ proportional terms,}\\ \Sigma^{\mathrm{NL}}_{R,\alpha\beta} =\;& \partial^X_\alpha\beta_\beta F_1\left(p^2,u\cdot p,\beta^2\right)+\partial^X_\alpha\beta_\rho u_\beta p^\rho F_2\left(p^2,u\cdot p,\beta^2\right)\\ &+\partial^X_\alpha\beta_\rho u_\beta u^\rho F_3\left(p^2,u\cdot p,\beta^2\right)-(\alpha\leftrightarrow\beta)\\ &+p_{\alpha,\beta}\text{ proportional terms.}\end{aligned}$$

$\Sigma(x_1,x_2)\sim\partial_{x_1}S(x_1,x_2)\partial_{x_2}S(x_2,x_1)$

$O(\partial^1)$:

$$\begin{cases}\partial^\mu_{x_1}\to\frac{1}{2}\partial^\mu_X-ip^\mu\\ S(X,p)\sim f^L_\pi(X,p)+f^{NL}_\pi(X,p)\end{cases}$$

$$f^L_\pi \to \frac{1}{e^{\beta(X)\cdot p}-1}$$

Solutions(I)

$O(\partial^0)$:

$$G^{\mu\nu}_{R,L} = -i\sum_{a=T,L}\Delta^{\mu\nu}_a\rho_a + i\frac{p^\mu p^\nu}{p^2 m_V^2},$$
$$G^{<,L}_{\mu\nu} = \sum_{a=T,L}\Delta^a_{\mu\nu}\rho_a\rho_a^*\Sigma^a_<.$$

$$\rho_{L/T} = \frac{1}{p^2 - m_V^2 - i\Sigma_{L/T}},$$

$\rho_{L/T}$: function of $p^2, u\cdot p, \beta^2$

specific relation for $\rho - \pi$ interaction:

$$\Sigma^{\mu\nu}_{>,L}(X,p) = e^{\beta p\cdot u}\Sigma^{\mu\nu}_{<,L}(X,p),$$
$$\Sigma^{\mu\nu}_{>,L} - \Sigma^{\mu\nu}_{<,L} = \Sigma^{\mu\nu}_{R,L} - \Sigma^{\mu\nu}_{A,L} = 2Re\left(\Sigma^{\mu\nu}_{R,L}\right),$$
$$\Sigma^a_< = 2n_B Re\left(\frac{i}{\rho_a}\right).$$

$$G^{<,L}_{\mu\nu} = 2n_B\sum_{a=T,L}\Delta^a_{\mu\nu}Re\left(i\rho_a^*\right),$$

Fluctuation-dissipative theorem

Solutions(II)

$O(\partial^1)$:

$$G^{R,\mathrm{NL}}_{\mu\nu}(X,p) = G^{R,\mathrm{L}}_{\mu\rho}(X,p)\,\Sigma^{\rho\sigma}_{R,\mathrm{NL}}(X,p)\,G^{R,\mathrm{L}}_{\sigma\nu}(X,p)$$
$$+i\frac{p_\alpha}{(p^2-m^2)}G^{R,\mathrm{L}}_{\mu\rho}(X,p)\,\partial^\alpha_X\left[\Sigma^{\rho\sigma}_{R,\mathrm{L}}(X,p)\,G^{R,\mathrm{L}}_{\sigma\nu}(X,p)\right]$$
$$+\frac{1}{2}\frac{p_\mu}{m^2(p^2-m^2)}\partial^X_\rho\left[\Sigma^{\rho\sigma}_{R,\mathrm{L}}(X,p)\,G^{R,\mathrm{L}}_{\sigma\nu}(X,p)\right]$$
$$+\frac{i}{2}G^{R,\mathrm{L}}_{\mu\rho}(X,p)\left[\Sigma^{\rho\sigma}_{R,\mathrm{L}}(X,p),G^{R,\mathrm{L}}_{\sigma\nu}(X,p)\right]_{P.B.},$$

$$G^{<,\mathrm{NL}}_{\mu\nu}(X,p) = G^{R,\mathrm{L}}_{\mu\rho}(X,p)\,\Sigma^{R,\rho\sigma}_{\mathrm{NL}}(X,p)\,G^{R,\mathrm{L}}_{\sigma\beta}(X,p)\,\Sigma^{<,\beta\alpha}_{\mathrm{L}}(X,p)\,G^{A,\mathrm{L}}_{\alpha\nu}(X,p)$$
$$+G^{R,\mathrm{L}}_{\mu\rho}(X,p)\,\Sigma^{<,\rho\sigma}_{\mathrm{NL}}(X,p)\,G^{A,\mathrm{L}}_{\sigma\nu}(X,p)$$
$$+G^{R,\mathrm{L}}_{\mu\rho}(X,p)\,\Sigma^{<,\rho\sigma}_{\mathrm{L}}(X,p)\,G^{A,\mathrm{L}}_{\sigma\rho}(X,p)\,\Sigma^{A,\rho\alpha}_{\mathrm{NL}}(X,p)\,G^{A,\mathrm{L}}_{\alpha\nu}(X,p)$$
$$-\frac{i}{2}G^{R,\mathrm{L}}_{\mu\rho}(X,p)\left[G^{-1,\rho\sigma}_{R,\mathrm{L}}(X,p),G^{<,\mathrm{L}}_{\sigma\nu}(X,p)\right]_{P.B.}$$
$$-\frac{i}{2}\left[G^{<}_{\mathrm{L},\mu\alpha}(X,p),G^{-1,\alpha\sigma}_{A,\mathrm{L}}(X,p)\right]_{P.B.}G^{A,\mathrm{L}}_{\sigma\nu}(X,p)$$
$$-\frac{i}{2}G^{R,\mathrm{L}}_{\mu\rho}(X,p)\,G^{<}_{\mathrm{L},\beta\alpha}(X,p)\left[G^{-1,\rho\beta}_{R,\mathrm{L}}(X,p),G^{-1,\alpha\sigma}_{A,\mathrm{L}}(X,p)\right]_{P.B.}G^{A,\mathrm{L}}_{\sigma\nu}(X,p)$$
$$+p_{\mu/\nu}\ \text{proportional terms.}$$

Self energy correction

Poisson bracket

Non-dissipative distribution (no contribution to spin alignment!)

MVSD

No-interaction case:

$$G^{\mu\nu}_{<,0}(X,p) = 2\pi\delta(p^2-m^2)\theta(-p^0)\epsilon^*_\mu(\lambda_1,-p)\epsilon_\nu(\lambda_2,-p)$$
$$+\theta(p^0)\epsilon_\mu(\lambda_1,p)\epsilon^*_\nu(\lambda_2,p)f^{V,(0)}_{\lambda_1\lambda_2}(X,p)$$
$$+\theta(-p^0)\epsilon^*_\mu(\lambda_1,-p)\epsilon_\nu(\lambda_2,-p)f^{V,(0)}_{\lambda_2\lambda_1}(X,-p)$$

$$f^{V,(0)}_{\lambda_1\lambda_2}(x,p) = \epsilon^*_\mu(\lambda_1,p)\,G^{\mu\nu}_{<,0}(x,p)\,\epsilon_\nu(\lambda_2,p)\,,$$

Assuming this relation works for interaction case

$$f^V_{\lambda_1\lambda_2}(x,p) \quad = \quad \epsilon^*_\mu(\lambda_1,p)\,G^{\mu\nu}_<(x,p)\,\epsilon_\nu(\lambda_2,p)$$

Spin alignment:

$$\rho_{00} \quad = \quad \frac{\int dp_0 p_0 \epsilon^*_\mu(0,p)\,G^{\mu\nu}_{<,\mathrm{L+NL}}(x,p)\,\epsilon_\nu(0,p)}{\Sigma_{\lambda_1=0,\pm}\int dp_0 p_0 \epsilon^*_\mu(\lambda_1,p)\,G^{\mu\nu}_{<,\mathrm{L+NL}}(x,p)\,\epsilon_\nu(\lambda_1,p)}$$

Spin alignment

$$\rho_{00}(x,\mathbf{p})-\frac{1}{3} \approx \frac{\int \frac{dp^0}{2\pi}p^0\left[f_{00}^{V,\mathrm{L}}-\frac{1}{3}Tr\left(f_{\lambda_1\lambda_2}^{V,\mathrm{L}}\right)\right]}{\int \frac{dp^0}{2\pi}p^0 Tr\left(f_{\lambda_1\lambda_2}^{V,\mathrm{L}}\right)}$$

Pure interaction effect

$$+\frac{\int \frac{dp^0}{2\pi}p^0\left[f_{00}^{V,\mathrm{NL}}-\frac{1}{3}Tr\left(f_{\lambda_1\lambda_2}^{V,\mathrm{NL}}\right)\right]}{\int \frac{dp^0}{2\pi}p^0 Tr\left(f_{\lambda_1\lambda_2}^{V,\mathrm{L}}\right)} - \frac{\int \frac{dp^0}{2\pi}p^0 Tr\left(f_{\lambda_1\lambda_2}^{V,\mathrm{NL}}\right)}{\int \frac{dp^0}{2\pi}p^0 Tr\left(f_{\lambda_1\lambda_2}^{V,\mathrm{L}}\right)} \frac{\int \frac{dp^0}{2\pi}p^0\left[f_{00}^{V,\mathrm{L}}-\frac{1}{3}Tr\left(f_{\lambda_1\lambda_2}^{V,\mathrm{L}}\right)\right]}{\int \frac{dp^0}{2\pi}p^0 Tr\left(f_{\lambda_1\lambda_2}^{V,\mathrm{L}}\right)}$$

Rearrange particle number correction

$$= \delta\rho_{00}^{(0)} + \xi^{\rho\sigma}X_{(\rho\sigma)} + \omega^{\rho\sigma}Y_{[\rho\sigma]} + \partial^\rho\beta_0 Z_\rho,$$

$$X'_{(\rho\sigma)} = X_{(\rho\sigma)} + \eta_{(\rho}u_{\sigma)}$$

$$Y'_{[\rho\sigma]} = Y_{[\rho\sigma]} + \eta_{[\rho}u_{\sigma]}$$

$$Z'_\rho = Z_\rho - \eta_\rho,$$

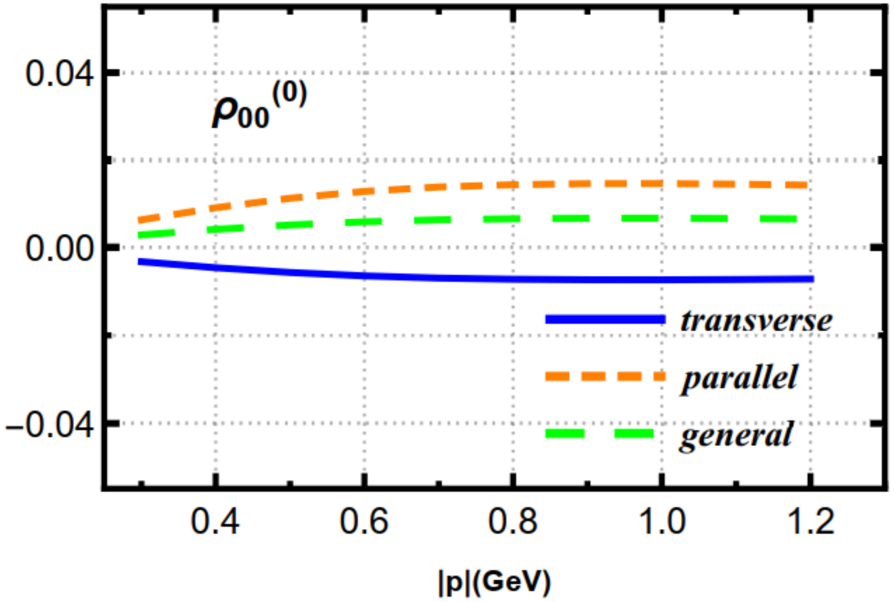
$\delta\rho_{00}^{(0)}, X, Y, Z$: relate with $T-L$,
when $T=L$ or $g\to 0$, vanish

Flow rest frame

Choice:

$m_\rho = 770MeV, m_\pi = 140MeV, g_{\rho\pi} = 6.067,\ T_f = 120MeV,$

$u = (1, \vec{0})$ and $Z^\rho = 0$



parallel: $(p_0, 0, |p|, 0)$
transverse: $(p_0, 0,0, |p|)$
general: $(p_0, 0,0.8|p|, 0.6|p|)$

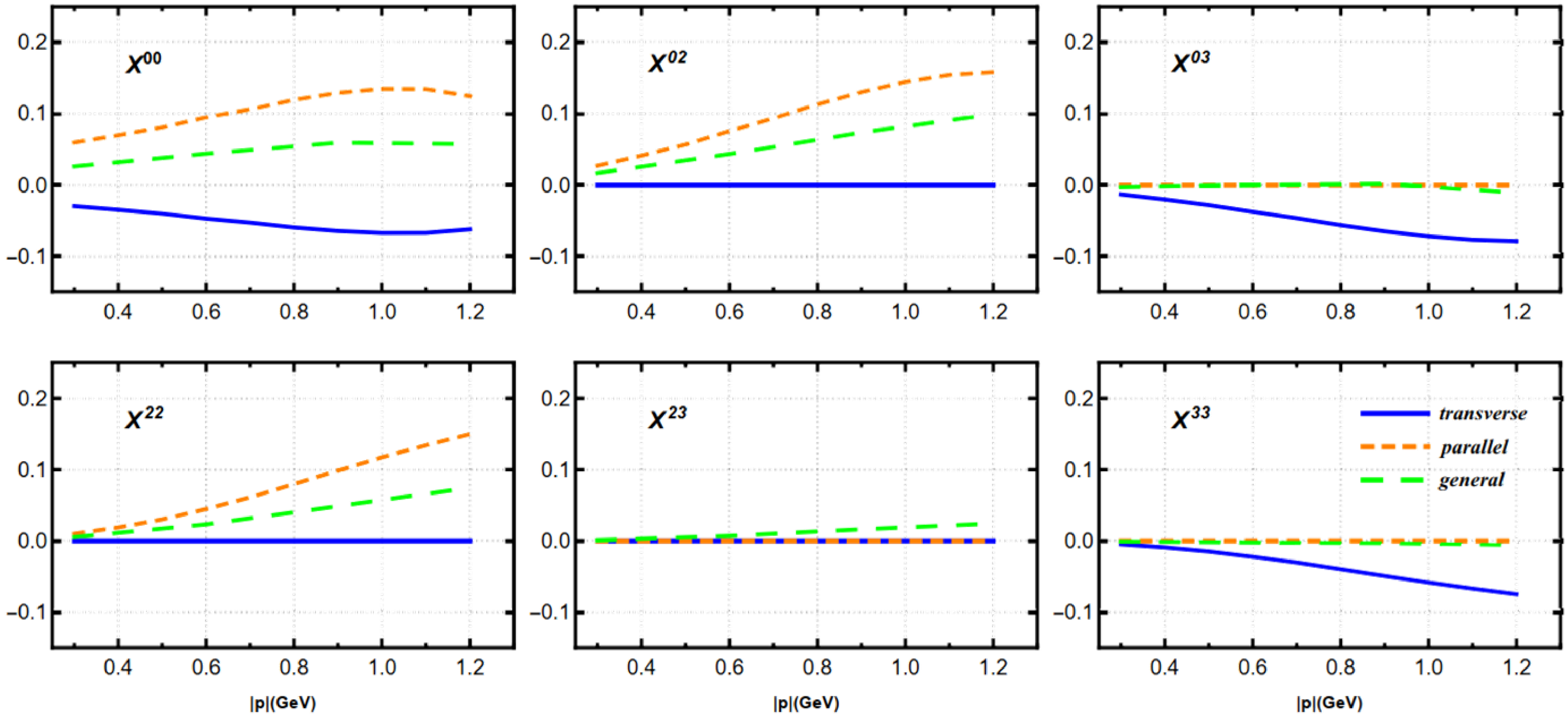
No global spin alignment at $O(\partial^0)$

$$\Delta^L_{\mu\nu}\epsilon^\mu_0\epsilon^\nu_0 + \frac{1}{3} = \frac{1}{3} - \frac{\mathbf{p}_y^2}{|\mathbf{p}|^2}.$$

$$Tr\left(f^{V,L}_{\lambda_1\lambda_2}\right) = 2n_B\left[3Im\left(\rho_T^*\right) + Im\left(\rho_L^* - \rho_T^*\right)\right],$$

$$\delta\rho^{(0)}_{00} = -\frac{2}{\int \frac{dp^0}{2\pi}p^0 Tr\left(f^{V,L}_{\lambda_1\lambda_2}\right)}\int\frac{dp^0}{2\pi}p^0\left(\frac{1}{3} + \epsilon^\mu_0\epsilon^\nu_0\Delta^L_{\mu\nu}\right)n_B Im\left(\rho_L^* - \rho_T^*\right).$$

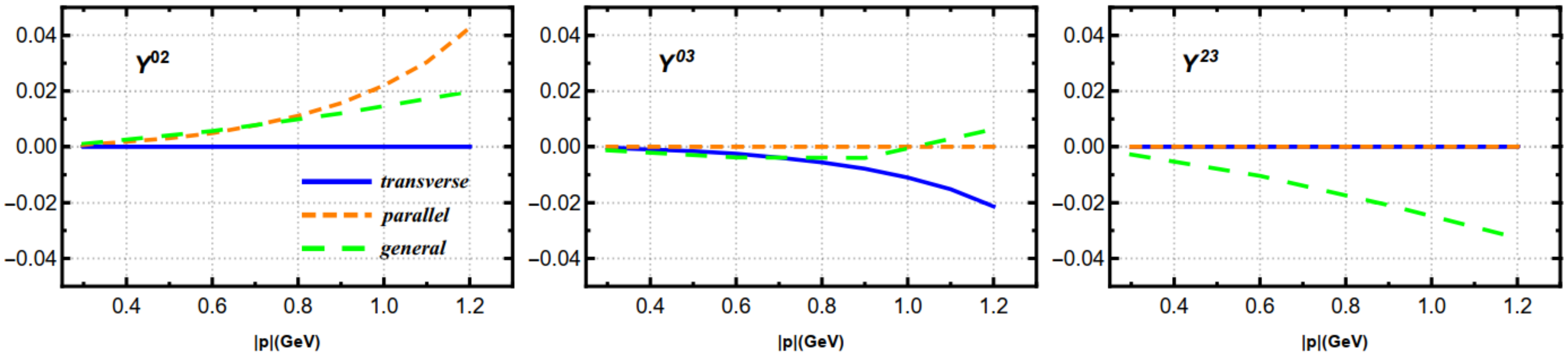
Coefficient to thermal shear



$X_{00}^{par} = -2X_{00}^{tran}, X_{02}^{par} = -2X_{03}^{tran}, X_{22}^{par} = -2X_{33}^{tran}, X \sim 0.1$

Inhomogeneous $\xi_{\mu\nu}$ induce global spin alignment

Coefficient to thermal vorticity



$Y_{02}^{par} = -2Y_{03}^{tran}, Y \sim 0.01$

Inhomogeneous $\omega_{\mu\nu}$ induce global spin alignment

To induce a global spin alignment at $O(\partial^1)$:

Interaction(deviation between T and L) + inhomogeneous $\xi_{\mu\nu}$ or $\omega_{\mu\nu}$

Summary

1. We present a new formalism to calculate the spin polarization phenomena based on DSE on CTP contour. It can introduce the effect of interaction or the off-shell correction.

2. We apply this formalism to the ρ meson's spin alignment in a pion gas. Coupling between the thermal shear/vorticity and spectral difference induce the global spin alignment.

3. We study the self energy correction at $O(\partial^1)$ and it gives a non-trivial contribution to the spin alignment.

Outlook

1. Spin-1/2 particle has a non-dissipative distribution at $O(\partial^1)$, it will contribute to $O(\partial^1)$ self energy. For other vector mesons (i.e. J/Ψ, D), this effect may be important.

2. We can solve the spin-1/2 DSE on CTP and study the relation between quark's polarization and its spectral function.

3. Other $O(\partial^1)$ effects($\partial_\mu \mu_5, \partial_\mu \mu_s$)

4. Particle number correction contains $\xi^{\mu\nu} p_\mu p_\nu n_B (1 + n_B) \rho_a Im(\rho_a)$, at fixed p^2, increasing $|\boldsymbol{p}|$, ratio between NL with L larger than 1, spatial gradient fails (general question for spatial gradient!)

Thanks for your time