A Cascade Simulation for BEC in Gluon Transport

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Outline

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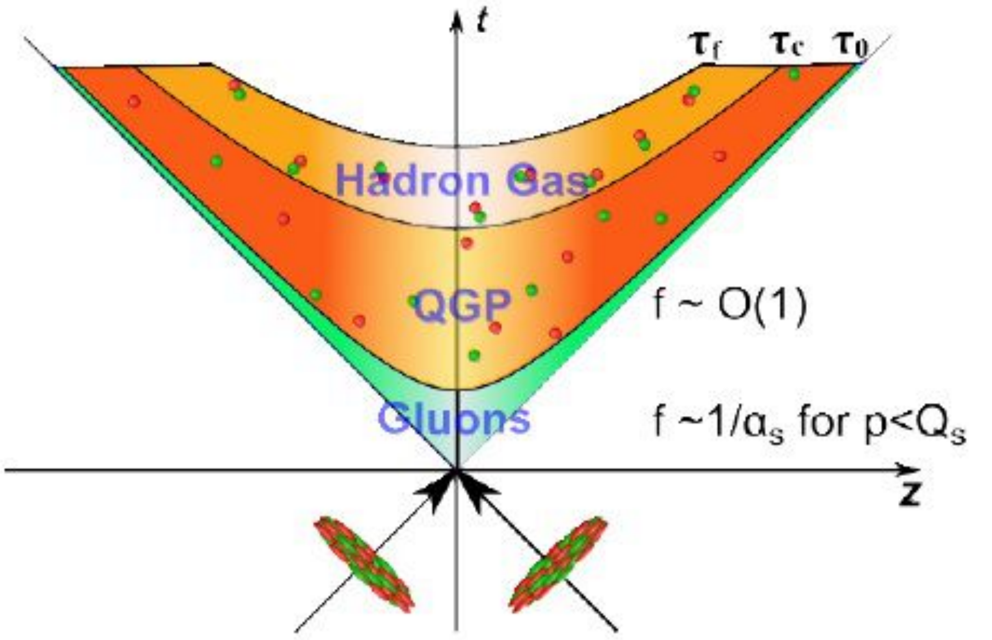
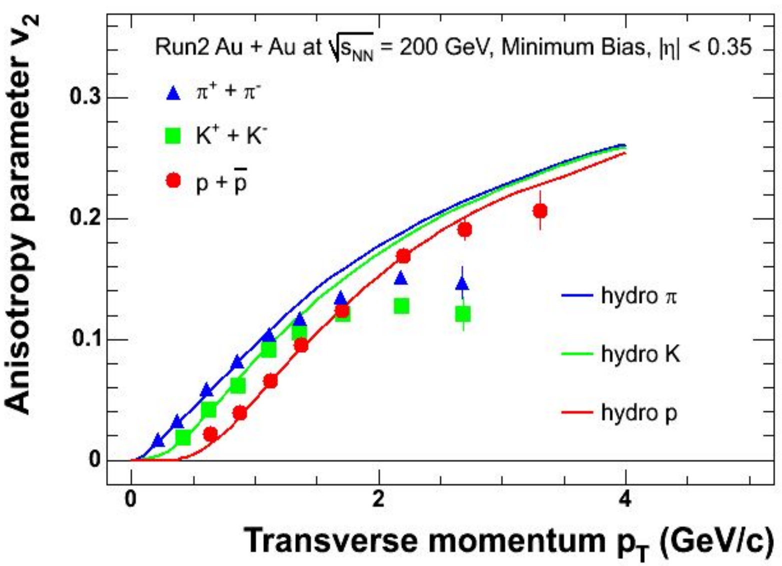
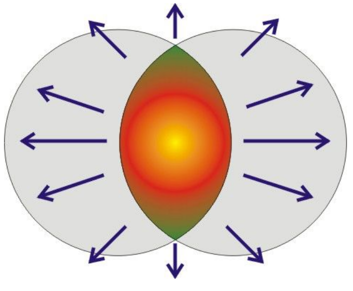
Introduction : Fast thermalization

how the system evolve to thermal eq. in a very short time scale



Initial: far-from equilibrium





Hydro onset time ~ 1fm/c

Introduction : Color Glass Condensate

CGC=effective field theory for hadrons at high energy limit

- high energy (time dilation)
  - fluctuation(sea partons) lifetime
  - internal interaction time scales
  - small x ($x \sim p_z / E_{hadron}$)

  ⇒ **gluon number increase at small x with increasing energy**

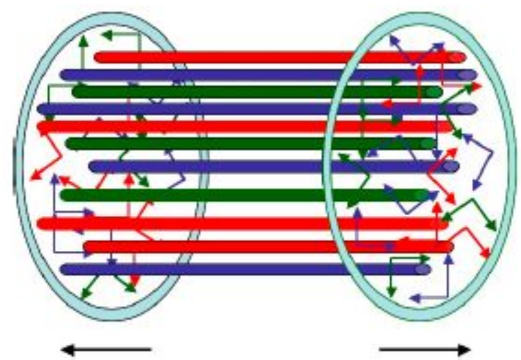
- gluon fusion $f \sim f^2 \alpha_s \Longrightarrow f \sim 1/\alpha_s$ **Saturation**

- **Saturation momentum Qs(~GeV)** $xG(x,Q^2)/Q^2 \to 1/\alpha_s$

⇒ $f \sim 1/\alpha_s (Q < Q_s)$ this feature can be inherited by initial Glasma through indrect way

Introduction : Glasma

Glasma=non-equilibrium state in between CGC and QGP



- using CGC as initial states for nuclei

$$\Longrightarrow \quad T^{\mu\nu} = diag(\varepsilon, \varepsilon, \varepsilon, -\varepsilon)$$

highly anisotyopic initial glasma fields

(won't last longer than 1/Qs)

- **Instabilities** -->wide range of unstable modes (up to Qs) grow exponentially untill saturation density

$\Longrightarrow$
- redistribute momentum ---> **isotropization**
- free up quanta from classical field

$$f_0 = 1/\alpha_s (p < Q_s)$$

initial condition amenable for kinetic theory

Introduction : Overpopulation

Overpopulation= the system contain more gluons than can be accommodated by a Bose-Einstein distribution

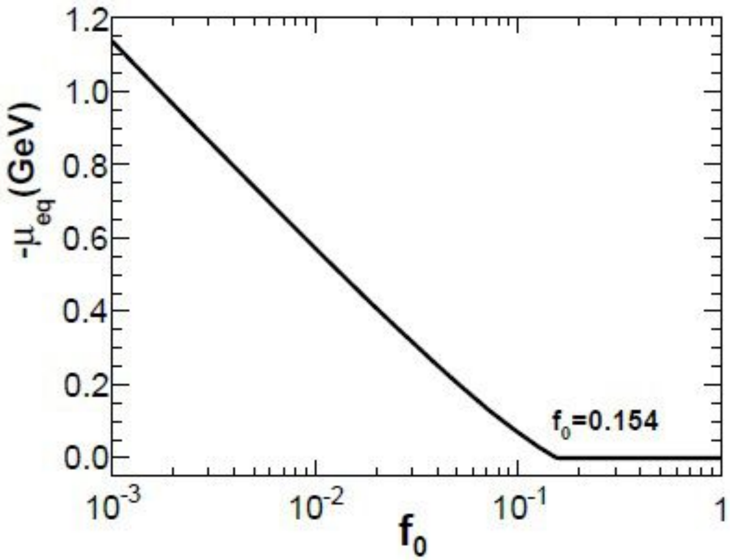
initially: $f_{init}(p) = f_0\theta(1 - p/Q_s)$ expected final: $f_{eq}(p) = 1/(e^{(p-\mu_{eq})/T_{eq}} - 1)$

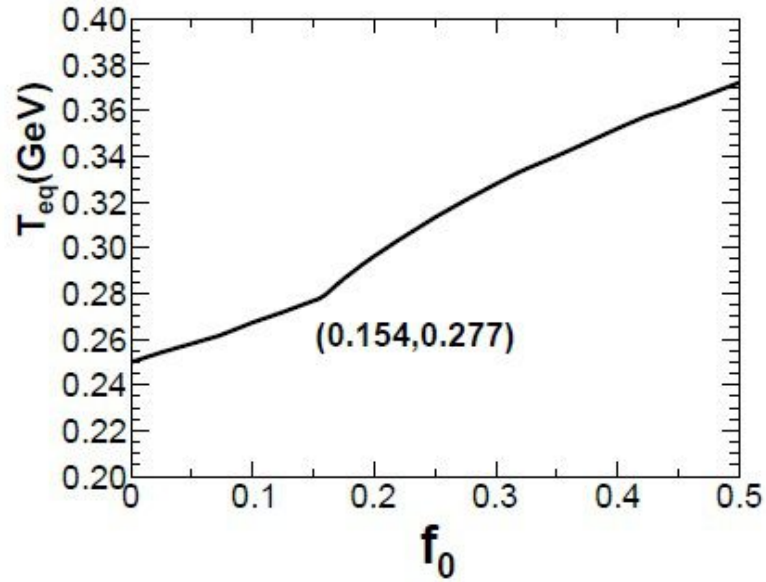
$$\varepsilon_{eq}(T_{eq}, \mu_{eq}) = \varepsilon_0 = \int \frac{d^3p}{(2\pi)^3} \frac{p}{e^{(p-\mu_{eq})/T_{eq}} - 1}$$

$$n_{eq}(T_{eq}, \mu_{eq}) = n_0 = \int \frac{d^3p}{(2\pi)^3} \frac{1}{e^{(p-\mu_{eq})/T_{eq}} - 1}$$

$f_{critical} = f_0 = 0.154$

even for $\alpha_s \sim 0.3$, still highly overpopulated

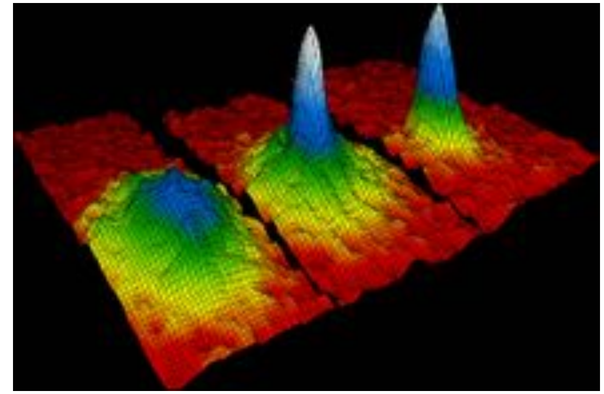




Introduction : Bose-Einstein Condensation?

BEC= macroscopic population of ground state

In principle, condensate behaves more like wave(field) & less like particles,since there quantum wave length overlap with inter particle scale



overpopulation <=> coherence : BEC ?

$$f_{eq}(\vec{p}) = 1/(e^{p/T} - 1) + \rho_c (2\pi)^3 \delta^{(3)}(\vec{p})$$

Study within classical field theory(no number conservation):

- Epelbaum, Gelis, NPA 872 (2011)
- Berges, Sexty, PRL 108 (2012)
- Kurkela, Moore, PRD 86 (2012)
- Berges,etc, arXiv:1408.1670

Can kinetic theory works ? (only before onset for BEC)

- Blaizot, Gelis, Liao, NPA 873 (2012)
- Blaizot, Liao, McLerran, NPA 920 (2013)
- Huang, Liao, arXiv:1303.7214
- Blaizot, Wu, Yan, arXiv:1402.5049
- Greco, arXiv:1408.1313

Transport equation with BEC

Boltzmann Equation:

$$\left(\partial_t + \frac{\vec{p}_1}{E_1}\cdot\vec{\nabla}\right) f_1(x,p_1) = \frac{1}{2E_1}\int \frac{d^3p_2}{(2\pi)^3 2E_2}\frac{d^3p_3}{(2\pi)^3 2E_3}\frac{d^3p_4}{(2\pi)^3 2E_4}\frac{1}{\nu}|\mathcal{M}_{12\to34}|^2$$

$$\times\ (2\pi)^4\delta^{(4)}(p_1+p_2-p_3-p_4)$$

$$\times\ [f_3 f_4(\mathbf{1}+\boldsymbol{f_1})(\mathbf{1}+\boldsymbol{f_2}) - f_1 f_2(\mathbf{1}+\boldsymbol{f_3})(\mathbf{1}+\boldsymbol{f_4})]$$

$$f = f^{gas} + f^{c}, \qquad f^{c} = (2\pi)^3 n_c \delta^{(3)}(\vec{p})$$

g: gas particle

c: condensate particle

Included	Not included
$g+g\leftrightarrow g+g$	$g+c\leftrightarrow g+c$
$g+g\leftrightarrow g+c$	$c+c\leftrightarrow c+c$

Transport equation with BEC

For gas particles:

$$\left(\partial_t + \frac{\vec{p}_1}{E_1}\cdot\vec{\nabla}\right) f_1^{gas}(x,p_1) = \frac{1}{2E_1}\int \frac{d^3p_2}{(2\pi)^3 2E_2}\frac{d^3p_3}{(2\pi)^3 2E_3}\frac{d^3p_4}{(2\pi)^3 2E_4}|\mathcal{M}_{12\to 34}|^2$$

$$\times\ (2\pi)^4\delta^{(4)}(p_1+p_2-p_3-p_4)$$

$$\times\ [\tfrac{1}{2}f_3^{gas}f_4^{gas}(1+f_1^{gas})(1+f_2^{gas}) - \tfrac{1}{2}f_1^{gas}f_2^{gas}(1+f_3^{gas})(1+f_4^{gas})$$

$$+f_3^{gas}f_4^{gas}(1+f_1^{gas})f_2^c - \tfrac{1}{2}f_1^{gas}f_2^c(1+f_3^{gas})(1+f_4^{gas})$$

$$+\ \tfrac{1}{2}f_3^c f_4^{gas}(1+f_1^{gas})(1+f_2^{gas}) - f_1^{gas}f_2^{gas}f_3^c(1+f_4^{gas})]$$

Transport equation with BEC

For condensate particles:

$$\partial_t f_1^c(x,p_1) = \frac{1}{2E_1}\int \frac{d^3p_2}{(2\pi)^3 2E_2}\frac{d^3p_3}{(2\pi)^3 2E_3}\frac{d^3p_4}{(2\pi)^3 2E_4}|\mathcal{M}_{12\to 34}|^2$$

$$\times\ (2\pi)^4\delta^{(4)}(p_1+p_2-p_3-p_4)$$

$$\times\ \left[f_3^{gas} f_4^{gas} f_1^c(1+f_2^{gas}) - \frac{1}{2} f_1^c f_2^{gas}(1+f_3^{gas})(1+f_4^{gas})\right]$$

$$f^c = (2\pi)^3 n_c \delta^{(3)}(\vec{p})$$

A small phase space volume competes a δ function.

$$\int \frac{d^3p_1}{(2\pi)^3}\partial_t f_1^c = \frac{\partial n_c}{\partial t} = R_c^{gain} - R_c^{loss}$$

Transport equation with BEC

Whether the condensation occurs?

$$R_c^{gain} = \frac{n_c}{(4\pi)^3} \int dp_3 dp_4 f_3 f_4 (1+f_2) \frac{p_3 p_4}{E_3 E_4} E \left\{ \frac{|\mathcal{M}_{34\to12}|^2}{s} \right\}_{E-\sqrt{p^2+m^2}=m}$$

$$E = E_3 + E_4, \qquad p = |\vec{p}_3 + \vec{p}_4|, \qquad s = E^2 - p^2$$

m: particle mass

similar derivation also see:
D.V.Semikoz, I.I.Tkachev,PRL74,3093(1995)

The kinematic constraint $E - \sqrt{p^2+m^2} = m$ leads to $s = 2Em$.

For massless particles $s = 0$, i.e., $\vec{p}_3$ and $\vec{p}_4$ are parallel.

$\left\{ \frac{|\mathcal{M}_{34\to12}|^2}{s} \right\}_{s=0}$ can be zero, infinity, or has a finite value, which depends on the form of the scattering matrix.

Transport equation with BEC

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Whether the condensation occurs?

$$F = \left\{ \frac{|\mathcal{M}_{34\to 12}|^2}{s} \right\}_{s=0} = ?$$

Case 1: interactions with the isotropic distribution of the collision angle.

$|\mathcal{M}_{34\to 12}|^2 \sim s\sigma$, F can be a finite value, if the cross section is not diverge at $s = 0$.

Case 2: interactions with the pQCD cross section of gluons

$$|\mathcal{M}_{34\to 12}|^2 \sim \frac{s^2}{t^2} \approx \frac{s^2}{(t - m_D^2)^2}, \quad F = 0 \text{ at } s = 0.$$

$$|\mathcal{M}_{34\to 12}|^2 \sim \frac{s^2}{t^2} \approx \frac{s^2}{t(t - m_D^2)}, \quad F \text{ has a finite value at } s = 0.$$

Kurkela, Moore, JHEP 1112 (2011) 044

Bose statistics in BAMPS : BAMPS

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- BAMPS: Boltzmann Approach of MultiParton Scatterings
  - **3+1 dimensional, fully dynamic parton transport model**
  - **solves the Boltzmann equations for on-shell partons**

Z.Xu & C.Greiner, PRC71,064901(2005) NPA774,787(2006)

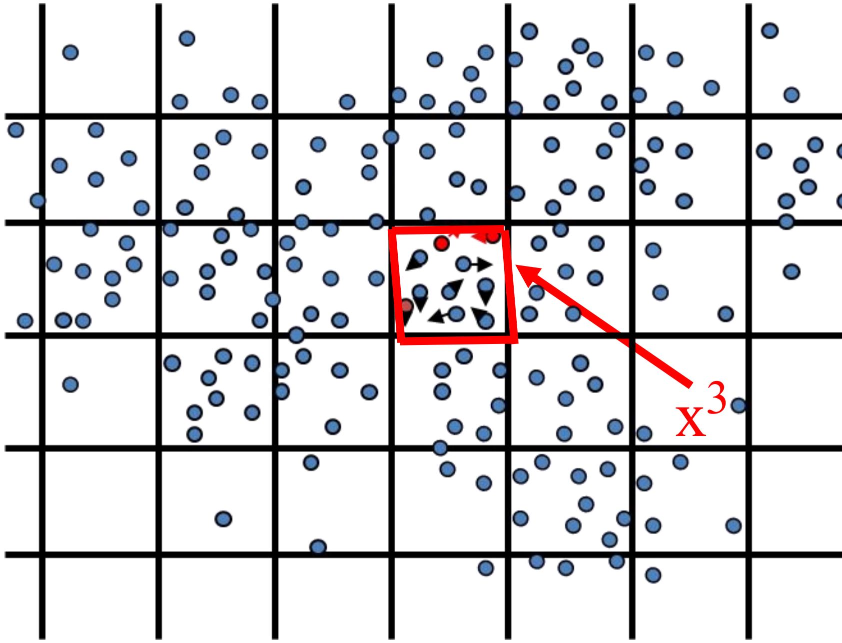
$$\left(\frac{\partial}{\partial t}+\frac{\mathbf{p}_i}{E_i}\frac{\partial}{\partial \mathbf{r}}\right) f_i(\mathbf{r},\mathbf{p}_i,t)=\mathcal{C}_i^{2\to 2}+\mathcal{C}_i^{2\leftrightarrow 3}+\ldots$$

- **Divide collision zone into cells**

$$f(x,p)=\sum_i \delta^{(3)}(\vec{x}-\vec{x}_i(t))\ \delta^{(3)}(\vec{p}-\vec{p}_i)$$

- **Stochastic interpretation for collision rates**

- **Test particle technique**



Bose statistics in BAMPS : Collision Prob.

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- g+g->g+g & g+c->g+g

For two particles within $(\vec{p}_1, \vec{p}_1 + d^3\vec{p}_1)$ and $(\vec{p}_2, \vec{p}_2 + d^3\vec{p}_2)$ in $(\vec{x}, \vec{x} + d^3\vec{x})$,
<u>the collision rate for such pair</u> can be derived from collision term as:

$$\frac{dN^{2\to 2}_{\vec{p}_1+\vec{p}_2}}{dt d^3\vec{x}} = \frac{d^3\vec{p}_1}{(2\pi)^3 2E_1} \frac{d^3\vec{p}_2}{(2\pi)^3 2E_2} f_1 f_2 \frac{1}{\nu} \int \frac{d^3\vec{p}_3}{(2\pi)^3 2E_3} \frac{d^3\vec{p}_4}{(2\pi)^3 2E_4} |M_{12\to 34}|^2 (2\pi)^4 \delta^{(4)}(p_1 + p_2 - p_3 - p_4) \tilde{f}_3 \tilde{f}_4$$

Substitute

$$\sigma_{22}^{eff} = \frac{1}{2s} \frac{1}{\nu} \int \frac{d^3\vec{p}_3}{(2\pi)^3 2E_3} \frac{d^3\vec{p}_4}{(2\pi)^3 2E_4} |M_{12\to 34}|^2 (2\pi)^4 \delta^{(4)}(p_1 + p_2 - p_3 - p_4) \tilde{f}_3 \tilde{f}_4$$

$$f(\vec{x}, \vec{p}_i) = dN_i / (d^3\vec{x} \frac{d^3\vec{p}_i}{(2\pi)^3})$$

$$(\tilde{f}_i = 1 + f(\vec{x}, \vec{p}_i))$$

⇨ Collision Probability for the given pair:

$$P^{2\to 2}_{\vec{p}_1+\vec{p}_2} = \frac{dN^{2\to 2}_{\vec{p}_1+\vec{p}_2}}{dN_1 dN_2} = v_{rel} \cdot \sigma_{22}^{eff} \cdot \frac{dt}{d^3\vec{x}} \to v_{rel} \cdot \frac{\sigma_{22}^{eff}}{N_{test}} \cdot \frac{\Delta t}{\Delta V}$$

$$\left(v_{rel} = \frac{\sqrt{(p_1 \cdot p_2)^2 - m_1^2 m_2^2}}{E_1 E_2} \Rightarrow \frac{s}{2E_1 E_2}\right)$$

Bose statistics in BAMPS : eff. Cross section

- If $\tilde{f} \approx 1$,it is easy to get the total cross section and then total probability:

$$\sigma_{22} = \frac{1}{128\pi^2}\frac{1}{s}\int d\Omega^* \left|M_{12\to 34}\right|^2 = \int d\Omega^* \frac{d\sigma_{22}}{d\Omega^*} \xrightarrow{isotropic} \frac{1}{32\pi}\frac{\left|M_{12\to 34}\right|^2}{s}$$

- Now we need to excatly consider the **bose statistic effects** :

$$\sigma_{22}^{gluon} = \frac{1}{4s}\int \frac{d^3\vec{p}_3}{(2\pi)^3 2E_3}\frac{d^3\vec{p}_4}{(2\pi)^3 2E_4}\left|M_{12\to 34}\right|^2 (2\pi)^4 \delta^{(4)}(p_1+p_2-p_3-p_4)\underline{(1+f_3)(1+f_4)}$$

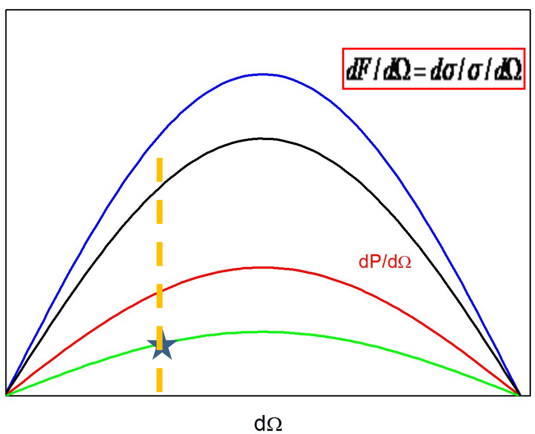
$$= \frac{1}{128\pi^2}\frac{1}{s}\int d\Omega^* \left|M_{12\to 34}\right|^2 \underline{(1+f_3)(1+f_4)} = \int d\Omega^* \frac{d\sigma_{22}}{d\Omega^*}\underline{(1+f_3)(1+f_4)}$$

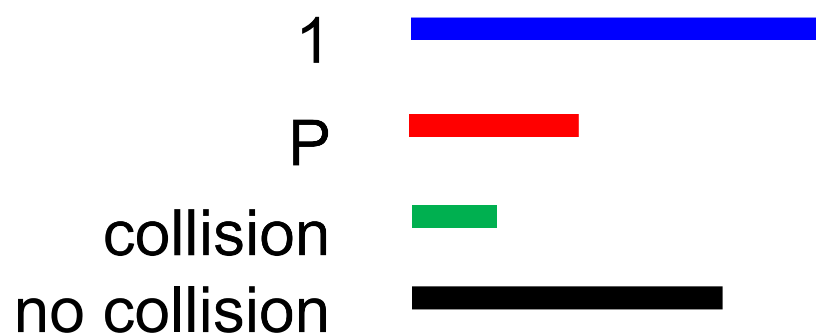
time consuming and uncertainties from extraction of distribution f !

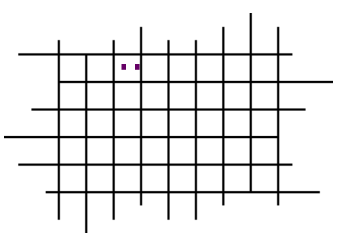
Bose statistics in BAMPS : New Scheme

$$P^{2\to2}_{\vec p_i+\vec p_j} = v_{rel}\frac{\int d\Omega^* \frac{d\sigma_{22}}{d\Omega^*}(1+f_3)(1+f_4)}{N_{test}}\frac{\Delta t}{\Delta V} = \int d\Omega^* \frac{dP^{2\to2}_{ij}}{d\Omega^*}$$

$$\frac{dP^{2\to2}_{ij}}{d\Omega^*} = v_{rel}\frac{1}{N_{test}}\frac{d\sigma_{22}}{d\Omega^*}(1+f_3)(1+f_4)\frac{\Delta t}{\Delta V}$$







1, Sample first the collision angle-->final states--> $dP^{2\to2}_{ij}/d\Omega^*$

2, Sample a random number x in $[0, dF/d\Omega^*]$

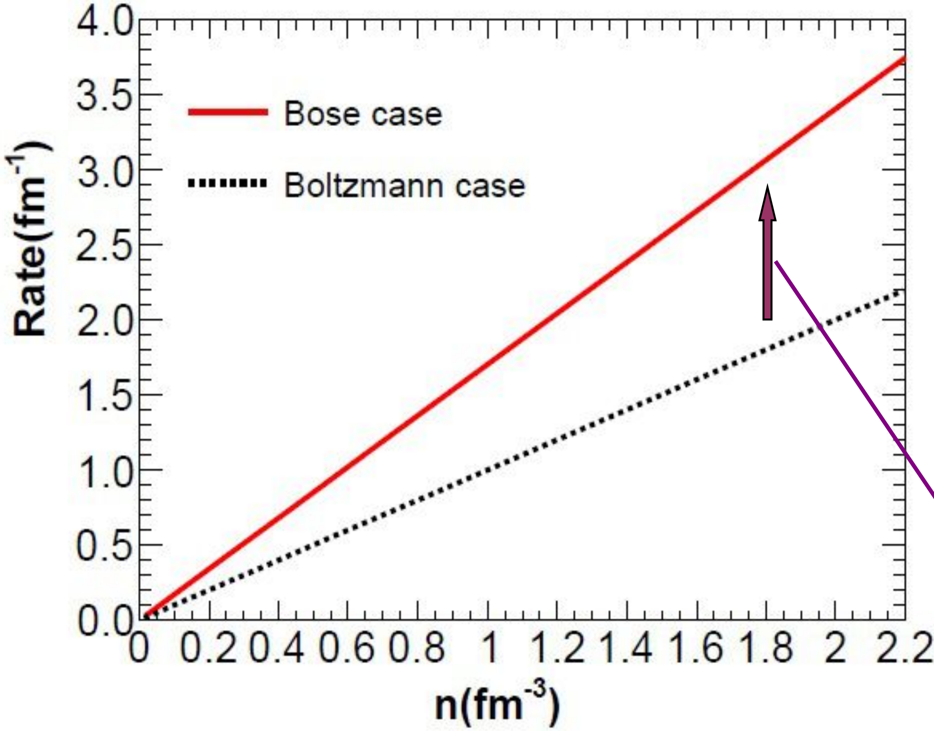
if x<dP : **Assign** the sampled final mom.(p) to outgoing partcl.
if not : Abandon the sampled mom..(p)

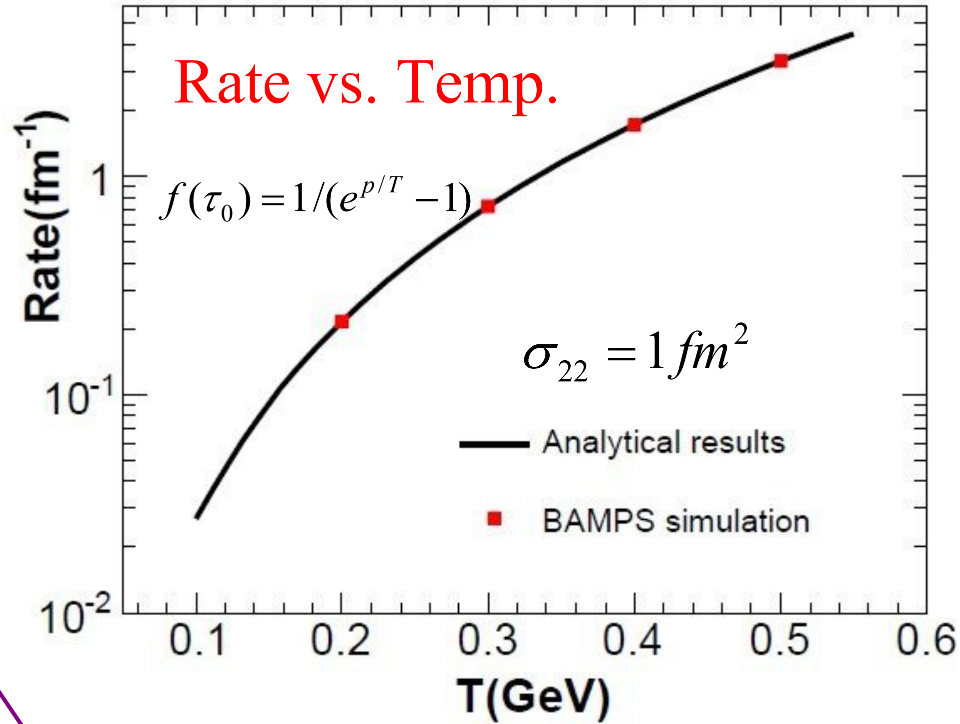
$dF/d\Omega = d\sigma/\sigma/d\Omega$ in isotropic case will be $1/4\pi$

Bose statistics in BAMPS : Rate test

$$Rate = n \cdot \langle v_{rel} \sigma_{22} \rangle^{medium} = \frac{1}{n} \int \frac{d^3\vec{p}_1}{(2\pi)^3} \frac{d^3\vec{p}_2}{(2\pi)^3} f_1 f_2 \int d\Omega^* \frac{d\sigma_{22}}{d\Omega^*} v_{rel} (1+f_3)(1+f_4)$$

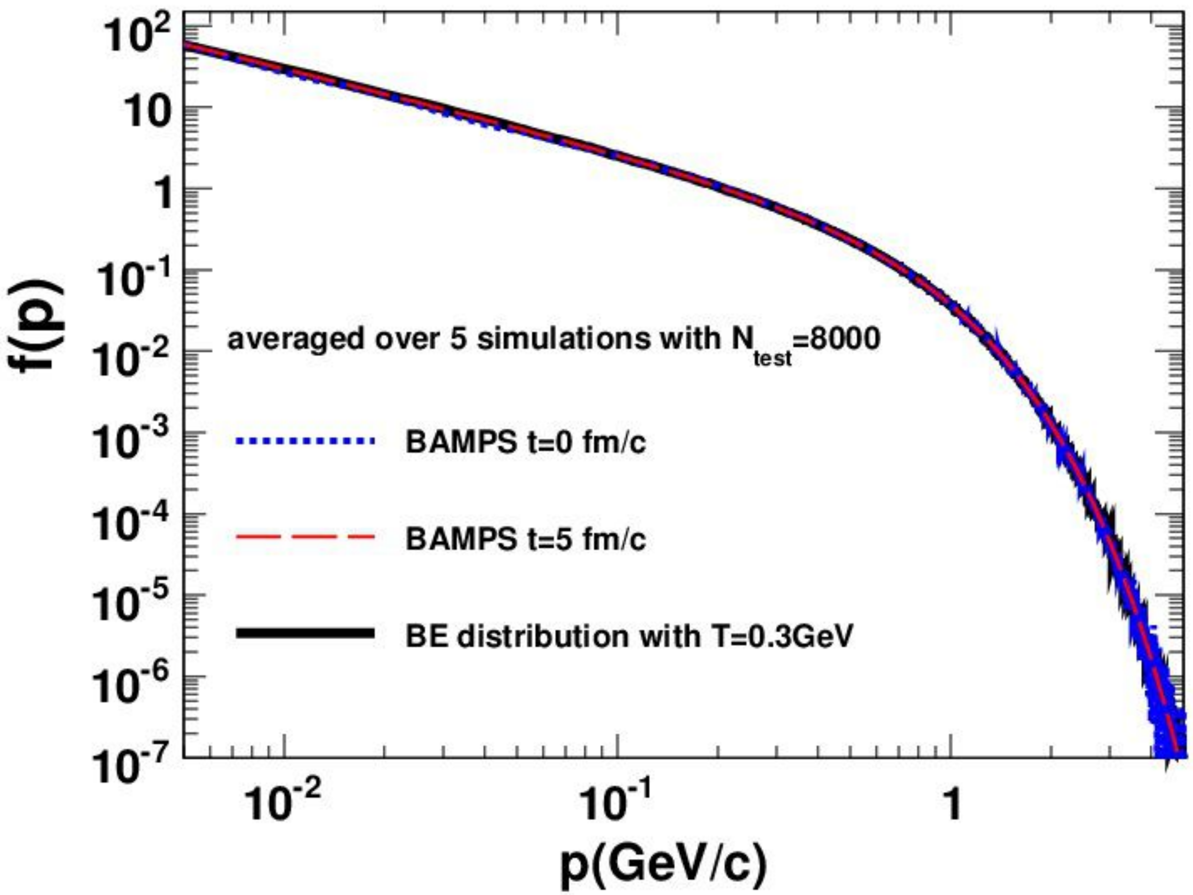
$$= N_{coll} / (t \cdot N_g / 2)$$





Bose Enhancement effect

Bose statistics in BAMPS : Equilibrium test



$$f(\tau_0) = 1/(e^{p/T} - 1)$$

$$T = 300 MeV$$

within acceptable fluctuation , the equilibrium state can be maintained <=>detailed balance ok & BE distribution is fix-point of the syst-em from simulation

Bose statistics in BAMPS : BEC Growth

- g+g->g+c   consider 1+2——--->3+c "c" is condensate

$$\sigma_{gain}^{eff}=\frac{1}{2s}\int\frac{d^3\vec{p}_c}{(2\pi)^3 2E_c}\frac{d^3\vec{p}_3}{(2\pi)^3 2E_3}\left|M\right|^2(2\pi)^4\delta^4(p_1+p_2-p_3-p_c)\underline{\rho_c(t)(2\pi)^3\delta^3(\vec{p}_c)}(1+f_3)$$

$$=\frac{\pi}{2}\rho_c(1+f_3)\frac{1}{P}\frac{|M_{34\to12}|^2}{s}\delta[(E-P)^2]$$

$$\begin{pmatrix} E=E_1+E_2 \\ \vec{P}=\vec{p}_1+\vec{p}_2 \end{pmatrix}$$

By keeping net Rate invariant, make Approximation:

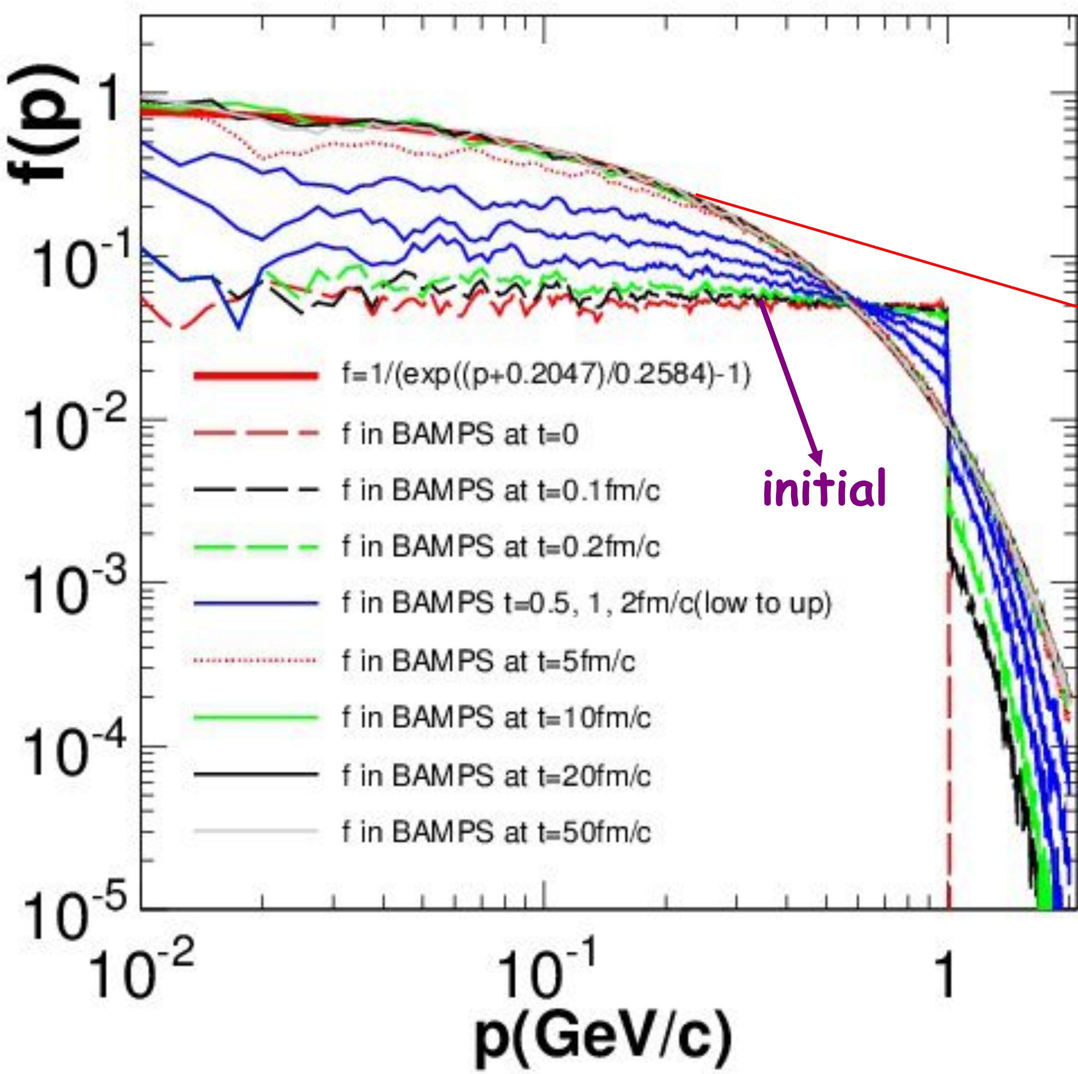
$$f_c=\rho_c(2\pi)^3\delta^{(3)}(\vec{p}_c)\approx\rho_c(2\pi)^3\frac{1}{p_c^2}\frac{1}{4\pi\Delta E}\theta(\Delta E-p_c) \qquad \rho_c(\tau_c)=\rho_{E<\Delta E}(\tau_c)$$

$$\Longrightarrow \quad \sigma_{gain}^{eff}\approx\frac{\pi}{2}\rho_c(1+f_3(E))\frac{1}{P}\frac{|M_{34\to12}|^2}{s}\frac{1}{4\Delta E}\left(\frac{2}{E-P}-\frac{1}{\Delta E}\right)\theta\left(\Delta E-\frac{E-P}{2}\right)$$

We checked that different chosen for ΔE (as long as small enough) can give the same right collisoin rate because of the ensurence for giving right integr--ation over delta function.---0.0025GeV in this work.

when effective chemical potential (fit by soft mode) becomes zero, we set in the above approximation.---> onset for condensate

f_0 = 0.05 simulation results



$3\times 3\times 3\, fm^3$ static box

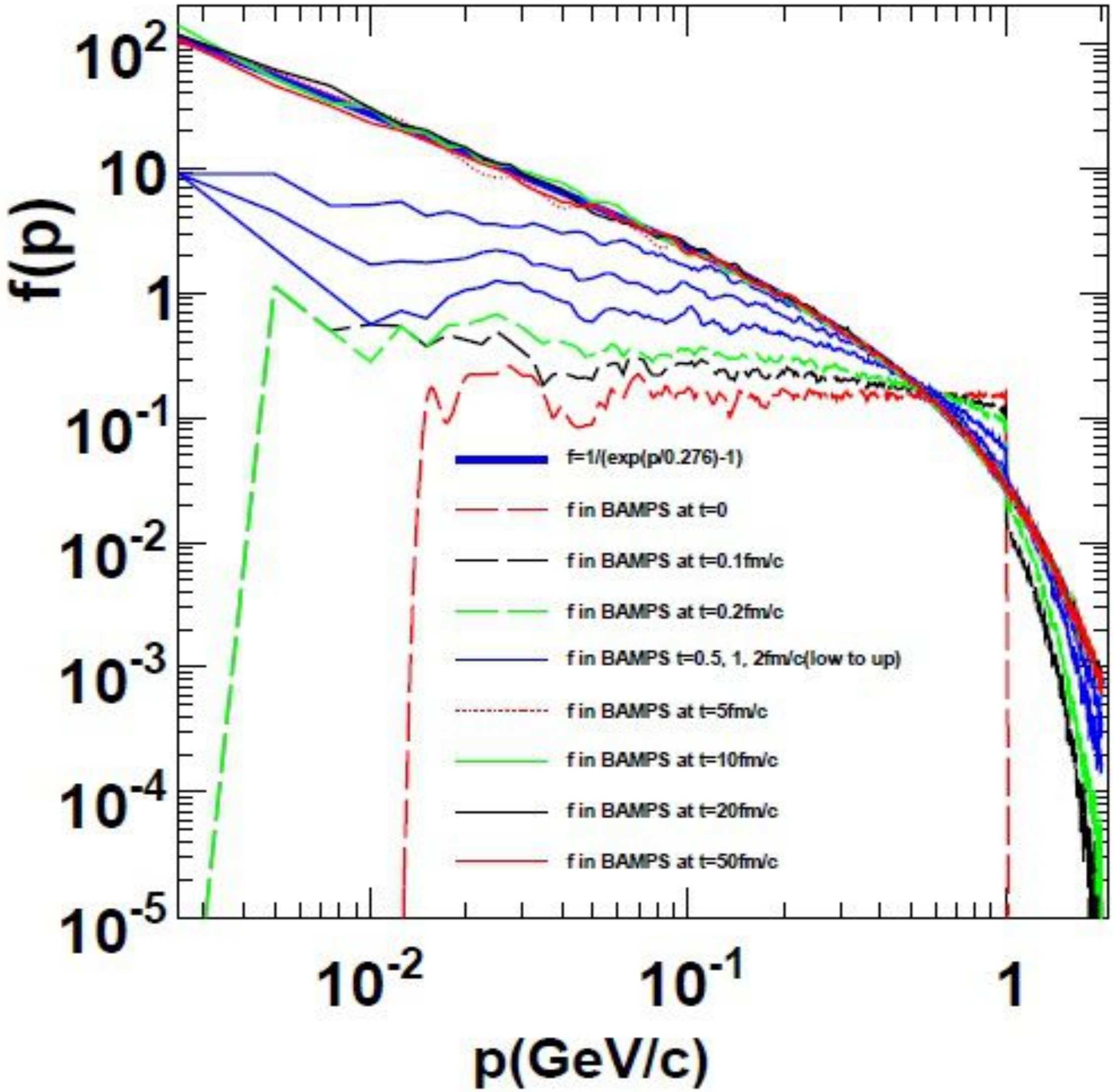
$f_0 = 0.05 < f_{critical}$

$T_{eq} = 0.258 MeV$

$\mu_{eq} = -0.205 MeV$

the system thermalizes to the fix-point: thermal BE distribution

f_0 = 0.154 simulation results



$3\times3\times3\,fm^3$ static box

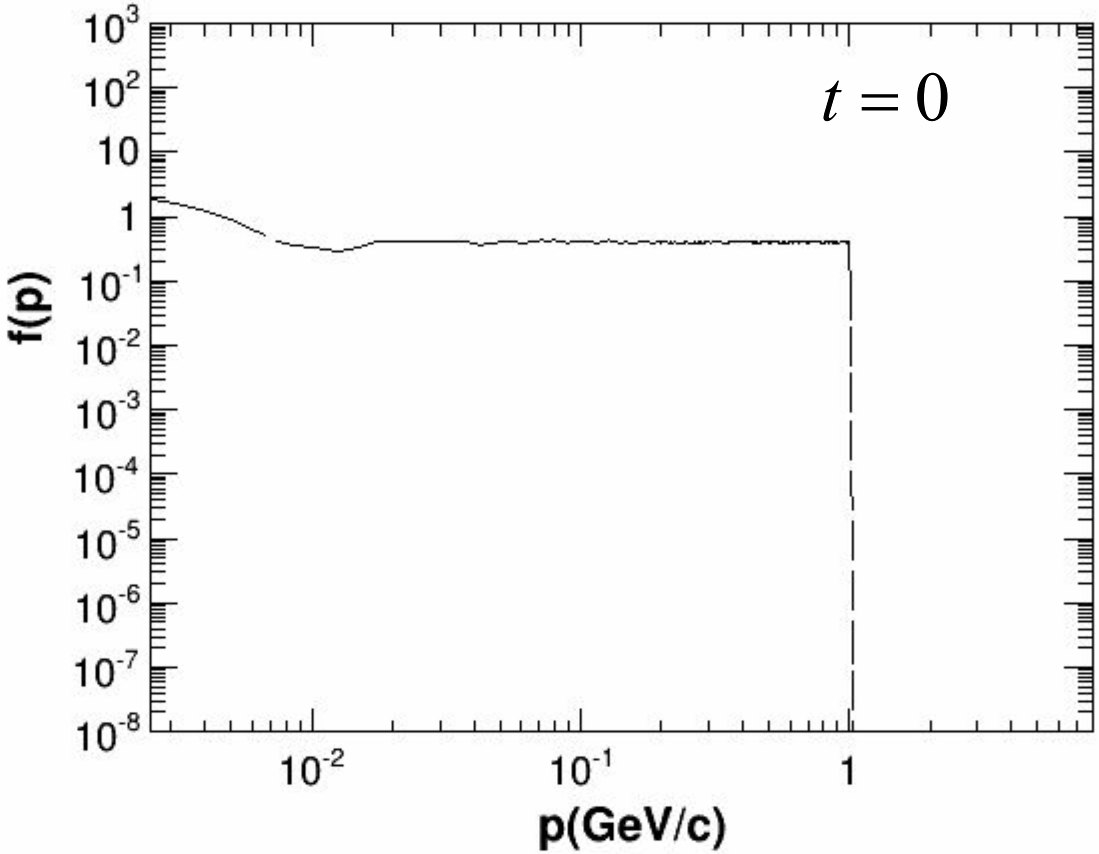
$$f_0 = 0.154 = f_{critical}$$

$$T_{eq} = 0.276 MeV$$

$$\mu_{eq} = 0 MeV$$

the system thermalizes to the fix-point: thermal BE distribution

f_0 = 0.4 simulation results



$3 \times 3 \times 3\, fm^3$ static box

$$f_0 = 0.4 > f_{critical}$$

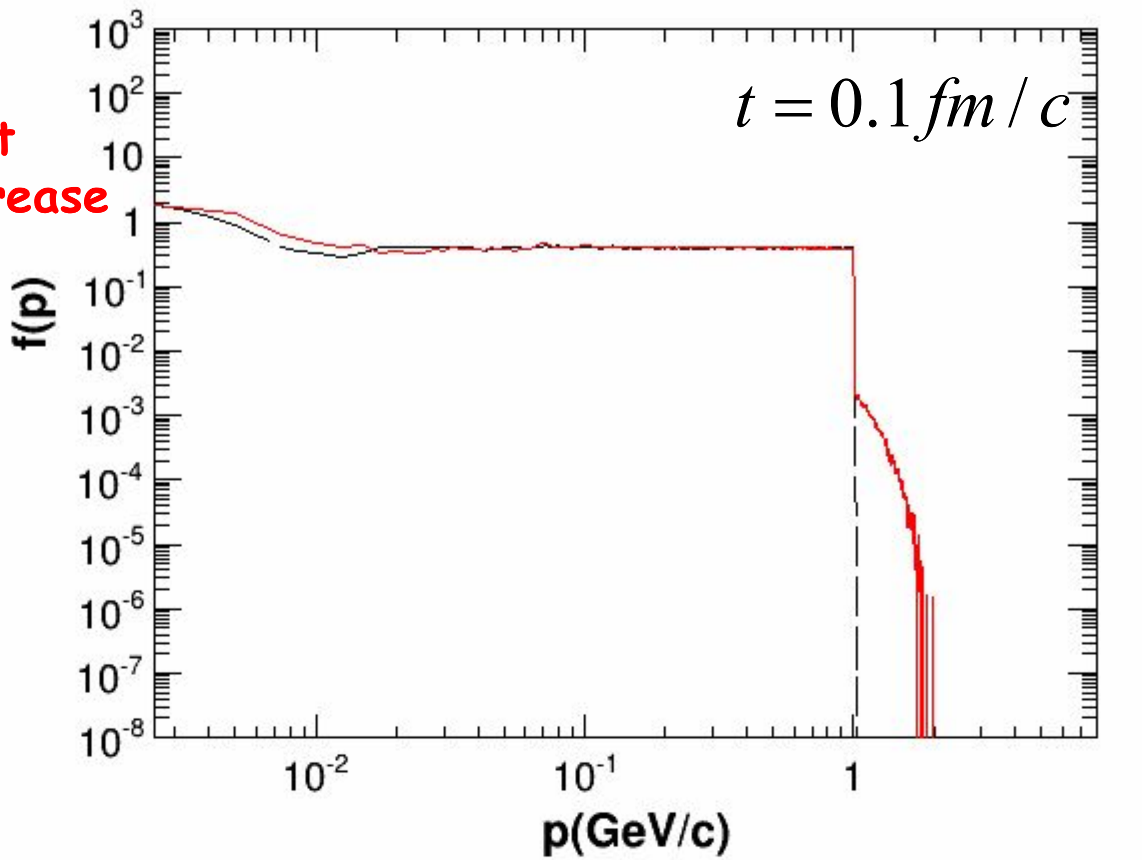
$$T_{eq} = 0.352 MeV$$

$$\rho_c^{eq} = 0.186\, fm^{-3}$$

the initial condition far-from-equlibrium

f_0 = 0.4 simulation results

soft increase



$3 \times 3 \times 3\, fm^3$ static box

$$f_0 = 0.4 > f_{critical}$$

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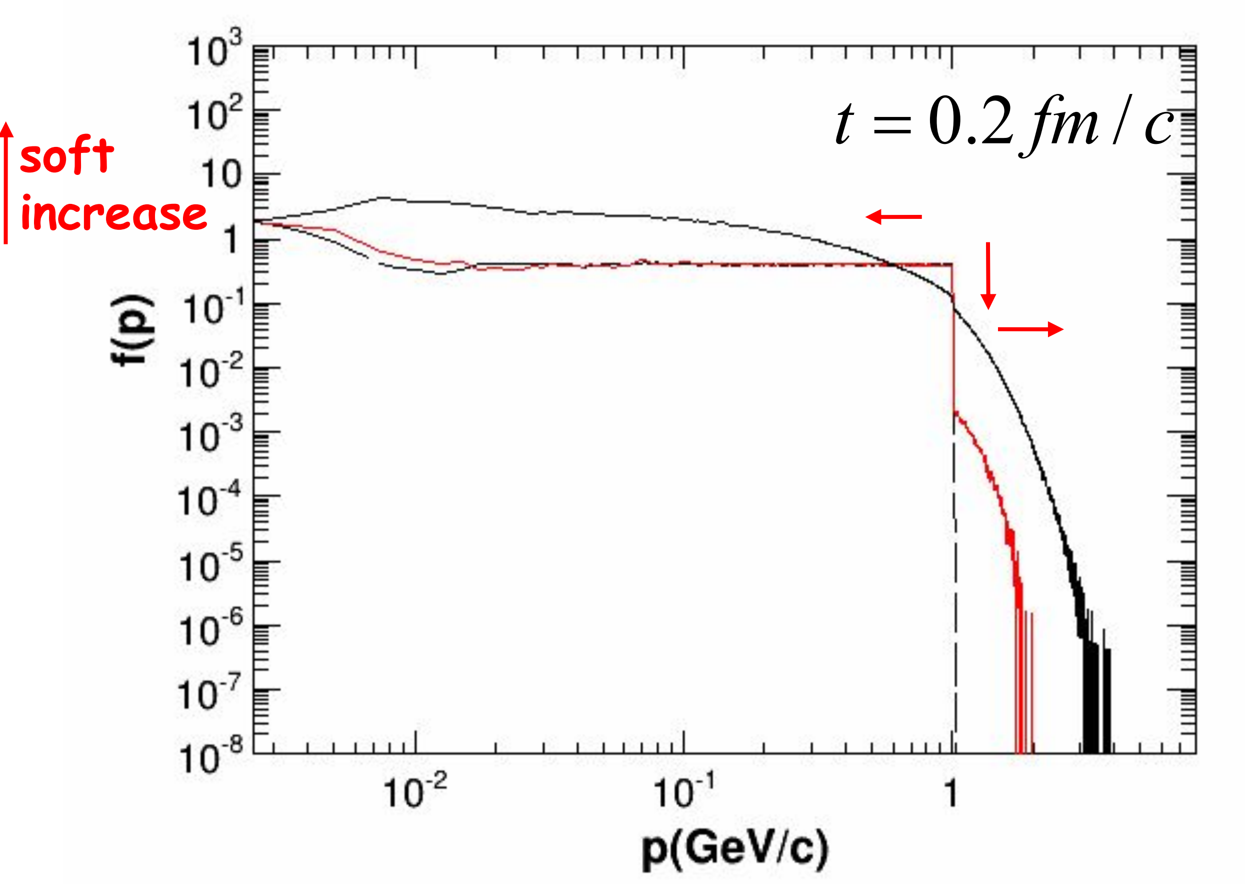
f_0 = 0.4 simulation results



$3\times3\times3\, fm^{3}$ static box

$$f_0 = 0.4 > f_{critical}$$

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f_0 = 0.4 simulation results

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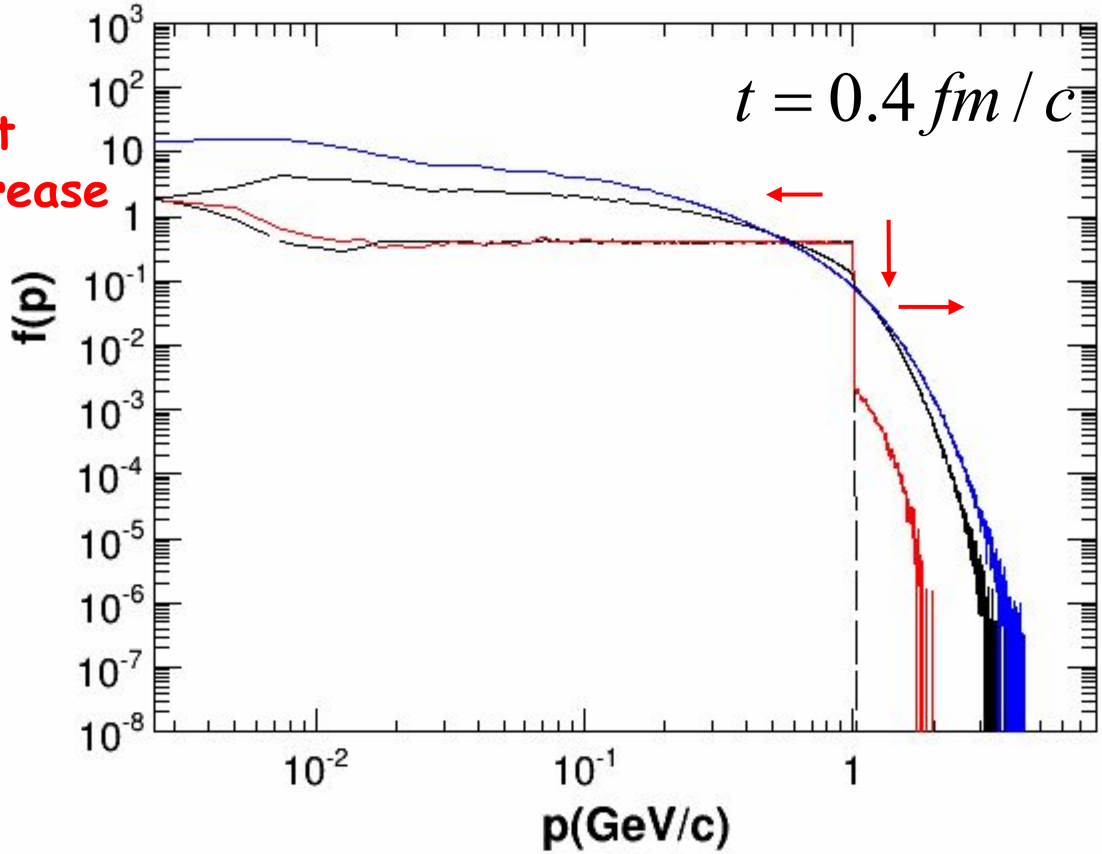
soft increase



$T^* \uparrow \quad -\mu^* \downarrow$

$3 \times 3 \times 3\, fm^3$ static box

$f_0 = 0.4 > f_{critical}$

$T_{eq} = 0.352 MeV$

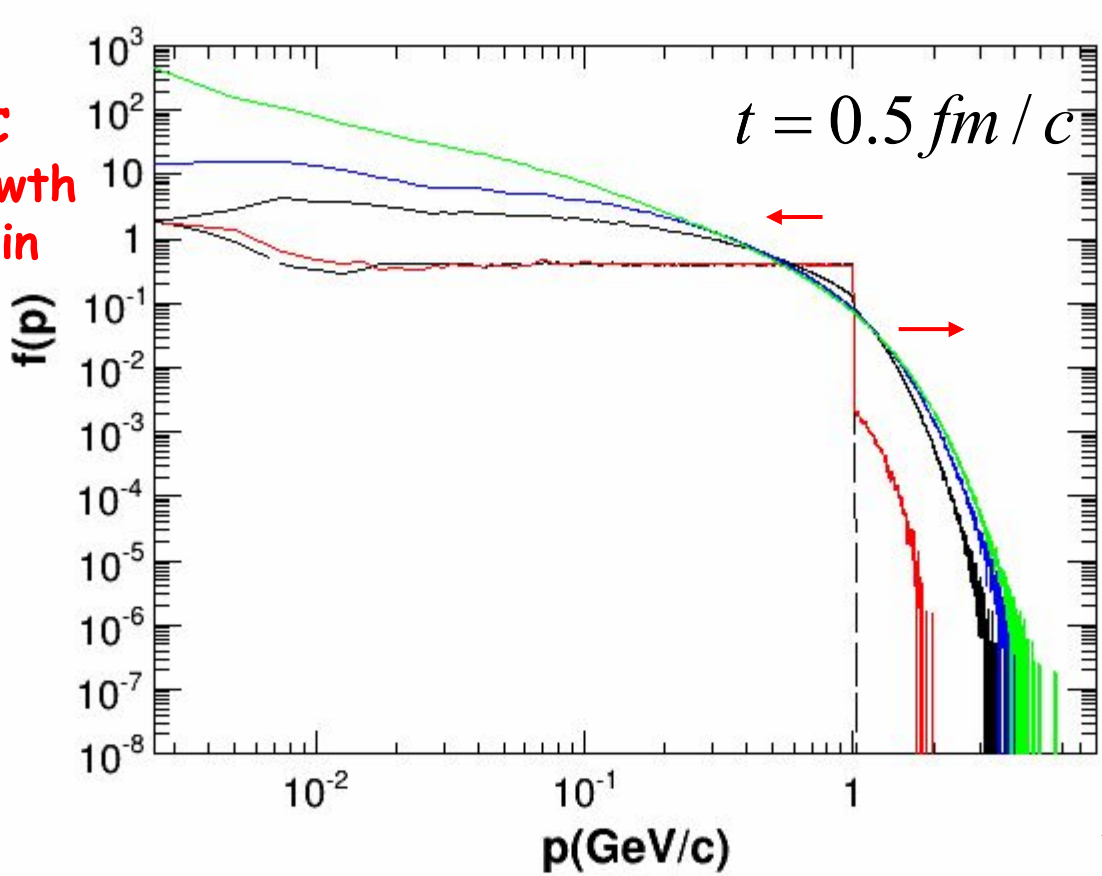
$\rho_c^{eq} = 0.186\, fm^{-3}$

$$f_{IR} = \frac{T^*}{p - \mu^*} (\mu^* < 0)$$

thermalization for soft part

f_0 = 0.4 simulation results

BEC growth begin



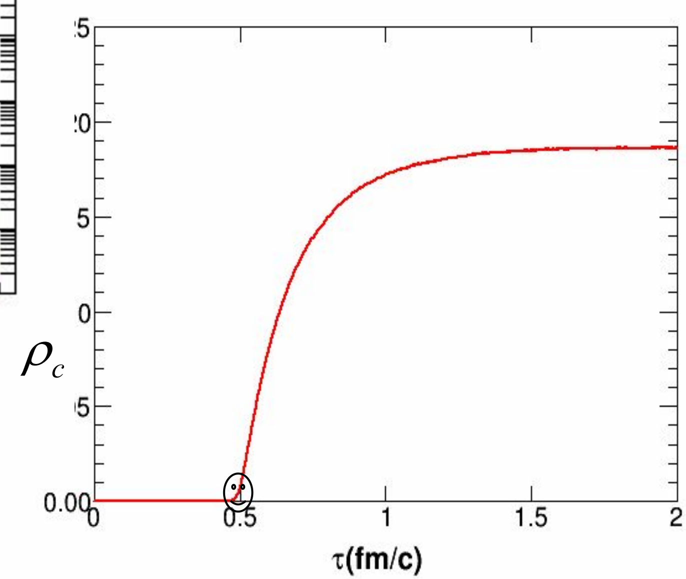
$-\mu^* \to 0$

$3\times3\times3\,fm^3$ static box

$f_0 = 0.4 > f_{critical}$

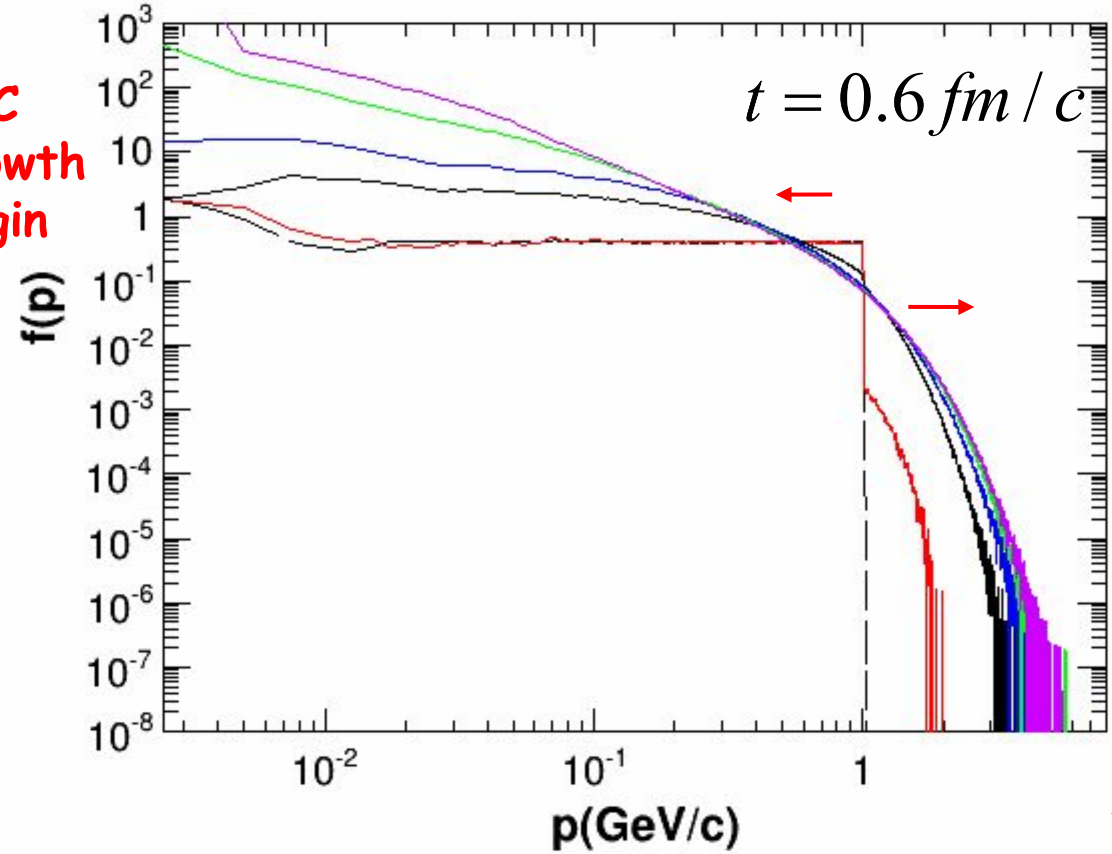
$T_{eq} = 0.352 MeV$

$\rho_c^{eq} = 0.186\,fm^{-3}$



f_0 = 0.4 simulation results

BEC
growth
begin



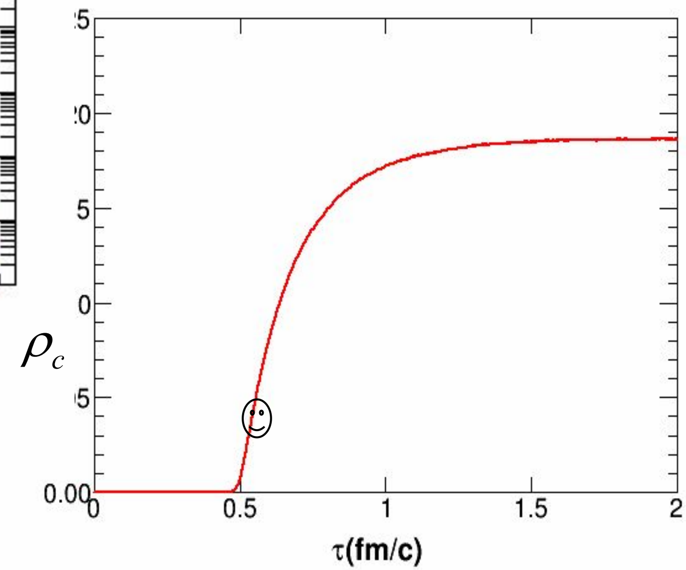
$T^* \downarrow \qquad -\mu^* = 0$

$3\times3\times3\,fm^3$ static box

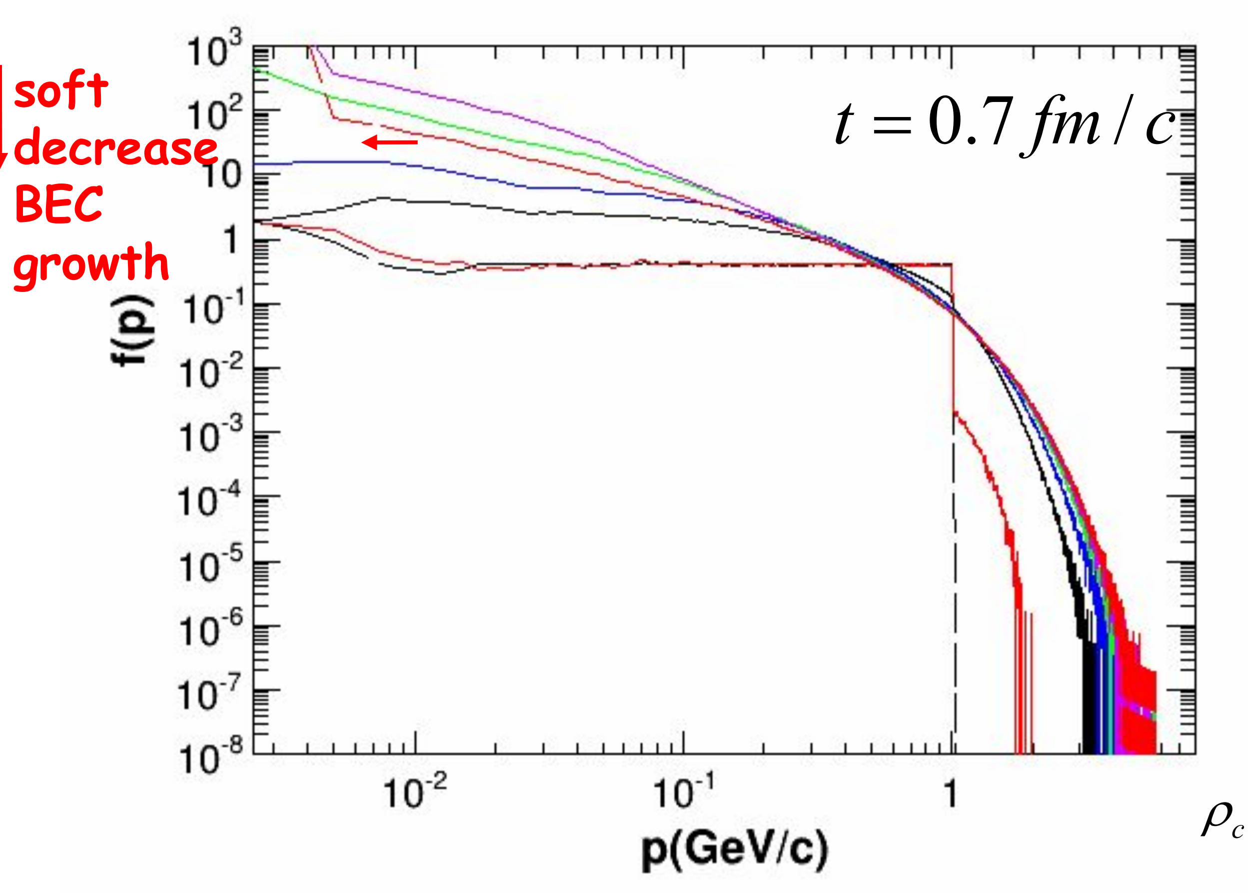
$f_0 = 0.4 > f_{critical}$

$T_{eq} = 0.352 MeV$

$\rho_c^{eq} = 0.186\,fm^{-3}$



f_0 = 0.4 simulation results



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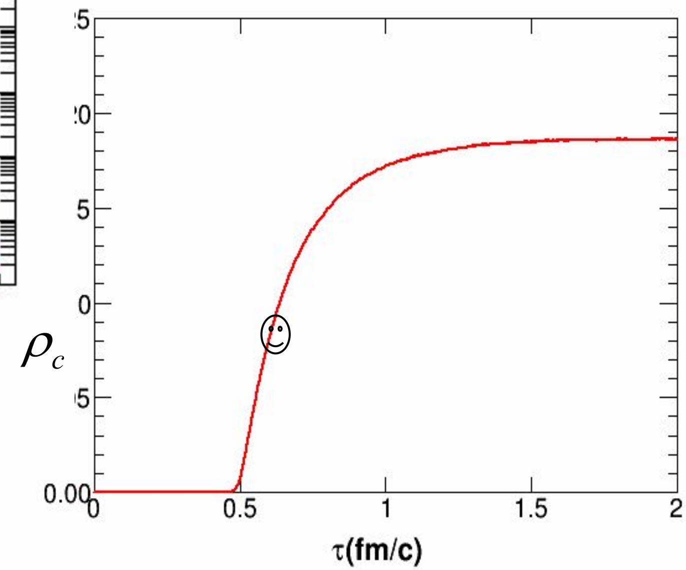
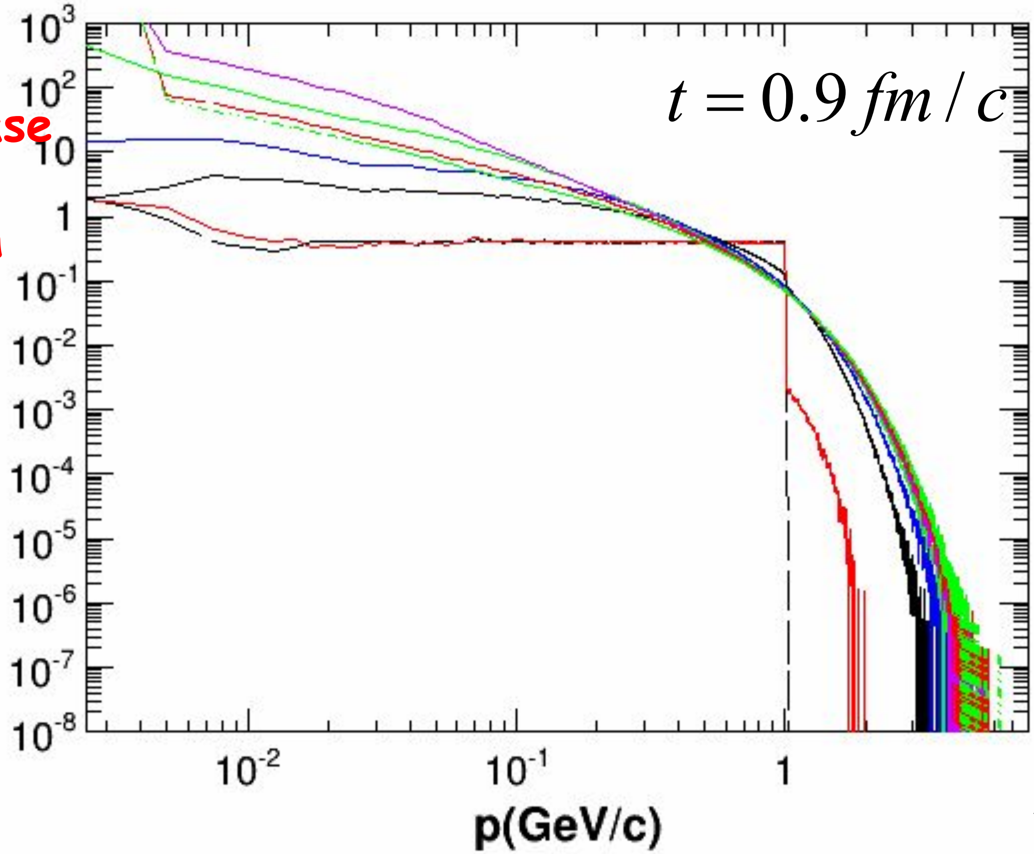


f_0 = 0.4 simulation results

soft
decrease
BEC
growth



$3 \times 3 \times 3\, fm^3$ static box

$$f_0 = 0.4 > f_{critical}$$

$$T_{eq} = 0.352 MeV$$

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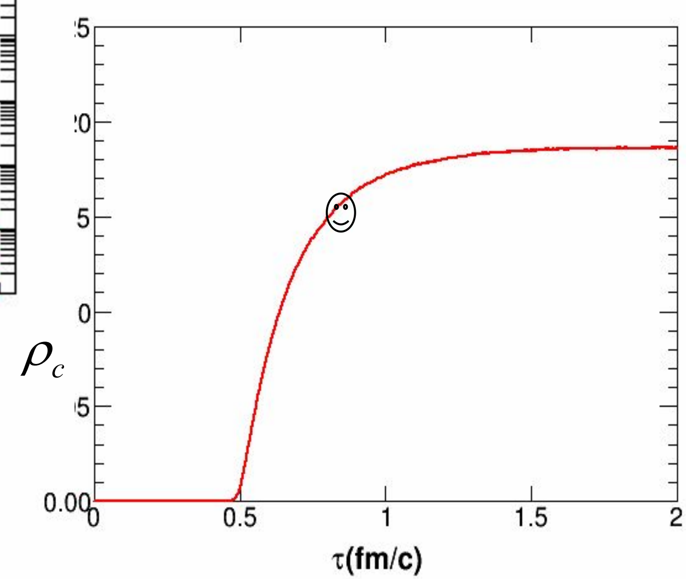
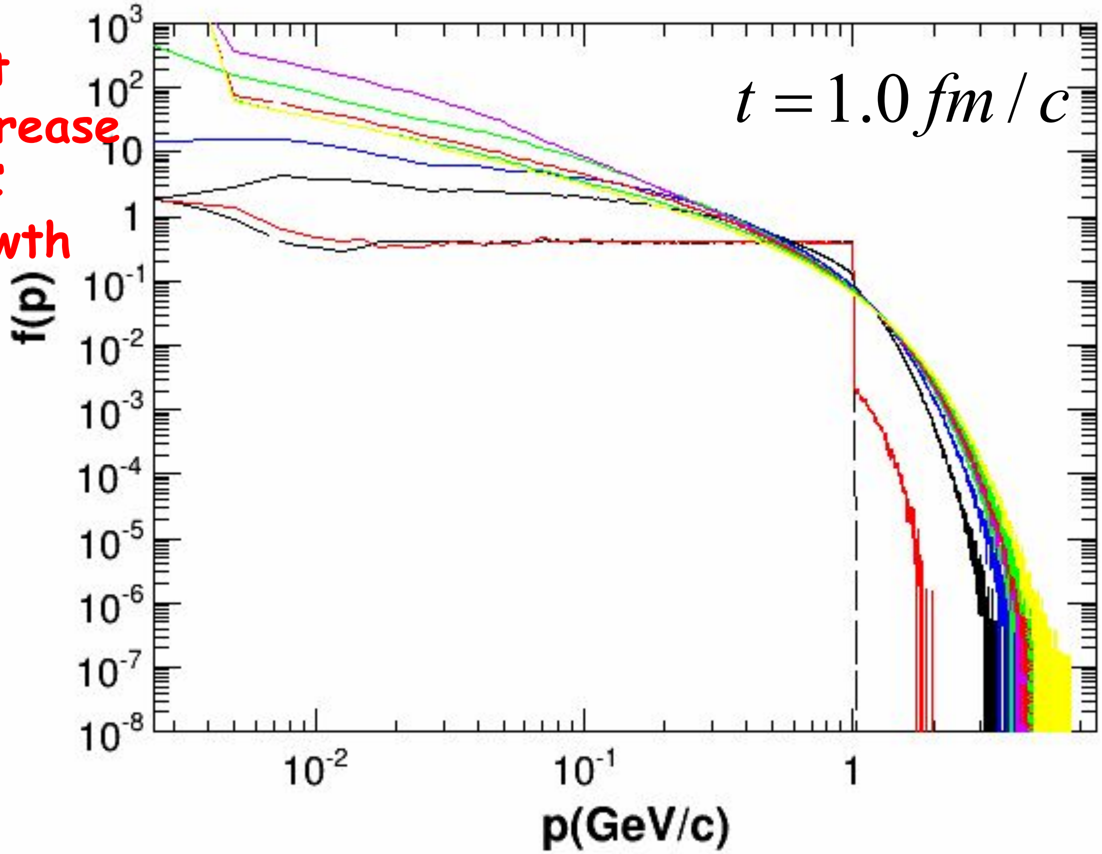


f_0 = 0.4 simulation results

soft decrease BEC growth



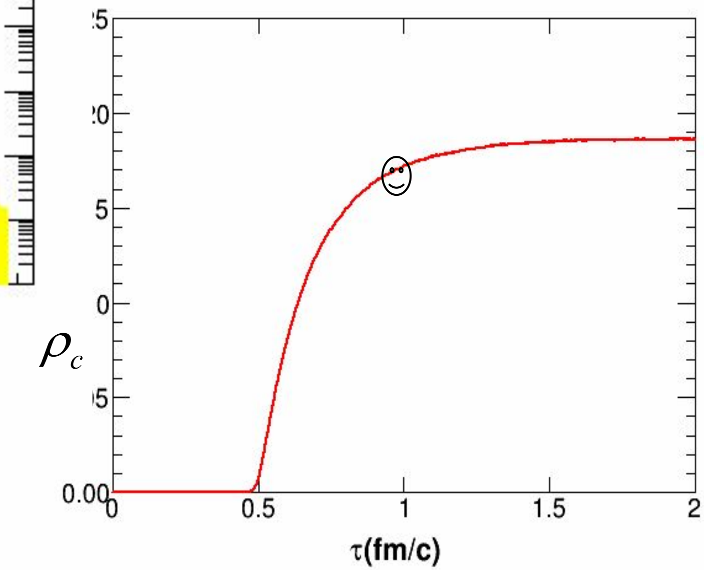
$T^* \to T_{eq}$

$3\times3\times3\, fm^3$ static box

$f_0 = 0.4 > f_{critical}$

$T_{eq} = 0.352 MeV$

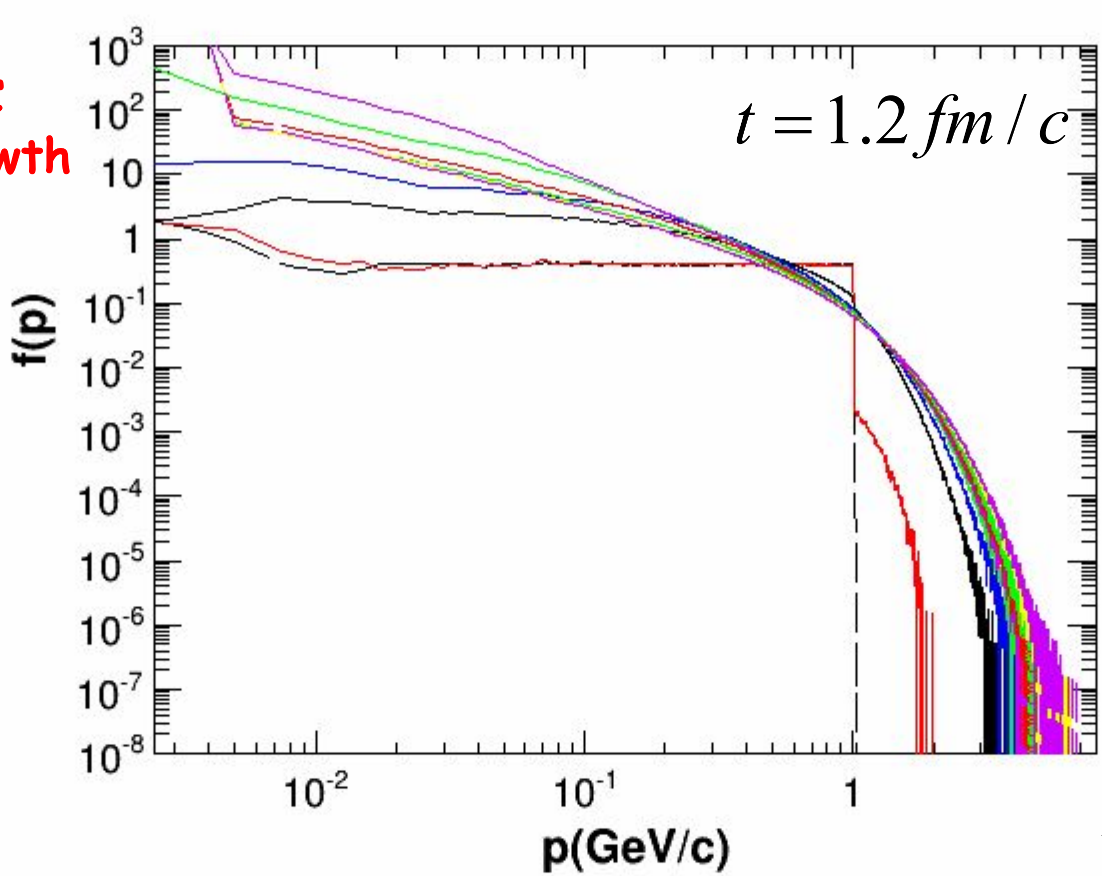
$\rho_c^{eq} = 0.186\, fm^{-3}$



f_0 = 0.4 simulation results

$3\times3\times3\,fm^3$ static box

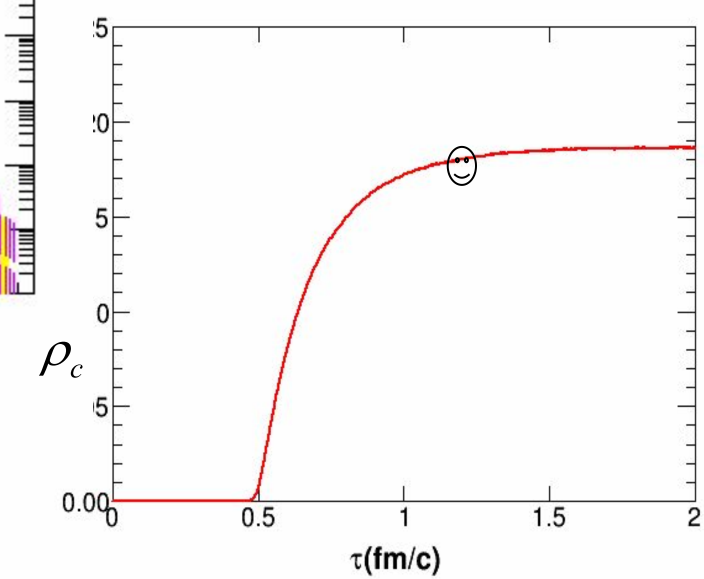
BEC growth



$$f_0 = 0.4 > f_{critical}$$

$$T_{eq} = 0.352 MeV$$

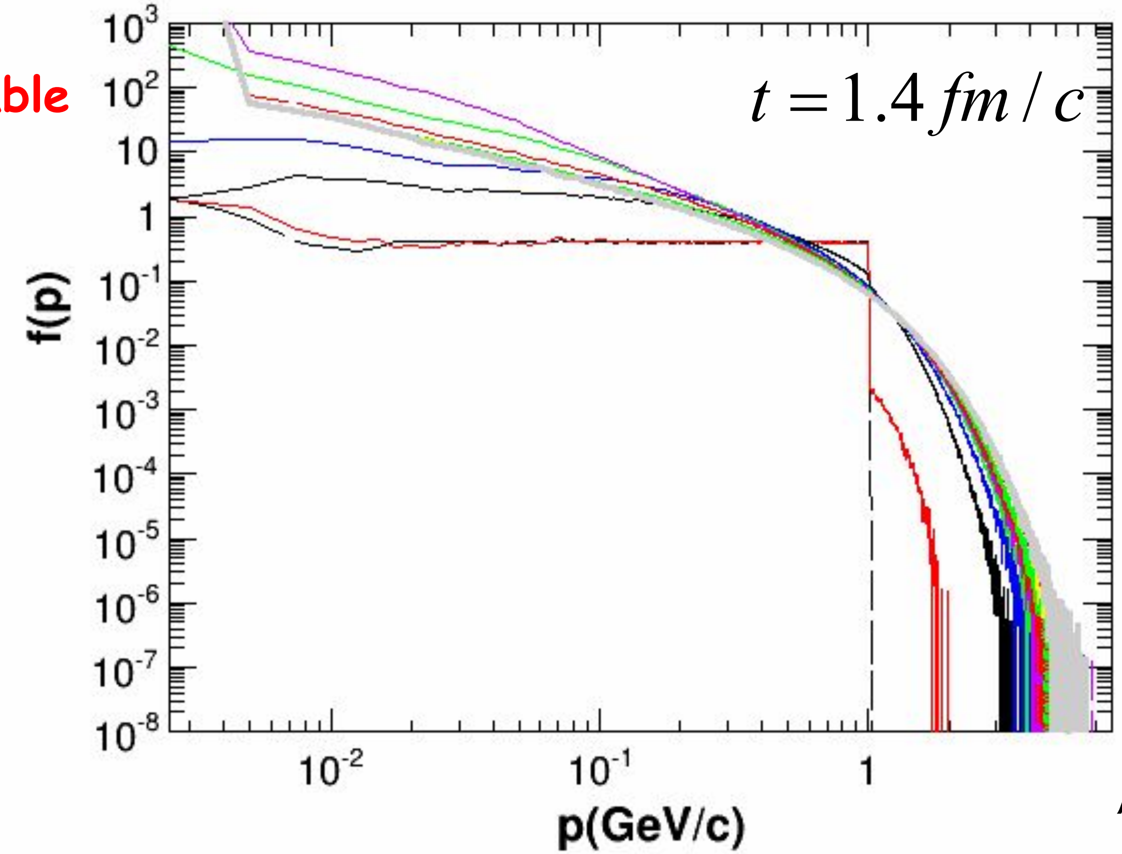
$$\rho_c^{eq} = 0.186\,fm^{-3}$$



$$T^* = T_{eq}$$

f_0 = 0.4 simulation results

stable

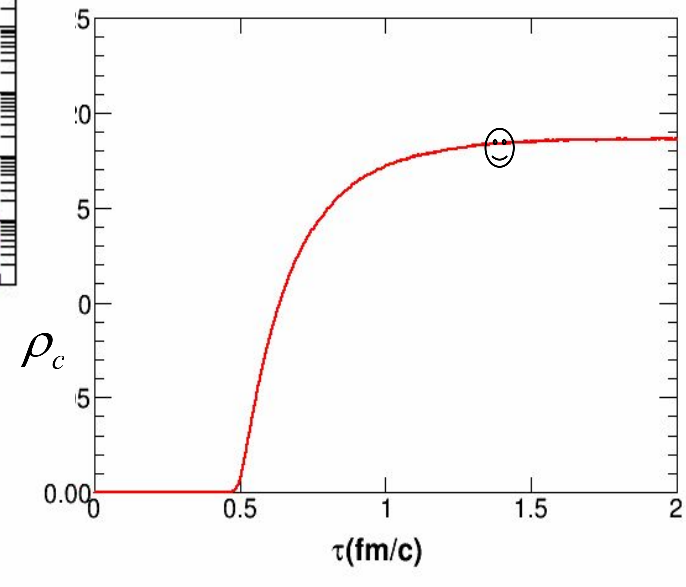


$3\times3\times3\, fm^3$ static box

$f_0 = 0.4 > f_{critical}$

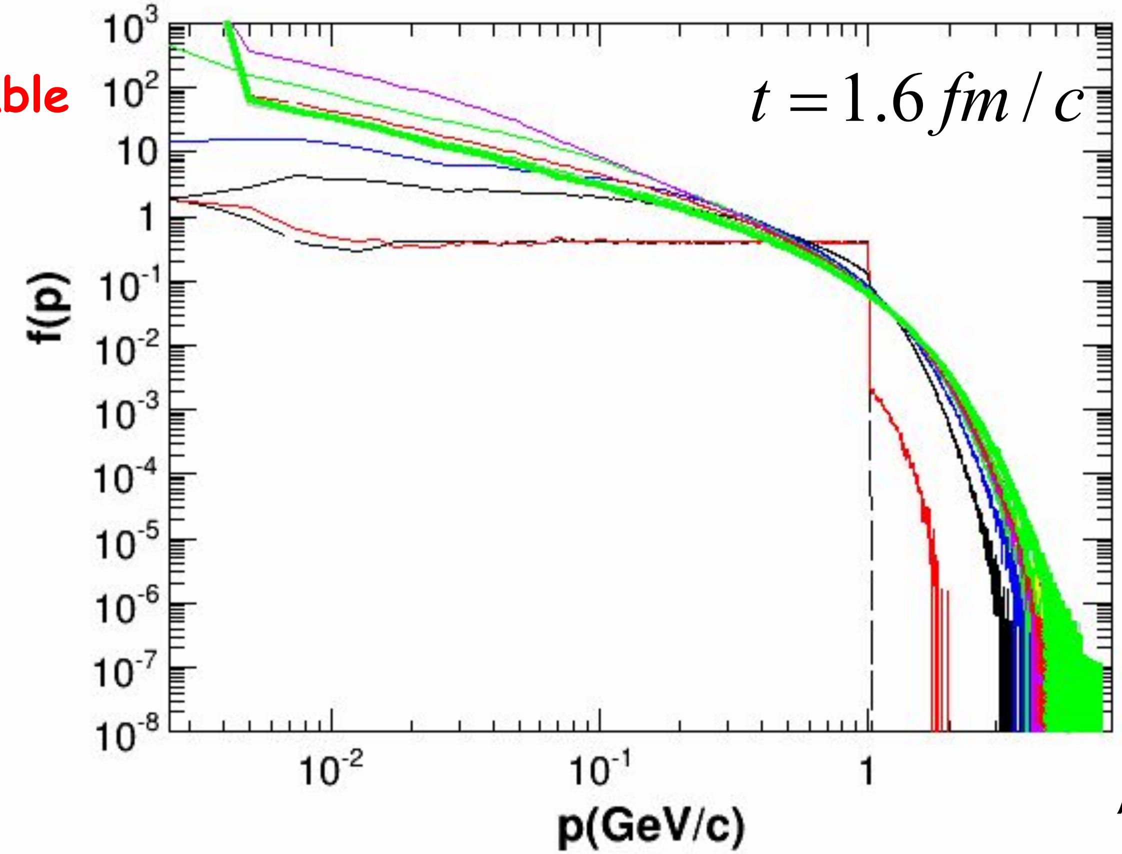
$T_{eq} = 0.352 MeV$

$\rho_c^{eq} = 0.186\, fm^{-3}$



f_0 = 0.4 simulation results

stable

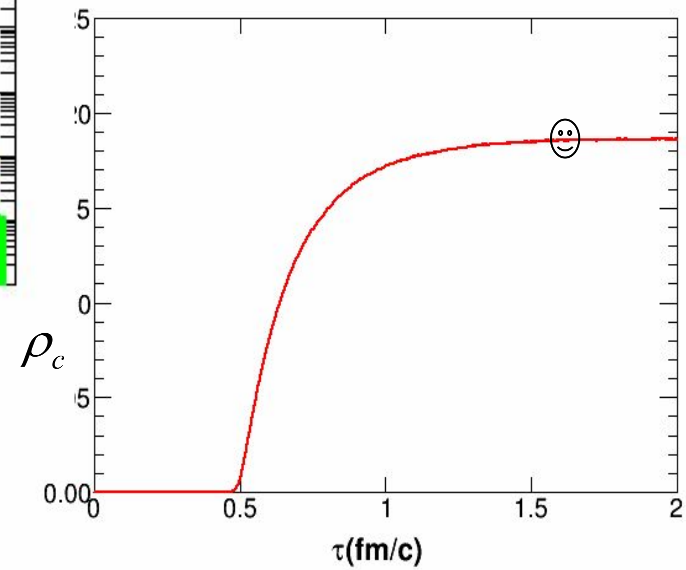


$3 \times 3 \times 3\, fm^3$ static box

$$f_0 = 0.4 > f_{critical}$$

$$T_{eq} = 0.352 MeV$$

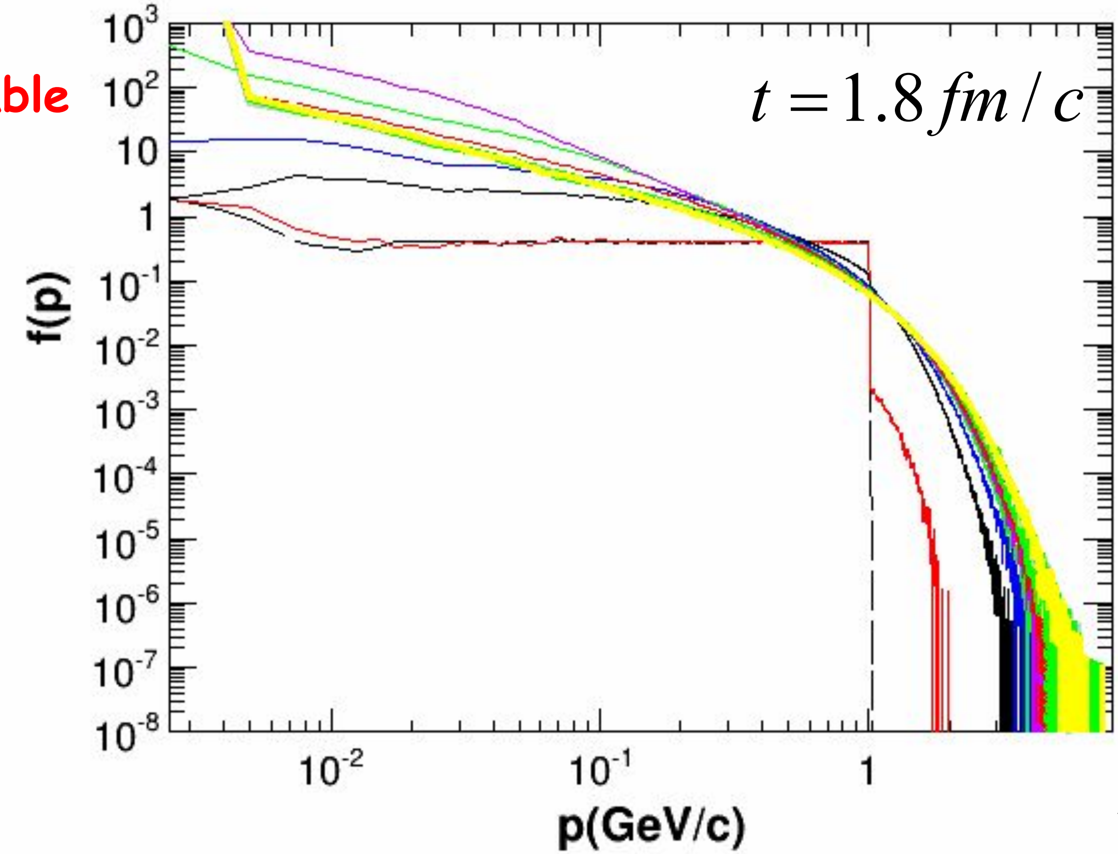
$$\rho_c^{eq} = 0.186\, fm^{-3}$$



f_0 = 0.4 simulation results

$3 \times 3 \times 3\, fm^3$ static box

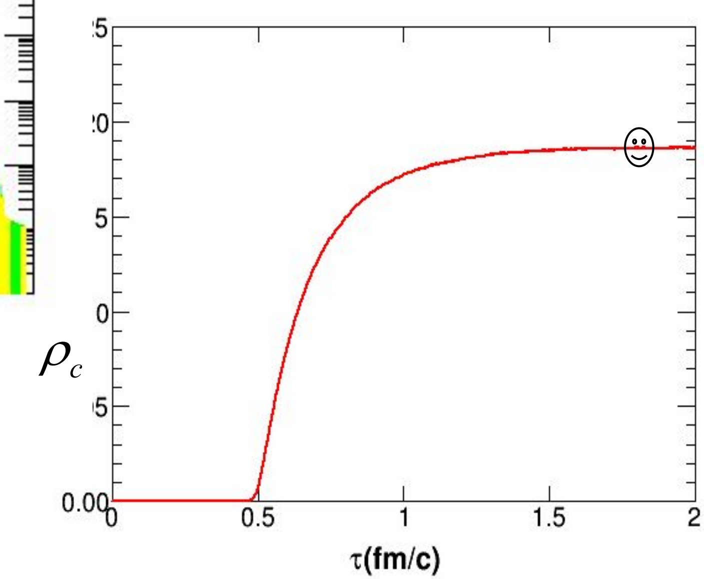
stable



$f_0 = 0.4 > f_{critical}$

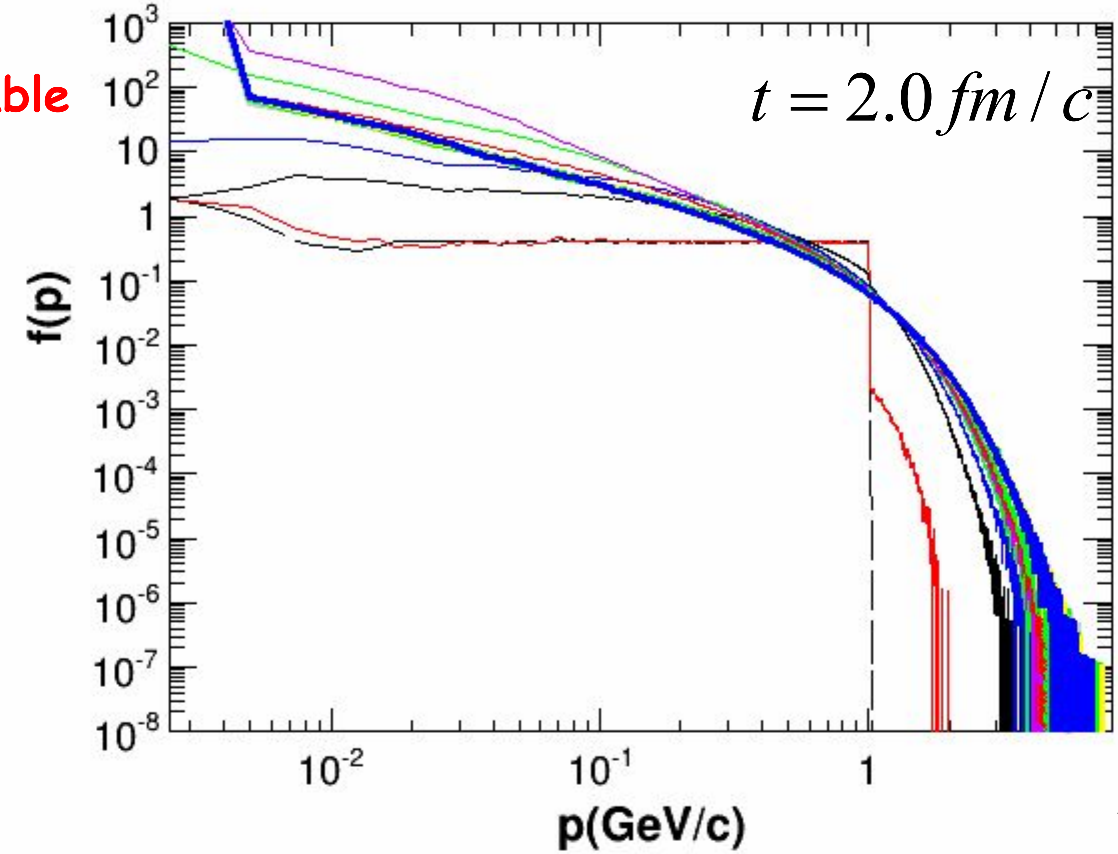
$T_{eq} = 0.352 MeV$

$\rho_c^{eq} = 0.186\, fm^{-3}$



f_0 = 0.4 simulation results

stable



$3 \times 3 \times 3\, fm^3$ static box

$$f_0 = 0.4 > f_{critical}$$

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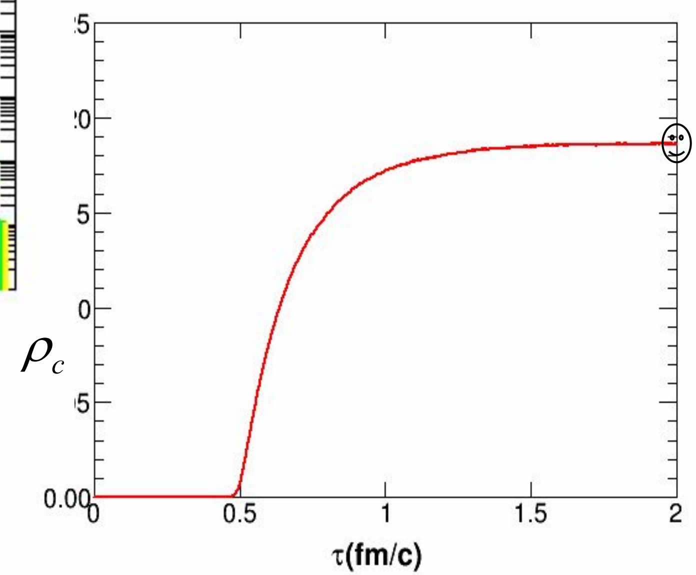

f_0 = 0.4 simulation results

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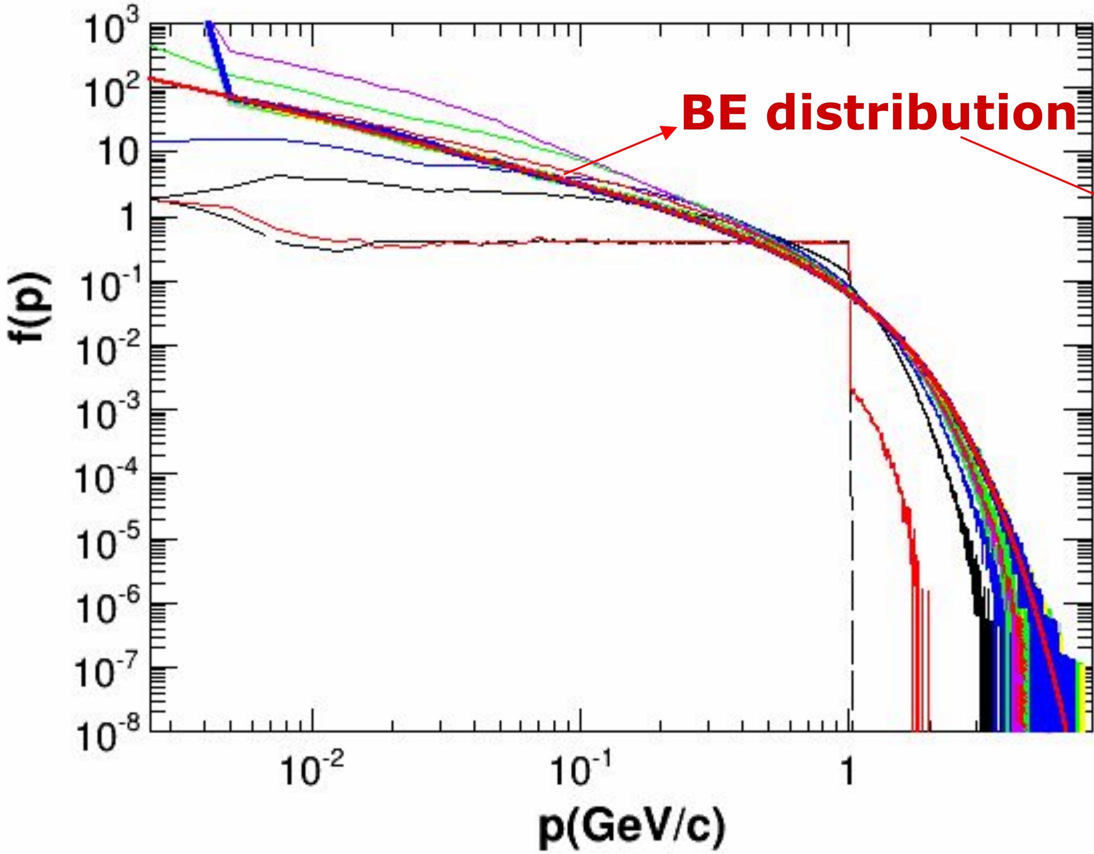
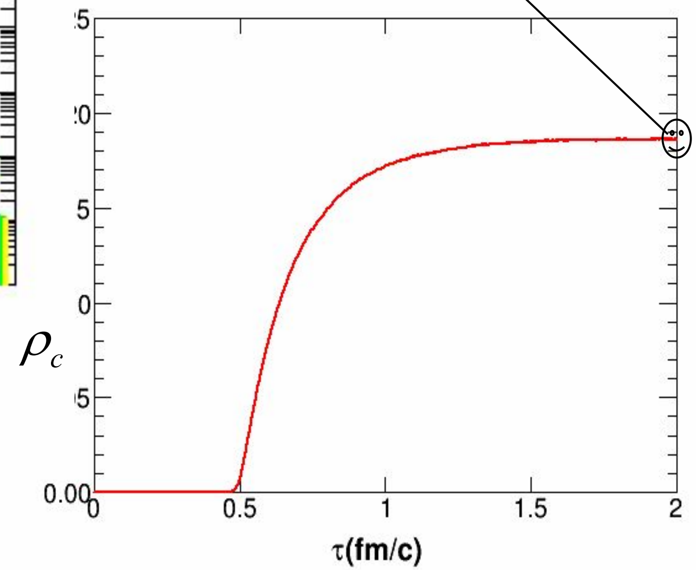


$3\times3\times3\,fm^3$ static box

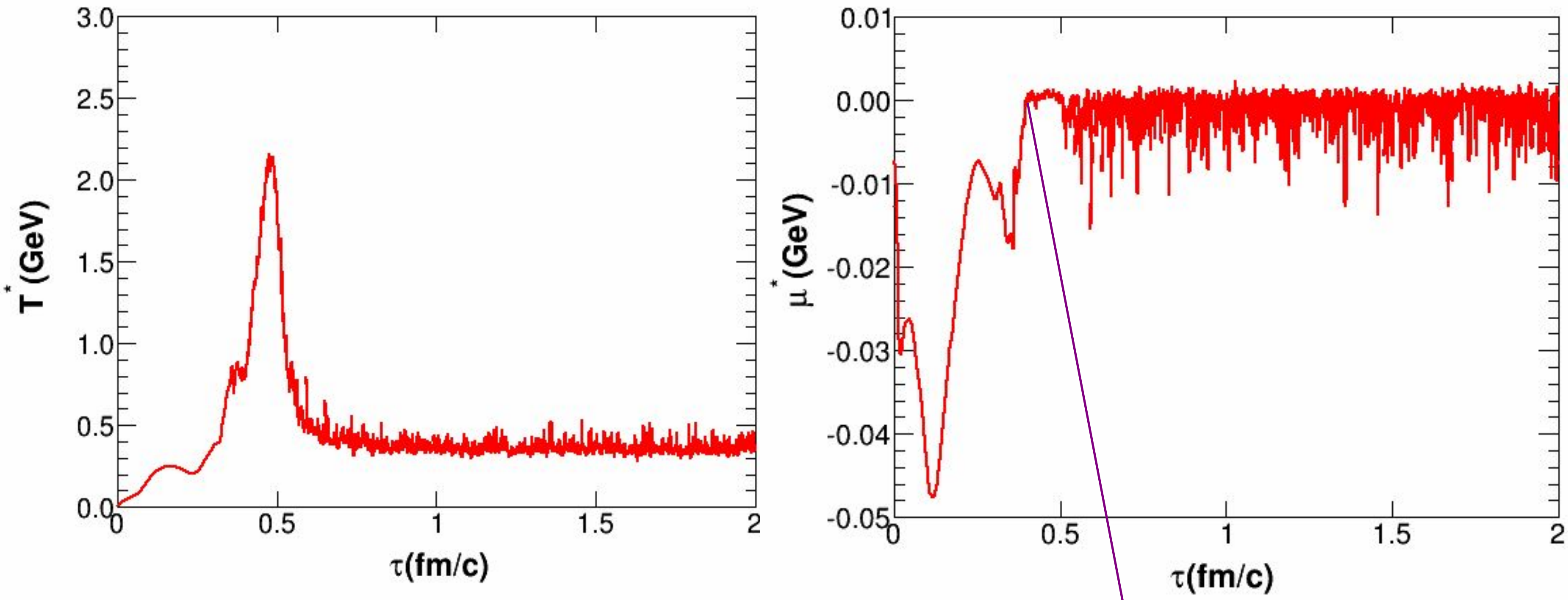
$f_0 = 0.4 > f_{critical}$

$T_{eq} = 0.352 MeV$ ✓

$\rho_c^{eq} = 0.186\,fm^{-3}$ ✓



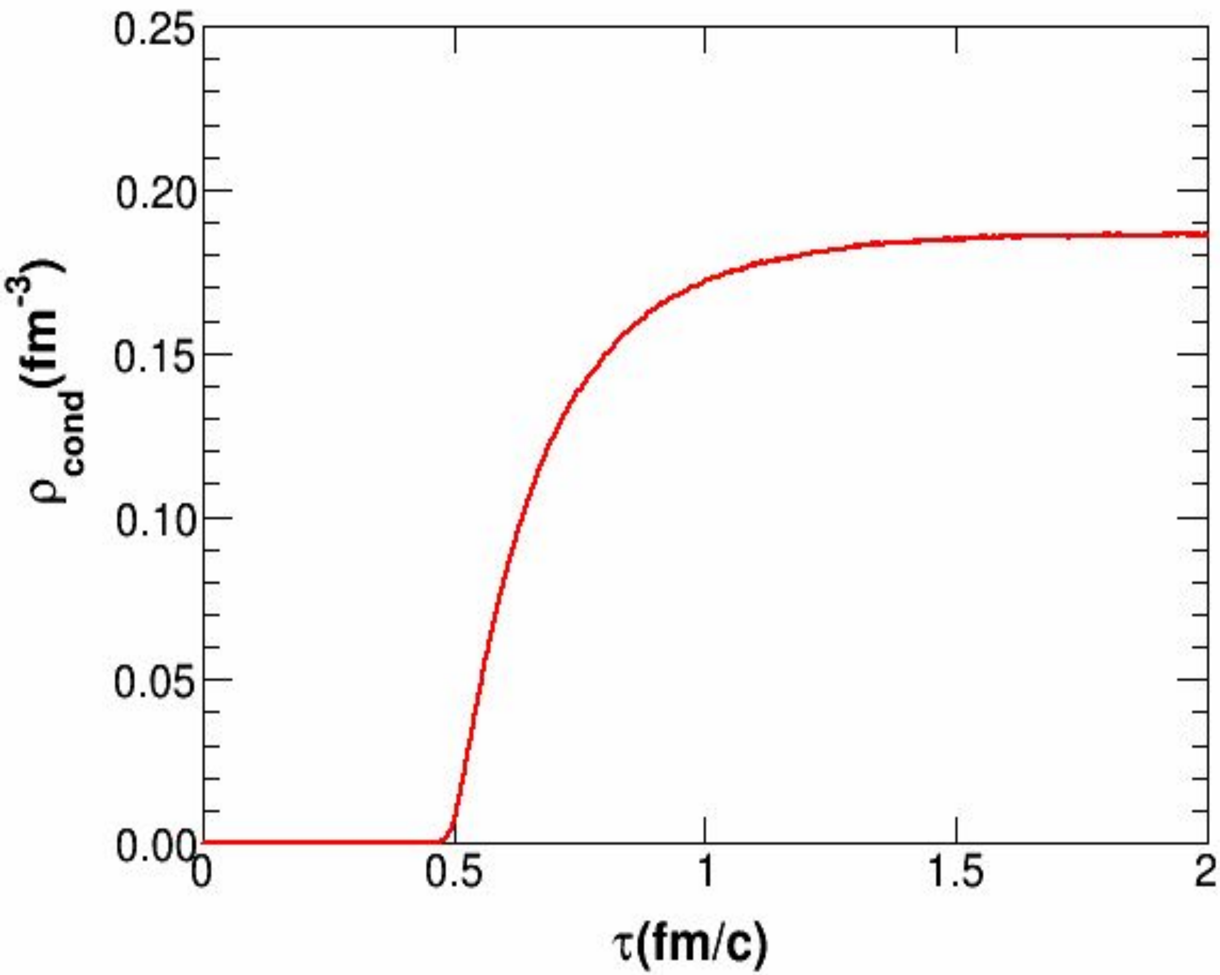
f_0 = 0.4 soft mode rapid thermalization



$$f_{IR} = \frac{T^*}{p - \mu^*} (\mu^* < 0)$$

gluon BEC arise

f_0 = 0.4 the growth for condensate



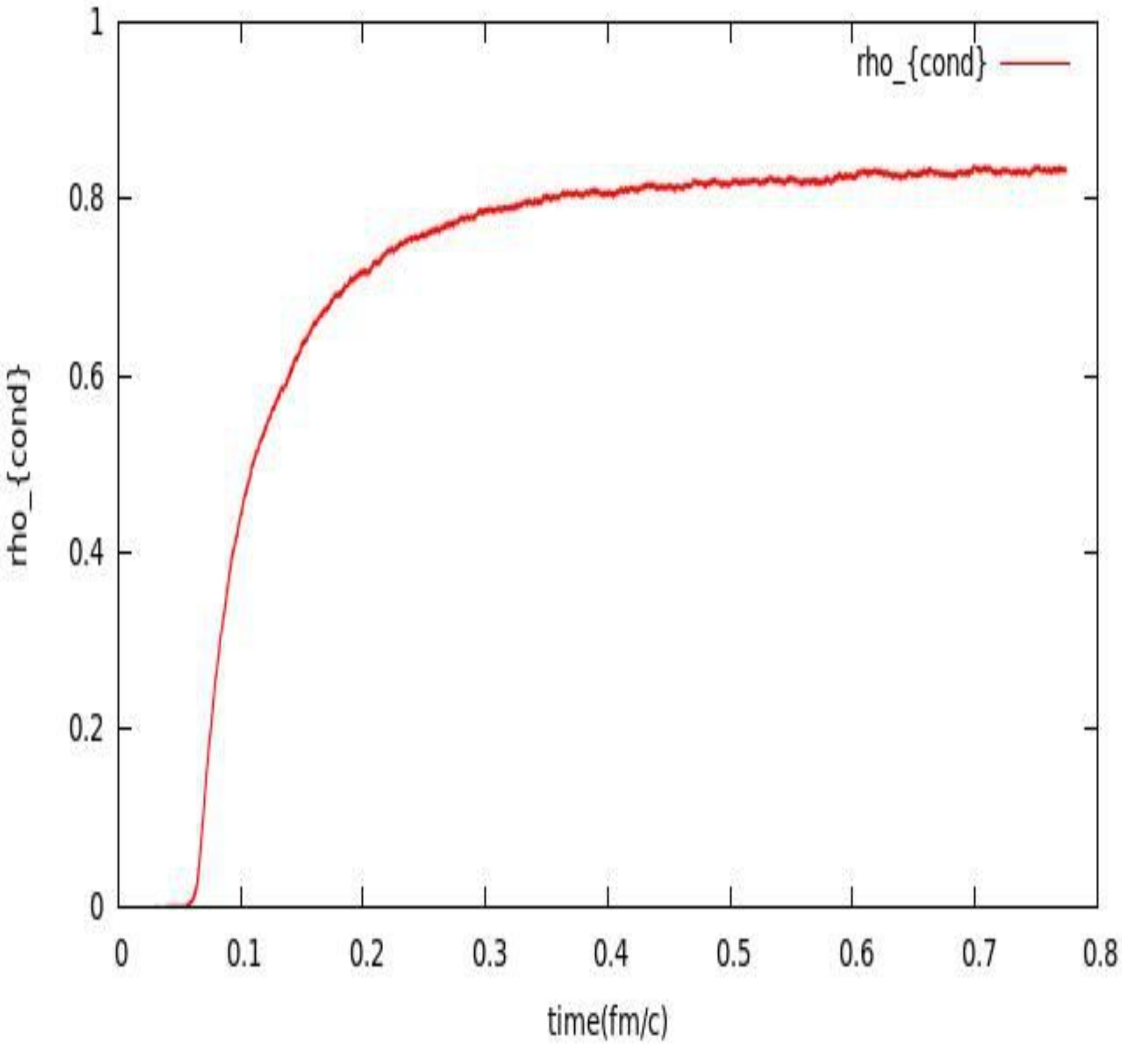
$$f_0 = 0.4 > f_{critical}$$

$$T_{eq} = 0.352 MeV$$

$$\rho_c^{eq} = 0.186\, fm^{-3}$$

gluon BEC arise

f_0 = 1 the growth for condensate



$$f_0 = 1 > f_{critical}$$

$$T_{eq} = 0.443 MeV$$

$$\rho_c^{eq} = 0.82 fm^{-3}$$

gluon BEC arise

Conclusion & Outlook

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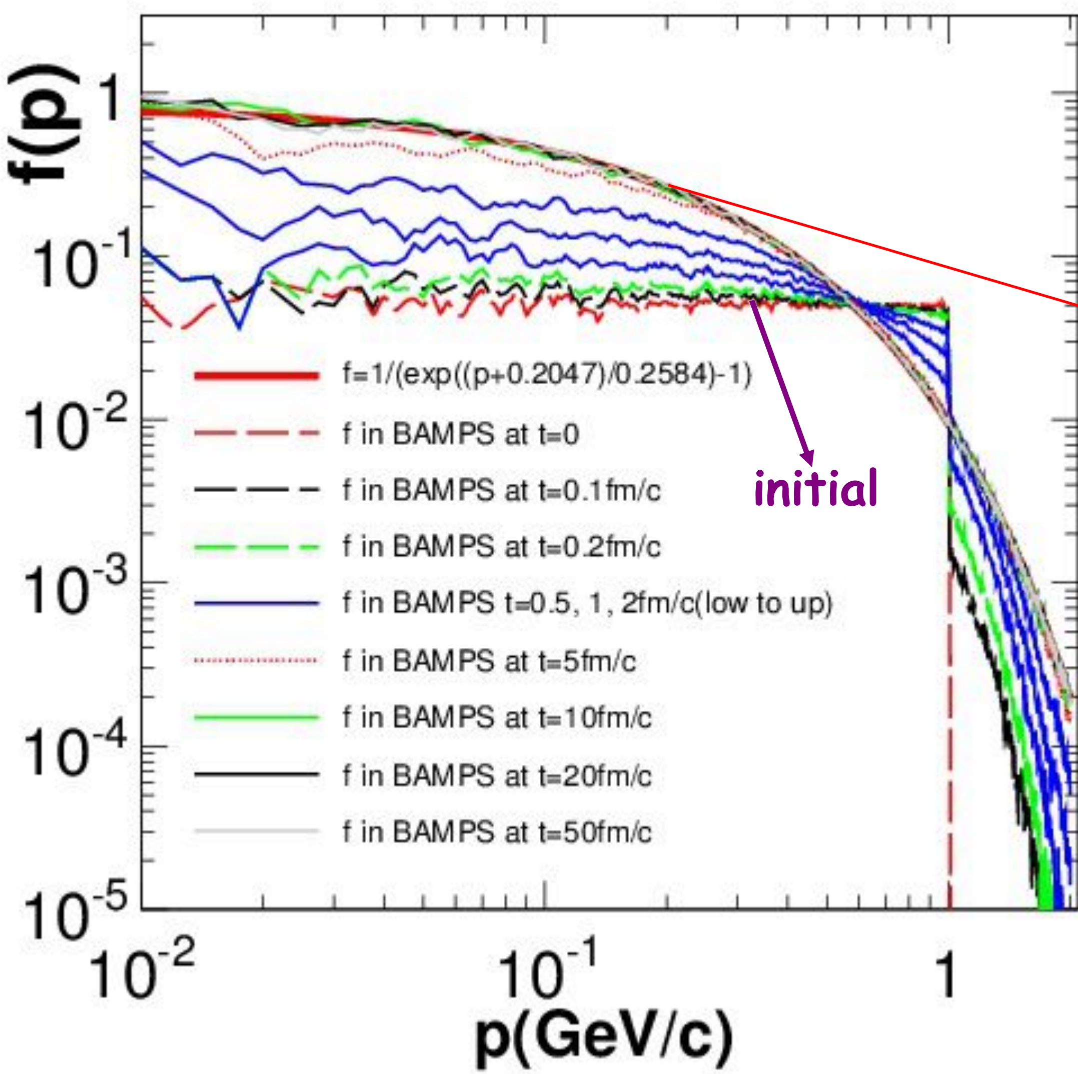
- Include **bose statistical factor:** Rate and Equilibrium Test
- **A new Cascade simulation** is developed, BEC growth is considered.
- **A non-zero Condensate** will arise under overpopulated initial condition when there's only elastic collisions.

- **Inelastic collision**'s effect for BEC and Thermalization ?
- How about a **expanding** system ?
- Add Gribov to BEC?

Couple a kinetic description and also field evolution for BEC

Thank you !

f_0 = 0.05 simulation results



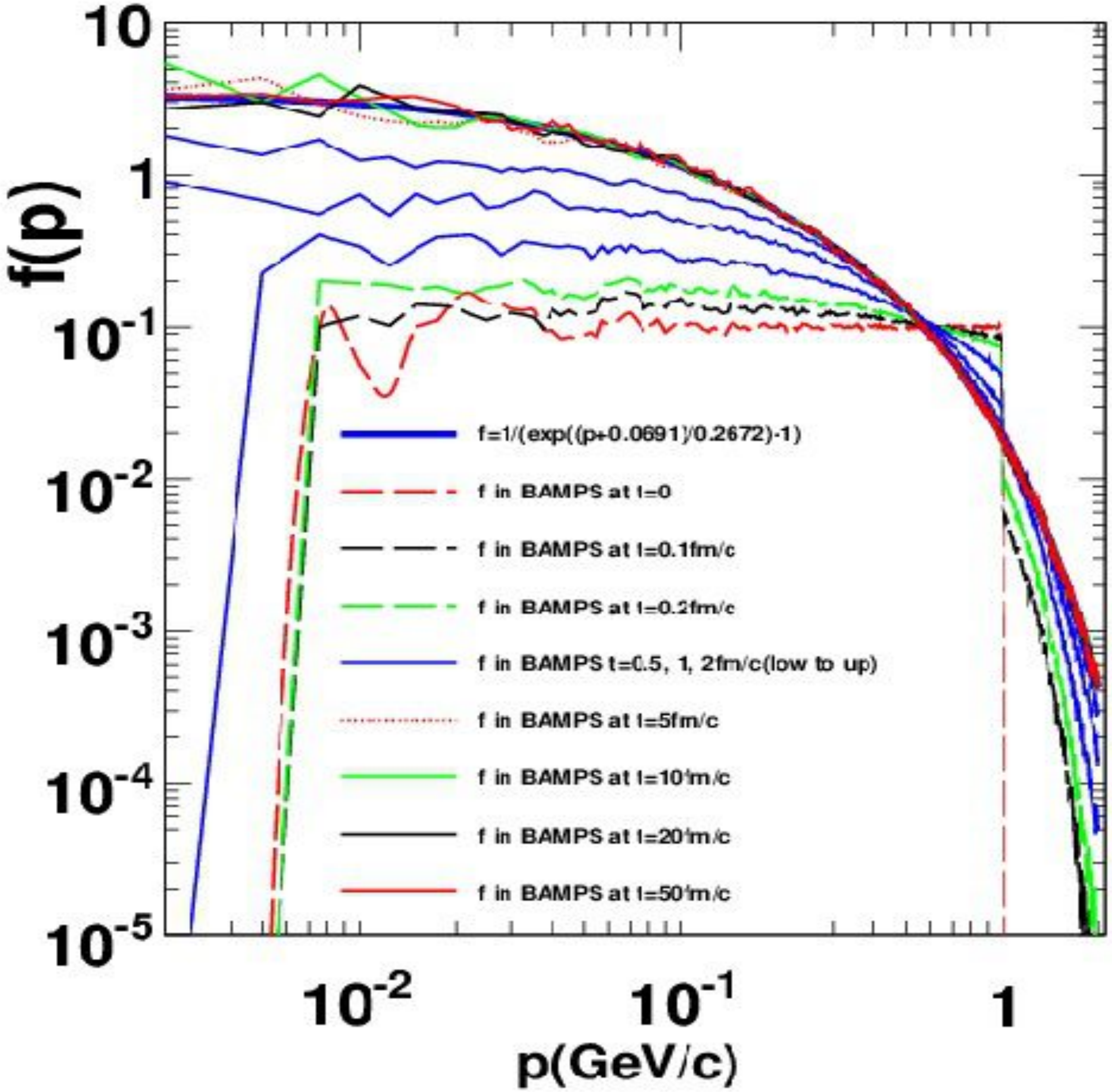
$3\times3\times3\,fm^3$ static box

$f_0 = 0.05 < f_{critical}$

$$\begin{cases} T_{eq} = 0.258MeV \\ \mu_{eq} = -0.205MeV \end{cases}$$

the system thermalizes to thermal BE distribution

f_0 = 0.1 simulation results



$3\times3\times3\,fm^3$ static box

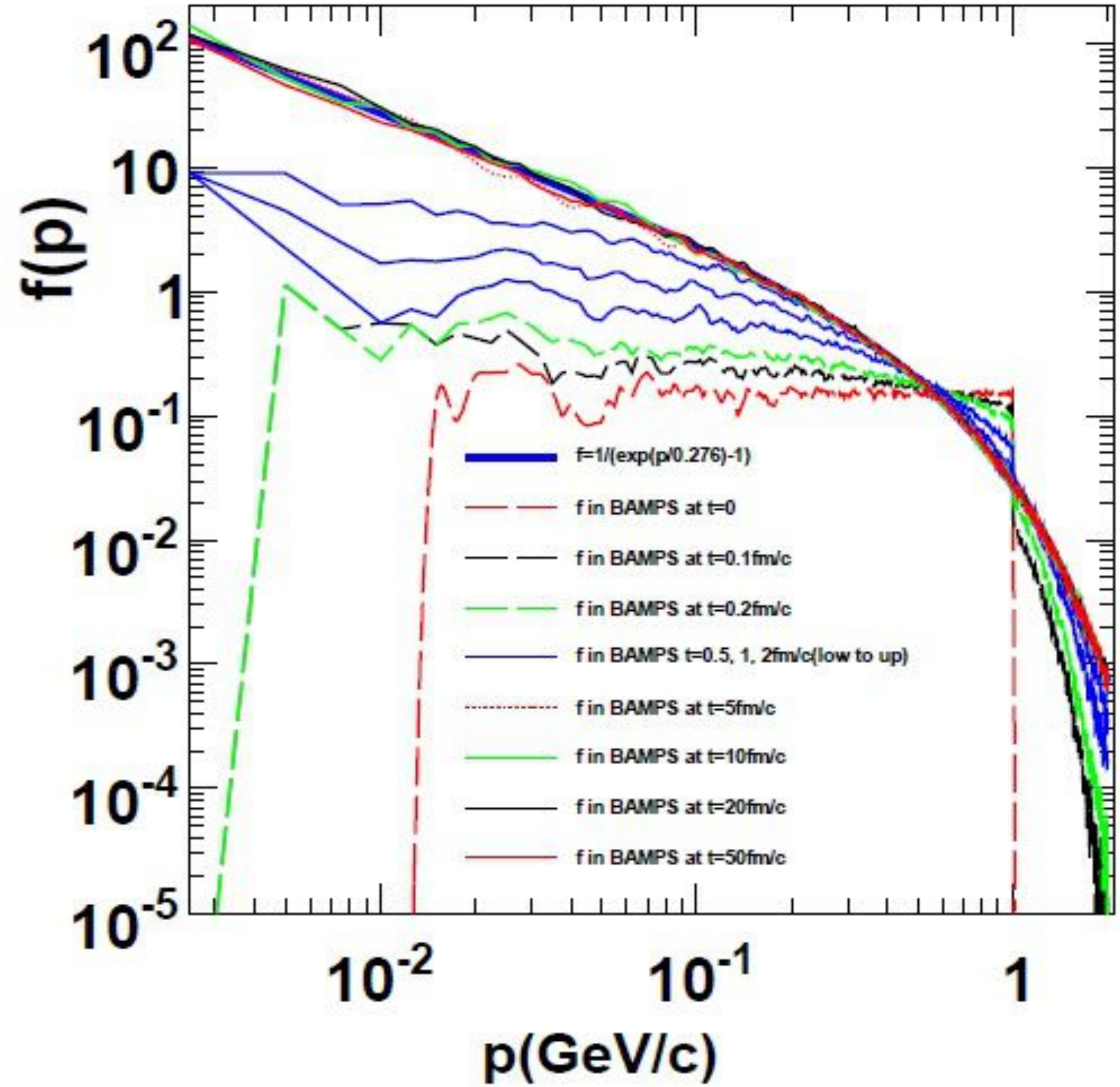
$f_0 = 0.1 < f_{critical}$

$T_{eq} = 0.267 MeV$

$\mu_{eq} = -0.07 MeV$

the system thermalizes to thermal BE distribution

f_0 = 0.154 simulation results



$3 \times 3 \times 3\, fm^3$ static box

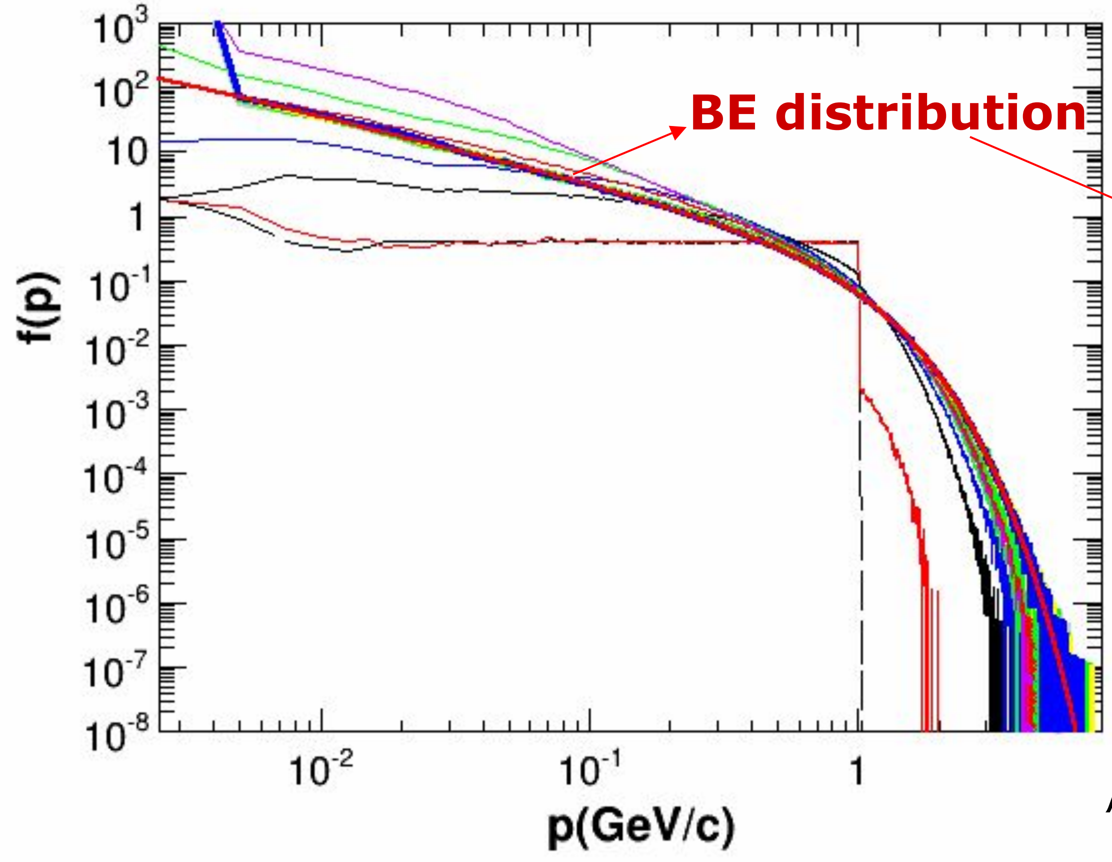
$$f_0 = 0.154 = f_{critical}$$

$$T_{eq} = 0.276 MeV$$

$$\mu_{eq} = 0 MeV$$

the system thermalizes to thermal BE distribution

f_0 = 0.4 simulation results

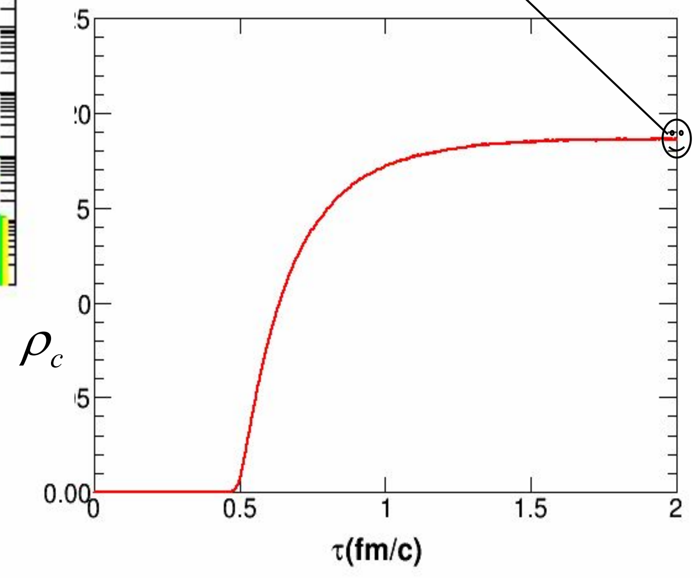


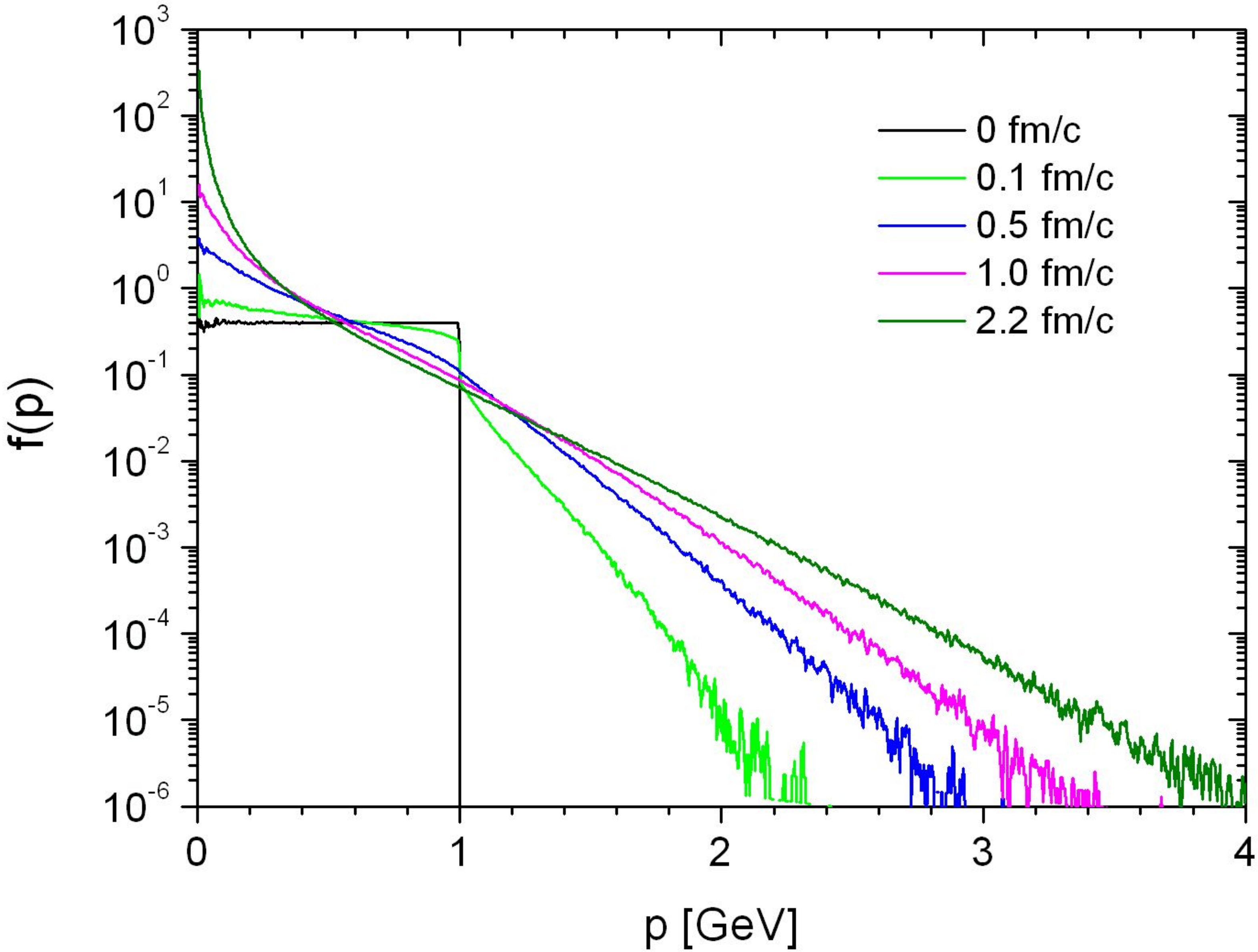
$3 \times 3 \times 3\, fm^3$ static box

$f_0 = 0.4 > f_{critical}$

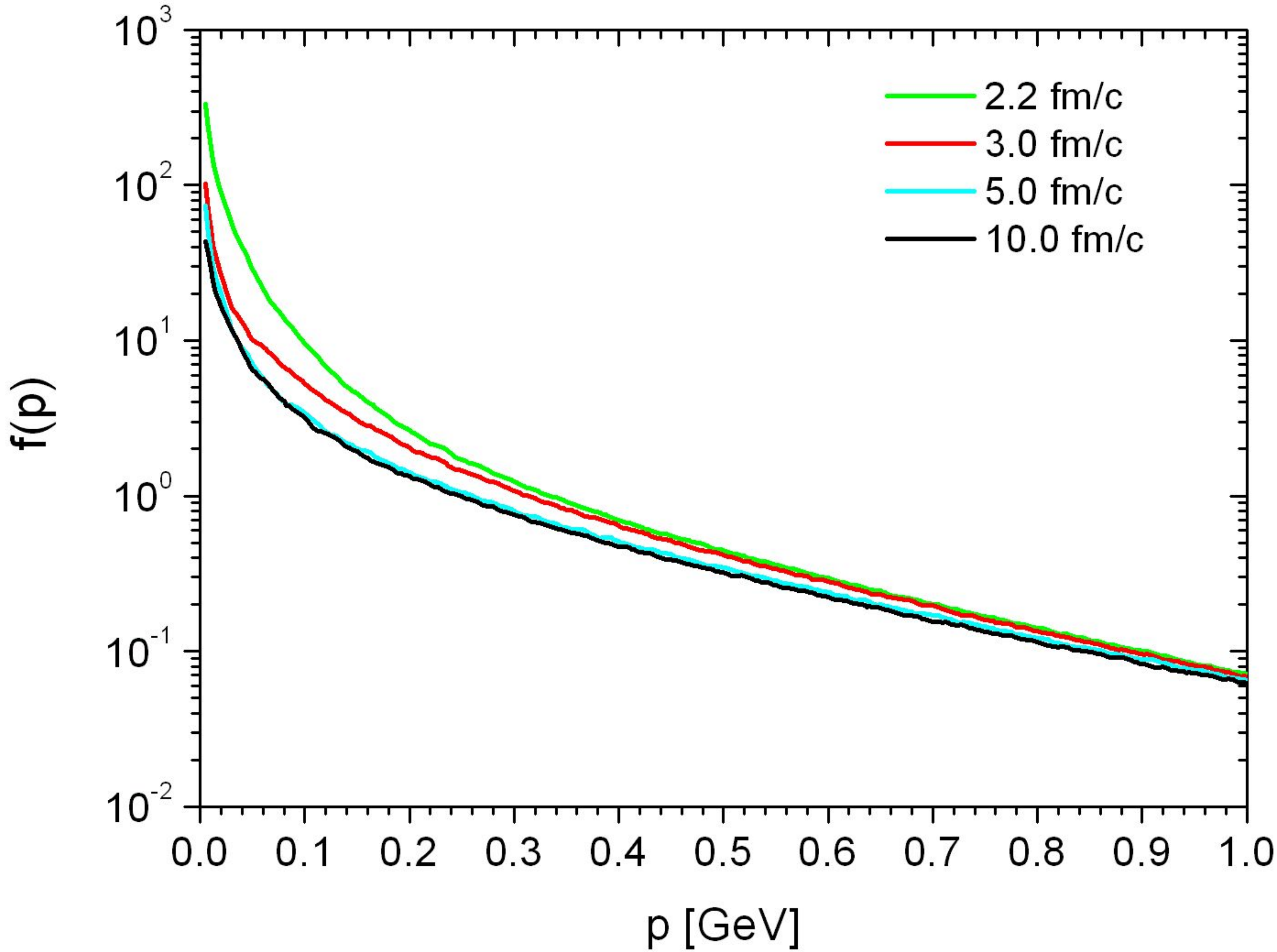
$T_{eq} = 0.352 MeV$ ✓

$\rho_c^{eq} = 0.186\, fm^{-3}$ ✓

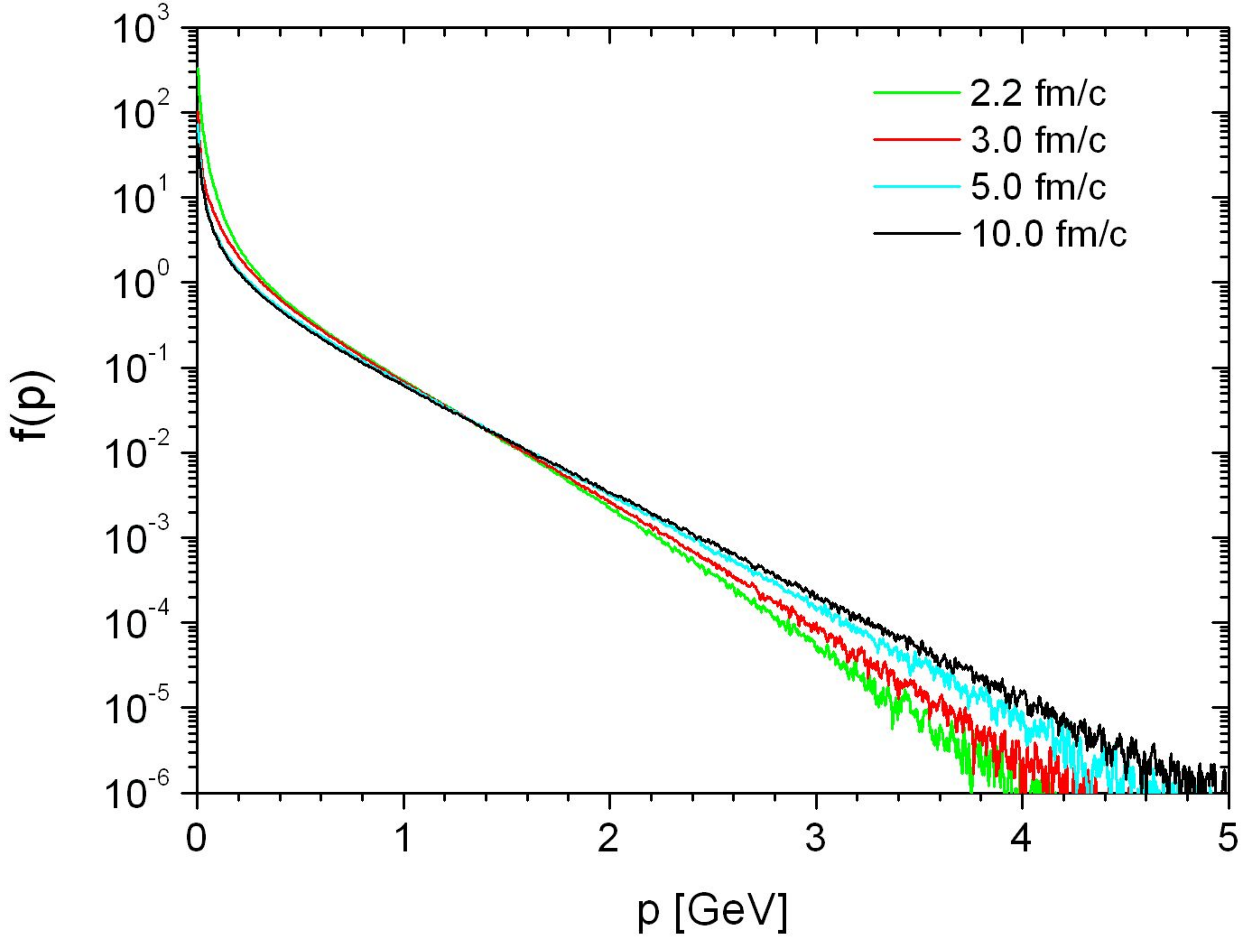




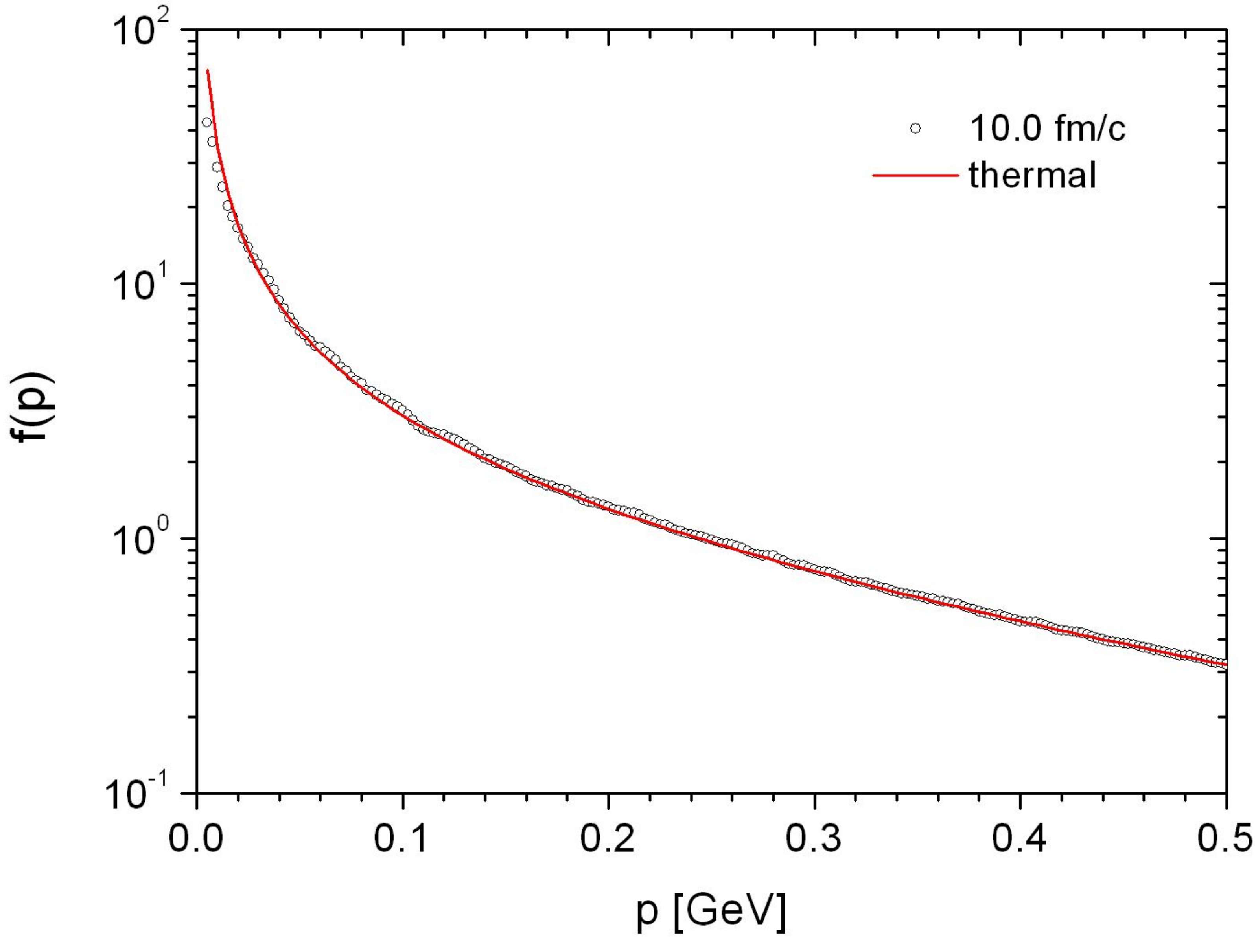
f(p)
p [GeV]
10^{3}
10^{2}
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
0
1
2
3
4
0 fm/c
0.1 fm/c
0.5 fm/c
1.0 fm/c
2.2 fm/c



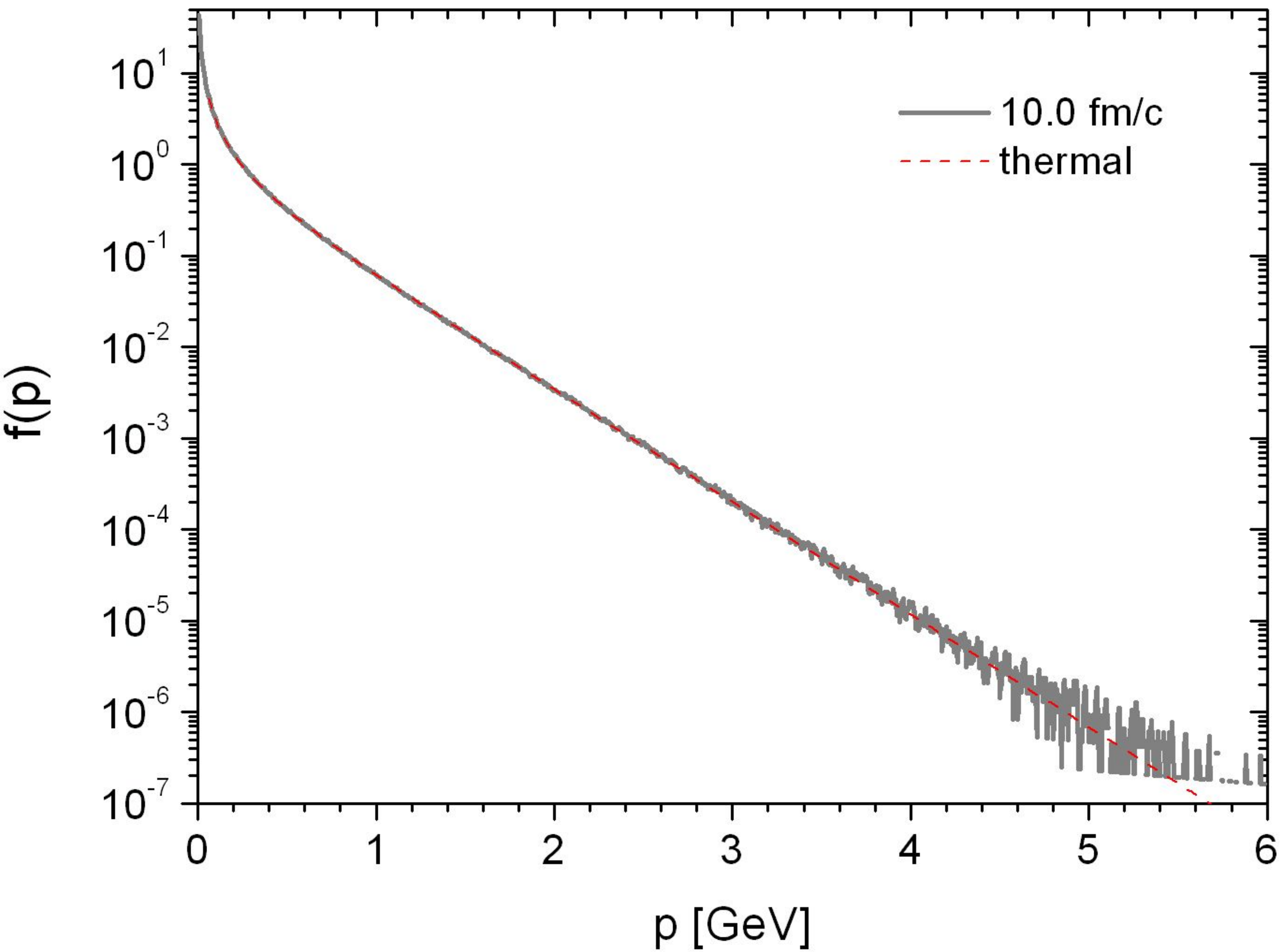
2.2 fm/c
3.0 fm/c
5.0 fm/c
10.0 fm/c
f(p)
p [GeV]
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0



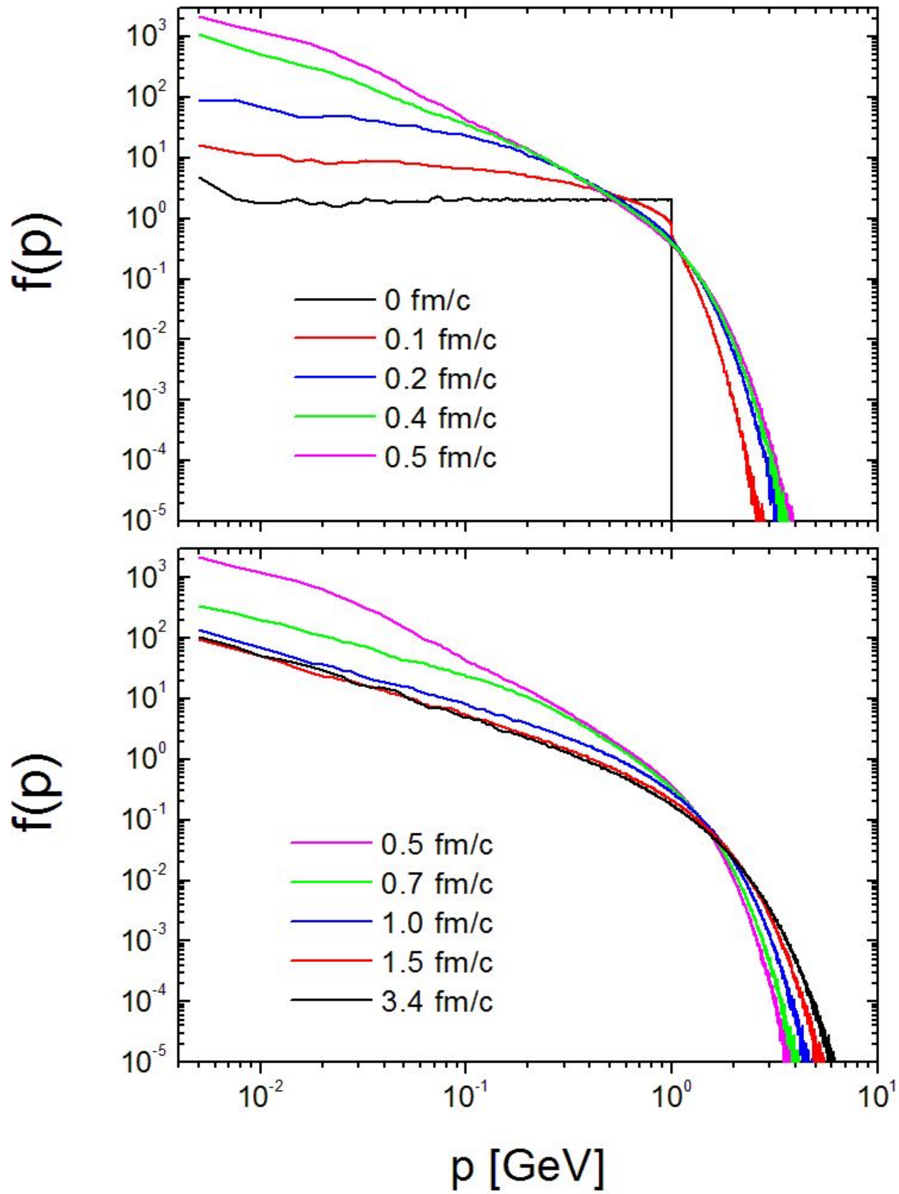
f(p)
p [GeV]
2.2 fm/c
3.0 fm/c
5.0 fm/c
10.0 fm/c



10.0 fm/c
thermal
f(p)
p [GeV]
10^2
10^1
10^0
10^{-1}
0.0
0.1
0.2
0.3
0.4
0.5



10.0 fm/c
thermal
f(p)
p [GeV]
10^1
10^0
10^-1
10^-2
10^-3
10^-4
10^-5
10^-6
10^-7
0
1
2
3
4
5
6



f0=2 The condensation begins at 0.376 fm/c.
The distribution at small p is increasing until 0.5 fm/c

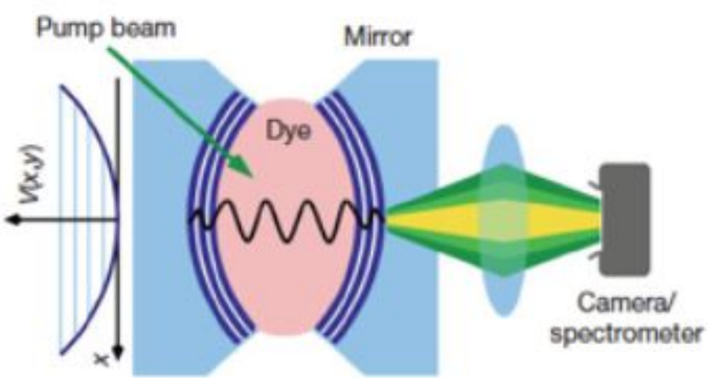
BEC for Non-Conserved Particles

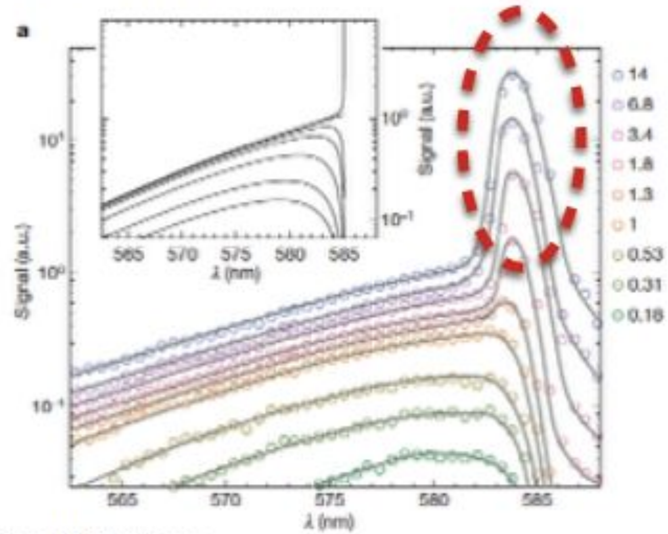
LETTER

doi:10.1038/nature09567

Bose–Einstein condensation of photons in an optical microcavity

Jan Klaers, Julian Schmitt, Frank Vewinger & Martin Weitz





increasing the photon density, we observe the following BEC signatures: the photon energies have a Bose–Einstein distribution with a massively populated ground-state mode on top of a broad thermal wing; the phase transition occurs at the expected photon density and exhibits the predicted dependence on cavity geometry; and the ground-state mode emerges even for a spatially displaced pump spot.

Another example:
idea of overcooled pion gas
in heavy ion collisions.

Key point: under suitable conditions, non-conserved particles may become effectively or transiently conserved.

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