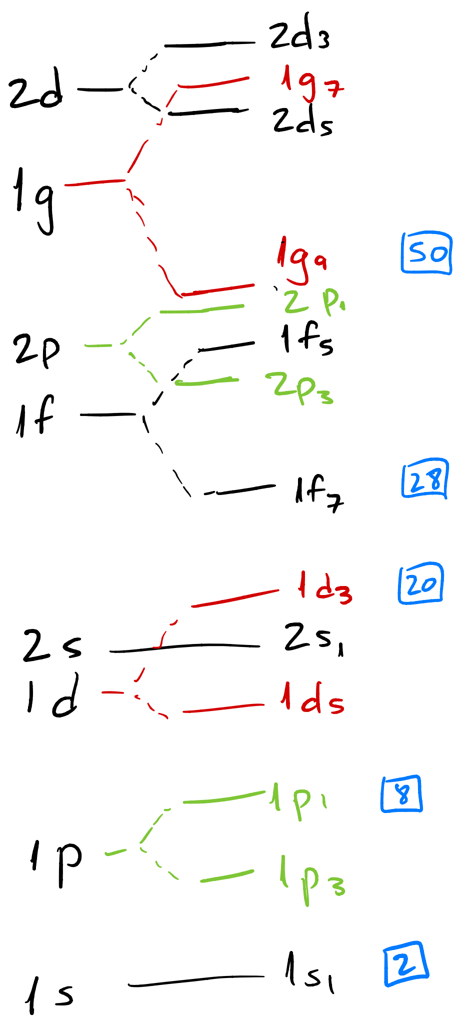
Solutions 4

VTHKP 1 – WS 2025

Renan Hirayama
Carl Rosenkvist

3.1 Shell model: spins and parity.

a) Find the shell config. for $^{7}_{3}Li$, $^{93}_{41}Nb$, $^{33}_{16}S$.



→ $^{7}_{3}Li$: $Z=3$, $N=4$

protons $1s_1^2\, 1p_3^{1}$

neutrons $1s^2\, 1p_3^{2}$

Acc. to the pairing hypothesis, since the neutrons are in a closed subshell,

$$\vec{L}_n = \vec{S}_n = 0 \Rightarrow \vec{J}_n = 0$$

So the configuration is determined by the single proton in $1p_{3/2}$:

$$\boxed{j_{^{7}_{3}Li} = 3/2}$$

→ $^{93}_{41}Nb$: $Z=41$, $N=52$

neutrons: $[50]\ 2d_5^{2}$ ← closed subshell

protons: $[28]\ 2p_3^4\ 1f_5^6\ 2p_1^2\ 1g_9^{1}$ open

$$\Rightarrow j_{^{93}_{41}Nb} = 9/2$$

$\rightarrow\ ^{33}_{16}S$: $Z=16$, $N=17$

protons: $[8]\,1d_5^6\,2s_1^2$

neutrons $[8]\,1d_5^6\,2s_1^2\,1d_3^1$

$\Big\}\ j_{^{33}_{16}S} = 3/2$

b) The two most likely excited states for ^7_3Li if a proton is excited ~~as~~

$$1p_3 \longrightarrow 1p_1 \quad \text{or} \quad 1s_1 \longrightarrow 1p_3$$

c) A state with total angular momentum $j\,(=\ell \pm 1/2)$ contains $N_j = 2(2j+1)$ nucleons,

so if $N_j = 16$, $j = 7/2$ 5

and $\ell = 3$ or 4. Since the state is odd,

$P=-1$ and $\ell = 3$ 5

3.2 shell model : magnetic moment

$$\vec{\mu}_{nucl} = g_{nucl}\, \mu_N \frac{\langle \vec{J} \rangle}{\hbar}, \quad \text{with}$$

$$g_{nucl} = \frac{\langle J M_J | g_\ell \vec{L}\cdot\vec{J} + g_s \vec{S}\cdot\vec{J} | J M_J \rangle}{\langle J M_J | J^2 | J M_J \rangle},$$

$* \ \hat{J}^2 | J M_\ell \rangle = j(j+1)$

$* \ \hat{\vec{J}} = \hat{\vec{S}} + \hat{\vec{L}}$

$\hookrightarrow \hat{S}^2 = \hat{J}^2 + \hat{L}^2 - 2\hat{\vec{J}}\cdot\hat{\vec{L}} \Rightarrow \vec{J}\cdot\vec{L} = \frac{1}{2}(\hat{J}^2 + L^2 - S^2)$

$\hookrightarrow \hat{L}^2 = \hat{J}^2 + S^2 - 2\hat{\vec{J}}\cdot\hat{\vec{S}}$ and similar

$$\Rightarrow \hat{\vec{J}}\cdot\underset{[\vec{S}]}{\hat{\vec{L}}} |JM\rangle = \frac{1}{2}\left[j(j+1) \underset{[-]}{+} \ell(\ell+1) \underset{[+]}{-} s(s+1)\right]$$

$$g_{nucl} = \frac{\frac{g_\ell}{2}\left[j(j+1) + \ell(\ell+1) - s(s+1)\right] + \frac{g_s}{2}\left[j(j+1) + s(s+1) - \ell(\ell+1)\right]}{j(j+1)}$$

For a single nucleon $j = l \pm \frac{1}{2}$, so

$\boxed{+}$: $g_{nucl} = \Big\{ g_l \left[j(j+1) + (j-\tfrac{1}{2})(j+\tfrac{1}{2}) - \tfrac{1}{2}(\tfrac{1}{2}+1) \right] +$
$+ g_s \left[j(j+1) + \tfrac{1}{2}(\tfrac{1}{2}+1) - (j-\tfrac{1}{2})(j+\tfrac{1}{2}) \right] \Big\} \Big/ 2j(j+1)$

$$= \frac{g_l}{2j(j+1)}\left[2j^2 + j - 1\right] + \frac{g_s}{2j\cancel{(j+1)}}\left[\cancel{j+1}\right]$$

$$= \frac{g_l}{2j}(2j-1) + \frac{g_s}{2j} = \underline{g_l + \frac{1}{2j}(g_s - g_l)}$$

$\boxed{-}$: $g_{nucl} = \frac{g_l}{2j(j+1)}\left[j(j+1) + (j+\tfrac{1}{2})(j+\tfrac{3}{2}) + \tfrac{1}{2}(1-\tfrac{1}{2}) \right] +$
$+ \frac{g_s}{2j(j+1)}\left[j(j+1) - \tfrac{1}{2}(1-\tfrac{1}{2}) - (j+\tfrac{1}{2})(j+\tfrac{3}{2}) \right]$

$$= \frac{g_l}{2j(j+1)}\left[2j^2 + 5j + 1\right] + \frac{g_s}{2j\cancel{(j+1)}}\left[\cancel{-j-1}\right]^{-1}$$

$$= \frac{g_l}{2j(j+1)}\left[(2j+1)(j+1) + 2j\right] - \frac{g_s}{2j}$$

$$= \frac{g_l}{2j}(2j+1) + \frac{g_l}{j+1} - \frac{g_s}{2j} \quad \leftarrow \quad g_l\left(1 + \frac{1}{j+1}\right) + \frac{g_l - g_s}{2j}$$

$$= \underline{\left(\frac{j+2}{j+1}\right) g_l + \frac{1}{2j}(g_l - g_s)}$$

Maximally aligned spins have $\langle J \rangle = J\hbar$, so

$$\mu_{nucl} = g_{nucl}\mu_N j, \quad \text{and}$$

$$\frac{\mu_{nucl}}{\mu_N} \overset{+}{=} j g_\ell + \frac{g_s - g_\ell}{2}$$

$$\overset{-}{=} \left(\frac{j+2}{j+1}\right) j g_\ell + \frac{g_\ell - g_s}{2}$$

where $g_\ell = 1$, $g_s = 5.5858$ for a proton and $g_\ell = 0$, $g_s = -3.8263$ for a neutron

$\rightarrow$ ^7Li : $j_p = \frac{3}{2}$, $\ell = 1 \Rightarrow s = \frac{1}{2}$

$$\Rightarrow \frac{\mu_{nucl}}{\mu_N} = \frac{2g_\ell^{(p)} + g_s^{(p)}}{2} \approx 3.79 \quad (\times 3.26 \text{ exp})$$

$\rightarrow$ ^{93}Nb : $j_p = \frac{9}{2}$, $\ell = 4 \Rightarrow s = \frac{1}{2}$

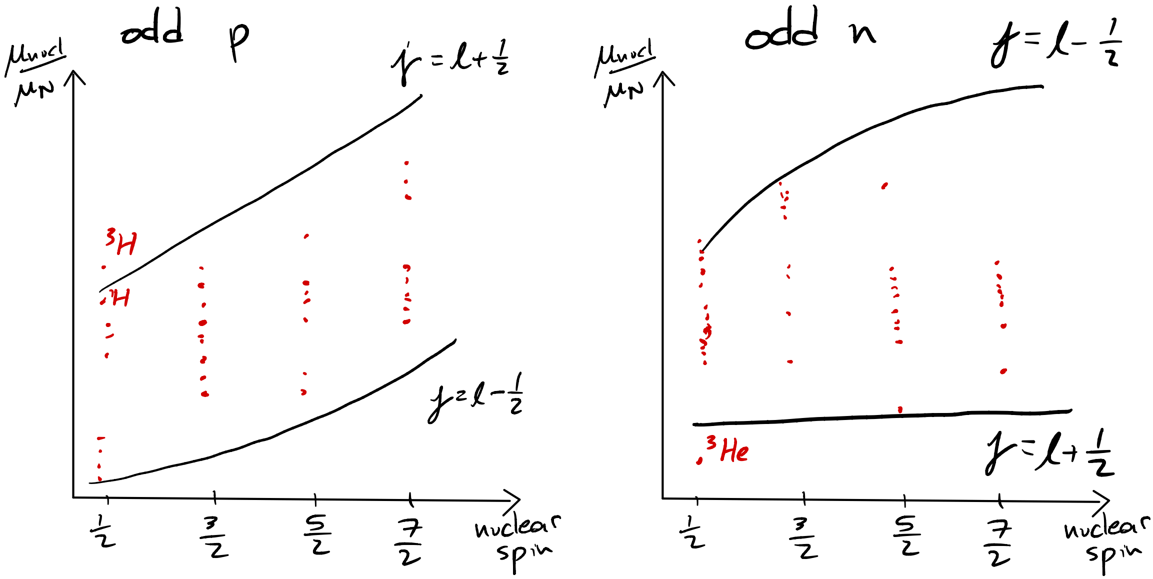
$$\Rightarrow \frac{\mu_{nucl}}{\mu_N} = \frac{8g_\ell^{(p)} + g_s^{(p)}}{2} \approx 6.79 \quad (\times 6.17 \text{ exp})$$

$\rightarrow$ ^{33}S : $j_n = \frac{3}{2}$, $\ell = 2 \Rightarrow s = -\frac{1}{2}$

$$\Rightarrow \frac{\mu_{nucl}}{\mu_N} = 0^{(n)} - \frac{g_s^{(n)}}{2} = 1.91 \quad (\times 0.64 \text{ exp})$$

why so bad?

"Schmidt Limits"



The shell model predicts broad trends of magnetic moments, but not very well.

The wavefunction is more complicated, and the nucleons interact, changing the shape of the potential.