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Exercise Sheet 1

Exercise 1.1: Fundamental Forces

What are the known fundamental interactions? Consider the following objects and discuss which interaction is essential for both its interaction and structure:
Galaxy, Solar System, Planet, Basket-ball, Bacterium, Molecule, Atom, Atomic Nucleus, Proton, Quark.

Exercise 1.2: Natural Units

In the course, so-called natural units are used. That means we set $c = \hbar = k_{\mathbf{B}} = 1$. As the name suggests, this does not change the physics, but only the units of our quantities. This exercise is about determining conversion factors between natural units and SI units.

For this you need $\hbar = h/(2\pi) \simeq 1.055\text{J s} \simeq 6.582\text{GeV s}$ and thus $\hbar c = 0.197\text{GeV fm}$.

a) What is a second in GeV^{-1}?
b) What is a meter in GeV^{-1}?
c) What is the unit of momentum in the SI system and in natural units? Determine the conversion factor.
d) What is the unit of temperature in natural units? What is a Kelvin in this unit?
e) What is a second in fm? Use the results of parts a) and e).

Exercise 1.3: Form factor and charge radius

The form factor for the scattering of an electron with a nucleus is given by

$$F(\vec{q}) = \frac{1}{Ze} \int_{\mathbb{R}^3} \mathrm{d}^3 r \exp(\mathrm{i}\vec{r} \cdot \vec{q}) \rho(\vec{r}),$$

where ρ is the charge density of the nucleus, and $\vec{q}$ is the momentum transfer in the scattering. Assume a spherically symmetric charge distribution, i.e., $\rho(\vec{r}) = \rho(r)$ (with $r = |\vec{r}|$) and that the typical scale of the nucleus's size R_{n} is such that one can assume $qR_{\text{nucl}} \ll 1$ within the integral. Show that for these small momentum transfers, the form factor is given by

$$F(\vec{q}) = F(|\vec{q}|) = 1 - \frac{1}{6}\vec{q}^2 \langle r^2 \rangle$$

with

$$\langle r^2 \rangle = \int_{R^3} \mathrm{d}^3 r r^2 \rho(r),$$

i.e., from measuring the form factor you can deduce the "root-mean-square charge radius" $R_{\text{rms}} = \sqrt{\langle r^2 \rangle} \simeq R_{\text{nucl}}$.