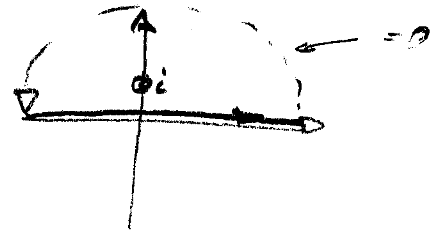
$$\int_{-\infty}^{+\infty} \frac{dx}{1+x^2} = ?$$



$\frac{1}{1+z^2}$: · $\left(\frac{1}{1+z^2}\right)_{z\to\infty} \to \infty$ (doppelte Nullstelle)

∘ außer i. $z=i$ für $\mathrm{Im}(z)>0$ analytisch

$$\oint f(\xi)\,d\xi = 2\pi i \sum_a \mathrm{Res}(f(z), a)$$

$$\mathrm{Res}(f(z), a) = \frac{1}{(m-1)!} \lim_{z\to a} \frac{d^{m-1}}{dz^{m-1}} \left((z-a)^m f(z)\right)$$

m-facher Pol

hier: $m=1$, $\mathrm{Res}\left(\frac{1}{1+z^2}, i\right) = \lim_{z\to i} (z-i)\frac{1}{1+z^2}$

$$= \lim \frac{1}{z+i} = \frac{1}{2i}$$

$$\Rightarrow \int_{-\infty}^{+\infty} \frac{dx}{1+x^2} = \oint_C \frac{dz}{1+z^2} = 2\pi i \cdot \frac{1}{2i} = \pi$$

$\square$