The renormalizability for massive Abelian gauge field theories re-visited

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We give a simplified proof for the perturbative renormalizability of theories with massive vector particles. For renormalizability it is sufficient that the vector particle is treated as an gauge field, corresponding to an Abelian gauge group. Contrary to the non-Abelian case one does not need the Higgs mechanism to create the appropriate mass terms. The proof uses "Stueckelberg's trick" and the Ward-Takahashi identities from local Abelian gauge invariance. The simplification is due to the fact that, again contrary to the non-Abelian case, no BRST analysis is needed.

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I. INTRODUCTION

We like to show that the theory of an interacting massive Abelian gauge field is renormalizable and that this proof can be significantly simplified, compared to the BRST method, discussed in [1]. As we shall see, we start with an explicitly gauge invariant classical action and then can use Ward-Takahashi identities (WTIs), directly obtained from this gauge invariance of the classical action. For the Abelian theory it is not necessary to use BRST symmetry.

It is also shown that one can choose a gauge, in which both, the Faddeev-Popov and the Stueckelberg ghost, are free fields and need no renormalization.

It should be mentioned that these results themselves are not new, since Kroll, Lee and Zumino [2] showed the renormalizability of the S-matrix, making use of the Proca version of the Lagrangian.

Here we show that this analysis of renormalizability can be much simplified by introducing the Stueckelberg ghost, because then it becomes a gauge theory which can be treated in manifestly renormalizable gauges. In this paper we use a gauge, which is a specialization of the well known R_ξ gauges, used for theories with spontaneously broken gauge symmetries[3]. Then we need only standard methods to prove the renormalizability of the model making use of the WTIs[4].

II. THE MODEL

We introduce a scalar field, the *Stueckelberg ghost* [5, 6, 7], into the action of a vector field with a "naive mass term":

$$\mathscr{L}_V = -\frac{1}{4}V_{\mu\nu}V^{\mu\nu} + \frac{m^2}{2}V_\mu V^\mu + \frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) + m\phi\partial_\mu V^\mu. \tag{1}$$

Here V_μ is a real vector field and $V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$ the usual field-strength tensor.

The advantage is that this Lagrangian leads to a manifestly gauge invariant action

$$S_V[V_\mu, \phi] = \int \mathrm{d}^4x \mathscr{L}_V(x). \tag{2}$$

The local gauge transformation for the fields are given by

$$V'_\mu = V_\mu + \partial_\mu\chi, \quad \phi' = \phi + m\chi, \tag{3}$$

where χ is an *arbitrary* scalar field.

The invariance of the action under infinitesimal transformations is easily checked:

$$\delta S_V = \delta \int \mathrm{d}^4 x \mathscr{L}_V = \int \mathrm{d}^4 x \left[m^2 V_\mu \partial^\mu + m \partial_\mu \phi \partial^\mu + m^2 \partial_\mu V^\mu + m\phi \partial_\mu \partial^\mu \right] \delta\chi = 0 \tag{4}$$

since the integrand is a total derivative.

It is clear that this gauge transformation remains a symmetry, if "matter fields" are introduced and the vector field is minimally coupled to conserved currents of these fields. As an example we consider the charged pions[1] as a complex scalar field in the following way:

$$\begin{aligned} &\mathscr{L}_\pi = (\mathrm{D}_\mu \pi)^* (\mathrm{D}^\mu \pi) - m_\pi^2 \pi^* \pi - \frac{\lambda}{8} (\pi^* \pi)^2, \\ &\text{with } \mathrm{D}_\mu \pi = (\partial_\mu + \mathrm{i} g V_\mu \pi). \end{aligned} \tag{5}$$

It should be mentioned that the restriction to those minimal couplings of the vector field to a conserved current is necessary for renormalizability. While a massless vector field is necessarily a gauge field, due to the Poincaré invariance (see, e.g., [4]) a massive vector field does not need to be a gauge field. Nevertheless, as far as is known today, there exist no renormalizable theories with massive vector fields, which are not treated as gauge fields.

The pion fields transform under gauge transformations as follows:

$$\pi' = \exp(-\mathrm{i} g \chi) \pi, \ \pi^{*\prime} = \exp(+\mathrm{i} g \chi) \pi^*. \tag{6}$$

The four-π-field interaction term in (5) is introduced in order to keep the quantized theory renormalizable. For our renormalizability proof, it is important that this is the only superficially renormalizable self-interaction term for the π which is consistent with gauge invariance.

With the path-integral approach, the quantization is straight forward: In the following we assume that a gauge invariant regularization, e.g., dimensional regularization, is applied. As for any gauge theory, one has to fix the gauge and introduce Faddeev Popov ghosts. The final expression for the generating functional of Green's functions is

$$Z[J] = N \int \mathrm{D}\Xi \exp[\mathrm{i} S_{\text{eff}} + J_k \Xi^k]. \tag{7}$$

Here we introduce Ξ^k as an abbreviation for all the fields, introduced so far, including the Faddeev-Popov ghosts.

We choose the following gauge fixing function, inspired by the R_ξ-gauges [3] used for the treatment of models with spontaneously broken gauge symmetries, like the standard model. One of its advantages is the vanishing of the mixing of the Stueckelberg ghost field with the vector field, introduced into the classical Lagrangian to obtain a gauge invariant Lagrangian with a massive gauge boson. Another advantage is that it contains the Proca formulation of the model as the limit $\xi \to \infty$ which thus is shown to be just the "unitary gauge" for our Abelian gauge theory.

$$g = \partial_\mu V^\mu + \xi m \phi. \tag{8}$$

In this gauge the effective Lagrangian is given by

$$\begin{aligned} \mathscr{L}_{\text{eff}} = &\mathscr{L}_\pi - \frac{1}{4} V_{\mu\nu} V^{\mu\nu} + \frac{m^2}{2} V_\mu V^\mu - \frac{1}{2\xi} (\partial_\mu V^\mu)^2 \\ &+ \frac{1}{2} (\partial_\mu \phi)(\partial^\mu \phi) - \frac{\xi m^2}{2} \phi^2 + (\partial_\mu \eta)^* (\partial_\mu \eta) - \xi m^2 \eta^* \eta. \end{aligned} \tag{9}$$

[1] To have a specific physical model in mind, one might look at the model as a vector-meson dominance model for interacting neutral ρ-mesons and pions[2, 8, 9]. It is clear that the addition of more Abelian vector fields, as done in these papers, does not change the argument. Of course, one has to introduce a Stueckelberg ghost for each massive gauge field.

The fields η and η^* are Grassmann fields (Faddeev-Popov ghosts), and the J_k are external currents.

The path integral can be read as a usual path integral for vacuum quantum field theory or at finite temperature. In the latter case one the time variable has to be read as an imaginary quantity, running from 0 to $-\mathrm{i}\beta$. Thereby the path integral is taken over all field configurations with the Kubo-Martin-Schwinger (KMS) conditions:

$$\begin{aligned} &\pi(-\mathrm{i}\beta,\vec{x}) = \exp(-\beta\mu)\pi(0,\vec{x}), \quad \pi^*(-\mathrm{i}\beta,\vec{x}) = \exp(\beta\mu)\pi^*(0,\vec{x}), \\ &\phi(-\mathrm{i}\beta,\vec{x}) = \phi(0,\vec{x}), \quad V_\mu(-\mathrm{i}\beta,\vec{x}) = V_\mu(0,\vec{x}). \end{aligned} \tag{10}$$

Here μ is a chemical potential for the conserved charge of the pions.

Thereby, it is important to note that the KMS condition for the Faddeev-Popov ghosts is periodic rather than anti-periodic, because they are not introduced by the path integral treatment of physical fermions (see, e.g., [4]), but to treat the appearance of the Faddeev-Popov determinant in the path-integral measure as a contribution to the effective classical action for a fixed gauge:

$$\eta(-\mathrm{i}\beta,\vec{x}) = \eta(0,\vec{x}), \quad \eta^*(-\mathrm{i}\beta,\vec{x}) = \eta^*(0,\vec{x}). \tag{11}$$

Concerning renormalization, it is sufficient to look at the vacuum case, since the renormalization of the model is completely done at $T = 0$. Going to finite temperature does not introduce new UV divergences (see, e.g., [10, 11]).

The Lagrangian (9) is superficially renormalizable since it contains only terms of momentum power 4 and less. So we know that the counterterms, necessary to render the quantum action finite, are local and also of momentum power 4 and less, since thanks to our choice of gauge the propagator of the gauge field goes like $1/p^2$ for large momenta.

Of course, one has to show that the model is really renormalizable since the classical action is restricted to gauge invariant couplings and a superficially necessary counterterm like, e.g., $(V_\mu V^\mu)^2$ is not needed to renormalize the effective action. On the other hand (9) is the most general superficially renormalizable Lagrangian, which is consistent with the local gauge invariance and the field content taken into account.

To show the renormalizability one can use the BRST invariance as described in [1]. This has the advantage that it is also applicable to non-Abelian gauge fields. For our purposes it is more convenient to use directly the local gauge invariance of the classical action. The reason is that both, the scalar auxiliary (or Stueckelberg) field and the Faddeev-Popov ghosts, are free fields in (9). Thus, we can define the gauge transformation to act trivially on the ghost fields, i.e., leaving them unchanged. As we shall show in a moment, this prevents the WTIs from gauge invariance for the generating functional Z to be free of higher order derivatives with respect to the external currents J_k. It is clear that for non-Abelian theories this holds no longer true, since the Faddeev-Popov ghosts, at least, are coupled to the gauge fields. Then the analysis through the BRST formalism is more appropriate for the renormalizability proof (see, e.g., [12]).

III. THE WARD-TAKAHASHI IDENTITIES

As shown in the previous section, the effective classical action (9), appearing in the path integral (7), is given by the classical gauge invariant Lagrangian, cf. (1) and (5), the gauge-fixing, and the source part:

$$\begin{aligned} \mathscr{L}_{\text{fix}} &= -\frac{1}{2\xi}(\partial_\mu V^\mu + \xi m\phi)^2, \\ \mathscr{L}_{\text{src}} &= J_k \Xi^k := j_\mu V^\mu + k\phi + \eta^*\alpha + \alpha^*\eta + a^*\pi + \pi^* a. \end{aligned} \tag{12}$$

For completeness we note also the Lagrangian for the free Stueckelberg and the Faddeev-Popov fields:

$$\mathscr{L}_{\text{FP}\phi} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{\xi m^2}{2}\phi^2 + (\partial_\mu \eta^*)(\partial_\mu \eta) - \xi m^2(\eta^* \eta). \tag{13}$$

In this notation the path integral (9) reads

$$Z[J] = N \int \mathrm{D}\Xi \exp[\mathrm{i}(S_V + S_\pi + S_{\text{fix}} + S_{\text{src}} + S_{\text{FP}\phi})]. \tag{14}$$

The integral over the Faddeev-Popov and the Stueckelberg fields yields just the generating functional for free fields. This is only important for the partition sum at finite temperature, since it cancels contributions from the unphysical degrees of freedom of the vector field, leading to the partition sum of three bosonic field degrees of freedom with mass m, as it should be[2].

For the proof of renormalizability we use the invariance of the path-integral measure under the gauge transformations, given by the Eqs. (3) and (6). By definition the Faddeev-Popov ghosts are unchanged under the here considered gauge transformations. Then only $S_{\text{fix}} + S_{\text{src}}$ is gauge dependent. Thus, substituting gauge transformed fields into the path integral (14), for infinitesimal gauge transformations one obtains the Ward-Takahashi identities (WTIs) which are summarized by the one identity for the generating functional Z:

$$-\frac{\Box + \xi m^2}{\xi}\left[\partial^\mu \frac{\delta Z}{\mathrm{i}\delta j^\mu} + \xi m \frac{\delta Z}{\mathrm{i}\delta k}\right] - \partial_\mu j^\mu Z + kmZ - \mathrm{i}g\left[a^* \frac{\delta Z}{\mathrm{i}\delta a^*} - a\frac{\delta Z}{\mathrm{i}\delta a}\right] = 0. \tag{15}$$

The next step is to introduce the generating functional for connected Green's functions $W = -\mathrm{i}\ln Z$. From (15) one immediately obtains

$$-\frac{\Box + \xi m^2}{\xi}\left[\partial^\mu \frac{\delta W}{\delta j^\mu} + \xi m \frac{\delta W}{\delta k}\right] - \partial_\mu j^\mu + km - \mathrm{i}g\left[a^* \frac{\delta W}{\delta a^*} - a\frac{\delta W}{\delta a}\right] = 0. \tag{16}$$

Finally, we define the generating functional for one-particle irreducible (1PI) truncated Green's functions (proper vertex functions) by a functional Legendre transform:

$$\begin{aligned} \Gamma[\bar{\Xi}] &= W[J] - \int \mathrm{d}^4 x J_k(x) \bar{\Xi}^k(x), \\ \bar{\Xi}^k &= \frac{\delta W}{\delta J_k} \Leftrightarrow \frac{\delta \Gamma}{\delta \bar{\Xi}^k} = -J_k. \end{aligned} \tag{17}$$

From (16) we obtain the WTIs for Γ

$$-\frac{\Box + \xi m^2}{\xi}(\partial_\mu \bar{V}^\mu + \xi m \bar{\phi}) + \partial^\mu \frac{\delta \Gamma}{\delta \bar{V}^\mu} - m\frac{\delta \Gamma}{\delta \bar{V}^\mu} - m\frac{\delta \Gamma}{\delta \bar{\phi}} + \mathrm{i}g\left[\frac{\delta \Gamma}{\delta \bar{\pi}}\bar{\pi} - \frac{\delta \Gamma}{\delta \bar{\pi}^*}\bar{\pi}^*\right] = 0. \tag{18}$$

IV. PROOF OF RENORMALIZABILITY

To prove the renormalizability of the model we use (18) to show that it is sufficient to introduce a counterterm Lagrangian of the same form as (9), except that we do not need counterterms for

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[2] Of course, for the model of free massive vector fields, this can be derived directly by using the Proca Lagrangian without the gauge theoretical treatment.

the Faddeev-Popov and Stueckelberg ghosts, since these are free fields and thus not involved in divergent loop integrals:

$$
\begin{aligned}
\mathscr{L}_{\text{ct}} = & -\frac{1}{4}\delta Z_3 V_{\mu\nu}V^{\mu\nu} + \frac{\delta m^2}{2}V_\mu V^\mu - \delta(1/\xi)(\partial_\mu V^\mu)^2 \\
& + \delta Z_2 (\partial_\mu \pi^*)(\partial^\mu \pi) - \frac{\delta m_\pi^2}{2}\pi^*\pi \\
& + \mathrm{i} g \delta Z_1 [\pi^* \overleftrightarrow{\partial}_\mu \pi] V^\mu + g^2 \delta Z_1' V_\mu V^\mu \pi^* \pi - \frac{\delta\lambda}{8}(\pi^*\pi)^2 .
\end{aligned}
\tag{19}
$$

We have to show that these counterterms are sufficient to render all divergences finite. Additionally, due to the gauge invariance of the classical action $S_V + S_\pi$, the counterterms have to satisfy the *Ward identities*

$$
\delta Z_1 = \delta Z_2 = \delta Z_1' \tag{20}
$$

For sake of simplicity, we consider only the case of unbroken gauge symmetry. It is no problem to generalize the proof for the case of spontaneous symmetry breaking, i.e., the massive model with a Higgs field[3].

The proof is by induction in the loop order L. At tree level, $L = 0$, all diagrams are finite. Now we suppose that (19), with the restrictions by the Ward identities (20), is sufficient to render the diagrams up to loop order L finite. We have to show that then the same holds true also at loop order $L + 1$. Then the renormalized effective action up to loop order L fulfills the WTI (18), including the counterterms up to this order.

By assumption, for any diagram of loop order $L + 1$ we can subtract the *proper subdivergences*, which come from proper subdiagrams with at most L loops, with counterterms of the structure (19,20). After this subtraction, due to Weinberg's theorem [14], the only divergences left are the overall divergences, which can appear only for diagrams with at most 4 external legs. This means that, in momentum representation, the divergent parts are polynomials in the external momenta. The counterterms for Γ are local and polynomials in the fields up to order 4. We have to prove that these polynomials, for the contributions to Γ at loop order $L + 1$, are of the form (19) and fulfilling the Ward identities (20).

We start the proof with the remark that there is no ϕV-mixing in Γ, i.e.,

$$
\left. \frac{\delta^2 \Gamma}{\delta V^\mu \delta \phi} \right|_{\bar{\Xi}=0} = 0. \tag{21}
$$

Indeed, thanks to our choice of the gauge fixing functional (8), there is no such term at tree level, and the field ϕ is free, so that there are no proper vertex functions involving ϕ. Thus, all proper vertex functions vanish except the two-point vertex, which is the inverse full propagator for the Stueckelberg ghost, which has to be identical to the free one to all loop orders.

Indeed, taking the functional derivative of the WTI (18) with respect to ϕ, one finds after setting the mean fields to 0 and making use of (21):

$$
G_\phi^{-1}(x, y) = \left. \frac{\delta^2 \Gamma}{\delta \phi(x) \delta \phi(y)} \right|_{\bar{\Xi}=0} = -(\Box_x + \xi m^2)\delta^{(4)}(x - y). \tag{22}
$$

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[3] The most simple realization would be to set $m_\pi^2 < 0$, analogous to the Higgs fields in the minimal standard model. Then one should impose a mass-independent renormalization scheme [13]. The possibility of such a choice proves that one can subtract all divergences in the symmetric phase, thereby introducing a renormalization π-mass scale.

This means that G_ϕ is indeed the free propagator for a scalar field with mass $\sqrt{\xi}m$. Thus (21) is consistent with the underlying gauge invariance of the classical action: In this gauge, the Stueckelberg field needs no renormalization at all, neither the wave function nor the mass.

The WTI for the vector-boson propagator is obtained by taking the derivative of the WTI (18) with respect to the vector field:

$$\partial_{x\mu}(G_V^{-1})^{\mu\nu}(x,y) = \frac{\Box + \xi m^2}{\xi} \partial_x^\nu \delta^{(4)}(x-y). \tag{23}$$

Introducing the polarization (self-energy) of the vector field by $G_V^{-1} = \Delta_V^{-1} - \Sigma_V^{-1}$, where Δ_V is its free propagator, and Fourier transforming (23) gives the transversality of the vector self-energy:

$$p_\mu \Sigma_V^{\mu\nu}(p) = 0. \tag{24}$$

Thus, at loop order $L+1$ the term, quadratic in V_μ, needs a counterterm of the form

$$\frac{\delta Z_3}{2} V^\mu (\partial_\mu \partial_\nu - \Box g_{\mu\nu}) V_\nu. \tag{25}$$

In other words, to render the V self-energy finite, the subtraction of a wave-function renormalization term is sufficient. Thus, the self-energy can be written in the form

$$\Sigma_V^{\mu\nu}(p) = \left(-p^2 g^{\mu\nu} + p^\mu p^\nu\right) \Sigma_V(p), \tag{26}$$

where Σ_V is a scalar function of p which is only logarithmically divergent[4]. Thus, one neither needs a vector mass nor a gauge fixing constant renormalization:

$$\delta m^2 = 0, \quad \delta(1/\xi) = 0. \tag{27}$$

For the π-propagator, there is no restriction by gauge invariance. Since it is quadratically divergent, the counter-terms in (18) are sufficient and consistent with the requirements of gauge invariance.

Now we have to consider the vertices. First, we look for those types which are already present at tree level: Taking the derivative of the WTI (18) with respect to $\bar{\pi}^*(y)$ and $\bar{\pi}(z)$ we obtain

$$\partial_x^\mu \Gamma_\mu^{V,\pi^*,\pi}(x,y,z) = g G_\pi^{-1}(y,z)[\delta^{(4)}(x-z) - \delta^{(4)}(x-y)]. \tag{28}$$

Here we have used the general definition for a proper vertex function:

$$\Gamma^{\Xi_1,\ldots,\Xi_k}(x_1,\ldots,x_k) = \mathrm{i} \left. \frac{\delta^k \Gamma}{\delta\bar{\Xi}_1(x_1)\cdots\delta\bar{\Xi}_k(x_k)} \right|_{\bar{\Xi}=0}. \tag{29}$$

Fourier transformation of (28) yields

$$\mathrm{i} p^\mu \Gamma_\mu^{V,\pi^*,\pi}(p,q,r) = g[G_\pi^{-1}(q) - G_\pi^{-1}(r)]. \tag{30}$$

Since the superficial degrees of divergence for the involved vertex functions are $\delta(\Gamma^{V,\pi^*,\pi}) = 1$ and $\delta(G_\pi^{-1}) = 2$, the overall divergences for these quantities are of the form

$$\Gamma_{\text{div}}^{V,\pi^*,\pi}(p,q,r) = -\mathrm{i}(C_1 q^\mu + C_2 r^\mu), \quad (G_\pi^{-1})_{\text{div}}(p) = \delta Z_2 p^2 - \delta m_\pi^2. \tag{31}$$

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[4] Of course, here also Lorentz invariance was used. At finite temperature, the Lorentz invariance is broken by the existence of the rest frame of the heat bath. Then we have two scalar functions for the polarization tensor, one which is transverse and one which is longitudinal with respect to the three-momentum. This does not affect the form (25) of the counter term since, after subtraction of the subdivergences of a diagram, the remaining overall divergent parts are independent of temperature and thus take the corresponding vacuum values.

For the vertex, here we used q and r as the independent momenta[5]. Applying the WTI (28) to the diverging part gives

$$(q_\mu - r_\mu)(C_1 q^\mu + C_2 r^\mu) = \delta Z_2 g(q^2 - r^2). \tag{32}$$

Comparing both sides of this equation yields

$$C_1 = C_2 := \delta Z_1 g = \delta Z_2 g \;\Rightarrow\; \delta Z_1 = \delta Z_2. \tag{33}$$

Thus the first of the Ward identities (20) is fulfilled also at loop order $L+1$.

Derivation of (18) with respect to $\pi^*(y)$, $\pi(z)$, and $A^\nu(x')$ gives, after a Fourier transformation, the WTI

$$p^\mu \Gamma^{V,V,\pi^*,\pi}_{\mu\nu}(p,p',q,r) = \Gamma^{V,\pi^*,\pi}_\nu(p,p-q,r) - \Gamma^{V,\pi^*,\pi}_\nu(p',q,p+r). \tag{34}$$

Since this vertex is logarithmically divergent, the overall diverging part at loop order $L+1$ is of the form $2\mathrm{i}\delta Z_1' g^{\mu\nu}$. Together with (31) this proves the second of the Ward identities (20):

$$\delta Z_1 = \delta Z_1'. \tag{35}$$

To show that the four-pion interaction counterterm has to be of the given form, even global gauge invariance is sufficient.

Now we have to show that the superficially divergent vertices, which are not contained in the original Lagrangian, are finite. First, we see that VVV-vertices are vanishing loop order by loop order, because the Lagrangian is invariant under charge-conjugation transformations. This is the same argument as used to prove Furry's theorem for QED, which says that all vertices with an odd number of vector-field legs (and no other external legs) vanish.

The symmetry under global gauge transformations also excludes πVV- and $\pi\pi\pi V$-vertices. Thus, we only need to show that the vertex with four V-legs is finite (in QED this is the famous Delbrück-scattering term), although it looks logarithmically divergent. For that we take the derivative of the WTI (18) with respect to $V_\nu(w)$, $V_\rho(y)$, and $V_\sigma(z)$, which leads, after taking the Fourier transform, to

$$p^\mu \Gamma^{VVVV}_{\mu\nu\rho\sigma}(p,q,r,s) = 0. \tag{36}$$

Since the vertex is logarithmically divergent, the overall divergent part at loop order $(L+1)$ must be of the form

$$D(g_{\mu\nu}g_{\rho\sigma} + g_{\mu\rho}g_{\nu\sigma} + g_{\mu\sigma}g_{\nu\rho}). \tag{37}$$

Applying the WTI (36) to the divergent part shows $D = 0$. This completes our proof of perturbative renormalizability.

As usual, the gauge independence of the S-matrix can be shown by making use of LSZ reduction, leading to the the so called *equivalence theorem* (see, e.g., the article by B. W. Lee in [15]). In our context this means that the renormalized S-matrix is independent of ξ and thus, after renormalization, remains finite for $\xi \to \infty$, which leads back to the theory in the Proca form. Thus, the S-matrix is renormalizable when calculated with the Proca form of the model which corresponds to the "unitary gauge" in Higgs models[6]. The same time, this shows that our gauge

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[5] Of course overall momentum conservation $p + q + r = 0$, with all currents running into the vertex, is implied.

[6] For a detailed discussion of renormalization in the unitary gauge see [16], where the Stueckelberg formalism is used to reconstruct the gauge invariant form of the standard model Lagrangian in unitary gauge.

model indeed describes a massive vector boson, interacting with matter particles (here described by the π-fields). In the limit of zero vector-boson mass one obtains Quantum electrodynamics (in our case "scalar electrodynamics"). Of course, taking the limit $m \to 0$, one has to take care of possible infrared problems. It is clear that the same analysis goes through with other matter contents like Dirac fields.

The gauge independence of observable thermodynamic quantities is seen simply by the fact that they are thermal expectation values of gauge invariant operators which can be calculated with help of the path integral for vanishing external sources. By construction, this path integral is independent of the chosen gauge fixing condition $g[A, \phi] = 0$.

V. CONCLUSION

It was shown that the renormalizability proof for massive Abelian gauge theories can be significantly simplified by using the Stueckelberg-field approach.

An extension of this proof to non-perturbative treatments, based on the 2PI formalism, which generalizes the analysis for theories with global symmetries [17], is in preparation for a later publication.

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