Exercise Sheet 4

Task 3.1: Nuclear shell model: spins and parity

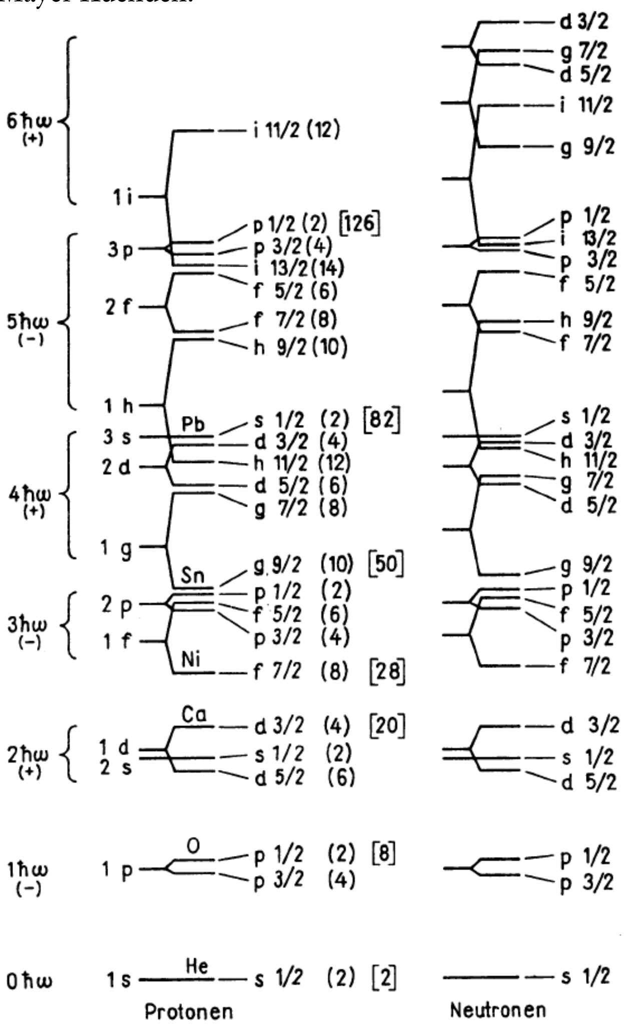
The nuclear shell model can be used to make predictions about the spins of ground states. Since magic number nuclides have closed sub-shells, such nuclides are predicted to have zero contribution to the nuclear spin from the neutrons or protons or both, whichever are magic numbers. Not only this fact is confirmed experimentally but also all even-Z/even-N nuclei have zero nuclear spin. Thus, we could argue that for ground state nuclei, pairs of neutrons and pairs of protons in a given sub-shell always couple to give a combined angular momentum of zero, even when the sub-shell is not filled. This is called the pairing hypothesis.

(a) Write down the shell-model configuration of the nuclei ${}^{7}_{3}\mathrm{Li}$, ${}^{93}_{41}\mathrm{Nb}$ and ${}^{33}_{16}\mathrm{S}$ and find their spin.

(b) In the case of ${}^{7}_{3}\mathrm{Li}$ give the two most likely configurations for the first excited state, assuming that only protons are excited.

(c) Following the pairing hypothesis, the total parity of a nucleus is found from the product of the parities of the last proton and the last neutron. A certain odd-parity shell-model state can hold up to a maximum of 16 nucleons; what are its values of j and ℓ?

Hint: Level scheme from Mayer-Kuckuck:



Task 3.2: Nuclear shell model: magnetic moments

Unless the nuclear spin is zero, we expect nuclei to have magnetic dipole moments, since both the proton and the neutron have intrinsic magnetic moments. Following Wigner-Eckart theorem that states that *the expectation value of any vector operator of a system is equal to the projection onto its total angular momentum*, the nuclear magnetic moment is given by

$$\vec{\mu}_{\text{nucl}} = g_{\text{nucl}} \mu_N \frac{\left\langle \vec{J} \right\rangle}{\hbar} \tag{1}$$

with

$$g_{\text{nucl}} = \frac{\langle J M_J | g_\ell \vec{L} \cdot \vec{J} + g_s \vec{S} \cdot \vec{J} | J M_J \rangle}{\langle J M_J | \vec{J}^2 | J M_J \rangle}, \tag{2}$$

where $\mu_N = e\hbar/(2m_{\text{p}}) \simeq 3.152 \cdot 10^{-8}\text{eV/T}$ is called *nuclear magneton* and ($g_s = 5.5858$, $g_\ell = 1$) for a proton whereas ($g_s = -3.8263$, $g_\ell = 0$) in the case of a neutron. Obtain an expression for g_{nucl} in terms of (j, l, s, g_ℓ, g_s). Assume that the nuclear spin $\vec{J}$ is defined by the $\vec{j}$ of the single unpaired nucleon.
The magnetic moment of the nucleus is defined as the value measured when the nuclear spin is maximally aligned, i.e. $M_J = J$. Then,
$\langle \vec{J} \rangle = J\hbar$ such that

$$\mu_{\text{nucl}} = g_{\text{nucl}} \mu_N J \tag{3}$$

Check the agreement between the nuclear shell model result for μ_{nucl}/μ_N and its experimental value for the nuclei of Task 3.1: $^{7}\text{Li}: 3.26$, $^{93}\text{Nb}: 6.17$, $^{33}\text{S}: 0.64$.