

Exercise Sheet 13

How do we know the quantum numbers of the (charged) pions?

As stressed in the lecture, we cannot deduce the intrinsic quantum numbers of the hadrons from the (approximate) chiral symmetry $SU(2)_L \times SU(2)_R$ of QCD, particularly not, how to spontaneously break it, i.e., that in the chiral limit of vanishing quark masses it must be the pseudoscalar mesons that are the (in this limit) massless Goldstone modes, and that these are the pions, while the scalar chiral partner, the σ meson takes a non-vanishing vacuum expectation value. This is known from the empirically determined $s_\pi = 0$ and $P_\pi = -1$.

To understand this we consider two historical experiments and analyze them with respect to the relevant conservation laws of angular momentum (which must hold for all Poincaré-covariant field theories) as well as the discrete symmetries P (space reflections) and T (time reversal) under the strong (as well as the electromagnetic) interaction¹.

- (a) Consider time-reversal symmetry, which is represented by an anti-unitary transformation \mathbf{T} in any QFT. The Hamiltonian of the strong interaction (on the fundamental level for QCD but also for any effective hadronic model and their strong interaction) commutes with T , $[\mathbf{T}, \mathbf{H}] = 0$, i.e., time reversal is a symmetry.

Now consider the asymptotic free states $|\vec{p}, \sigma\rangle$. What is $\mathbf{T}|\vec{p}, \sigma\rangle$ (just think about, how momentum and angular momentum change when making $T\vec{x} = \vec{x}$ and $Tt = -t$ in classical point-particle mechanics).

Now consider the time evolution of states in the Schrödinger picture,

$$|\psi(t)\rangle = \exp[-i\mathbf{H}(t - t_i)]|\psi(t_i)\rangle = \mathbf{U}(t - t_i)|\psi(t_i)\rangle. \quad (1)$$

Calculate the time-reversed time-evolution operator $\mathbf{U}_T(t) = \mathbf{T}\mathbf{U}(t)\mathbf{T}^\dagger$.

Now the scattering operator is defined formally as

$$\mathbf{S} = \lim_{t_i \rightarrow -\infty} \lim_{t_f \rightarrow +\infty} \mathbf{U}(t_f - t_i) \quad (2)$$

Now consider the transition-matrix element \mathcal{M}_{fi} , defined in Lect. 9. Argue, why for any scattering process $|i\rangle \rightarrow |f\rangle$ (two asymptotic free particles in $|i\rangle$ scattering under influence of the strong interaction into asymptotic free particles in $|f\rangle$) $|\mathcal{M}_{fi}|^2 = |\mathcal{M}_{ij}|^2$ must hold.

- (b) **Determination of the spin of the charged pion.** For charged pions the spin has been determined by the reversible reaction $p + p \leftrightarrow \pi^+ + d$ (Proton-Proton reaction to a positive pion and a deuteron and the reverse reaction, respectively). Now use the $2 \rightarrow 2$ cross-section formula

$$\sigma_{AB \rightarrow CD} \propto (2s_C + 1)(2s_D + 1) \overline{|\mathcal{M}_{fi}|^2} \frac{p_{\text{cm}}^{(f)}}{p_{\text{cm}}^{(i)}}$$

for the two reactions mentioned above (note that for the “back reaction” one has two identical particles in the final state, so that there is an extra factor $1/2$). Calculate the ratio of the cross sections, using time-reversal invariance of the strong interaction and explain, how from that (model independently) s_π can be determined (assuming that $s_p = 1/2$ and $s_d = 1$ are known). The experiment lead to $s_\pi = 0$ [DLS51].

¹Note that the weak interaction violates all the symmetries P , T , and C (charge conjugation, i.e., the exchange of any particle with its anti-particle), CP , PT , and CT . Only CPT must be necessarily a symmetry for any local Poincaré-covariant QFT.

- (c) **Parity of the charged pion.** For the determination of the parity of the π^- it's sufficient to know that the pion-capture in deuterium $\pi^- + d \rightarrow n + n$ is observed for slow pions via an s-wave scattering channel [?].

Argue from the final state that the total angular momentum is $J = 1$.

The neutrons in the final state are non-relativistic, i.e., the intrinsic two-neutron wave function can be described as $\Psi_{nn} = \psi(x_{\text{extrel}})\psi(\text{spin})$, and this must be antisymmetric under exchange of the neutrons, because they are fermions. The total spin S can be 0 or 1. Now argue that under exchange of the neutrons the wave function changes by a factor $(-1)^{L+S+1}$ (where L is the orbital angular momentum of the relative-space wavefunction ψ), which implies that $L + S = \text{even}$.

Finally discuss all possible values for L and S given that total angular momentum is J (note that $\vec{J} = \vec{L} + \vec{S}$ and remember, how angular momenta are added in quantum mechanics). From this deduce the parity of the final two-neutron state.

From this and given that the intrinsic parity of both n and d are $+1$, what follows for the parity of the pion, given that the strong interaction preserves parity?

References

- [CS54] W. Chinowsky and J. Steinberger, Absorption of Negative Pions in Deuterium: Parity of the Pion, Phys. Rev. **95**, 1561 (1954), <https://doi.org/10.1103/PhysRev.95.1561>.
- [DLS51] R. Durbin, H. Loar and J. Steinberger, The Spin of the Pion via the Reaction $\pi^+ + D \leftrightarrow P + P$, Phys. Rev. **83**, 646 (1951), <https://doi.org/10.1103/PhysRev.83.646>.