

# Rate equation approach to $J/\psi$ equilibration



**MainCampus**

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# Outline

1.  $J/\psi$  in the statistical hadronization model
2. Production and dissociation processes
3. Rate equation approach
4. Results

## Statistical hadronization

Multiplicity given by T, μ, V:

$$N_i^{th} = \frac{d_i VT}{2\pi^2} m_i \sum_{n=1}^{\infty} (\bar{\mp} 1)^{n+1} \frac{\lambda_i^n}{n} K_2 \left( \frac{nm_i}{T} \right)$$

T, μ: ratio fits

V: charged hadron multiplicity

# Exact charm conservation (hadron picture)

Balance equation:

$$N_{c\bar{c}}^{dir} = \frac{1}{2} g_c N_{oc}^{th} \frac{I_1(g_c N_{oc}^{th})}{I_0(g_c N_{oc}^{th})} + g_c^2 N_{\psi}^{th}$$

$g_c$ : charm fugacity (hadronic picture)

$$N_{J/\psi} = g_c^2 \frac{d_{J/\psi} VT}{2\pi^2} m_{J/\psi} \sum_{n=1}^{\infty} \frac{\lambda_{J/\psi}^n}{n} K_2\left(\frac{nm_{J/\psi}}{T}\right)$$

## Exact charm conservation (QGP picture)

Balance equation:

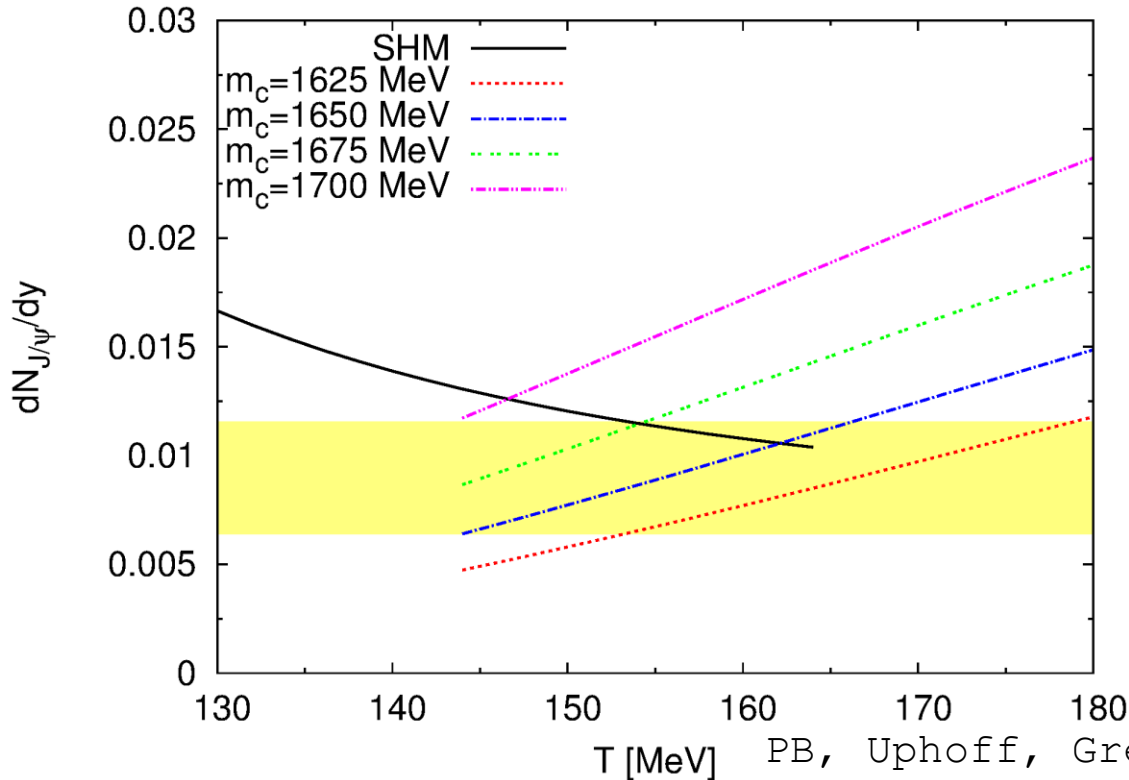
$$N_{c\bar{c}}^{dir} = \lambda_{c\bar{c}} N_c^{th} = \lambda_{c\bar{c}} \frac{d_c VT}{2\pi^2} m_c \sum_{n=1}^{\infty} (-1)^{n+1} \frac{\lambda_c^n}{n} K_2\left(\frac{nm_c}{T}\right)$$

$\lambda_{c\bar{c}}$ : charm fugacity (QGP picture)

$$N_{J/\psi} = \lambda_{c\bar{c}}^2 \frac{d_{J/\psi} VT}{2\pi^2} m_{J/\psi} \sum_{n=1}^{\infty} \frac{\lambda_{J/\psi}^n}{n} K_2\left(\frac{nm_{J/\psi}}{T}\right)$$

# Production equivalency

$g_c$  and  $\lambda_{c\bar{c}}$  are connected measures of chemical equilibrium



Strong dependence on  $m_c$ !

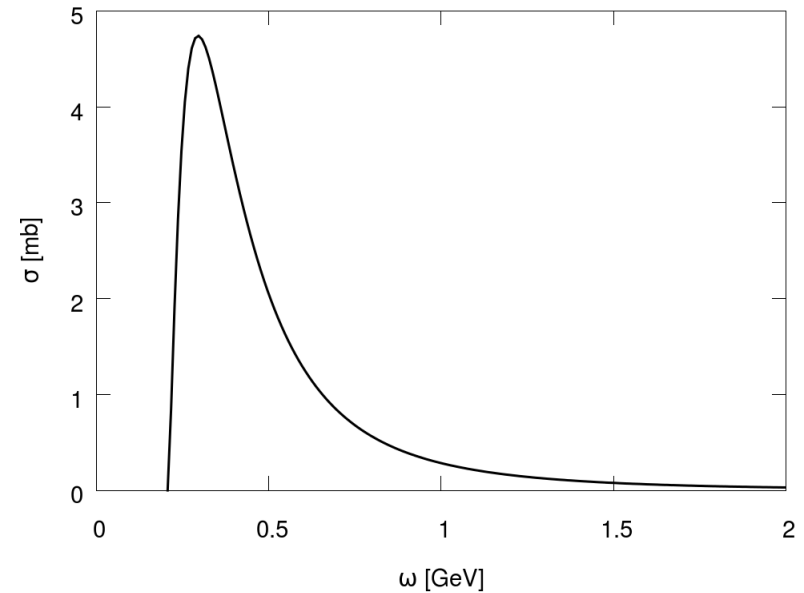
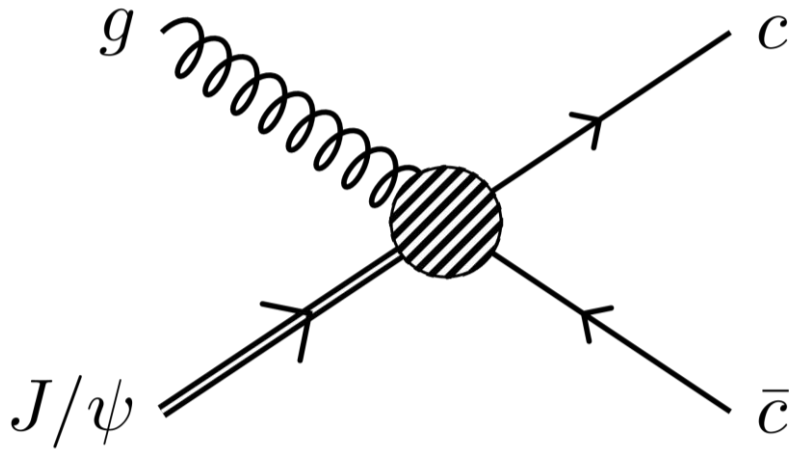
PB, Uphoff, Greiner: PRC 90 (2014) 6, 061901

## Motivation

- Simultaneous production of hadrons fits  $J/\psi$
- $T_{dis} > T_{FO}$ : QGP picture should hold

Can QGP scatterings produce statistical  $J/\psi$  multiplicities?

## Processes



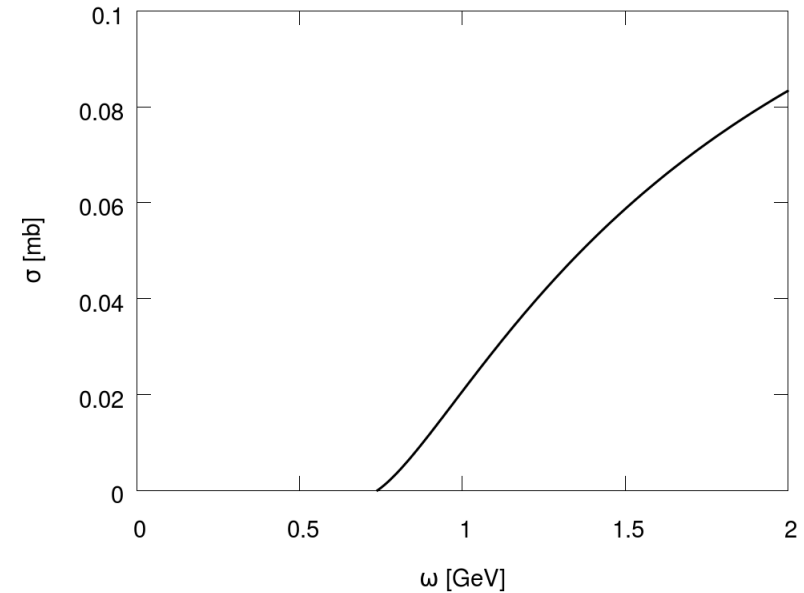
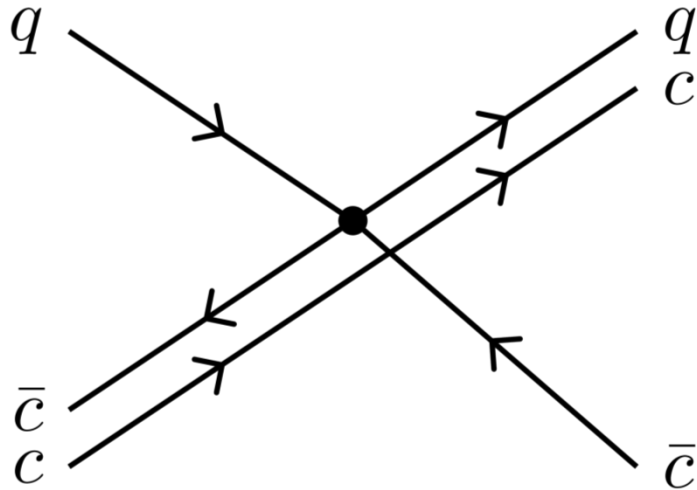
## Bhanot-Peskin gluon dissociation

$$\frac{2^{11}}{27} \frac{1}{\sqrt{m_c^3 \epsilon}} \frac{(\omega/\omega_0 - 1)^{3/2}}{(\omega/\omega_0)^5}$$

Bhanot, Peskin: Nucl. Phys. B 156 (1979) 391



## Processes

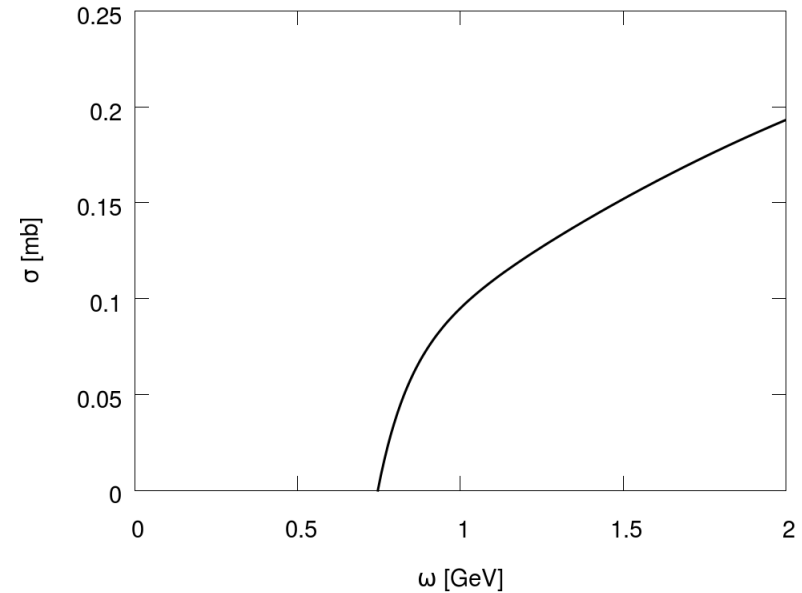
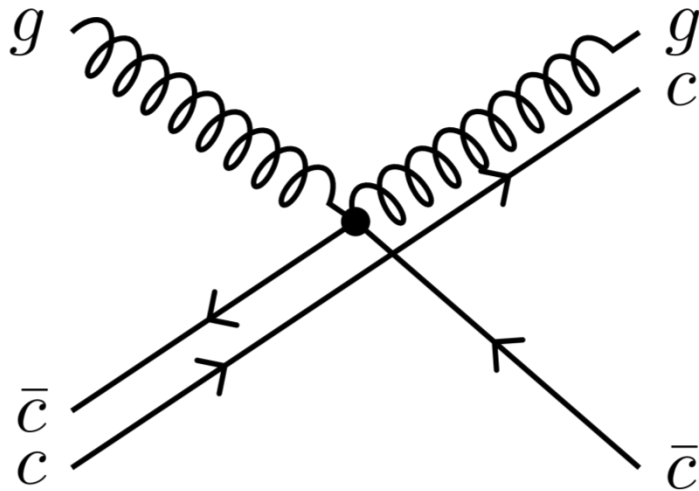


Quasi-free quark dissociation

$$\frac{4\pi\alpha^2}{9(2m_c\omega)^2} \left[ \left( 1 + \frac{2(2m_c\omega + m_c^2)}{Q_0^2} \right) \left( \frac{(2m_c\omega)^2}{2m_c\omega + m_c^2} - Q_0^2 \right) - 2(2m_c\omega + m_c^2) \log \frac{(2m_c\omega)^2}{(2m_c\omega + m_c^2)Q_0^2} \right]$$

Combridge: Nucl. Phys. B 151 (1979) 429

## Processes



## Quasi-free gluon dissociation

$$\frac{2\pi\alpha^2}{(2m_c\omega)^2} \left[ \left( 1 + \frac{4}{9} \left( \frac{m_c + \omega}{\omega} \right)^2 \right) (L - Q_0^2) + \frac{Q_0^4 - L^2}{9m_c\omega} + 4m_c(m_c + \omega) \log \frac{Q_0^2}{L} \right. \\ \left. + \frac{2m_c^3 - 5m_c^2\omega}{9\omega} \log \frac{2m_c\omega - Q_0^2}{2m_c\omega - L} + 8m_c^2\omega^2 \left( \frac{1}{Q_0^2} - \frac{1}{L} \right) + \frac{16m_c^4}{9} \left( \frac{1}{2m_c\omega - L} - \frac{1}{2m_c\omega - Q_0^2} \right) \right]$$

Combridge: Nucl. Phys. B 151 (1979) 429

## Potential model

Binding radius from screened potential:

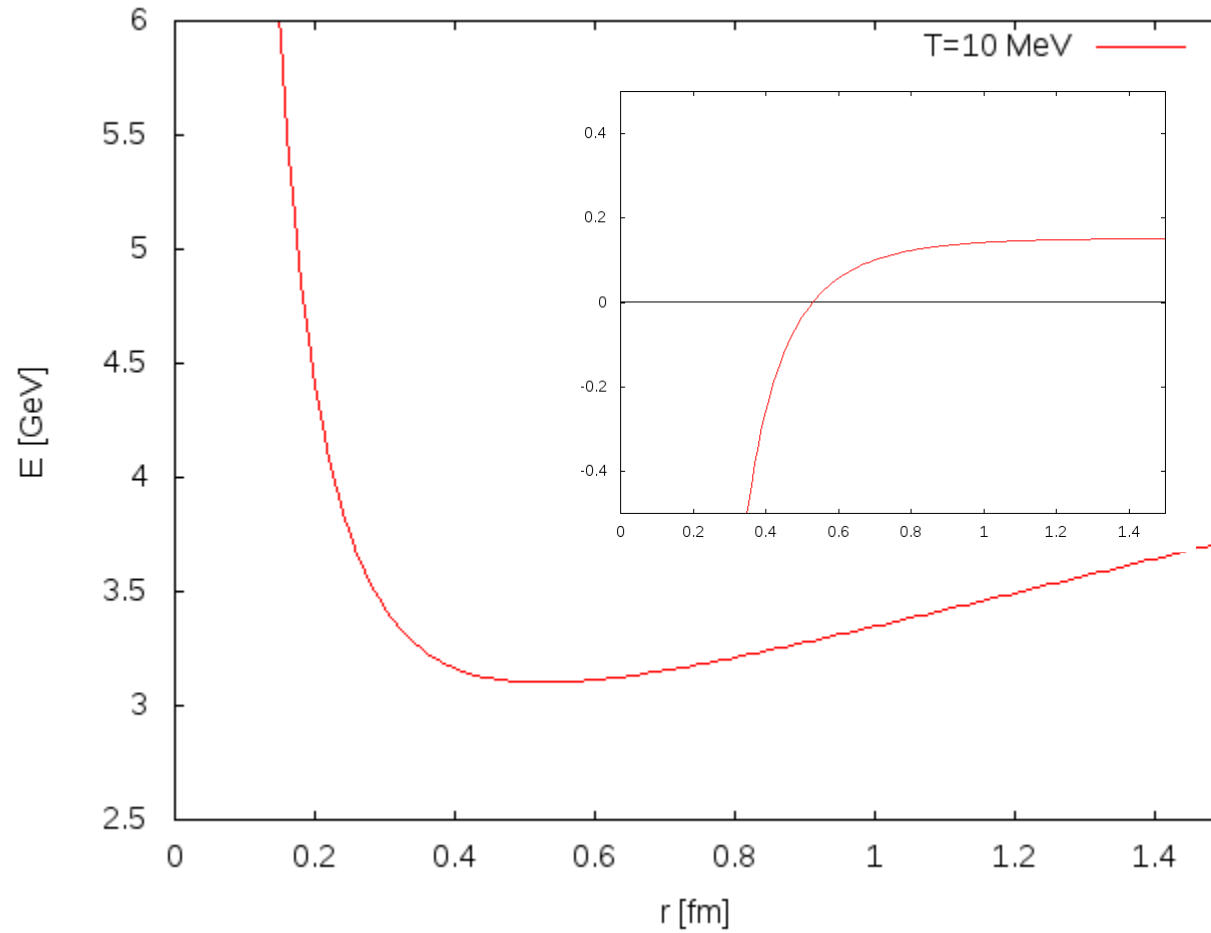
$$V = -\frac{4\alpha}{3r} e^{-m_D r} + \frac{\sigma}{m_D} (1 - e^{-m_D r})$$

$$E = 2m_c + \frac{b}{m_c r^2} - \frac{4\alpha}{3r} e^{-m_D r} + \frac{\sigma}{m_D} (1 - e^{-m_D r})$$

$$-\frac{2b}{m_c r^3} + \left( \sigma + \frac{4\alpha}{3r} \left( \frac{1}{r} + m_D \right) \right) e^{-m_D r} = 0$$

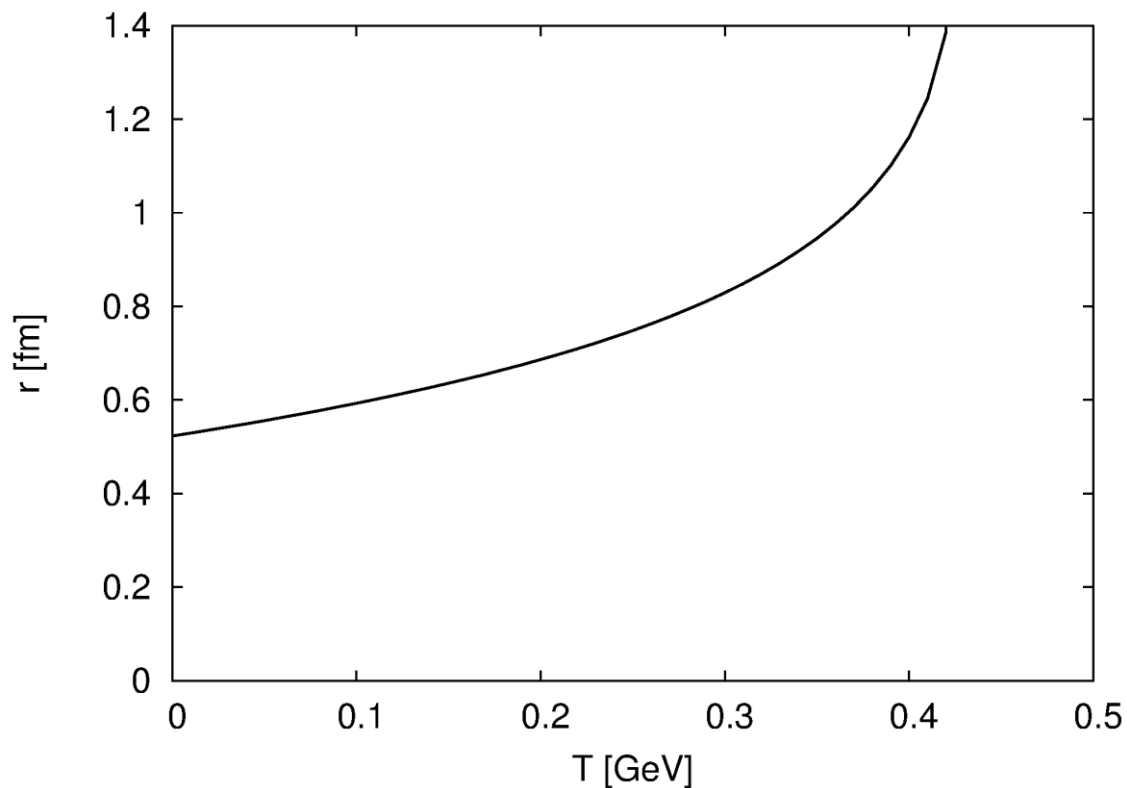
Karsch, Mehr, Satz: Z. Phys. C 37, 617 (1988)

# Melting



# Modifications

Cross sections increase with radius



$$\sigma(T) = \sigma(0) \frac{\langle r^2 \rangle(T)}{\langle r^2 \rangle(0)}$$

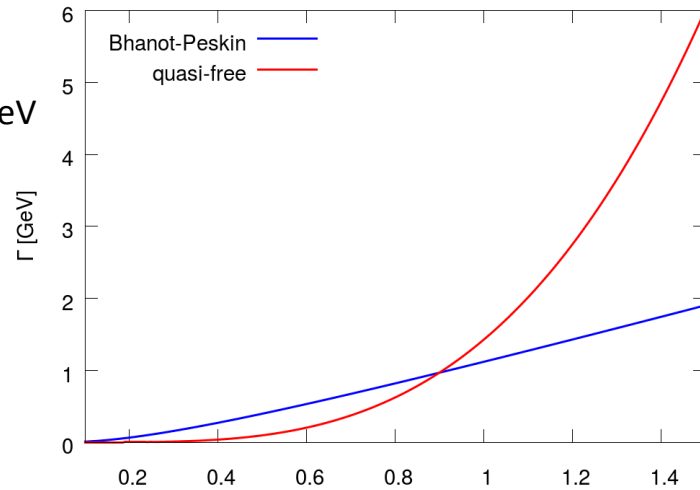
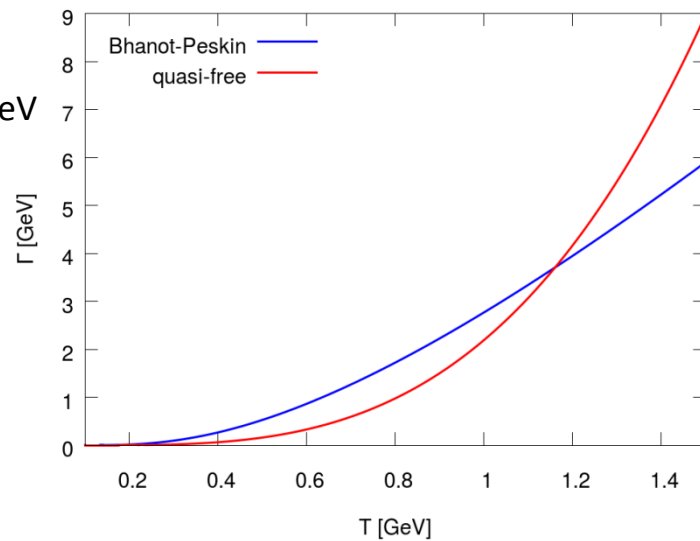
## Dissociation rate

$$\Gamma = \int \frac{d^3\mathbf{p}}{(2\pi)^3} d_g f_g \sigma_{gJ/\psi} v_{rel}$$

$$\Gamma(u_r, T) = \frac{d_g T}{(2\pi)^2 |\mathbf{u}_r|} \int_{\omega_0}^{\infty} d\omega \omega_{gJ/\psi} \log \frac{1 - e^{-(\sqrt{|\mathbf{u}_r|^2+1}+|\mathbf{u}_r|)\omega/T}}{1 - e^{-(\sqrt{|\mathbf{u}_r|^2+1}-|\mathbf{u}_r|)\omega/T}}$$

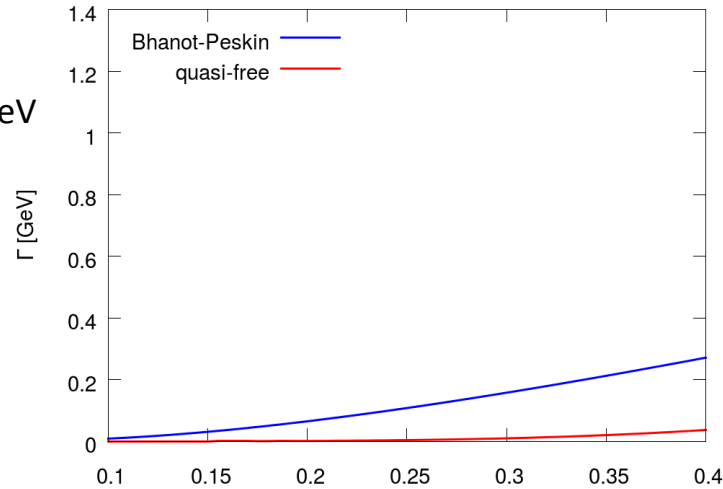
$$\Gamma(T) = \frac{d_g}{2\pi^2} \int_{\omega_0}^{\infty} d\omega \frac{\omega^2 \sigma_{gJ/\psi}}{e^{\omega/T} - 1} \text{ for } |\mathbf{u}_r| = 0$$

## Rates

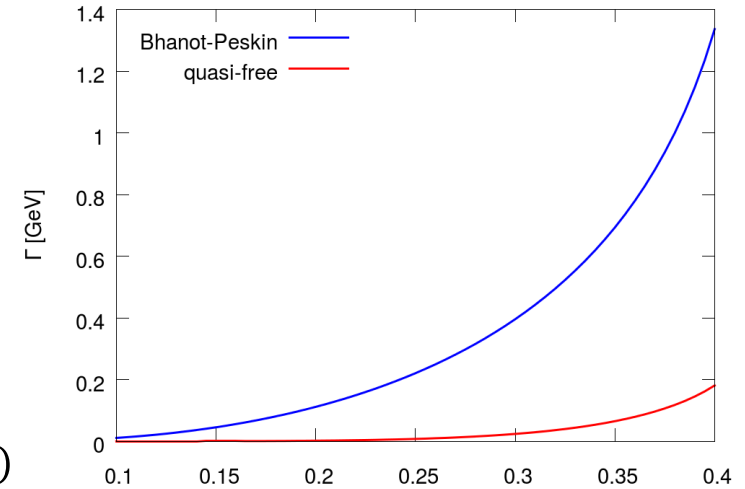
 $m_c = 1.65 \text{ GeV}$  $m_c = 1.87 \text{ GeV}$ 

# Rates

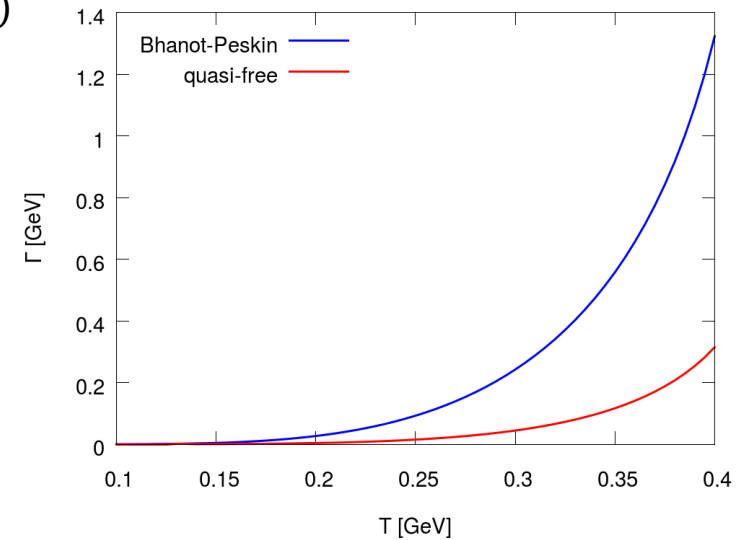
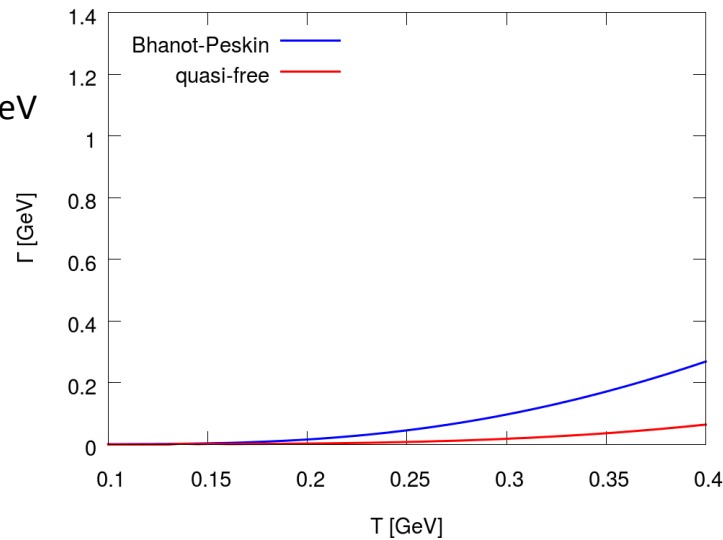
$m_c = 1.65 \text{ GeV}$



$$\times \frac{\langle r^2 \rangle(T)}{\langle r^2 \rangle(0)} =$$



$m_c = 1.87 \text{ GeV}$





# Rate equation

$$dN = \text{gain rate} - \text{loss rate}$$

$$\text{loss} \leftrightarrow \Gamma N(t)$$

$$N = N_{eq} \leftrightarrow dN = 0 \leftrightarrow \text{gain} = \Gamma N_{eq}$$

$$\frac{dN}{dt} = -\Gamma (N(t) - N_{eq})$$

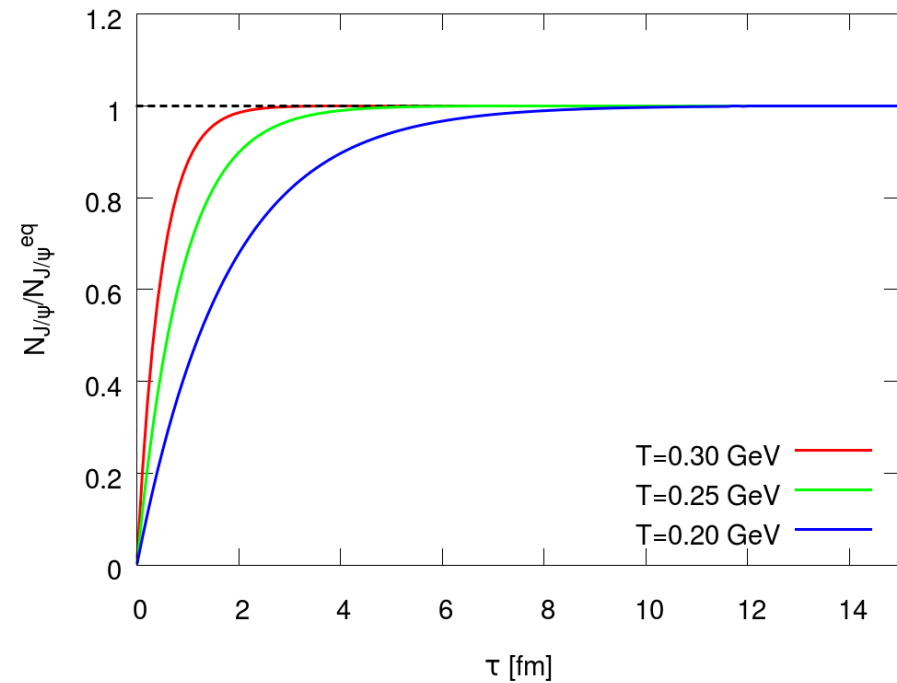
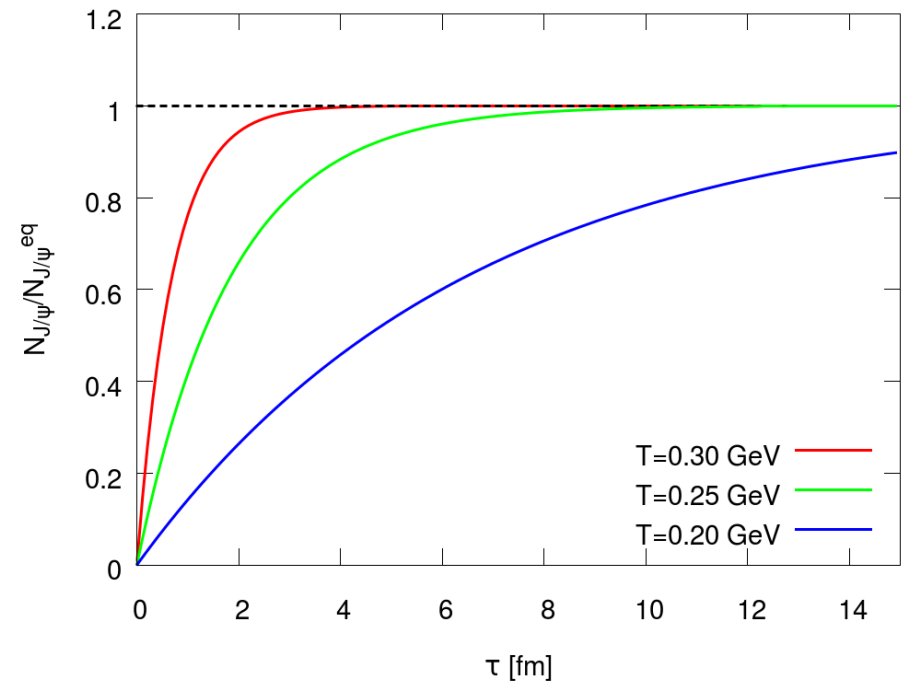
## Explicit form

$$N_c = \lambda_{c\bar{c}}(T) V(T) n_c^{th}(T) = \text{const.}$$

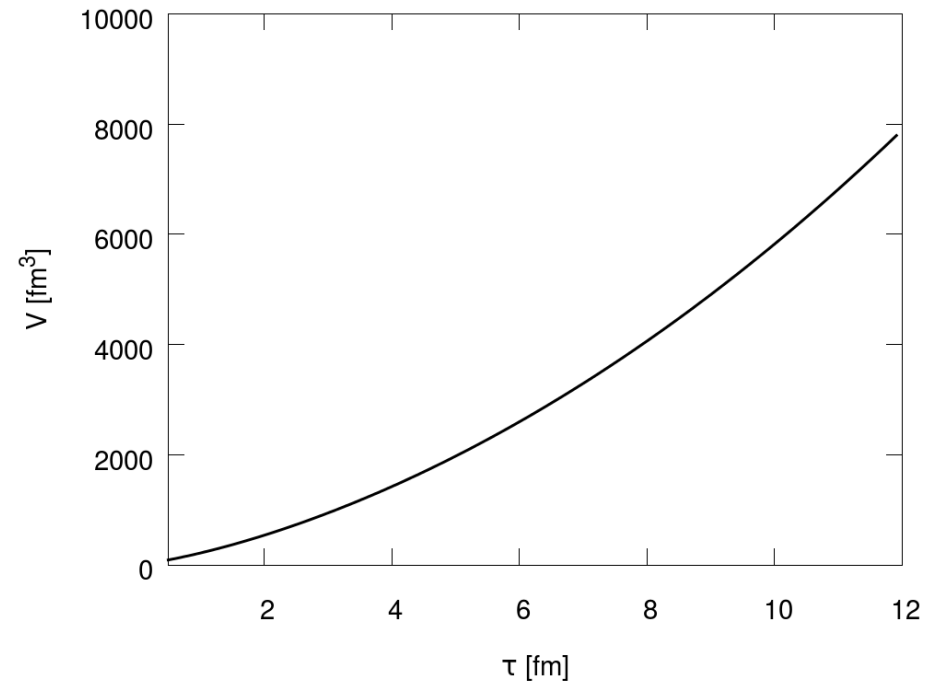
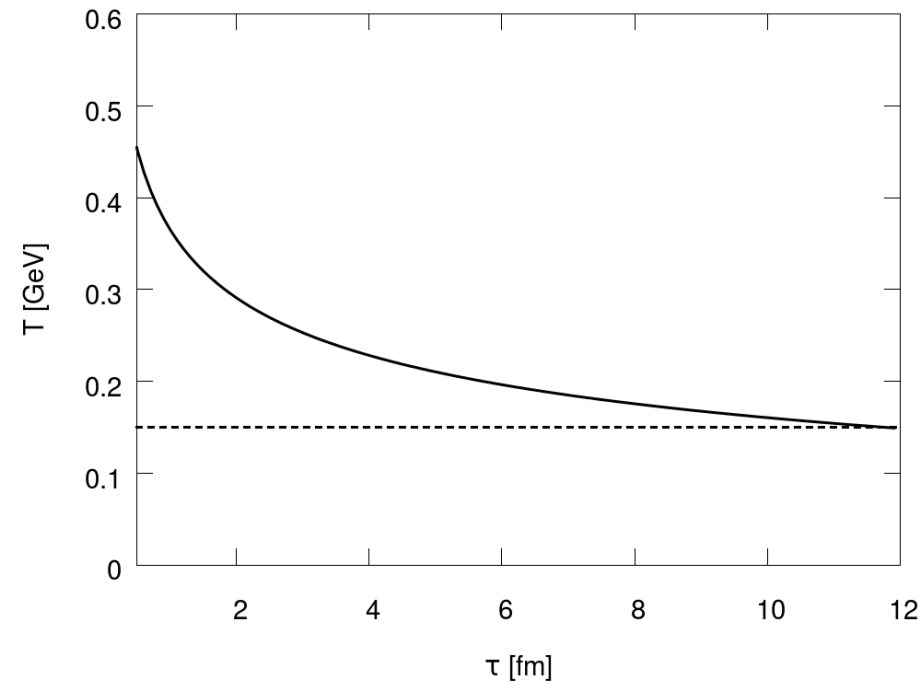
$$N_{eq} = \lambda_{c\bar{c}}^2 N_{J/\psi}^{th} = \frac{1}{V n_c^2} N_c^2 n_\psi^{th}$$

$$\frac{dN}{dt} = -\Gamma(T) \left( N - \frac{N_c^2}{V(T) n_c^2(T)} n_\psi^{th}(T) \right)$$

## Static box

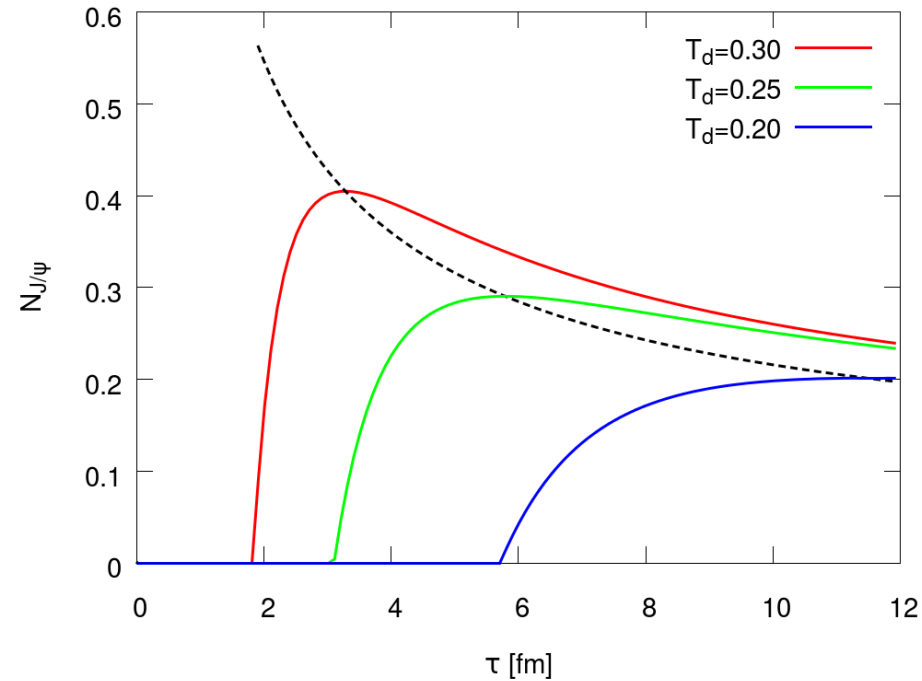
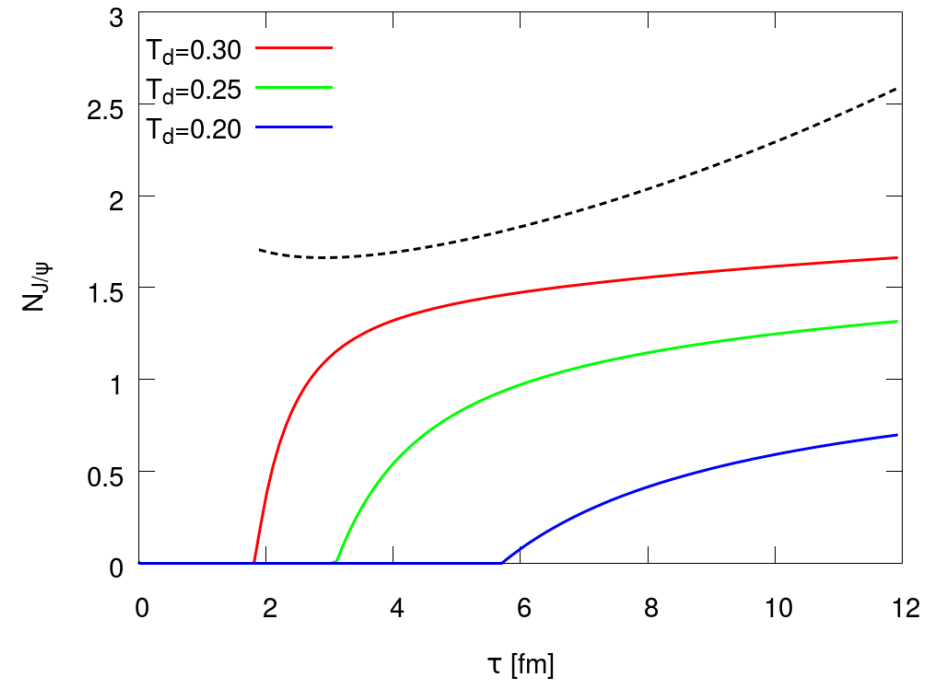
 $m_c = 1.65$  GeV $m_c = 1.87$  GeV

# Cooling and expanding



Schenke, Jeon, Gale: Phys. Lett. B 702 (2011) 59

# Dynamic equilibration

 $m_c = 1.65 \text{ GeV}$  $m_c = 1.87 \text{ GeV}$ 

## Summary

- Systematic approach cross sections  $\rightarrow$  equilibration times
- External temperature profiles
- Equilibration possible for  $m_c = 1.65$  GeV
- To do: external flow profiles

