Influence of the QCD critical point on the nucleon-nucleon potential and nuclear correlations

GOETHE UNIVERSITÄT FRANKFURT AM MAIN

Juan M. Torres-Rincon
(Goethe University Frankfurt)

ITP Institute for Theoretical Physics

arXiv:1805.04444 [hep-ph]
with E. Shuryak (Stony Brook Uni.)

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Outline

- **Motivation:** QCD critical point and fluctuations
- **Main idea:** Critical mode affects *NN* interaction
- **Example:** Strongly-correlated systems
- **Results:** Nuclear correlations close to the critical region
- **Results:** Clustering close to the critical point
- **Summary**

QCD phase diagram and critical point

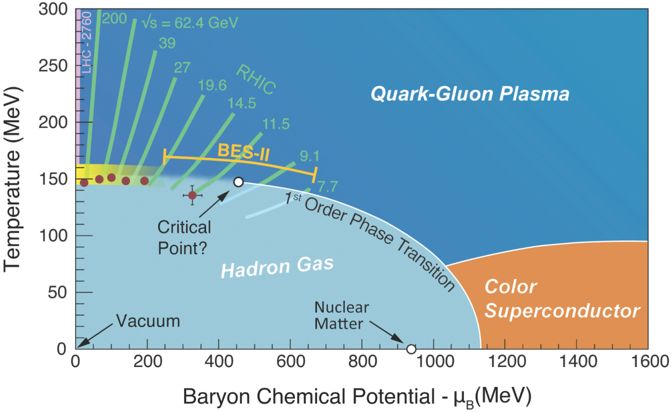
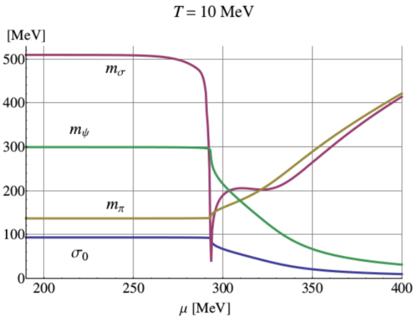


Image: S. Mukherjee (Brookhaven National Lab)

σ as a critical mode

σ mass decreases close to the phase transition/critical point (correlation length ξ increases)

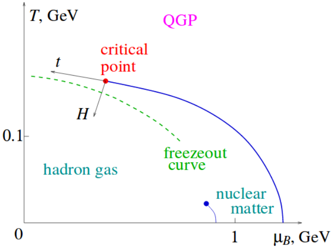


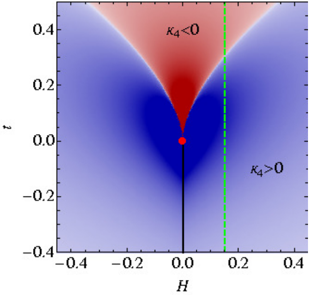
R.-A. Tripolt, Ph. Thesis, 2015
(quark-meson model with FRG approach)

$$m_\sigma \sim \frac{1}{\xi} \sim \left(\frac{|T - T_c|}{T_c} \right)^\nu$$ (with ξ limited by finite lifetime effects)

Moments of the σ probability distribution

M. Stephanov, 2008 and 2011





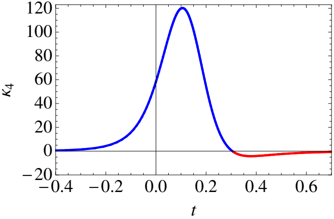
$$P[\sigma] \sim \exp\left(-\Omega/T\right)$$

$$\Omega = \int d^3x \, \frac{(\nabla\sigma)^2}{2} + \frac{m_\sigma^2}{2}\sigma^2 + \frac{\lambda_3}{3}\sigma^3 + \frac{\lambda_4}{4}\sigma^4$$

$$\kappa_2 = \langle \sigma_0^2 \rangle$$

$$\kappa_4 = \langle \sigma_0^4 \rangle - 3\langle \sigma_0^2 \rangle^2$$

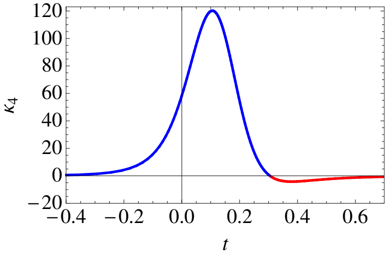
$$Kurtosis = \kappa_4/\kappa_2^2$$

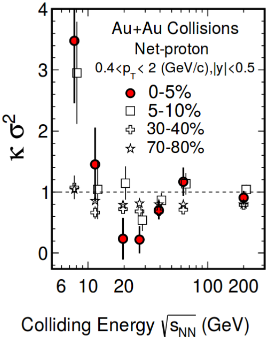


Critical mode coupled to baryons

M.Stephanov, 2011

$$\mathcal{L}_{eff} = g\sigma p\bar{p}$$





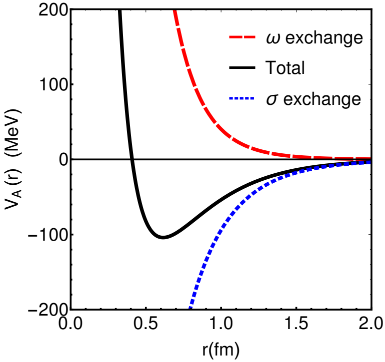
$$C_2 = \langle \delta N^2_{p-\bar{p}} \rangle$$

$$C_4 = \langle \delta N^4_{p-\bar{p}} \rangle - 3\langle \delta N^2_{p-\bar{p}} \rangle^2$$

$$\kappa\sigma^2 = C_4/C_2$$

STAR Coll., 2015

Simple-as-possible (but not simpler) model for *NN* interaction due to **Serot-Walecka (1984)**



$$V_A(r) = -\frac{\alpha_\sigma}{r} e^{-m_\sigma r} + \frac{\alpha_\omega}{r} e^{-m_\omega r}$$

mean-field parameters:

$$\alpha_\sigma = 6.0$$

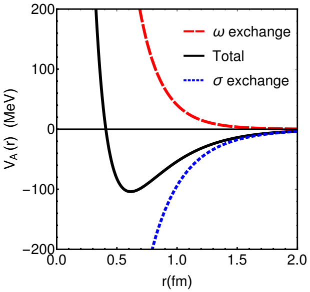
$$\alpha_\omega = 10.8$$

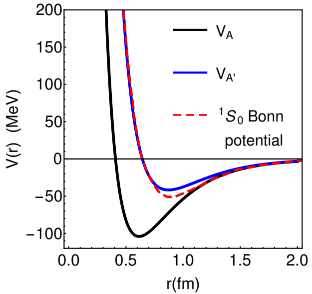
$$m_\sigma = 500 \text{ MeV}$$

$$m_\omega = 782 \text{ MeV}$$

Near cancellation

- Almost **cancellation** between attraction and repulsion.
- Additional Fermi motion provides stable nuclear matter with $E/N = -16$ MeV
- Small imbalance would strongly modify the potential

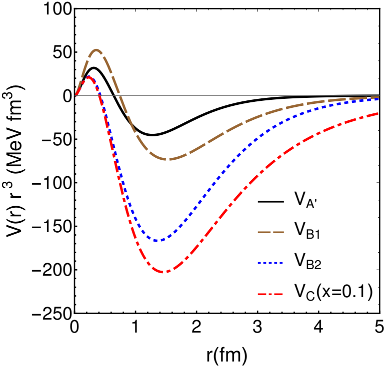




In $V_{A'}(r)$ we allow for extra repulsion to match the Bonn NN potential (Machleidt, 2000)

NN potential modifications

- Close to T_c, a very light σ controls the NN attraction
- ***NN* potential should be affected by the presence of the QCD critical point!**
- We consider more and more attractive potentials:



- V_A: Serot-Walecka with MF parameters
- $V_{A'}$: extra repulsion $\alpha_\omega \to 1.4\alpha_\omega$
- V_{B1}: $V_{A'}$ with $m_\sigma^2 \to m_\sigma^2/2$, $\alpha_\sigma \to \alpha_\sigma/2$
- V_{B2}: $V_{A'}$ with $m_\sigma^2 \to m_\sigma^2/2$
- V_C: very light critical mode $V_C(x) = (1-x)V_{B2} + xV_{A'}(m_\sigma^2 \to m_\sigma^2/6)$

Numerical study: Molecular Dynamics + Langevin

NN potential in a classical nonrelativistic Molecular Dynamics scheme

$$\begin{cases} \dfrac{d\vec{x}_i}{dt} = \dfrac{\vec{p}_i}{m_N} \\ \dfrac{d\vec{p}_i}{dt} = -\displaystyle\sum_{j\neq i} \dfrac{\partial V(|\vec{x}_i - \vec{x}_j|)}{\partial \vec{x}_i} - \lambda \vec{p}_i + \vec{\xi}_i \end{cases}$$

with Langevin dynamics,

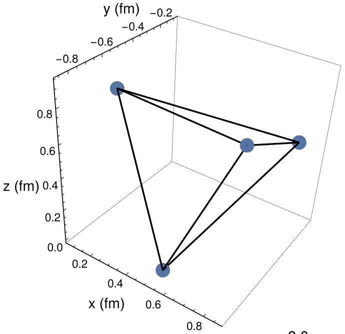
$$\langle \vec{\xi}_i(t) \rangle = 0$$

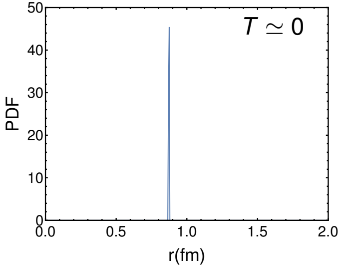
$$\langle \xi_i^a(t) \xi_j^b(t') \rangle = 2T\lambda m_N \delta^{ab} \delta_{ij} \delta(t - t')$$

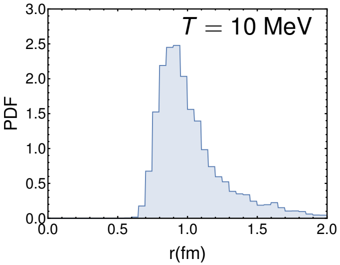
with $a, b = 1, 2, 3$ and $\lambda = T/(m_N D_B)$
λ: drag force, D_B: baryon diffusion coefficient

- Quantum effects neglected at high temperature T ("localization quantum potential" added for infinite nuclear matter at saturation to account for Fermi motion)

$V_{A'}$ potential

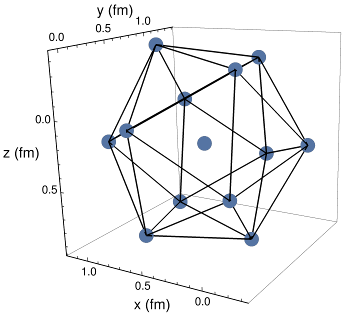


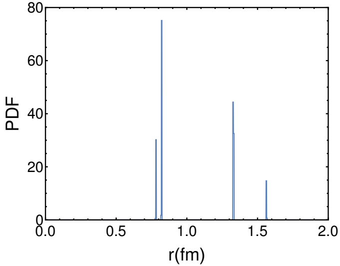




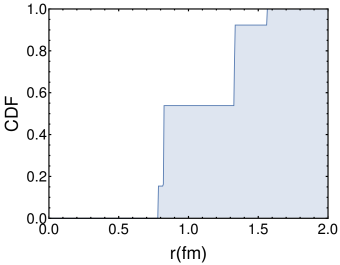
Medium-size clusters, $N = 13$

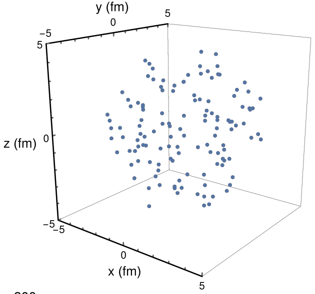
$V_{A'}$ potential





$T \simeq 0$





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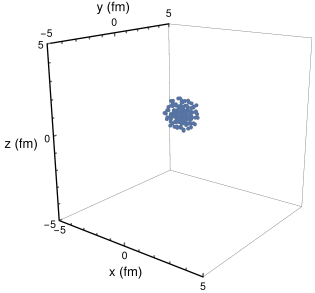
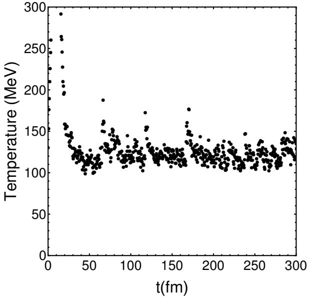


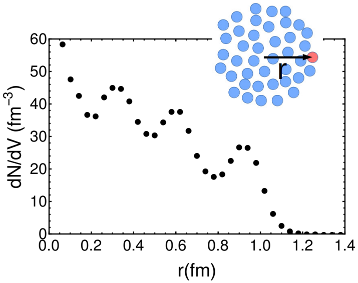


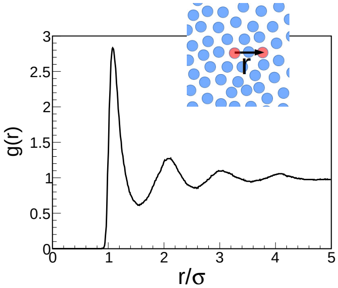
$T = 120$ MeV

Huge potential energy well when N is large: **clustering**.

Strongly-correlated systems

- Strongly correlated system ($P/K \simeq \mathcal{O}(N) > 1$): well beyond mean field





- Infinite systems: internal structure is described by the **pair correlation function** $g(r)$ e.g. liquid Argon ($N = 108$) via Lennard-Jones potential

- Approaches based on Boltzmann assumptions do not capture this

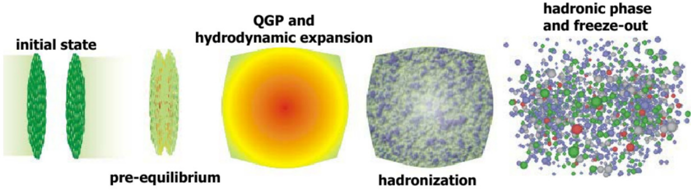
Approaching the physical case

Previous examples were unrealistic for heavy-ion collisions

Effects preventing clustering

- Expansion, radial collective flow
- Freeze-out temperatures $T \sim 150$ MeV
- Finite time effects (in particular, duration of hadronic phase)

We need to address these for RHIC collisions at the Beam Energy Scan



BES STAR freeze out

We try to mimic as much as possible experimental situation in BES I, as measured by STAR @ RHIC (STAR Collab. 2016 & 2017)

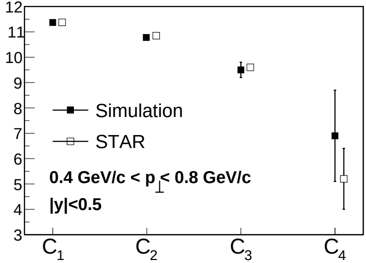
- Temperature $T \simeq 150$ MeV
- Densities: 1-2 n_0
- Finite time evolution: $t = 5$ fm
- Non-relativistic nucleon dynamics
- Fireball expansion: mapping of y and p_T distributions to experimental measured distributions
- Simulations: 32 nucleons, 10^5 events (similar to experiment for 5% most central events)
- Antinucleons: For $\sqrt{s_{NN}} < 19.6$ GeV they are suppressed, at least, a factor of 10 w.r.t. protons

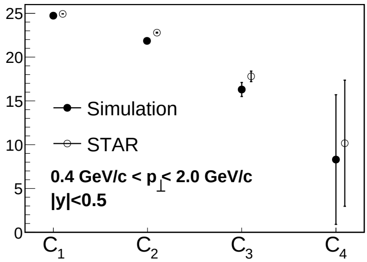
Note: It is a crude model and several effects not covered.
Understand as a first approximation to the physical situation.

Poisson distribution at $\sqrt{s_{NN}} = 19.6$ GeV $\leftrightarrow$ Noncritical potential $V_{A'}$

- $|y| < 0.5$, 0.4 GeV$< p_\perp < 0.8$ GeV
- $|y| < 0.5$, 0.4 GeV$< p_\perp < 2$ GeV

protons

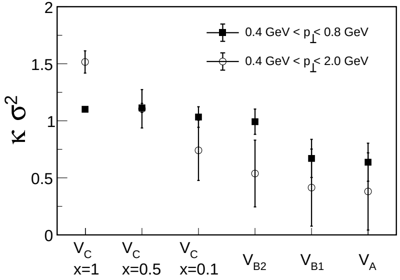


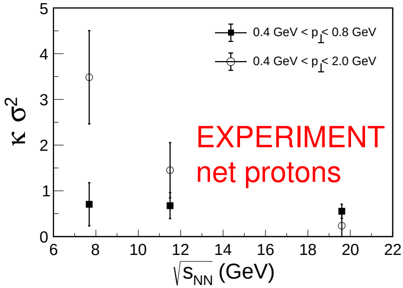


$$C_1 = \langle N_p \rangle \ , \quad C_2 = \langle \delta N_p^2 \rangle \ , \quad C_3 = \langle \delta N_p^3 \rangle \ , \quad C_4 = \langle \delta N_p^4 \rangle - 3 \langle \delta N_p^2 \rangle^2$$

Few-body correlations should contribute to net-proton moments.

Scaled kurtosis: $\kappa\sigma^2 = C_4/C_2$

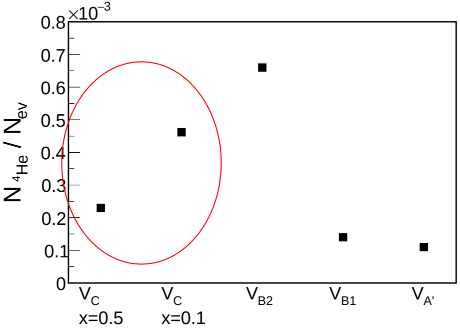




Expected qualitative increase with enhanced attraction.

Aggregation of few nucleons (**pre-nuclei**) can be formed within few fm/c.
We search 4 isolated nucleons close in phase space in the same simulation

Nucleons belong to bigger clusters for these potentials



Close to T_c, we expect an **excess of light nuclei** over thermal expectations.

Triton-proton/deuteron ratio

$$\left.\frac{N_t N_p}{N_d^2}\right|_{ideal} = g \qquad (g = 0.29)$$

(ratio considered by Sun,Chen,Ko,Xu, 2017)

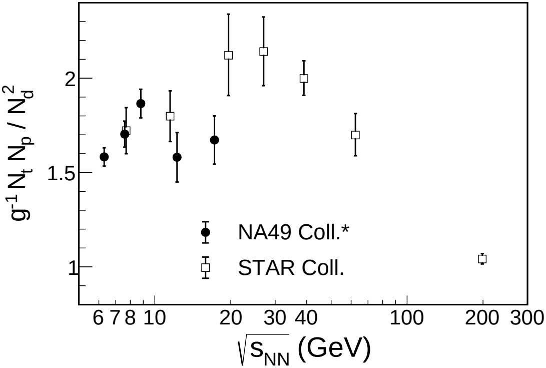
$$N = Vol\, \frac{(2S+1)}{2\pi^2} m^2 T\, K_2(m/T) \exp\left(\frac{B\mu_B + q\mu_q}{T}\right)$$

Triton-proton/deuteron ratio

$$g^{-1}\frac{N_t N_p}{N_d^2} \sim \left\langle e^{-\frac{V}{T}} \right\rangle \qquad (g = 0.29)$$

V is the interparticle potential

$$g^{-1}\frac{N_t N_p}{N_d^2} \sim \left\langle e^{-\frac{V}{T}} \right\rangle \qquad (g = 0.29)$$



*Sun, Chen, Ko, Xu 2017, based on NA49 Coll. data

STAR Collaboration, preliminary 0%-10% (QM2018)

Pre-nuclei vs nuclei

Pre-nuclei

- Formed by strong correlation until kinetic freeze-out.
- $T \sim 150$ MeV. Mainly Classical. Modified *NN* potential.
- Characterized by Wigner distribution $W_{\mathrm{pre-nuclei}}(x, p)$

Nuclei

- Final state, measured by experiment.
- Vacuum. Quantum states.
- Characterized by Wigner transform of the wave function Ψ^W_{nuclei}

How to continuously interpolate between a classical description at high temperatures and a quantum system in vacuum (ground state)?

Summary

- Critical mode σ becomes light close to T_c

$\downarrow$

- *NN* potential significantly attractive and long-ranged
  Modifications from cold nuclear matter potential

$\downarrow$

- Strong correlations among nucleons (increase $\kappa\sigma^2$,...)
  Mean field/*Stosszahlansatz* **not enough** to capture the effect

$\downarrow$

- We predict enhanced formation of light nuclei (t, ^{4}He...) close to T_c

Summary

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$$\downarrow$$

- *NN* potential significantly attractive and long-ranged
  Modifications from cold nuclear matter potential

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  Mean field/*Stosszahlansatz* **not enough** to capture the effect

$$\downarrow$$

- We predict enhanced formation of light nuclei (t, ^{4}He...) close to T_c
- **OUTLOOK**: Flucton (semiclassical) method to consider both quantum and thermal fluctuations under an interaction potential

Influence of the QCD critical point on the nucleon-nucleon potential and nuclear correlations

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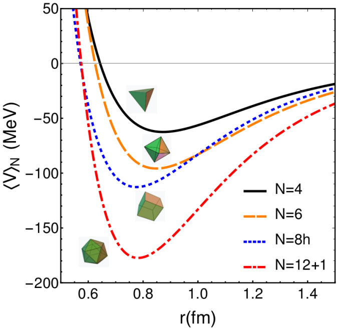
Platonic solids

Few-body systems usually follow geometry arguments.

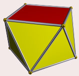


$V_{A'}$ potential

Curious fact

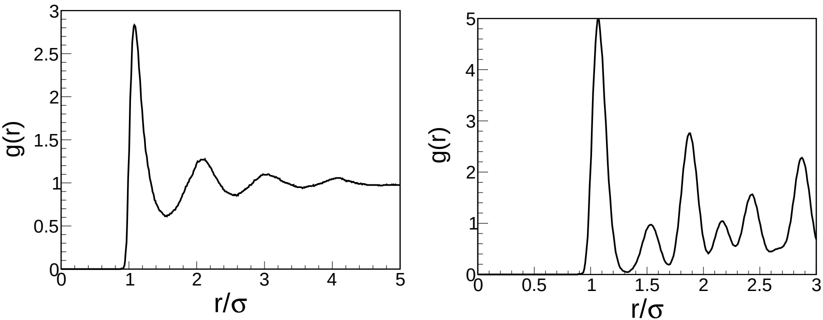
For $N = 8$ the cube is **not** the equilibrium configuration.

In a good approximation it is a **square antiprism**



Strong correlations

Lennard-Jones potential, for N=108 Ar atoms, liquid vs solid

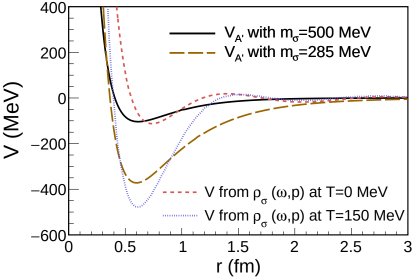


Boltzmann approximation assumes $g(r) = 1$ (dilute gas)
Correlations are important in our system!

Scalar meson with full spectral width

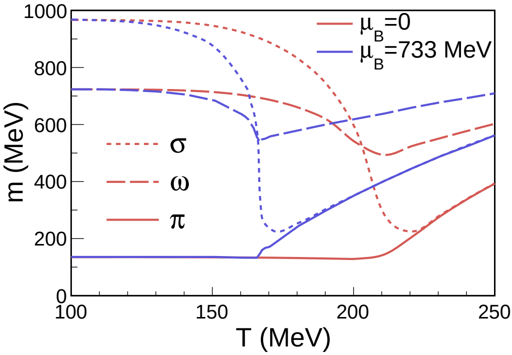
$$V_\sigma(\mathbf{r}) = g_\sigma^2 \int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} \frac{d^4p}{(2\pi)^4} e^{ip\cdot x}\, D_\sigma^R(p_0,\mathbf{p})$$

$$D_\sigma^R(p_0,\mathbf{p}) = -\int_{-\infty}^{\infty} d\omega \frac{\rho_\sigma(\omega,\mathbf{p})}{\omega - p_0 - i\epsilon}$$



Spectral function from quark-meson model using FRG.
R.-A. Tripolt, Ph.D. Thesis 2015

σ and ω pole masses in PNJL model

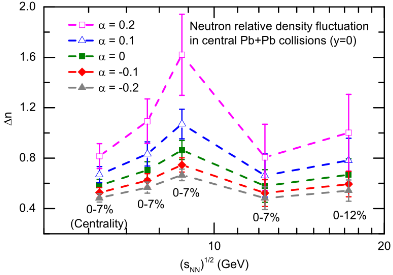


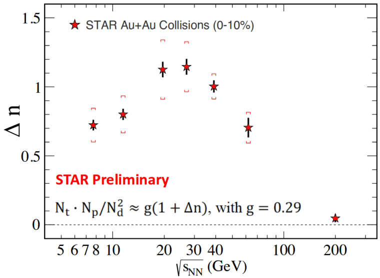
JMT-R, 2018 ($N_f = 3$ Polyakov-Nambu-Jona–Lasinio model)

- 80 % σ mass reduction at T_c
- 25 % ω mass reduction at T_c

Caveat: σ is to be identified with $f_0(980)$

$$\frac{N_t N_p}{N_d^2} = g(1+\Delta n) \qquad (\alpha = 0)$$





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