Thermal properties of UrQMD box-simlulations

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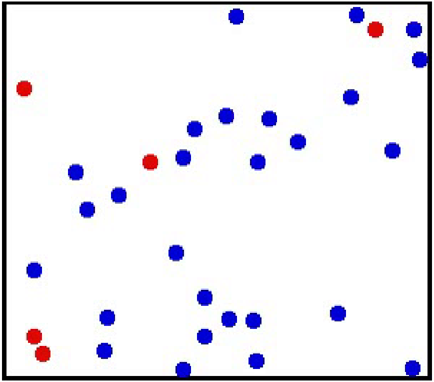
outline

short motivation

why box-calculations?

- final state at CERN, RHIC dominated by hadron-interaction $\rightarrow$ transport coefficients
- transport coefficients as input for inmedium models (dissipative fluid dynamics)
- low η/s values needed to reproduce elliptic flow
- lower bound for $\eta/s = 1/4\pi$ suggested (Kovtun Son Starinets bound KSS)



initialisation

use "infinite hadron matter":

- cubic box
- periodic boundary conditions:
  particle leaving at **r** reenters at -**r** with same momentum
- no string-excitations, no 1 to 3 decays:
  destroys detailed-balance
- input to initialise box:
  temperature T and volume V
  used to calculate number of particles N and total energy E

number of particles

first find out number of particles in the system
from relativistic kinetic theory: number of particles in a phase space volume

$$\mathrm{d}N = f(x,p)\frac{\mathrm{d}^3\mathbf{x}\mathrm{d}^3\mathbf{p}}{(\hbar c)^3(2\pi)^3}$$

use Maxwell-Jüttner distribution function f in local Lorentz rest frame at high temperatures
Number of particles:

$$N = g_\mathrm{s} g_\mathrm{I} \frac{V}{(2\pi)^3(\hbar c)^3} \int\limits_{-\infty}^{+\infty} p^0 e^{-\frac{p^0}{T}} \frac{\mathrm{d}^3\mathbf{p}}{p^0}$$

$$N = g_\mathrm{s} g_\mathrm{I} \frac{2Vm^2 T}{(2\pi)^2(\hbar c)^3} \kappa_2\left(\frac{m}{T}\right)$$

energy of particles

Get energy from energy-momentum tensor:

$$T^{\mu\nu} = \int p^\mu p^\nu f \frac{\mathrm{d}^3\mathbf{p}}{p^0},$$

energy density is T^{00} and thus:

$$E = g_{\mathrm{s}} g_{\mathrm{I}} \frac{V}{(2\pi)^3 (\hbar c)^3} \int\limits_{-\infty}^{\infty} p^0 e^{-\frac{p^0}{T}} \mathrm{d}^3\mathbf{p}$$

solve analytically:

$$E = g_{\mathrm{s}} g_{\mathrm{I}} \frac{3}{2} \frac{V m^2 T^2}{\pi^2 (\hbar c)^3} \left(\kappa_2 \left(\frac{m}{T} \right) + \frac{1}{3} \frac{m}{T} \kappa_1 \left(\frac{m}{T} \right) \right)$$

check thermal equilibrium

Initialize a box with π, η, ρ and a_1 mesons.
Calculate particle numbers and energies at T = (80,) 100, 110, 120, 130, 140, 150, 160, 170, 180, 200 and 220 MeV.
The η-meson is stable.
The allowed inelastic channels:

$$\begin{aligned} \rho &\leftrightarrow \pi + \pi \\ a_1 &\leftrightarrow \pi + \rho \end{aligned}$$

check thermal equilibrium

temperature from thermal pion-distribution:

$$\frac{\mathrm{d}N}{\mathrm{d}|\mathbf{p}|} = g_{\mathrm{I}} g_{S} \cdot \frac{2V}{(2\pi)^2 \hbar^3} \cdot |\mathbf{p}|^2 \cdot \exp\left(-\frac{\sqrt{m_\pi^2 + |\mathbf{p}|^2}}{T} \right)$$

linearize this equation:

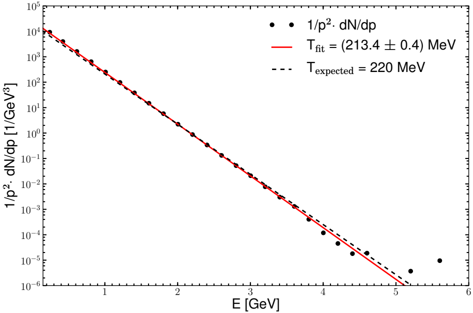
$$\ln\left(\frac{1}{|\mathbf{p}|^2} \cdot \frac{\mathrm{d}N}{\mathrm{d}|\mathbf{p}|} \right) = A \cdot E + B$$

A: inverse of the temperature and

$$B = \ln\left(\frac{g_{\mathrm{I}} g_{S} 4\pi V}{(2\pi)^3 \hbar^3} \right)$$

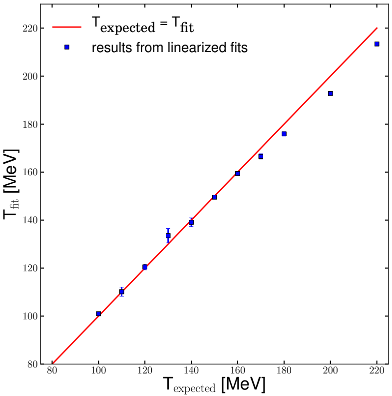
check thermal equilibrium

extract temperature T from thermal pion distribution



check thermal equilibrium

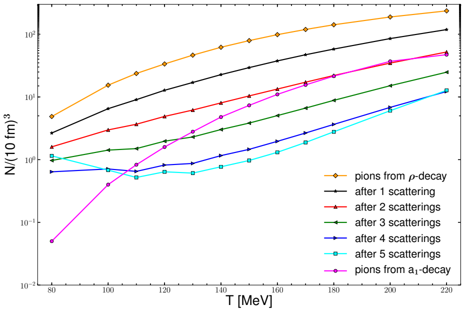
Results for temperature-fits for different temperatures:



Why is the temperature (of the pions) too low for high temperatures?

check thermal equilibrium

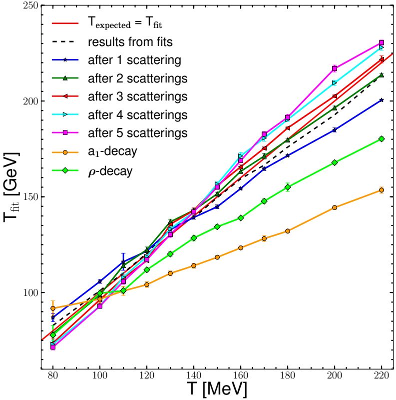
What are the sources of the pions?



- main-pion source: ρ-decay
- a_1 decay relevant for higher temperatures
- large number of pions with $\geq$ 5 scatterings for low T: a_1 channel starts to play role for higher T

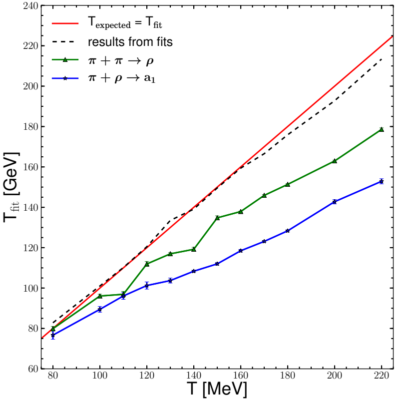
check thermal equilibrium

temperatures from different sources



→ Pions stemming from ρ and a_1 decay are colder than medium.
But:
Why are pions getting hotter?

check thermal equilibrium



$\rightarrow$ Pions incoming in inelastic scatterings are colder than medium.

results of first part

- linear fitting itself works good (compared to exponential fitting)
- a_1 channel only relevant for high T
- gap between temperature of pions from decay and medium inceases with higher overall-T
- additional effect: pions, that form ρ and a_1, are colder than medium

analysis of further thermodynamic quantities

We use temperature T and volume V to calculate:

- energy E
- particle number N

We extract

- the energy density ϵ
- the isotropic pressure P
- the entropy using Gibbs formula $s = 1/T(\epsilon + P - \mu_B \rho_B)$.
- check Gibbs formula for whole box and virtual box inside the whole
- shear viscosity η, viscosity to entropy ratio η/s and thermal conductivity κ and compare to analytic calculations in the ultra-relativistic limit using Green-Kubo formulas.
- extract η (and η/s) for baryonic medium

isotropic pressure and entropy

Isotropic pressure from diagonal part of energy-momentum-tensor:

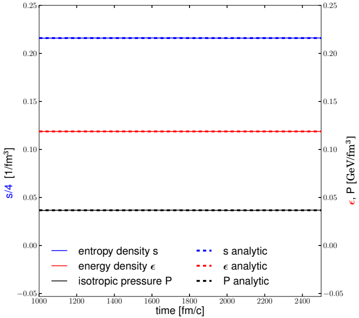
$$P = \frac{1}{3} \int |\mathbf{p}|^2 f \frac{\mathrm{d}^3\mathbf{p}}{p^0} = \ldots = g_\mathrm{s} g_\mathrm{I} \frac{2}{(2\pi)^2} \frac{1}{(\hbar c)^3} m^2 T^2 \kappa_2 \left(\frac{m}{T}\right)$$

Entropy:

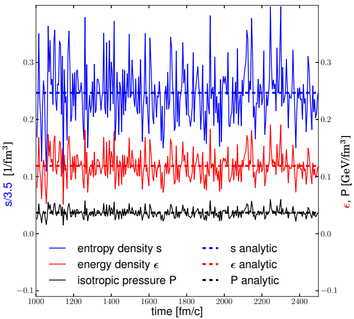
$$\begin{aligned} S &= -4\pi V \int_0^\infty f \left(\ln \left(\frac{(\hbar c)^3}{g_\mathrm{s} g_\mathrm{I}} f \right) - 1 \right) |\mathbf{p}|^2 \mathrm{d}|\mathbf{p}| \\ &= \ldots = g_\mathrm{s} g_\mathrm{I} \frac{V}{2\pi^2 (\hbar c)^3} m^3 \cdot \left(\kappa_1 \left(\frac{m}{T}\right) + 4 \frac{T}{m} \kappa_2 \left(\frac{m}{T}\right) \right) \end{aligned}$$

fluctuation of the quantities

whole box



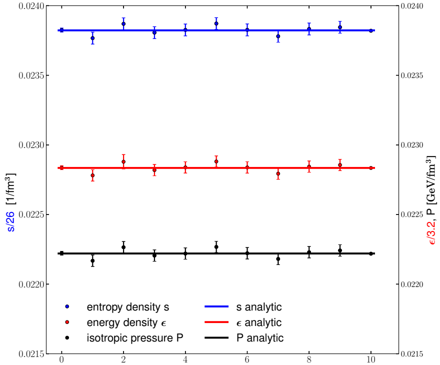
box in center



T = 180 MeV

position of smaller box

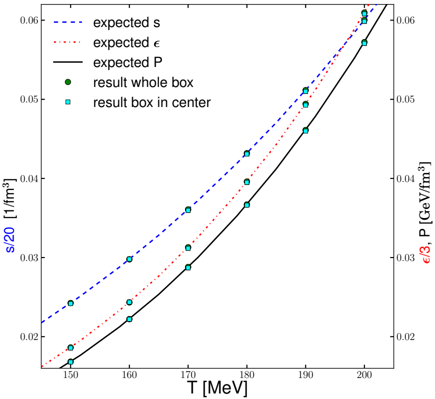
Is it important where to put smaller box?



- 1-8: box in each corner
- 9: box in center
- 0: average over 1-9
- 10: whole box

T = 160 MeV

results for different temperatures T



calculation:

$$\epsilon = \frac{1}{V} \sum_{i=1}^{N_{particles}} p_i^0$$

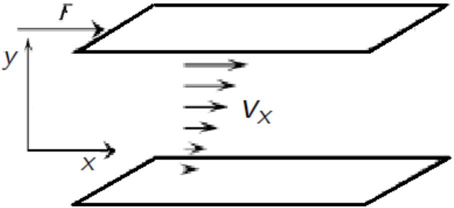
$$P = \frac{1}{3V} \sum_{i=1}^{N_{particles}} \frac{|\mathbf{p}_i|^2}{p_i^0}$$

$$s = 1/T \sum_{i=1}^{N_{particles}} (\epsilon_i + P_i)$$

η: mechanical definition

shear force:
→ velocity flow field
→ non-zero P^{xy}
shear viscosity (coefficient) η:

$$P^{xy} = -\eta \frac{\partial v_x}{\partial y}$$



in transport models:
use linear response theory (Green Kubo) to extract η:

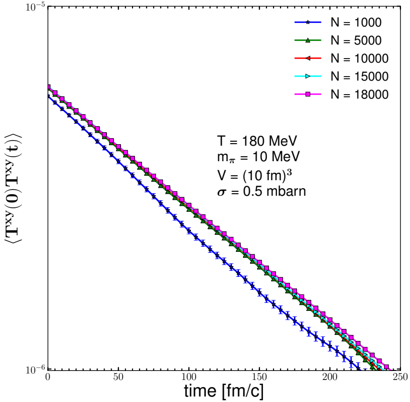
$$\eta = \frac{V}{T} \int_0^\infty \langle \pi^{xy}(0) \pi^{xy}(t) \rangle_{\mathrm{eq.}} \mathrm{dt}$$

$$\pi^{xy} = T^{xy} = \sum_{i=1}^{N_{particles}} \frac{p_i^x p_i^y}{p_i^0}$$

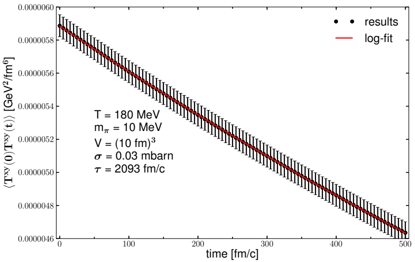
ensemble average

$$\langle \pi^{xy}(0)\pi^{xy}(t)\rangle_{\text{eq.}} = \frac{1}{N_{events}} \sum^{N_{events}} \frac{1}{N} \sum_{i=1}^{N} T^{xy}(i\Delta t) T^{xy}(i\Delta t + t)$$

- How many timesteps N? We use 18000
- How many independent box-calculations ? question of storage volume; at least 150 N_{events}



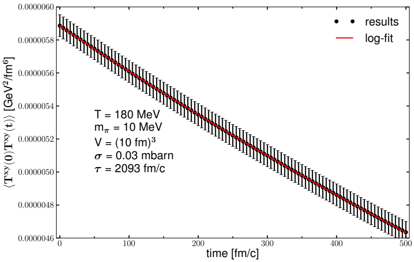
integrate correlation function



$$\int_0^\infty \langle \pi^{xy}(0)\pi^{xy}(t)\rangle_{\mathrm{eq.}}\mathrm{dt}$$

How to integrate the correlation function over time?
$\rightarrow$ correlation function decays exponentially

integrate correlation function



Hence we can write:

$$\eta = \frac{V}{T}\int_0^\infty \langle T^{xy}(0)T^{xy}(t)\rangle_{\mathrm{eq.}}\mathrm{dt}$$

$$= \frac{V}{T}\int_0^\infty e^{\left(\frac{-t}{\tau}\right)} \cdot \langle T^{xy}(0)^2\rangle_{\mathrm{eq.}}\mathrm{dt}$$

$$= \frac{V}{T}\cdot\tau\cdot\langle T^{xy}(0)^2\rangle_{\mathrm{eq.}}$$

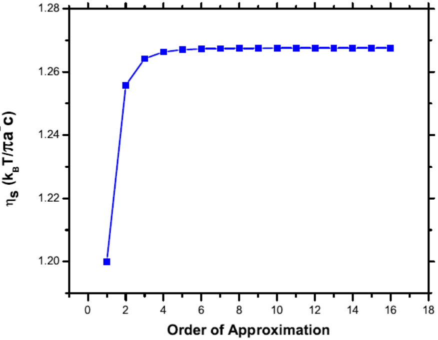
τ: relaxation time

comparing with analytic results

Now, compare to the results from A. Wiranata and M. Prakash, arXiv:1203.0281 (2012)
For a ultra-relativistic hard sphere gas one can write:

$$\eta \approx 1.27 \cdot \frac{T}{\sigma}$$

T: temperature
σ: total cross section



ultrarelativistic-limit?

have to satisfy ultra-relativistic and hard-sphere-limit

Ultra-relativistic means massless particles and coldness $\varsigma = m/T = 0$.
We try:

- coldness $\varsigma = m/T \ll 1$
- mass: m_π 138 MeV $\rightarrow$ 10 MeV
- temperature: T = 180 MeV

additional assumptions:

- isotropic cross-section
- one-component gas, binary, elastic scatterings
- hard-sphere approx.:
  const. differential cross-section: $\sigma = a^2/4$
  total cross-section: $\sigma_T = \pi a^2$
  a: radius of sphere

comparing with Green Kubo

compare Green Kubo

$$\eta = \frac{V}{T} \cdot \tau \cdot \langle T^{xy}(0)^2 \rangle_{\mathrm{eq.}}$$

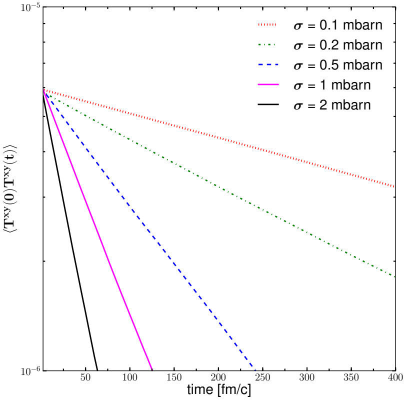
with

$$\eta \approx 1.27 \cdot \frac{T}{\sigma}$$

has T-, σ-dependence

σ-screening:

$\langle T^{xy}(0)^2 \rangle_{\mathrm{eq.}}$ stays the same, but τ changes



σ-screening

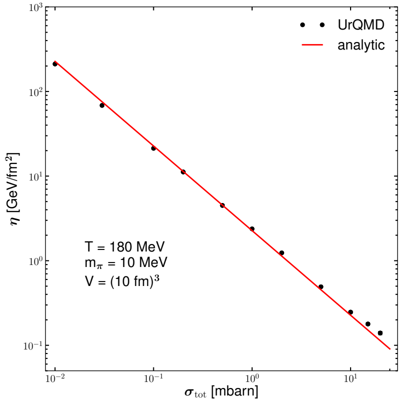
compare Green Kubo

$$\eta = \frac{V}{T} \cdot \tau \cdot \langle T^{xy}(0)^2 \rangle_{\mathrm{eq.}}$$

with

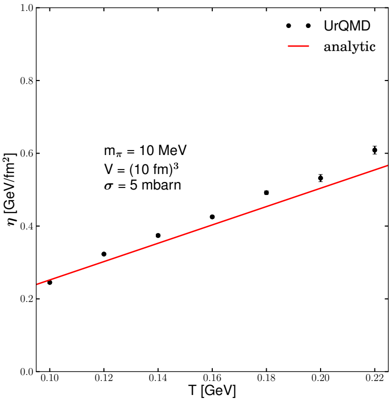
$$\eta \approx 1.27 \cdot \frac{T}{\sigma}$$

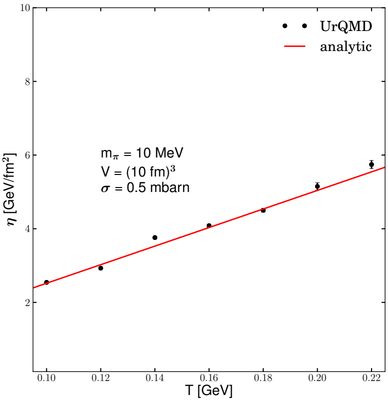
here σ-dependence



temperature screening

η vs. temperature T





η/s

calculate η/s using

- Green Kubo results
- extracted value for s from

$$s = 1/T \sum_{i=1}^{N_{particles}} (\epsilon_i + P_i)$$

compare with analytic:

- viscosity from:

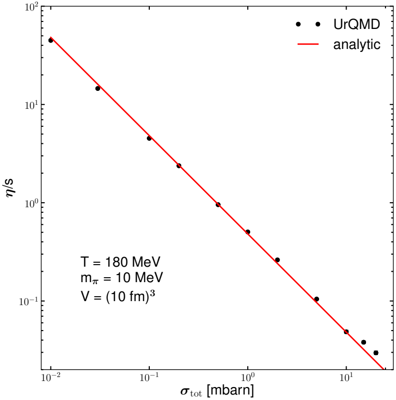
$$\eta \approx 1.27 \cdot \frac{T}{\sigma}$$

- entropy from:

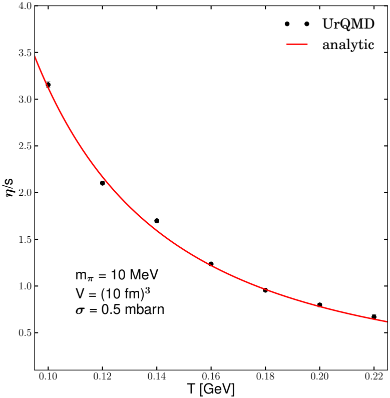
$$s = g_{\mathrm{s}} g_{\mathrm{I}} \frac{1}{2\pi^2 \hbar^3} m_i^3 \cdot \left(\kappa_1 \left(\frac{m}{T} \right) + 4 \frac{kT}{m_i c^2} \kappa_2 \left(\frac{m}{T} \right) \right)$$

η/s results

η/s vs. cross-section σ



η/s vs. temperature T



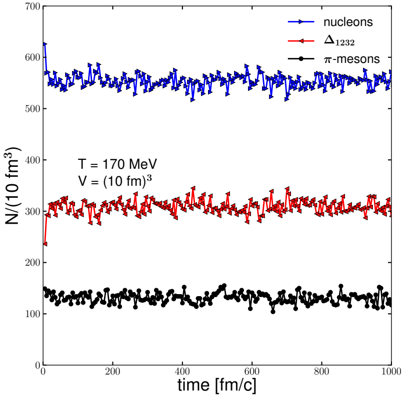
η for box including baryons

different setting:
box with nucleons, π-mesons and Δ_{1232}-resonances.

$$\pi + n \leftrightarrow \Delta_{1232}$$

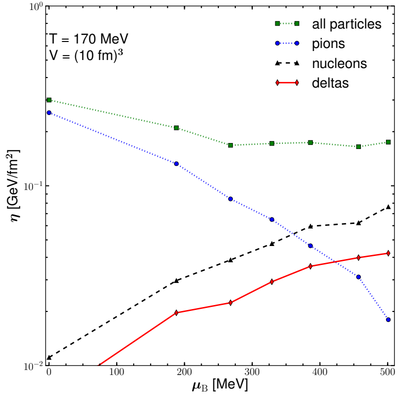
Motivation:
since $\eta \sim \frac{|\bar{\mathbf{p}}|}{\sigma}$, one might see where $N_{pions}=N_{nucleons}$
$\rightarrow$change baryon number by introducing baryo-chemical potential μ_B

thermalization



- chemical-equilibrium realized very fast
- we assume thermal-equilibrium is realized after 1000 fm/c

results η



- note:
  use extracted temperature T
- η falls and saturates
- to do: extract s for η/s
  difficulty:
  get real μ_B

thermal conductivity κ

isotropic medium: heat flux **q**, thermal conductivity κ:

$$\mathbf{q} = -\kappa \nabla T$$

q: heat flowing per second and per unit area

thermal conductivity κ (Green Kubo):

$$\kappa = \frac{V}{T^2} \int_0^\infty dt \langle q^\mu(0) q^\mu(t) \rangle_{\mathrm{eq.}}$$

relativistic heat flow q^μ:

$$q^\mu = (u_\nu T^{\nu\sigma} - hN^\sigma) h^\mu_\sigma .$$

$\rightarrow$ difference of energy flow and flow of enthalpy h carried by the particles
$h = (en + p)/n$: heat function per particle
u^μ: hydrodynamic velocity
$h^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$: projector $\rightarrow$ gives part $\perp$ u^μ

heat flow

heat flow depends strongly on decomposition

Eckart's definition:

$$u_{\mathrm{E}}^{\mu} = \frac{N^{\mu}}{\sqrt{N^{\nu} N_{\nu}}}$$

$$\rightarrow h^{\mu\nu} N_{\nu} = 0$$

$$q_{E}^{\mu} = u_{\nu} T^{\nu\sigma} h_{\sigma}^{\mu}.$$

Landau and Lifshitz's definition:

$$u_{L}^{\mu} = \frac{T^{\mu\nu} u_{\nu}}{\sqrt{u_{\rho} T^{\rho\sigma} T^{\sigma\tau} u_{\tau}}}$$

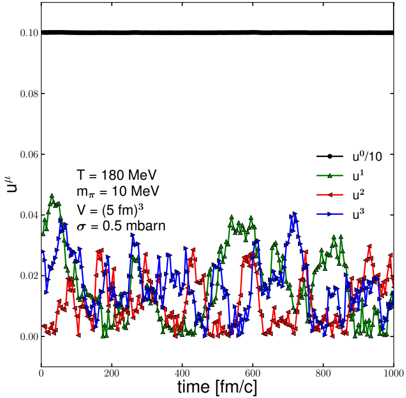
$$\rightarrow h^{\mu\nu} T_{\nu\sigma} u^{\sigma} = 0$$

$$q_{L}^{\mu} = -h N^{\sigma} h_{\sigma}^{\mu}.$$

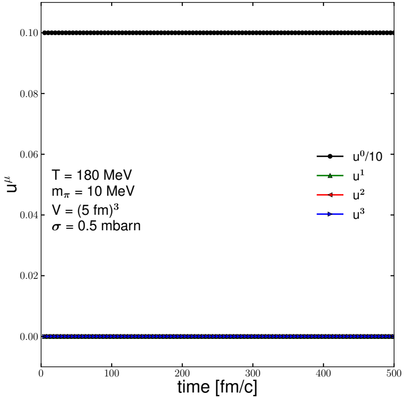
heat flow

In which local Lorentz rest frame ($u^\mu = (1,0,0,0)^T$) are we?

Eckart:



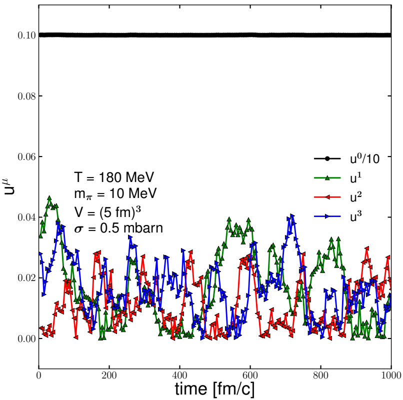
Landau Lifshitz:



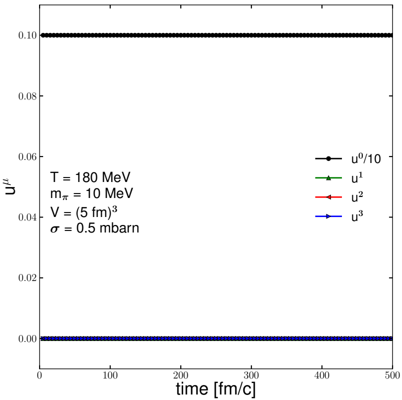
heat flow

In which local Lorentz rest frame ($u^\mu = (1,0,0,0)^{\mathrm{T}}$) are we?

Eckart:



Landau Lifshitz:



result heat flow and thermal conductivity

This means for heat flow:

$$q^{\mu} = -hN^{i}$$

N^{i}: flux of particle number per unit time and unit area in i-th spacial direction:

$$N^{i} = \int p^{i} f \frac{\mathrm{d}^{3}\mathbf{p}}{p^{0}}$$

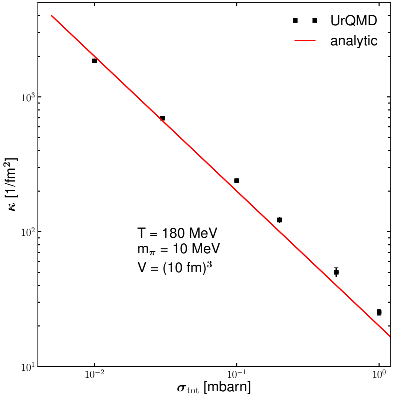
for UrQMD-calculations:

$$N^{i} = \frac{1}{V} \sum_{i=1}^{N_{particles}} \frac{p^{i}}{p^{0}} \quad \rightarrow \quad \kappa = \frac{V}{3T^{2}} \int_{0}^{\infty} \langle h\mathbf{N}(0)\; h\mathbf{N}(t) \rangle_{0} dt$$

result thermal conductivity

compare Green Kubo with ultra-relativistiv hard sphere approximation in first order from Carlo Cercignani and Gilberto Medeiros Kremer (Relativistic Boltzmann Equation):

$$\kappa \approx \frac{2}{\sigma}\left(1 + \left(\frac{m}{T}\right)^2 + ...\right)$$



summary and outlook

- extracted thermal properties like pressure P and entropy s
- extracted η and η/s using Green Kubo:
  successful T- and σ-screening
- η for baryonic-medium:
  μ_B-screening
- extracted κ using Green Kubo:
  successful σ-screening
- to do:
  try κ for Eckart decomposition
  extract $\mu_B \rightarrow \eta/s$ from baryonic-medium

Thank you all for your attention.