Selfconsistent Renormalization Schemes for Thermodynamic Potentials

Hendrik van Hees

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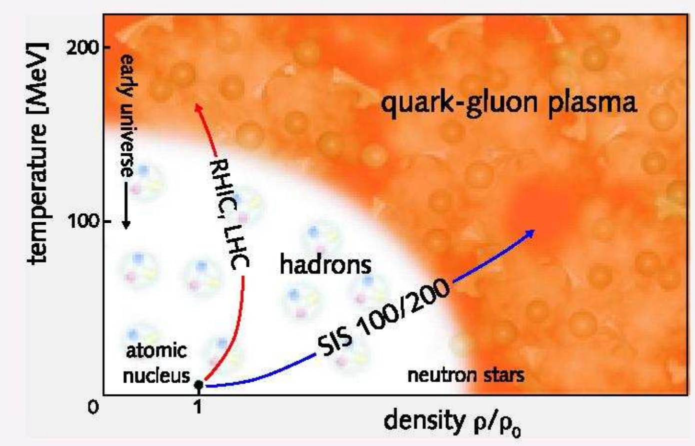
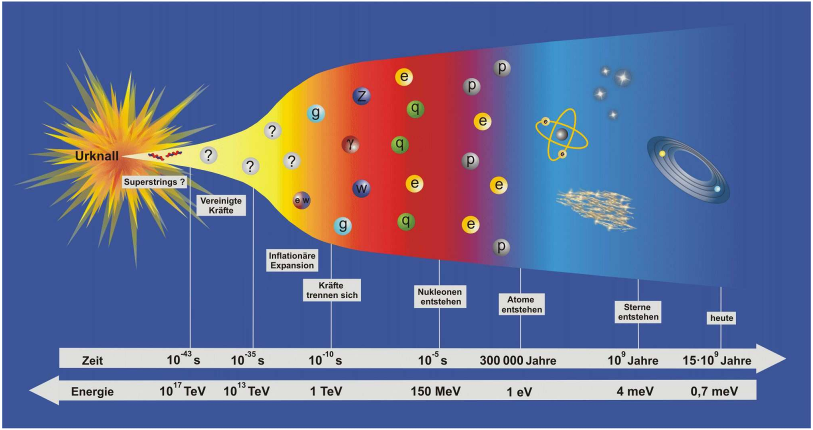
Outline

Heavy Ion Collisions and Phase Transitions



History of our Universe

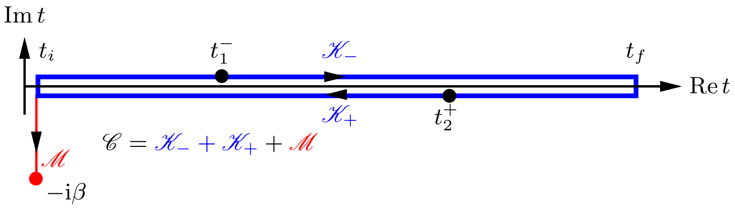


Schwinger-Keldysh time contour

- Aim: Calculate expectation values:

$$\langle \mathbf{O}(t)\rangle = \mathrm{Tr}[\boldsymbol{\rho}\mathbf{O}(t)], \quad \text{Equilibrium: } \boldsymbol{\rho} = \exp(-\beta \mathbf{H})/Z$$

- Introduce extended closed time-path, invented by Schwinger and Keldysh



- Green's function on the contour:

$$\mathrm{i}G_{\mathscr{C}}(x_1, x_2) = \langle \mathcal{T}_{\mathscr{C}}\phi(x_1)\phi(x_2)\rangle_\beta$$

Local and bilocal sources

- Generating functional for (disconnected) Green's functions

$$Z[J, B] = N \int \mathrm{D}\phi \exp \left[\mathrm{i} S[\phi] + \mathrm{i} \left\{ J_1 \phi_1 \right\}_1 + \frac{\mathrm{i}}{2} \left\{ B_{12} \phi_1 \phi_2 \right\}_{12} \right]$$

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$$W[J,B] = -\mathrm{i}\ln Z[J,B], \quad \frac{\delta W}{\delta J_1} = \varphi_1, \quad \frac{\delta W}{\delta B_{12}} = \frac{1}{2}\left(G_{12} + \varphi_1\varphi_2\right)$$

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- Legendre transform: 2PI generating functional

$$\Gamma[\varphi, G] = W[J, B] - \left\{J_1\varphi_1\right\}_1 - \frac{1}{2}\left\{(\varphi_1\varphi_2 + \mathrm{i}G_{12})B_{12}\right\}_{12}$$

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- Saddle point expansion of the path integral

$$\Gamma[\varphi, G] = S[\varphi] + \frac{\mathrm{i}}{2}\mathrm{Tr}\ln(\beta^2 G^{-1}) + \frac{\mathrm{i}}{2}\left\{D_{12}^{-1}(G_{12} - D_{12})\right\}_{12} + \Phi[\varphi, G]$$

with $$D_{12}^{-1} = \frac{\delta^2 S[\varphi]}{\delta\varphi_1\delta\varphi_2}$$

Equations of Motion

- Want to find φ and G at vanishing external sources $\Rightarrow$ Equations of motion:

$$\frac{\delta \Gamma}{\delta \varphi_1} = j_1 + \{B_{12}\varphi_2\}_2 \stackrel{!}{=} 0, \quad \frac{\delta \Gamma}{\delta G_{12}} = -\frac{\mathrm{i}}{2} B_{12} \stackrel{!}{=} 0$$

- Second equation:

$$D_{12}^{-1} - G_{12}{}^{-1} = 2\mathrm{i}\frac{\delta \Phi}{\delta G_{12}} = \Sigma_{12}$$

- Φ generates skeleton diagrams for self-energy
- Φ must be 2-particle irreducible (2PI)
- Saddle-point expansion of the path integral: Φ diagrams ≥ 2 loops

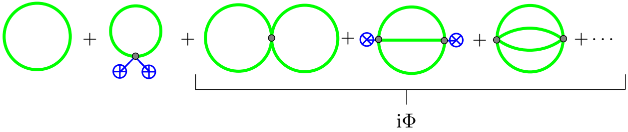
"Diagrammar"

Simple ϕ^4 model

$$\mathscr{L} = \frac{1}{2}(\partial_\mu \phi)(\partial_\mu \phi) - \frac{m}{2}\phi^2 - \frac{\lambda}{2}\phi^4, \quad S[\phi] = \{\mathscr{L}_1\}_1$$

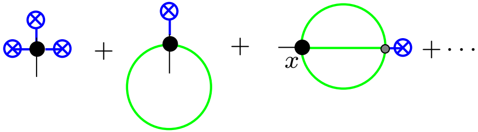
The functional:

$\mathrm{i}\Gamma[\varphi, G] = \mathrm{i}S[\varphi] +$



Field equation of motion:

$\mathrm{i}(\Box + m^2)\varphi =$



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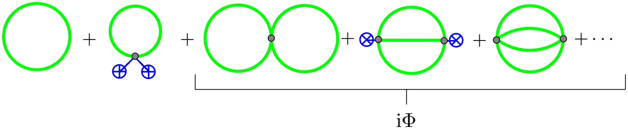
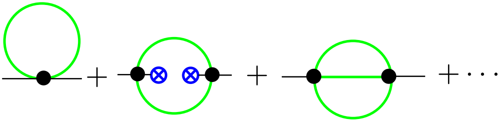


Self energy:

$-i\Sigma_{12} =$



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- To show now: Such approximations are renormalizable with local, temperature-independent counterterms

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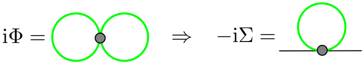
- Last but not least: Must be feasible for numerical calculations

Example: Tadpole in ϕ^4-model



Temperature dependent mass

$$M^2 = m^2 + \Sigma_{\text{ren}}$$

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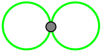
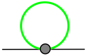
$\mathrm{i}\Phi =$  $\Rightarrow$ $-\mathrm{i}\Sigma =$ 

Eq. of motion $\Rightarrow$ Resummation of "daisy" and "super-daisy" diagrams:

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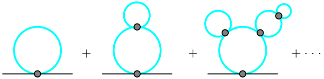

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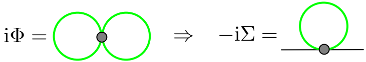

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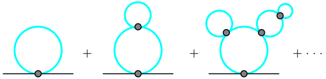
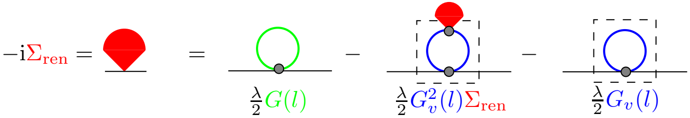


- Expand Green's function in vacuum part and temperature part
- Dyson equation: $G = G_v + G_v \Sigma G_v + \ldots$
- Subtract vacuum divergences and subdivergences only
- Counterterms: Vacuum-mass and coupling-constant counterterm



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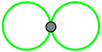
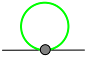
$\mathrm{i}\Phi =$  $\Rightarrow$ $-\mathrm{i}\Sigma =$ 

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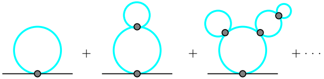

- Result: Finite gap equation

$$M^2 = m^2 + \Sigma_{\text{ren}} = m^2 + \frac{\lambda}{32\pi^2}\left(M^2 \ln \frac{M^2}{m^2} - \Sigma_{\text{ren}}\right) + \underbrace{+ \frac{\lambda}{2}\int \frac{\mathrm{d}^4 p}{(2\pi)^4} 2\pi\delta(p^2 - M^2) n(p_0)}_{\to 0 \text{ for } T \to 0}$$

with $n(p_0)$ Bose-Einstein distribution

Renormalization of general approximations

- The same strategy as in the tadpole example
- Renormalize vacuum first
- can be done with the BPHZ formalism
- power counting is the same for perturbative diagrams
- The temperature part of the self-energy is of power 0
- the asymptotic behavior is governed by the vacuum part alone

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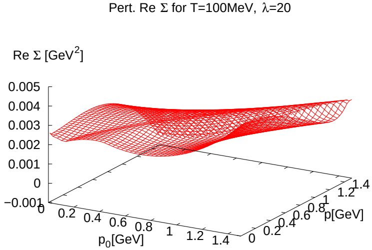
- expand Green's function due to Dyson equation

$$G = \underbrace{G_v}_{\delta=-2} + \underbrace{G_v \Sigma_T G_v}_{\delta=-4} + \underbrace{G_r}_{\delta=-6}$$

- coupling constant renormalization more difficult than for tadpole
- can be solved due to the 2PI properties of the Φ-functional!


Pert. Re Σ for T=100MeV, λ=20
Re Σ [GeV^2]
0.005
0.004
0.003
0.002
0.001
0
−0.001
0 0.2 0.4 0.6 0.8 1 1.2 1.4
p0[GeV]
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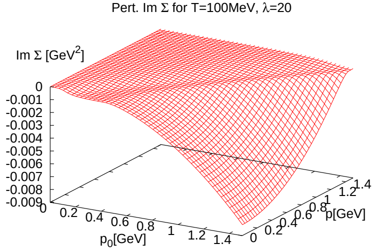
p[GeV]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-0.007
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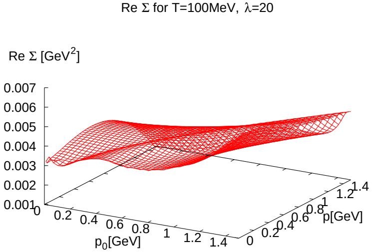
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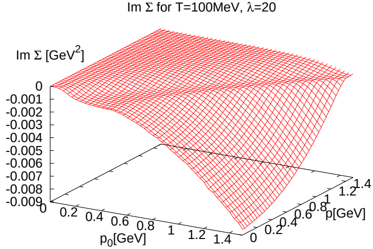
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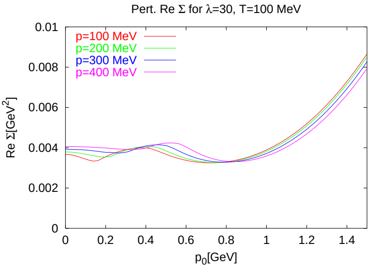
Pert. Re Σ for λ=30, T=100 MeV
Re Σ[GeV²]
p₀[GeV]
p=100 MeV
p=200 MeV
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0.01
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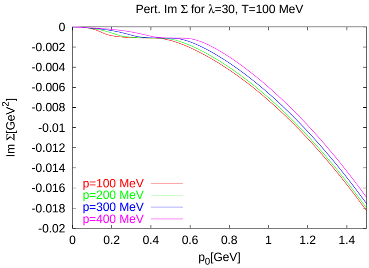
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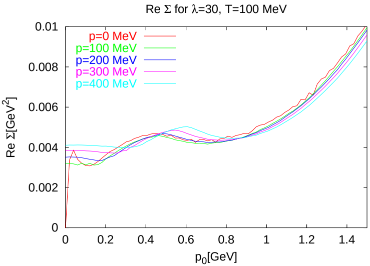
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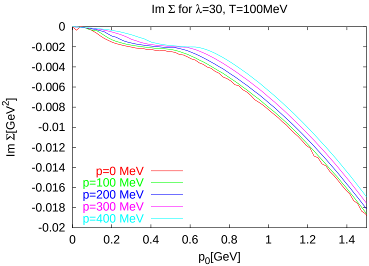
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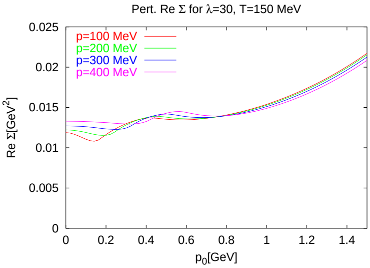
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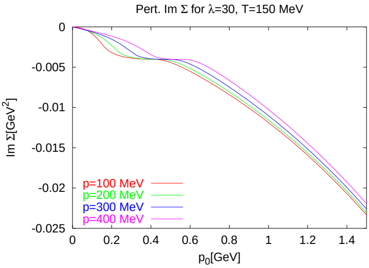
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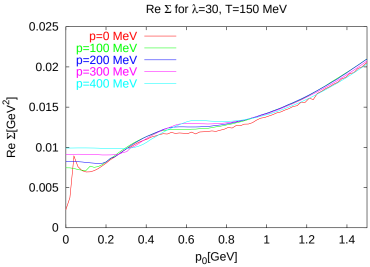
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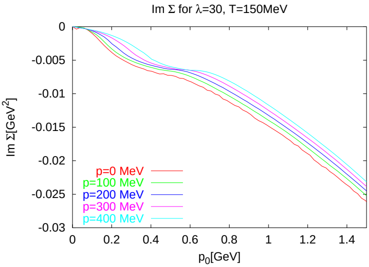
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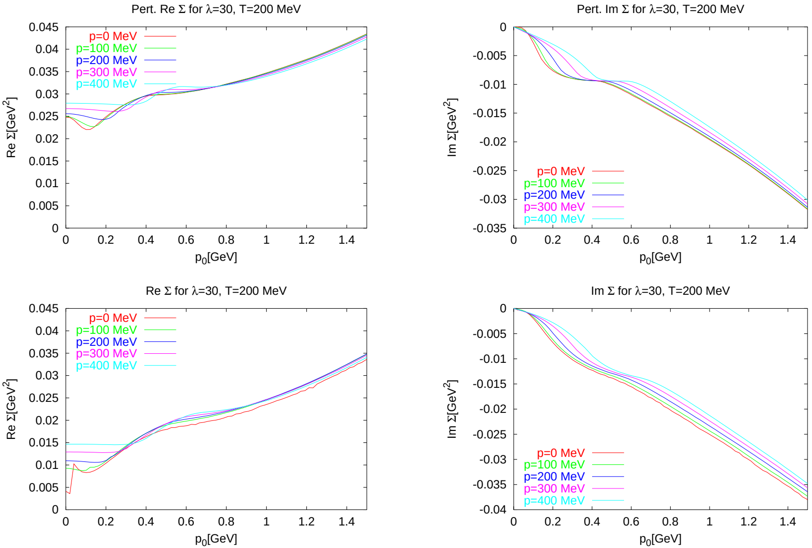
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p=400 MeV
Im Σ for λ=30, T=200 MeV
Im Σ[GeV^2]
p_0[GeV]
p=0 MeV
p=100 MeV
p=200 MeV
p=300 MeV
p=400 MeV

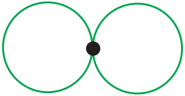
Breaking of symmetries: The O(N)-σ model

$$\mathscr{L} = \frac{1}{2}(\partial_\mu \vec{\phi})(\partial^\mu \vec{\phi}) - \frac{m}{2}\vec{\phi}^2 - \frac{\lambda}{8}\left(\vec{\phi}^2\right)^2$$

- Action symmetric under global O(N) rotations of $\vec{\phi}$
- Symmetry linear $\Rightarrow$ exact Quantum action also symmetric
- perturbative loop expansion = power expansion in $\hbar \Rightarrow$ also symmetric at any finite order of pert. theory
- If symmetry spontaneously broken ($m^2 < 0$), from this symmetry alone follows Goldstone's theorem: There are $N-1$ massless Goldstone bosons
- Long known (Baym, Grinstein 1977): Φ-derivable approximations break the symmetry explicitly!
- Goldstone's theorem also violated

Why is the symmetry broken?

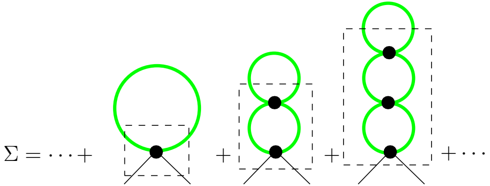
- Loop expansion of the functional is of certain order of $\hbar$
- but solutions are of arbitrary order of $\hbar$
- but, of course, not completely resummed
- Nevertheless expectation values of Noether currents are conserved
- Even crossing symmetry is violated: Four-point function is resummed only in certain channels
- Example: Tadpole Approximation for spontaneously broken O(N)-Model

$\mathrm{i}\Phi =$ 

- here: Put all mean-field interactions to the Φ-functional ⇒ provides possibility of self-consistenten MIR!

Why is the symmetry broken?

- Through self-consistency the four-vertex is intrinsically resummed:



- The "*t* and *u* channels" of the intrinsic four-point function are missing
- Way out: Calculate the corresponding approximation to the 1PI action
- extract proper vertex function from them
- This, by construction, restores crossing symmetry wrt. the external points

From 2PI back to 1PI approximations

- 2PI functional becomes the 1PI functional (i.e., the "effective action") by setting the bilocal source to 0
- For an arbitrarily given mean field $\vec{\varphi}$ we define a Green's function $\tilde{G}[\varphi]$ by

$$\left.\frac{\delta\Gamma[\varphi, G]}{\delta G}\right|_{G=\tilde{G}[\varphi]} = -\frac{\mathrm{i}}{2} B \stackrel{!}{=} 0$$

- The 1PI functional is then given by

$$\Gamma_{1\text{PI}}[\varphi] = \Gamma[\varphi, \tilde{G}[\varphi]]$$

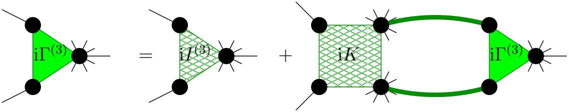
- For approximations to Γ ⇒ nonperturbative approximations to $\Gamma_{1\text{PI}}$
- generate proper vertex functions, which ...
  - ... are symmetric in their arguments ⇒ crossing symmetric
  - ... fulfil the Ward-Takahashi identities of linearly realized symmetries
  - Goldstone's theorem fulfilled for spontaneously broken symmetries

The “external” propagators

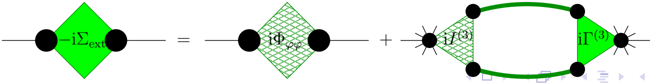
- Define the inverse propagator from 1PI as usual

$$(G_{\text{ext}}^{-1})_{12} = \frac{\delta^2 \Gamma_{1\text{PI}}}{\delta \varphi_1 \delta \varphi_2} := D_{12}^{-1} - (\Sigma_{\text{ext}})_{12}$$

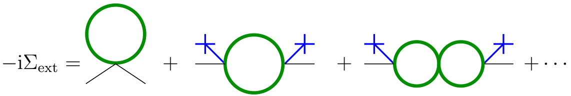
- Only for the exact 2PI functional $G_{\text{ext}} = G$
- For approximations, we have to resum the missing channels for the vertex:



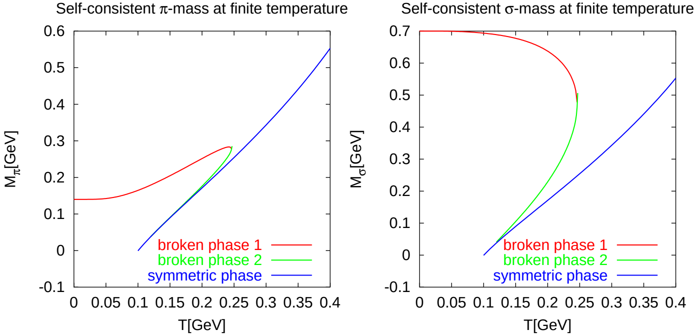
$$\mathrm{i}\Gamma^{(3)}_{123} = 2\frac{\delta \tilde{G}_{12}}{\delta \varphi_3}, \quad \mathrm{i}I^{(3)}_{123} := \mathrm{i}\frac{\delta^3 S[\varphi]}{\delta \varphi_1 \delta \varphi_2 \delta \varphi_3} + 2\frac{\delta^2 \Phi}{\delta G_{12} \delta \varphi_3}, \quad K_{12,34} := -2\frac{\delta^2 \Phi}{G_{12} G_{34}}$$



Example: The Hartree Approximation

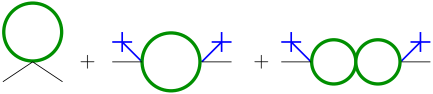


- The self-consistent solutions

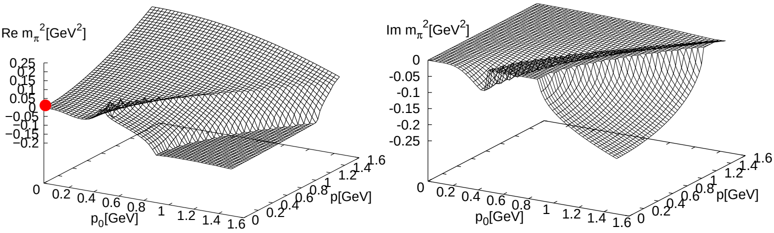


- Violate symmetries: Goldstone's theorem not fulfilled!

Example: The Hartree Approximation

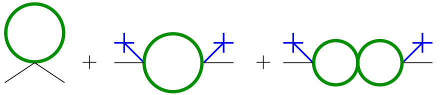
$-\mathrm{i}\Sigma_{\text{ext}} =$  $+\cdots$

- The inverse external propagators

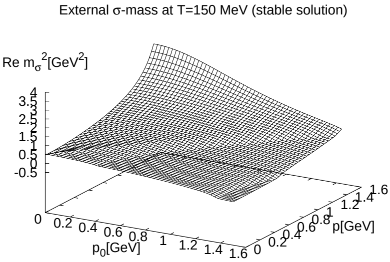


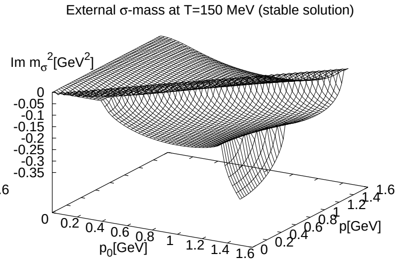
- External inverse propagator fulfills Goldstone's theorem

Example: The Hartree Approximation

$-\mathrm{i}\Sigma_{\mathrm{ext}} =$  $+ \cdots$

- The inverse external propagators:





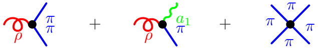
Schematic rough calculation

- Motivation: Check finite mass-widths effects of dressed propagators on dilepton spectra
- Used Kroll-Lee-Zumino type vector meson dominance model for π and ρ mesons
- Symmetry problem causes even worse trouble: Unphysical, acausal degrees of freedom become falsely populated
- Way out: Just projected to transverse propagators
- In this calculation only imaginary parts taken into account
- Thus: No mass shifts included

The model and approximation for Φ

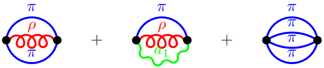
Lagrangian:

$\mathscr{L}_{\text{int}} =$



Φ–Funktional:

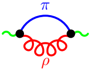
$\Phi =$



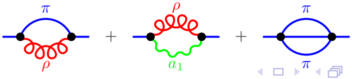
Self–energies:

$\Pi_\rho =$

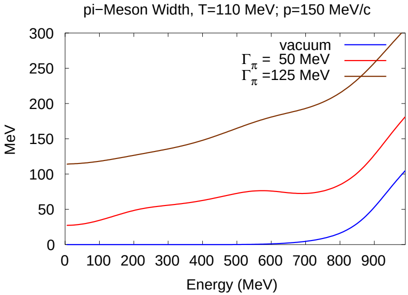
$\Pi_{a_1} =$

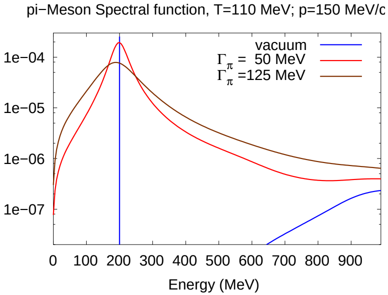


$\Sigma_\pi =$

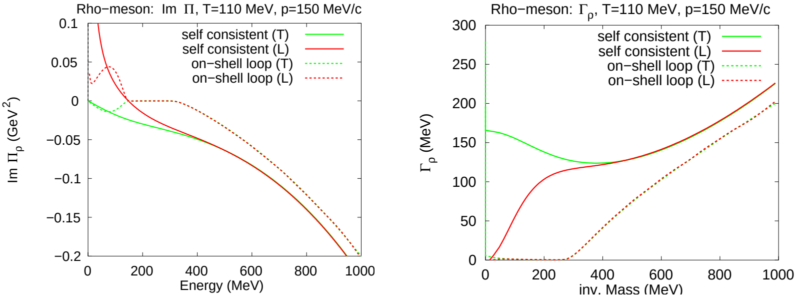


Results

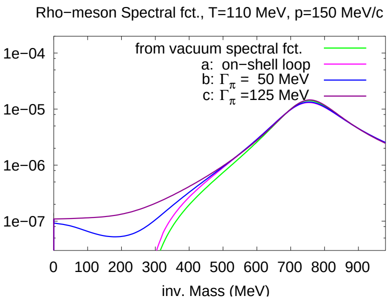

pi−Meson Width, T=110 MeV; p=150 MeV/c
MeV
Energy (MeV)
vacuum
Γπ = 50 MeV
Γπ =125 MeV
0
50
100
150
200
250
300
0 100 200 300 400 500 600 700 800 900

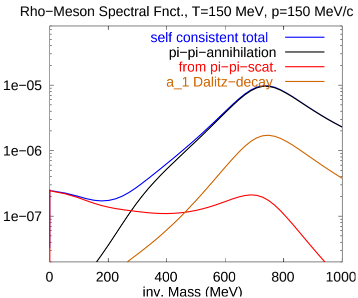

pi−Meson Spectral function, T=110 MeV; p=150 MeV/c
Energy (MeV)
vacuum
Γπ = 50 MeV
Γπ =125 MeV
1e−04
1e−05
1e−06
1e−07
0 100 200 300 400 500 600 700 800 900

Results

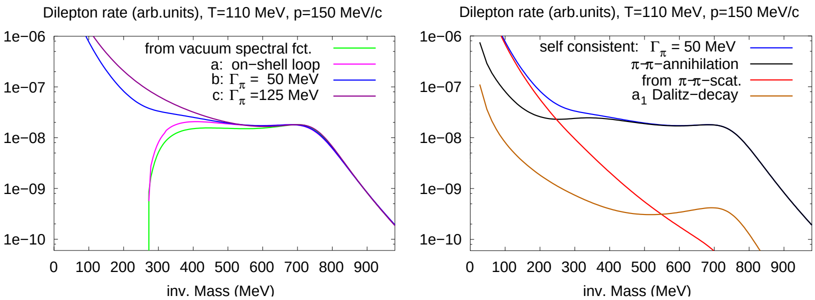

Rho−meson: Im Π, T=110 MeV, p=150 MeV/c
0.1
0.05
0
−0.05
−0.1
−0.15
−0.2
Im Π_ρ (GeV^2)
0
200
400
600
800
1000
Energy (MeV)
self consistent (T)
self consistent (L)
on−shell loop (T)
on−shell loop (L)
Rho−meson: Γ_ρ, T=110 MeV, p=150 MeV/c
300
250
200
150
100
50
0
Γ_ρ (MeV)
0
200
400
600
800
1000
inv. Mass (MeV)
self consistent (T)
self consistent (L)
on−shell loop (T)
on−shell loop (L)

Results


Rho−meson Spectral fct., T=110 MeV, p=150 MeV/c
from vacuum spectral fct.
a: on−shell loop
b: Γ_π = 50 MeV
c: Γ_π =125 MeV
1e−04
1e−05
1e−06
1e−07
0 100 200 300 400 500 600 700 800 900
inv. Mass (MeV)


Rho−Meson Spectral Fnct., T=150 MeV, p=150 MeV/c
self consistent total
pi−pi−annihilation
from pi−pi−scat.
a_1 Dalitz−decay
1e−05
1e−06
1e−07
0 200 400 600 800 1000
inv. Mass (MeV)

Results


Dilepton rate (arb.units), T=110 MeV, p=150 MeV/c
from vacuum spectral fct.
a: on−shell loop
b: Γπ = 50 MeV
c: Γπ =125 MeV
1e−06
1e−07
1e−08
1e−09
1e−10
0 100 200 300 400 500 600 700 800 900
inv. Mass (MeV)
Dilepton rate (arb.units), T=110 MeV, p=150 MeV/c
self consistent: Γπ = 50 MeV
π−π−annihilation
from π−π−scat.
a1 Dalitz−decay
1e−06
1e−07
1e−08
1e−09
1e−10
0 100 200 300 400 500 600 700 800 900
inv. Mass (MeV)

Conclusions

- Self-consistent Φ-derivable approximation schemes
- Renormalization problem is solved
- Symmetry problems analyzed
- Projector method for vector particles (or other fields with unphysical degrees of freedom like Δ)
- Class of numerically feasable approximations
- Can calculate the thermodynamic potential: Nonpert. study of phase transitions

Outlook

- Appl. to realistic models for in-medium properties of hadrons (the QGP)
- Self-consistent treatment of gauge theories: Abelian case formally understood
- Trouble remains for non-Abelian theories like QCD
- Also applicable to derive consistent transport equations for particles with broad mass widths

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