Reonances in the medium

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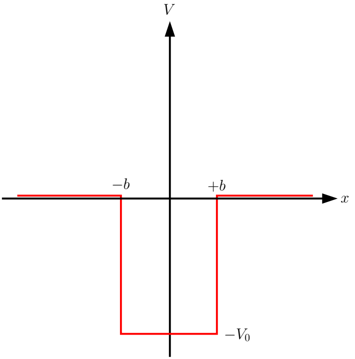
based on

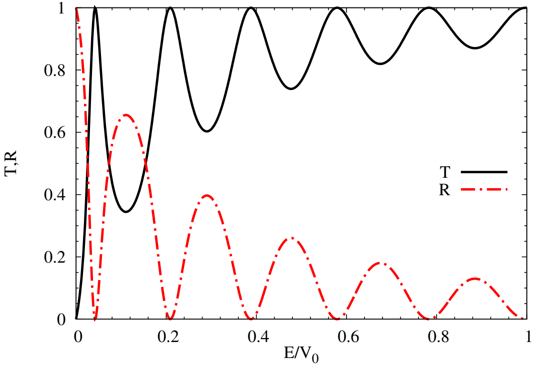
HvH, PRD **65**, 025010 (2001); PhD thesis 2000
Ivanov, Knoll, Voskresensky, NPA **657**, 413 (1999); NPA **672**, 313 (2000)
Knoll, Ivanov, Voskresensky, Ann. Phys. **293**, 126 (2001)

Outline

What's a resonance?

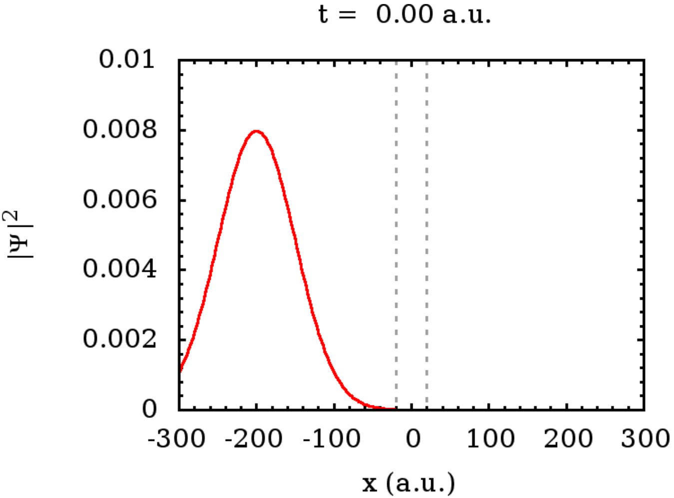
- quantum mechanics 101: Particle in a potential pot
- wave packet with energy around transmission-resonance peak
- nearly no reflection





What's a resonance?

- quantum mechanics 101: Particle in a potential pot
- wave packet with energy around resonance peak
- nearly no reflection; stays a while in pot

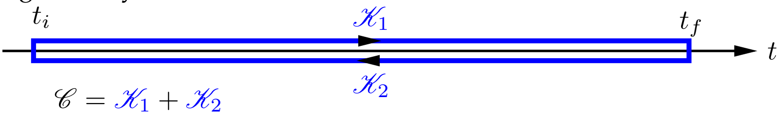


Schwinger-Keldysh real-time formalism

- calculate expectation values of observables
- statistical operator defines state at initial time, t_i ⇒ “in-in formalism”
- time evolution

$$\langle O \rangle (t) = \mathrm{Tr}\left[\hat{\rho}(t_i)\, \underbrace{\mathcal{T}_a \left\{\exp\left[+\mathrm{i}\int_{t_i}^{t} \mathrm{d}t' \mathbf{H}_I(t')\right]\right\}}_{\text{anti time–orderd}} \mathbf{O}_I(t) \underbrace{\mathcal{T}_c \left\{\exp\left[-\mathrm{i}\int_{t_i}^{t} \mathrm{d}t' \mathbf{H}_I(t')\right]\right\}}_{\text{time–ordered}}\right].$$

- Schwinger-Keldysh real-time contour:



Baym's Φ functional

- write generating functional for Green's functions as path integral
- introduce local and bilocal sources

$$Z[J,K] = N \int \mathrm{D}\phi \exp\left[\mathrm{i}S[\phi] + \mathrm{i}\left\{J_1\phi_1\right\}_1 + \left\{\frac{\mathrm{i}}{2}K_{12}\phi_1\phi_2\right\}_{12}\right]$$

- generating functional for connected Green's functions

$$W[J,K] = -\mathrm{i}\ln Z[J,K]$$

- functional Legendre transform

$$\Gamma[\varphi,G] = W[J,K] - \{\varphi_1 J_1\}_1 - \frac{1}{2}\{(\varphi_1\varphi_2 + \mathrm{i}G_{12})K_{12}\}_{12}$$

- loop expansion

$$\Gamma[\varphi,G] = S_0[\varphi] + \frac{\mathrm{i}}{2}\mathrm{Tr}\ln(-\mathrm{i}G^{-1}) + \frac{\mathrm{i}}{2}\left\{D_{12}^{-1}(G_{12} - D_{12})\right\}_{12}$$
$$+ \Phi[\varphi,G] \Leftarrow \text{all closed 2PI interaction diagrams}$$
$$D_{12}^{-1} = -\Box - m^2$$

Baym's Φ functional

- equations of motion

$$\frac{\delta \mathbb{\Gamma}}{\delta \varphi_1} = -J_1 - \{K_{12}\varphi_2\}_2 \stackrel{!}{=} 0, \quad \frac{\delta \mathbb{\Gamma}}{\delta G_{12}} = -\frac{\mathrm{i}}{2} K_{12} \stackrel{!}{=} 0$$

- mean field

$$-\Box \varphi - m^2 \varphi := j = -\frac{\delta \Phi}{\delta \varphi}$$

- "full" propagator $G \Rightarrow$ Dyson equation:

$$-\mathrm{i}(D_{12}^{-1} - {G_{12}}^{-1}) := -\mathrm{i}\Sigma = 2\frac{\delta \Phi}{\delta G_{21}}$$

- retarded Green's function for homogeneous system in momentum space

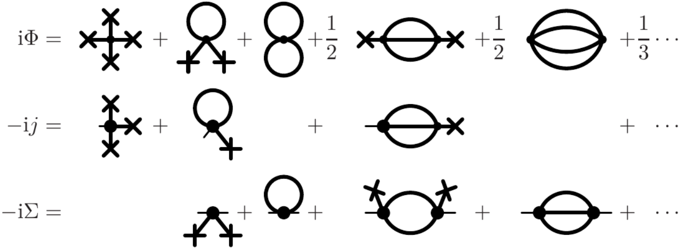
$$G_{\text{ret}}(p) = \frac{1}{p^2 - m^2 - \Sigma_{\text{ret}}(p)}$$

- spectral function

$$A(p) = -2 \operatorname{Im} G_{\text{ret}}(p) = -2 \frac{\operatorname{Im} \Sigma_{\text{ret}}(p)}{[p^2 - m^2 - \operatorname{Re} \Sigma_{\text{ret}}(p)]^2 + [\operatorname{Im} \Sigma_{\text{ret}}(p)]^2}$$

Properties of Φ-derivable approximations

- truncations of Φ functional ⇒ Φ-derivable approximations



- conservation laws for expectation values of conserved quantities
- in thermal equilibrium $\mathrm{i}\Gamma = \ln Z$
- thermodynamic consistency: bulk properties like pressure, energy, entropy in accordance with dynamics
- same result from partition sum as from Green's functions!
- "Φ derivability" sufficient and necessary scheme!

Transport equations

- start from Φ-derivable Dyson equation for Green's function

$$(\Box_1 - \Box_2) D^{12}(x_1, x_2) = \int_{\mathcal{C}} \mathrm{d}x_3 [\Sigma(x_1^1, x_3) D(x_3, x_2^2) - D(x_1^1, x_3) \Sigma(x_3, x_2^2)]$$
$$= \mathrm{Coll}(x_1^1, x_2^2)$$

- assume smallness of space-time gradients in "collective macroscopic" variable $R = (x_1 + x_2)/2$
- Wigner transform of any two-point function, F

$$F(x_1, x_2) = \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \exp[-\mathrm{i}p \cdot (x_1 - x_2)] \tilde{F}\left(\frac{x_1 + x_2}{2}, p\right).$$

- assume space-time gradients wrt. R to be "small" ⇒ gradient expansion ⇒ "coarse graining"

$$2p \cdot \frac{\partial}{\partial R} \mathrm{i}D^{12}(R, p) = \mathrm{Coll}^{12}(R, p)$$

Gradient expansion of collision term

$$2p \cdot \frac{\partial}{\partial R} \mathrm{i} D^{12}(R,p) = \mathrm{Coll}^{12}(R,p)$$

- if Φ beyond pure two-point level $\Rightarrow$ memory + spatial correlations
- simplify further by introducing $\mathrm{Coll}^{12}_{\mathrm{loc}}$:
  diagrams evaluated at reference point R
- usual momentum Feynman rules with $D_{12}(R,p)$
- to have exact conservation laws add 1st-order ∂_R correction

$$D\left(\frac{x_i+x_j}{2},p\right) \simeq D(R,p) + \frac{1}{2}[(x_i+x_j)-R] \cdot \frac{\partial}{\partial R} D(R,p)$$

- for local Green's functions and self-energies

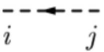
$$\mathrm{i}D^{12}(R,p) = f(R,p)A(R,p), \quad A(R,p) = -2\,\mathrm{Im}\,D_{\mathrm{ret}}(R,p)$$

- as in equilibrium with off-equilibrium phase-space distrib. $f(R,p)$
- usual local Dyson equation for retarded Green's function

$$D_{\mathrm{ret}}(R,p) = \frac{1}{p^2 - m^2 - \Sigma_{\mathrm{ret}}(R,p)}$$

Diagrammar for gradient expansion

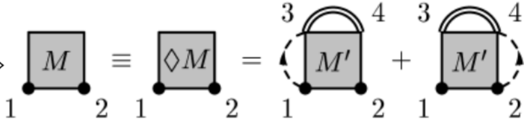
$$= \frac{1}{2}(\partial_i + \partial_j) G(i, j) \to \partial_X G(X, p),$$



$$= -i(x_i - x_j) \to -(2\pi)^4 \frac{\partial}{\partial p} \delta(p)$$

- arbitrary two-point function $M(x_1, x_2)$ with internal points $x_3, \ldots$

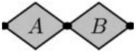
$$\Diamond\{M(1,2)\} = \Diamond$$



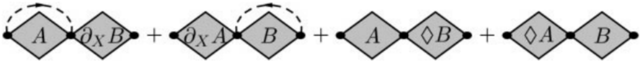
$$M'(x_1, x_2; x_3, x_4) = \frac{\delta M(x_1, x_2)}{\delta \mathrm{i}G(x_4, x_3)}$$

- collision term ⇒ convolution integral

$$\Diamond\{\mathcal{C}(X, p)\} = \Diamond$$



$$=$$



$$= \{A(X, p), B(X, p)\} + A(X, p)\Diamond\{B(X, p)\} + \Diamond\{A(X, p)\}B(X, p).$$

Diagrammar for gradient expansion

- transport equation in Kadanoff-Baym form

$$p \cdot \frac{\partial}{\partial R} \mathrm{i} D^{12}(R,p) + \left\{ \operatorname{Re} \Sigma_{\text{ret}}, \mathrm{i} D^{12} \right\}_{\text{pb}} + \left\{ \mathrm{i} \Sigma^{12}, \operatorname{Re} D_{\text{ret}} \right\}_{\text{pb}} = C_{\text{loc}}^{12} + C_{\text{mem}}^{12}$$

- then Noether currents exactly conserved also after gradient expansion
- problem: 2^{nd} Poisson bracket ("back-flow term") cannot be represented in test-particle Monte Carlo
- Botermans-Malfliet ansatz

$$\mathrm{i}\Sigma^{12}(R,p) = -f(R,p)\Gamma(R,p), \quad \Gamma(R,p) = -2 \operatorname{Im} \Sigma_{\text{ret}}$$

- valid up to 1^{st}-order gradients
- Caveat: in conservation laws from BM ansatz

$$A(R,p) \to \mathcal{B}(R,p) := \frac{\partial}{\partial p_0} \left[2 \operatorname{Im} \ln(D_{\text{ret}}^{-1}) - \operatorname{Re} G_{\text{ret}} \Gamma \right]$$

- for narrow resonances (BW approximation) $\mathcal{B} \simeq \frac{1}{2} A^2 \Gamma$
- for test-particle off-shell method ⇒ see W. Cassing's talk
- Caveat: possible trouble with tachyons
  - transition to semi-class. particle picture ↔ WKB/eikonal approximation
  - particle velocity ⇒ group velocity superluminal around resonance
  - no trouble in wave picture (see Sommerfeld+Brillouin 1913!)

Application: Lifetime of an "off-shell resonance"

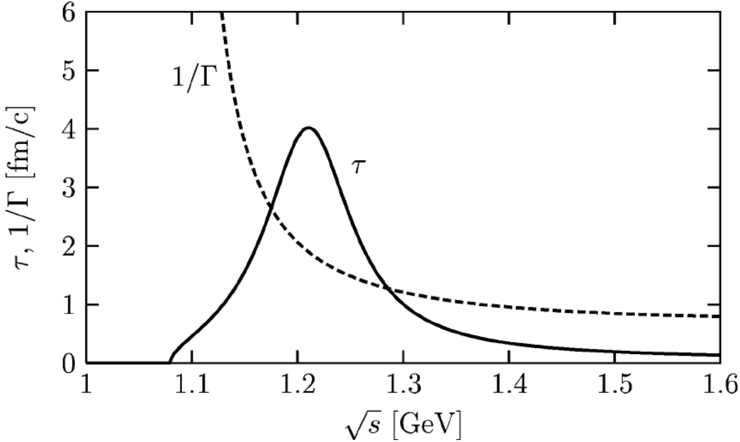
- in transport codes: resonances propagated as particles
- subject to decay with probability $\exp(-\Gamma \Delta t)$
- but $\Gamma = \Gamma(M)$ (vacuum) or even $\Gamma = \Gamma(M, \vec{p})$ (in med)
- in virial expansion (formally expansion of D around $D_{\text{vac}} \Rightarrow$ "thermodynamics" in terms of S matrix [Dashen, Ma, Bernstein 1969]
- correct lifetime from KB equations [Leupold, NPA **695**, 377 (2001)]

$$\tau = 2p_0 \mathcal{B}_{\text{vac}} = \frac{\partial \delta}{\partial p_0}$$

- also from resonant wave propagation [Danielewicz, Pratt, PRC **53**, 249 (1996)] $\Rightarrow$ "delay time": $\partial \delta / \partial E$

Application: Lifetime of an "off-shell resonance"

- example: $\Delta(1232)$ (from [Leupold, NPA **695**, 377 (2001)])



Summary

- propagation of instable resonances great challenge for transport
- start from self-consistent Φ derivable approximations
- approximate Kadanoff-Baym equations
  for Wigner transformed single-particle GF
- gradient expansion $\Rightarrow$ coarse-grained dynamics
  $\Rightarrow$ semi-classical transport equations
  $\Rightarrow$ positive phase-space distributions
- Kadanoff-Baym form: exact conservation laws for Noether currents for complete 1^{st}-order gradient expansion
- Botermans-Malfliet form: feasibility as test-particle MC
- finite width $\Rightarrow$ "off-shell potential"
- Caveat: danger of superluminal particles;
  pragmatically solved in GiBUU, pHSD (...?)
- has intuitive physical interpretation (at least in simplifying limits)